ABSTRACT

Investigation of Low Higgs Models in Weakly Coupled Free Fermionic Heterotic String Theory

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Superstring phenomenology explores classes of vacua which can reproduce the Standard Model at low energy. We consider Weakly Coupled Free Fermionic Heterotic String Theory (WCFFHST) which produces four dimensional Standard-like Models and allows for their $SO(10)$ embedding. In the models herein, we explore the removal of extra Higgs representations via free fermion boundary conditions directly at the string level, rather than in the low energy effective field theory. We focus on the flat direction analysis of four models with reduced number of Higgs, after flat direction analysis of a three generation reduced Higgs model revealed no stringent $F-$ and $D-$flat solutions to all order in the superpotential. Flat direction analysis of the four models presented herein shows the lack of $D-$ and $F-$flat solutions to all order is not a general property of low Higgs models, as stringent flat directions appear to all order for three of our four models.

Investigation of Low Higgs Models in Weakly Coupled Free Fermionic Heterotic String Theory

by

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A Thesis

Approved by the Department of Physics

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Submitted to the Graduate Faculty of
Baylor University in Partial Fulfillment of the
Requirements for the Degree
of
Master of Science

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August 2010

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ACKNOWLEDGMENTS

I have received much support from many sources throughout the preparation of this work. First and foremost, I would like express my gratitude to my advisor, Dr. Gerald Cleaver, for his willingness to introduce me to the fascinating world of String Theory research, even before I began graduate school, as well as all of his guidance and patience during this project and the preparation of this thesis. Thank you for being supportive of my future goals and endeavors, even though things have not turned out as either of us would have liked. Your often goofy sense of humor has been a source of light in these dark and frustrating days. Dr. Anzhong Wang has been instrumental in giving me a solid background in the areas of General Relativity and Cosmology, which will no doubt be invaluable to me in the future, and I am grateful to have been taught by him. The collaboration with Dr. Alon Faraggi at the University of Liverpool made the generation of these models possible, so that I was able to proceed with this project. I could not have completed this work without the assistance of the others in my research group. Thank you, Tim Renner, Jared Greenwald, and Doug Moore, not only for your hard work contributing to this project, but also for making our group meetings one of the highlights of my time here. Preet and Helena, you have been the two best friends I have made while here. Thank you for staying up late with me and discussing random things over various beverages. It has been a welcome break from my work. Preet, thank you for always being willing to answer a General Relativity question, no matter how late at night it was. Helena, much thanks for the help with the final formatting of this document. I would like to express my gratitude to my parents, who, though they never understood my fascination with physics, always encouraged me to persue my dreams. Though I am no longer an undergraduate, I am indebted to the man whom

I still consider a mentor, Phillip Anz-Meador, not only for the extra time he put in helping me with programming in FORTRAN, but for all his moral support along the way. Finally, I could have accomplished none of this without the unending support of my best friend, who would push me to finish even when I thought I didn't have it in me to keep going. I could not have completed this work without you by my side.

CHAPTER ONE

Introduction

1.1 Motivation

The Standard Model (SM) of Particle Physics correctly describes the physics of the elementary particles and their interactions, as confirmed by experiment, up to the electroweak scale, $M_W = 246\,GeV$. It combines three of the four fundamental forces of nature, the weak nuclear, the strong nuclear, and the electromagnetic interactions into a single, unique fundamental framework, a Yang-Mills gauge theory based on the symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where C, L, and Y denote the color, the weak isospin, and the hypercharge quantum number, respectively. In particular, the weak nuclear and the electromagnetic interactions are described by an $SU(2)_L \times U(1)_Y$ gauge symmetry, which is spontaneously broken to a $U(1)_{em}$ by the Higgs mechanism (1). The resulting massive gauge bosons, $W^{\pm}$ and Z^0, mediate the weak interactions, while the massless boson, γ, the photon, is the carrier of the electromagnetic force. The $SU(3)_C$ sector describes the Quantum Chromodynamics (QCD), which remains unbroken, and whose messengers are the eight massless gluons of the strong nuclear force. The SM content consists of three generations of quarks, in agreement with observed experiments. The predictability of the SM is a consequence of its renormalizability, which assures a consistent perturbative analysis of quantities related to particle physics, i.e, infinities that may appear in the calculations are consitently absorbed into a finite number of physical parameters. Despite the acheivements accomplished in this setup, however, several issues have not yet been resolved. In the next few paragraphs, we detail some of the most important shortcomings of the SM (2).

The SM does not include in its descrpition Newtonian gravity, which is 42 orders of magnitude smaller than the nuclear forces. Although General Relativity (GR) describes its infrared properties consistently, gravity is characterized by nonrenormalizable operators which produce divergences in the ultraviolet limit.

The Higgs boson, responsible for the electroweak symmetry breaking and for the generations of the masses for the elementary particles, has a mass of the order of 100 GeV if correctly predicted by the SM. This mass receives radiative corrections which can make the Higgs very heavy ($\approx 10^{19}\, GeV$), while its vacuum expectation value is of the order of the electroweak scale. The hierarchy between the two energy scales in the physics of the Higgs boson appears very unnatural, and certainly unappealing for a fundamental theory. The introduction of supersymmetry (a symmetry between fermionic and bosonic degrees of freedom in the theory) solves this problem by preventing the scalar particle from acquiring the dangerous contributions from the perturbation theory, thus stabilizing its mass.

The coupling constants for the electromagnetic and nuclear forces are parameters which depend on the energy scale. If their behavior is extrapolated at high energy, roughly $10^{16}\, GeV$, these values approach one point, but do not coincide. If supersymmetry is included, the final theory provides a unified description of the forces of the SM at high energy.

More than twenty free parameters describe the physics of the SM and their values are completely arbitrary. For instance, the fermion masses, the gauge and Yukawa couplings, the Kobayashi-Maskawa parameters, and many others have to be fixed by experiment and put by hand into the theory.

Most quantum field theories predict a very large cosmological constant, of the order M_{PL}^4, from the energy of the quantum vacuum. However, the measured cosmological constant is smaller than this by a factor of 10^{120}, and has been called one of the worst theoretical predictions in the history of physics.

There are many other open questions related to the physics of the SM, such as the non-existence of magnetic monopoles, which would explain charge quantization. Also, there is no reasonable explanation for the number of families. In addition, there is the issue of the non-zero neutrino masses, due to their oscillations, which does not fit into the description of leptonic physics of the SM. The attempts at overcoming all these inconsistencies lead to several different theoretical solutions in physics beyond the SM, for instance, the introduction of grand unified theories (GUTs) and supersymmetry. The main goal of GUTs (2; 3) is solving the unification problem previously mentioned, by extending the gauge symmetry group of the SM to a G_{GUT} characterized by only one gauge coupling. In principle, the strong and weak nuclear and the electromagnetic interaction merge together at some higher energy scale M_{GUT} where the theory has the larger gauge symmetry G_{GUT}. When the energy decreases below M_{GUT}, the GUT symmetry breaks to the SM gauge group $SU(3) \times SU(2) \times U(1)$ and the couplings associated with different factors evolve at at different rates. The smallest simple group which accomodates the SM is the $SU(5)$ with $M_{GUT} \approx 10^{15}\, GeV$. (4) A typical feature of GUTs is the mixing of quarks and leptons into the same group representation. Thus, in the case of an $SU(5)$ gauge group, a matter generation is confined to the two irreducible representations $\{10, \bar{5}\} \in SU(5)$. By considering a larger G_{GUT}, an $SO(10)$ symmetry, for example (5), it is possible to combine one generation into only one irreducible representation, precisely the **16** representation of $SO(10)$. In the latter case, the presence of a singlet state, the right-handed neutrino, and the absence of exotic particles makes the model highly predictive.

Unfortunately, there are several unsolved questions appearing in GUTs, most of which originated from the quark-lepton mixing. A first example is given by the existence of new interactions that violate lepton and baryon number, which are responsible for the instability of the proton. Another typical problem is the presence of color-triplet Higgs states which we do not expect to see in the low energy spectrum, called the double-triplet splitting problem. Additionally, GUTs do not provide a solution to the hierarchy problem, which already affects the physics of the SM. Finally, GUTs still suffer from the lack of gravity.

Several answers to the previous problems are presented by supersymmetric theories. In particular, the heirarchy problem is solved with the introduction of supersymmetry (SUSY), as anticipated earlier, which associates to each boson of the theory a fermionic superpartner with the same quantum numbers (since any internal symmetry commutes with SUSY). This symmetry is an extension of the Poincare algebra which includes the fermionic generators $Q^i, i = 1, ...N$, satisfying anticommutation relations. The way SUSY overcomes the heirarchy problem is by 'doubling' the spectrum, where each scalar coexists with its fermionic partner. Basically, the radiative corrections of the scalar Higgs at one-loop include a divergent scalar self-energy term. In SUSY theories, a quadratically divergent term from the bosonic superpartner arises, giving an exactly opposite contribution. Hence, we arrive at a cancellation of terms which stabilizes the scalar masses of the theory. At low energies, there is no experimental evidence of SUSY particles, implying that SUSY has to be broken at a low scale, while being an exact symmetry at high energies.

1.2 String Theory as a Theory of Unification

As mentioned before, the non-renormalizability of GR makes a consistent description of quantum gravity problematic. Therefore, the formulation of a quantum theory that includes gravity with the other three forces is very important. String theory seems to be the most successful candidate to date for a unified theory of all the forces in naure. The regularization of the gravitational interactions is realized courtesy of the introduction of an extended object, the string. The known particles are then associated with massless excitations of the string. Beside these particles, there is an infinite tower of fields with increasing masses and spins(6; 7) with typical mass of the order of the Planck scale, $M_P \sim 10^{19}\, GeV$. Among all excitation modes, the graviton, the quantum of the gravitational field, arises in the spectrum, and suggests the interpretation of string theory as a quantum theory of gravity. Moreover, the presence of only one parameter, the string coupling, g_s, used in the description of all phenomena, is considered a key feature in the prospect of a unifying theory. From a more technical point of view, string theory contains gauge symmetries which may incorporate the SM symmetry. Finally, supersymmetry arises in a natural way in this setup, despite the existence of consistent modular invariant string theories which are not supersymmetric. In the quantization procedure, the consistency of string theory requires spacetime to have a critical dimension, which corresponds to $D = 10$ for supersymmetric strings. In the table below, we present the five 10-dimensional perturbative superstring theories and some of their most important properties.

Table 1.1: The Five Different Types of String Theories.

Type	N_{SUSY}	String	Massless Bosonic Content
$H_{E_8 \times E_8}$	1	closed and oriented	$g_{\mu\nu}$, φ, $B_{\mu\nu}$, A_μ of $E_8 \times E_8$
$H_{SO(32)}$	1	closed and oriented	$g_{\mu\nu}$, φ, $B_{\mu\nu}$, A_μ of $SO(32)$
$I - SO(32)$	1	open + closed unoriented	$g_{\mu\nu}$, φ, $A_{\mu\nu}$, A_μ of $SO(32)$
IIA	2	closed and oriented	$g_{\mu\nu}$, φ, $B_{\mu\nu}$, $C_{\mu\nu\rho}$, A_μ of $U(1)$
IIB	2	closed and oriented	$g_{\mu\nu}$, φ, $B^N_{\mu\nu}$, $\varphi\prime$, $B^R_{\mu\nu}$, $D^\dagger_{\mu\nu\rho\sigma}$

In Table 1.1, $g_{\mu\nu}$, φ, $B_{\mu\nu}$, A_μ, represent the graviton, the dilaton, the anti-symmetric tensor, and the gauge bosons, respectively. The bosons A_μ belong to the adjoint representation of $E_8 \times E_8$ or $SO(32)$ for the first three cases, while they are bosons of $U(1)$ symmetries for the type IIA case. $C_{\mu\nu\rho}$, $\varphi\prime$, $B^{R}_{\mu\nu}$, and $D^{\dagger}_{\mu\nu\rho\sigma}$ represent, again respectively, a three-index tensor potential, a zero-form, a two-form, and a four-form potential, the latter with self-dual field strength. The five superstring models represent a single, unique theory, known as M-theory, but in different regimes. Thus, each of the five superstring models are connected by various types of equivalencies, the so-called string dualities (8). The underlying fundamental theory, whose low energy limit 11-dimensional supergravity (SUGRA) (9), is still poorly understood.

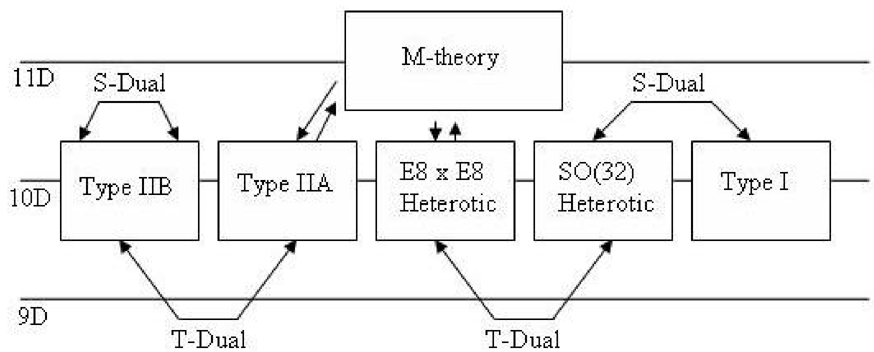

Figure 1.1: Supersymmetric perturbative consistent string theories in 10 dimensions.

As we can see from Figure 1.1, the duality transformations relate the superstring theories in nine and ten dimensions. T duality inverts the radius, R, of the circle S^1 along which a space direction is compactified, $R \to \frac{1}{R}$. In particular, this duality relates the weak-coupling limit of a theory compactified on a space with large volume to the corresponding weak-coupling limit of another theory compactified on a small volume. S duality instead provides the quantum equivalence of two theories which are perturbatively distinct. In fact, it inverts the string coupling, $g_s \to \frac{1}{g_s}$. The perturbative excitations of a theory are mapped to non-perturbative excitations of the dual theory and vice versa. Figure 1.1 summarizes the relevant information of the perturbative string theories and their network of dualities.

In order to make contact with the real world, the compactification of the six extra dimensions is needed. This procedure follows the Kaluza-Klein dimensional reduction of quantum field theory and is generalized to the case where a certain number of spacetime dimensions give rise to a compact manifold, invisible at low energy (10; 11). Demanding four-dimensional $N = 1$ spacetime supersymmetric models leads us to a special choice of internal manifolds, the so-called Calabi-Yau manifolds (12). Compactifications of this kind are characterized by some free parameters, the moduli, generally related to the size and shape of the extra dimensions. The low energy parameters often depend on these free values which spoil the predictivity of the theory. The moduli describe possible deformations of the theory, and their continuous changes allow us to go from one vacuum to another. So far, the problem of fixing the moduli has not been solved, since no fundamental principle is able to single out a unique physical vacuum. The study of Calabi-Yau manifolds is complicated since the computation of properties which are not of topological nature is difficult. A simpler class of compact manifolds is given by the toroidal compactification, although the resulting theory is not chiral. Hence, combining the desirable pictures of Calabi-Yau manifolds and toroidal compactifications, we arrive at the orbifold construction. The orbifold seems to provide a simple framework for the realization of $N = 1$ supersymmetric models in four dimensions, which contain chiral particles.

In this thesis, we discuss the free fermionic heterotic construction, one of the two main compactification schemes which offer complementary advantages in the understanding of semi-realistic heterotic string models. The free fermionic construction is based on an algebraic method to build consistent string vacua directly in four dimensions. In the fermionic formalism, all the world sheet degrees of freedom, required to cancel the conformal anomaly, are given by free fermions on the string worldsheet. This setup offers a convenient setting for experimentation of models, allowing a systematic classification of free fermion vacua and their phenomenological properties. Additionally, this setup has provided the most semi-realistic models to date.

We produced the following results in this thesis. We presented four semi-realistic models in the free fermionic formulation with a reduced Higgs spectrum. The truncation of the Higgs is content is realized in this setup at the level of the string scale, by the assignment of asymmetric boundary conditions to the internal right and left-moving fermions of the theory. The analysis of flat directions, performed with the standard methods, leads to very different results for each of the four models, with two of four models lacking any abelian singlet flat directions to all order. In addition, we present preliminary analysis of the dark matter content of each model via the hidden sector, which shows several hidden sector fields taking on mass at one tenth of the string scale for model 1. This has greater implications to cosmology, as the dark matter content of semi-realistic free fermionic models can be used not only to asses the similarity of a given model to our universe based on already known dark matter constrains, but to provide new results which may assist cosmologists in constraining dark matter parameters in our universe. Finally, we present a geometric variation on the Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) set of basis vectors which change the standard NAHE gauge group of $(SO(10) \otimes SO(6)^3)_{obs} \otimes (E_8)_{hid}$ to $(E_6 \otimes U(1)^5)_{obs} \otimes SO(22)_{hid}$. This setup provides three generations under the **27** representation of E_6. This NAHE variation also provides for the possibility of the investigation of mirror models, in which the observable and hidden sector gauge groups are the same.

1.3 Organization of the Chapters

We begin with a general introduction to the bosonic and fermionic string in order to provide perturbative superstring constructions in Chapter 2. A brief overview of the partition function which encodes the modular invariant properties of the theory is discussed. We explain the bosonization procedure necessary for the correspondence between fermionic and bosonic conformal field theories. We close the chapter with some generalities on the heterotic string, which will be analyzed in detail in the next chapters.

We present the main features of four dimensional semi-realistic models in the free fermionic construction and show the advantages of using this compactification scheme in Chapter 3. We fix the formalism to provide the consistency constraints and the model building rules for this framework and explain the general derivation of the spectrum before analyzing specific models in the next chapter.

In Chapter 4, we provide examples of four semi-realistic free fermionic models within the NAHE basis. Analysis and discussion of the matter content in both the observable and hidden sectors is presented. Flat direction analysis and discussion for each model is presented. Comparisons to the Standard Model are given, with emphasis on the observable sector matter. Additionally, where relevant, we discuss the implications for dark matter constraints and detection in cosmology.

Finally, in Chapter 5, we present a geometric variation on the NAHE set. We present an example of a model within this variation and discuss its matter content and gauge group representations. We close the chapter with discussion about the relevance of investigating this class of models.

We conclude in Chapter 6 underlining the main aim of our research, to obtain a semi-realistic free fermionic string model which accurately describes the Standard Model in the observable sector, and perhaps gives new insight into dark matter via the hidden sector. We present the main results obtained, including different motives for investigating the different classes of models presented, and we finally provide possible outlooks.

CHAPTER TWO

The Heterotic String

In this chapter we construct the heterotic string. We begin by describing the bosonic string, the simplest example of a string theory. We then discuss the quantization procedure of the theory, and show how bosons are related to fermions in a conformal field theory description. This will be one of the building blocks for the free fermionic description of the heterotic string. Additionally, the four different closed strings are discussed. We explain how the different string theories are related by dualities. In most cases, we restrict our discussion to closed strings, since our target is the construction of the heterotic string.

2.1 The Bosonic String

Strings are one dimensional objects whose propagation in a D dimensional spacetime gives rise to a two dimensional worldsheet, $X^{\mu}(\sigma,\tau), \mu = 0, ..., D-1$. The worldsheet is parameterized by the two real and independent coordinates, σ and τ, where σ is a space-like parameter spanning the interval $[0,\pi]$, and τ is a time-like parameter. In Figure 2.1, this surface is shown for the cases of both the free closed and free open string.

The simplest action which describes the motion of a string is the Nambu-Goto action,

$$S_{NG} = -T \int_M d^2\sigma \sqrt{-\gamma} \tag{2.1}$$

where γ is the determinant of the induced metric on the worldsheet,

$$\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}, \tag{2.2}$$

T is the string tension, $T = \frac{1}{2\pi\alpha'}$, and the integral is over the string worldsheet, M. The notation $d^2\sigma$ represents $\sigma = (\sigma^0, \sigma^1) = (\tau, \sigma)$. The action is proportional

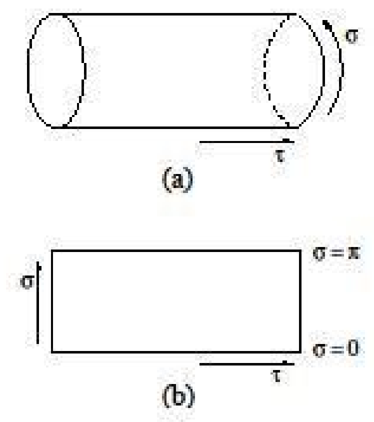

Figure 2.1: The worldsheets of both (a) a closed string and (b) an open string.

to the area swept from the worldsheet, and thus it provides a geometric and intuitive meaning of the string action. By introducing the independent metric on the worldsheet, $h^{\alpha\beta}$, we obtain the Polyakov action,[1]

$$S = -\frac{T}{2}\int_M d^2\sigma\sqrt{-h}h^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu, \quad (2.3)$$

where $h = det(h^{\alpha\beta})$, and we have replaced the general metric, $g_{\mu\nu}$, with the Minkowski metric, $\eta_{\mu\nu}$, for a flat D dimensional spacetime (13; 14). For the general background with $g_{\mu\nu}(X)$, (2.3) becomes the worldsheet action of D dimensional scalar fields X^μ coupled to the dynamical two dimensional metric, which is the theory of quantum gravity coupled to matter.

The Polyakov action has three symmetries:

- Poincare invariance in the target space, X^μ.
- Local reparametrization invariance.
- Conformal (Weyl) invariance.

The last two properties are local symmetries which can be used to fix the worldsheet metric in the conformal gauge, $h^{\alpha\beta} = e^{\phi(\tau,\sigma)}\eta_{\alpha\beta}$, obtaining a flat metric up to a

[1] We choose to work with the Polyakov action because it supplies the equations of motion in a simpler way than the Nambu-Goto action.

scaling function. The equations of motion (EOM) for the bosonic fields X^μ and for the metric $h^{\alpha\beta}$ are obtained via the variation of the action with respect to each of these fields, as is usual. Varying the Polyakov action with respect to the worldsheet metric gives the definition of the energy momentum tensor, $T_{\alpha\beta}$:

$$T_{\alpha\beta} = -\frac{2}{T\sqrt{-h}}\frac{\delta S}{\delta h^{\alpha\beta}} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2}h_{\alpha\beta}h^{\rho\gamma}\partial_\rho X^\mu \partial_\gamma X_\mu, \tag{2.4}$$

The energy momentum tensor is symmetric and tracless ($T_{\alpha\alpha} = 0$) as a result of the Weyl invariance. Then, requiring that the energy momentum tensor vanishes, $T_{\alpha\beta} = 0$, gives the EOM for $h^{\alpha\beta}$. The requirement that the energy momentum tensor vanishes is a condition called the Virasoro constraint, and is important in considering the physical states of a given model.

It is convenient to rewrite the Virasoro conditions in light cone coordinates, $\sigma^+ = \tau + \sigma,\ \sigma^- = \tau - \sigma$, where $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. Then, we can rewrite the Virasoro constraints as

$$T_{++} = \frac{1}{2}(\partial_+ X)^2 = 0;\ T_{--} = \frac{1}{2}(\partial_- X)^2 = 0;\ T_{\pm\mp} = 0. \tag{2.5}$$

The EOM for the fields X^μ take the form $\partial_+\partial_- X^\mu = 0$, whose general solution can be written as the sum of a 'right moving' solution plus a 'left moving' solution,

$$X^\mu(\tau,\sigma) = X^\mu_R(\tau - \sigma) + X^\mu_L(\tau + \sigma). \tag{2.6}$$

Together with the periodicity constraint, $X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma + 2\pi)$, we obtain the expansion

$$X^\mu_R(\tau - \sigma) = \frac{1}{2}x^\mu + \alpha' p^\mu(\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n e^{-2in(\tau-\sigma)}, \tag{2.7}$$

$$X^\mu_L(\tau + \sigma) = \frac{1}{2}x^\mu + \alpha' p^\mu(\tau + \sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}^\mu_n e^{-2in(\tau+\sigma)}, \tag{2.8}$$

From (2.7) and (2.8), we see that the classical motion of the string is described by the center of mass position, x^μ, the momentum, p^μ, and the oscillator modes.

For later convenience, we define the Virasoro operators as Fourier modes of the stress tensor, which in the right and left moving sectors, respectively, are given by

$$L_m = \frac{T}{2}\int_0^\pi d\sigma e^{2im(\tau-\sigma)}T_{--} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\alpha^\mu_{m-n}\cdot\alpha_{\mu n},\ (m\neq 0), \tag{2.9}$$

$$\tilde{L}_m = \frac{T}{2}\int_0^\pi d\sigma e^{2im(\tau-\sigma)}T_{++} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\alpha^\mu_{m-n}\cdot\alpha_{\mu n},\ (m\neq 0), \tag{2.10}$$

with $\alpha^\mu_0 = \sqrt{\frac{\alpha'}{2}}p^\mu_0$. The Virasoro operators satisfy $L_m = 0, \forall n \in Z$, and for the case $n = 0$, we obtain the mass equations for the right and left moving modes, to be discussed in more detail in the next section.

The oscillators, center of mass postion, and momentum all satisfy the standard commutation relations, while the Virasoro operators form what is called the Virasoro algebra. In the covariant canonical quantization procedure, these commutation relations become

$$[x^\mu, p^\mu] = i\eta^{\mu\nu}, \tag{2.11}$$

$$[\alpha^\mu_m, \alpha^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}, \tag{2.12}$$

$$[\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}, \tag{2.13}$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{12}m(m^2-1)\delta_{m+n}. \tag{2.14}$$

All other commutators between different combinations of operators vanish. D represents the central charge, and for the bosonic string, $D = \eta^{\mu\nu}\eta_{\mu\nu}$. The hermiticity of X^μ gives $(\alpha^\mu_n)^\dagger = \alpha^\mu_- n$; $(\tilde{\alpha}^\mu_n)^\dagger = \tilde{\alpha}^\mu_- n$. The same alegbra holds for the left operator, $\tilde{L}_m$, and from now on, we will assume implicitly when defining properties of operators in the right sector that analogous relations hold in the left sector. In the quantization of a classical system, an ambiguity is introduced in the definition of the operators, but this can be solved if we consider the corresponding normal-ordered expressions. In the case of the Virasoro operators, the correct definition is given by

$L_m = \sum_{n=-\infty}^{\infty} : \alpha^{\mu}_{m-n}\alpha_{\mu n} :$. The only term sensitive to normal ordering is L_0, where a normal ordering constant, a, is introduced.

In the covariant quantization, we obtain states with negative norm, which destroy the unitarity of the theory, but we can discharge those by imposing the constraints:

$$L_{m>0}|phys\rangle = 0, \; (L_0 - a)|phys\rangle = 0. \tag{2.15}$$

It has been shown that the subset of positive norm states exists only for $D \leq 26$ and $a \leq 1$. (15)

It is easier to solve the Virasoro constraints in the light cone quantization[2] where the states, obtained by solving the mass-shell equation, (2.15), are always positive. However, if unitarity is guaranteed in this procedure, we need to verify Lorentz invariance, which is not manifest. We have already mentioned that Lorentz invariance is preserved for $D = 26$ and $a = 1$. Thus, $D = 26$ is a special choice of spacetime dimensions, called the critical dimension of the bosonic string.

We use now a residual invariance, leftover after imposing the conformal gauge, which is a reparametrization invariance up to scaling, generally defined as

$$\sigma'_+ \rightarrow f(\sigma_+), \; \sigma'_- \rightarrow f(\sigma_-). \tag{2.16}$$

This invariance allows us to fix the value of X^+ as follows, leading to the light cone gauge,

$$X^+ = x^+ + 2\alpha' p^+ \tau. \tag{2.17}$$

The light cone coordinates are given by $X^{\pm} = (X^0 \pm X^{D-1})/\sqrt{2}$, and by using the Virasoro constraints, we can express X^- in terms of the transverse coordinates X^i, where i takes values in the transverse directions. This means that we are left only

[2] We have already defined the operators in terms of light cone coordinates.

with the transverse oscillators, while the light cone oscillators are given by

$$\alpha_n^- = \frac{1}{\sqrt{2\alpha' p^+}} \{ \sum_{m\in Z} : \alpha^i_{n-m}\alpha^i_m : -2a\delta_{n0} \}, \quad (2.18)$$

$$\alpha_n^+ = \sqrt{\frac{\alpha'}{2} p^+ \delta_{n0}}. \quad (2.19)$$

Analogous expressions hold for $\tilde{\alpha}_n^{\pm}$. The Virasoro constraints in the light-cone gauge define the mass-shell condition for the physical states

$$2p^+p^- = \frac{2}{\alpha'}(L_0 + \tilde{L_0} - \frac{D-12}{12}) \;;\; L_0 = \tilde{L_0}. \quad (2.20)$$

In the first equation of (2.20), the Riemann-Zeta function[3] $\zeta(-1) = \frac{-1}{12}$ has been used, as a result of the divergent sums of zero-point energies due to the normal ordering a of L_0 and $\tilde{L_0}$ (16). The second equation in (2.20) is the level matching condition, a relation which connects the left with the right excitation modes of the closed string. This constraint has to be imposed for the consistency of every closed string model, and it contains important information regarding the physical states of the model; the right and left modes provide the same contribution to the mass of the physical states of the model. The masses of the string excitations are obtained by the contributions of the transverse momenta, which for the right moving sector are provided by the formula $L_0 = \frac{\alpha'}{4} p^i p^i + N$. The mass operator is

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N} - \frac{D-2}{12}), \quad (2.21)$$

where $N = \sum_{m>0} \alpha_{-m} \cdot \alpha_m$. In the case at hand, $D = 26$, thus the first state obtained from (2.21) is the ground state, $|p^\mu\rangle$, with $N = \tilde{N} = 0$, whose mass is given by $M^2 = \frac{-4a}{\alpha'}$, where, as stated earlier, a takes the value 1 for consistency. Such a state is called the tachyon. The first excited state is the tensor $\alpha^i_{-1}\tilde{\alpha^j_{-1}}|p^\mu\rangle$. If we decompose this into irreducible representations of the group $SO(24)$, we obtain

[3] The infinite sum due to the zero-point energy is calculated by a regularization procedure which introduces the Riemann-Zeta function, $\zeta(s) = \sum_{k=1}^{\infty} k^{-s}$. It provides the value of a in terms of the space-time dimension D, which is exactly $a = \frac{D-2}{24}$, as show in (2.21) for $\zeta(-1) = \frac{-1}{12}$ (16).

a symmetric tensor, $g_{\mu\nu}$, a spin-2 particle which is the graviton, the antisymmetric tensor, $B_{\mu\nu}$, and a scalar, φ, the dilaton. At the next level, we obtain states which are organized into representations of $SO(25)$ and are massive.

2.2 The Superstring

As mentioned at the beginning of this chapter, the bosonic string suffers from two main problems: the absence of spacetime fermions, which are necessary for a realistic description of nature, and the presence of tachyons, which is a sign of an incorrect identification of the vacuum. The solution to these problems leads to the construction of the superstring, which comes about via the introduction of worldsheet supersymmetry, realized by including D two-dimensional Majorana fermions, $\Psi^\mu = (\psi_-^\mu, \psi_+^\mu), \mu = 0, ..., D-1$, on the worldsheet. From the spacetime point of view, these fields are vectors, but will provide spacetime fermions when combined with the appropriate boundary conditions. In the following, we will work in the Ramond-Neveu-Schwarz (RNS) formalism (17; 18), where the Gliozzi-Scherck-Olive (GSO) projections are introduced in order to obtain supersymmetry (19). The generalized action in the conformal gauge,

$$S_T = -\frac{T}{2}\int d^2\sigma(\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu) \tag{2.22}$$

is invariant under worlsheet global supersymmetric transformations

$$\partial_\epsilon X^\mu = \bar{\epsilon}\psi_\mu \ , \ \partial_\epsilon X^\mu = -i\rho^\alpha \partial_\alpha X^\mu \epsilon, \tag{2.23}$$

with ϵ a constant spinor and $\rho^\alpha, \alpha = 0, 1$, Dirac matrices which can be chosen as

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} . \tag{2.24}$$

In light cone coordinates, the fermionic contribution of (2.22) is

$$\psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+, \tag{2.25}$$

where the spacetime index μ has been supressed.

The equations of motion are simply the Dirac equations, $\partial_\pm \psi_\pm = 0$. Their solutions are of the form $\psi_- = \psi_-(\sigma_+)$ and $\psi_+ = \psi_+(\sigma_-)$, and we can thus say that ψ_- represents the right moving field while ψ_+ represents the left moving field. The boundary conditions arise from requiring that

$$(\psi_+ \partial \psi_+ + \psi_- \partial \psi_-)|_{\sigma=0}^{\sigma=\pi} = 0, \tag{2.26}$$

which is satisfied if ψ_+ and ψ_- are periodic or anti-periodic,

$$\psi_+^\mu(\sigma + \pi, \tau) = \pm \psi_+^\mu(\sigma, \tau), \tag{2.27}$$

$$\psi_-^\mu(\sigma + \pi, \tau) = \pm \psi_-^\mu(\sigma, \tau). \tag{2.28}$$

The periodic case is called the Ramond (R) boundary condition, while the anti-periodic case is called the Neveu-Schwarz (NS) boundary condition. The general solution in terms of mode expansion for the right moving states is given by

$$\psi_-^\mu = \sum_r b_r^\mu e^{-2i\pi(\sigma_-)}. \tag{2.29}$$

An analogous expression holds for the left moving states, ψ_+^μ, by replacing σ_- by σ_+, and b_r^μ by $\tilde{b}_r^\mu$. As a result of the boundary conditions, the frequency, r, is integer for R boundary conditions and half integer for NS boundary conditions.

The R boundary conditions and the integer modes describe string states which are spacetime fermions. If we consider the fundamental state, $b_0^i|0;p^\mu\rangle$, we see that it is massless and degenerate, as b_0 satisfies the Clifford algebra $\{b_0^i, b_0^j\} = \delta^{ij}$. This means that the Ramond vacuum is a spinor of $SO(8)$, and all the states obtained from the vacuum with the creation operators are fermionic as well. The NS boundary conditions, on the other hand, with half integer excitations, give bosons. The fundamental state, $|0; p^\mu\rangle$, has negative mass, which corresponds to a tachyon, and is a scalar. The first excited massless state, $b^i_{-\frac{1}{2}}|0; p^\mu\rangle$, is a vector of $SO(8)$, and all the states in this sector, created by half integer modes, provide bosons.

Because the superstring is an extension of the bosonic case, it is necessary to expand the algebra which describes the theory. The classical Virasoro constraints are now generalized to

$$J_\pm = 0 \ , \ T_{\pm\pm} = 0, \tag{2.30}$$

where the supercurrents and the energy momentum tensors are given in their light-cone gauge coordinates

$$J_+ = \psi^\mu_+ \partial_+ X_\mu, \ T_{++} = \partial_+ X^\mu \partial_+ X_\mu + \frac{i}{2}\psi^\mu_+ \partial_+ \psi_{+\mu}, \tag{2.31}$$

$$J_- = \psi^\mu_- \partial_- X_\mu, \ T_{--} = \partial_- X^\mu \partial_- X_\mu + \frac{i}{2}\psi^\mu_- \partial_- \psi_{-\mu}. \tag{2.32}$$

The quantization of the fermionic fields is obtained by imposing the anticommutation relations

$$\{b^\mu_r, b^\nu_s\} = \eta^{\mu\nu}\delta_{r+s} \ , \ \{\tilde{b}^\mu_r, \tilde{b}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s}. \tag{2.33}$$

The anticommutator of left and right oscillators vanishes. For $r < 0, b_r$ denotes the creation operators, and for $r > 0, b_r$ denotes the annihilation operators. The complete spectrum is provided by the action of the creation operators on the vacuum.

The mass-shell condition in (2.21) is now generalized by redefining N as the number of right bosonic plus right fermionic oscillators acting on the vacuum. The same redefinition applies to $\tilde{N}$. We need to take into account that real fermions can assume either R or NS boundary conditions, which will change the contribution to the zero-point energy, a. Each fermionic coordinate contributes with a $-1/48$ in the NS sector and a $1/24$ in the R sector, while each boson gives a contribution of $-1/24$ in both sectors. In D dimensions, in light cone gauge, we have $D-2$ transverse bosons, and $D-2$ transverse fermions, which give $a = 0$ in the R sector, but give $a = -1/16(D-2)$ in the NS sector.

After quantizing the supersymmetric theory, the Virasoro constraints become

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{8}m(m^2-1)\delta_{m+n}, \quad (2.34)$$

$$[L_m, G_r] = (\frac{m}{2} - r)G_{m+r}, \quad (2.35)$$

$$\{G_r, G_s\} = 2L_{r+s} + \frac{D}{2}(r^2 - \frac{a}{2})\delta_{r+s}, \quad (2.36)$$

where the operators are defined by their normal ordered expressions,

$$L_m = L_m^{a'} + L_m^{b'}, \quad (2.37)$$

$$L_m^{a'} = \frac{1}{2}\sum_{n\in Z} : \alpha_{-n} \cdot \alpha_{m+n} :, \quad (2.38)$$

$$L_m^{b'} = \frac{1}{2}\sum_{n\in Z+a} : (r - \frac{m}{2})b_{m-r} \cdot b_r :, \quad (2.39)$$

$$G_r = \sum_{n\in Z} : b_{r-n} \cdot \alpha_n : . \quad (2.40)$$

For completeness with respect to the bosonic case, we shall provide the light cone quantization for the superstring case. The theory is ghost free, but not explicitly covariant. However, we can assure Lorentz invariance if $D = 10$ and $a = 1/2$ (20).

The gauge is fixed via the relation $\psi_+ = 0$ and $X^+ = \alpha\prime p^+\tau$ and because we are fixing the longitudinal oscillator modes, the only independent degrees of freedom are the transverse ones.

A supersymmetric non-tachyonic theory is obtained when the spectrum is truncated by GSO projections (21). We will explain this truncation separately in the R and NS sectors. In the NS sector, the GSO projections, P_{GSO}, are defined by keeping states with an odd number of b^i_{-r} oscillator excitations and removing those with even number. The projection operator in the NS sector, and the fermion number are given by

$$P^{NS}_{GSO} = \frac{1}{2}(1-(-1)^F)\ ,\ F = \sum_{r=1/2}^{infty} b^i_{-r} \cdot b^i_r. \quad (2.41)$$

Thus, the bosonic ground state is now massless, and the spectrum no longer contains a tachyon, which has fermion number $F = 0$. In the R sector, the fundamental state, a Majorana spinor, lives in the spinorial representation of $SO(8)$, as mentioned before. If we introduce the projection operator,

$$P^R_{GSO} = \frac{1}{2}(1 - (-1)^F \Gamma_g), \tag{2.42}$$

where $\Gamma_g = b^1_0, ..., b^8_0$ is the chiral operator in the transverse dimensions, then the fundamental state becomes a Majorana-Weyl spinor of definite chirality. P^R_{GSO}, while projecting onto spinors of opposite chirality, guarantees spacetime supersymmetry of the physical superstring spectrum.[4]

The general procedure to obtain the massless spectrum is to solve the massless equations for the left and right sectors, apply level matching conditions and the particular GSO projections depending on the perturbative superstring model considered, and finally tensor the left with the right states. If we want to proceed with the explicit calculation of the spectrum, we need to specify the string theory we wish to analyze. Supersymmetric theories with only closed strings are type IIA, type IIB, and heterotic models. In types IIA and IIB, supersymmetry is realized in both sectors, while in the heterotic string, supersymmetry is realized only on the left (right) sector. By taking the tensor products of the right and left movers in types IIA and IIB, we get four distinct sectors: NS-NS, R-R, NS-R, and R-NS, where the former two sectors give bosons and the latter two sectors give fermion fields in the target space. The features and differences among these two models have been given in the introduction. In this thesis, we are interested in the heterotic string, and will hence focus on the technicalities of the heterotic string in Section 2.4.

2.3 Bosonization

In this section, we present the equivalence between fermionic and bosonic conformal field theories in two dimensions, a correspondence which allows the consistent construction of free fermionic models. Before entering into the details, we will give

[4] We note that the choice of sign of $(-1)^F \Gamma_g = \pm 1$, corresponding to different chirality projections on the spinors, is a matter of convention.

the definition of operator product expansions (OPEs) in conformal field theories in two dimensions, as it is from OPEs that we will show the boson-fermion equivalence.

2.3.1 Product Expansion Operator

In quantum field theory, the infinitesimal coordinate transformations,

$$z \to z + \epsilon(z)\ ,\ \ \bar{z} \to \bar{z} = \bar{\epsilon}(\bar{z}) \tag{2.43}$$

produce a variation of a field, $\Phi(z, \bar{z})$ given by the equal time commutator with the conserved charge, $Q = \frac{1}{2\pi i}\oint(dzT(z)\epsilon(z) + d\bar{z}\bar{T}(\bar{z})\bar{\epsilon}(\bar{z}))$, where T and $\bar{T}$ are the stress energy tensors in complex coordinates. The products of the operators is well defined only if time-ordered. A complete treatment of the complex tensor analysis can be found in (22; 23). Here we mention only the results which will be useful for our purposes.

The commutator of an operator A with a spatial integral of an operator B corresponds to

$$[\int d\sigma B, A] = \oint dz R(B(z)A(z)), \tag{2.44}$$

which leads to (23) the operator product expansions (OPEs) of the stress energy tensors $T(z)$ and $\bar{T}(\bar{z})$ with the field $\Phi(w, \bar{w})$

$$R(T(z)\Phi(w,\bar{w})) = \frac{h}{(z-w)^2}\Phi + \frac{1}{z-w}\partial_w \Phi + ..., \tag{2.45}$$

$$R(\bar{T}(\bar{z})\Phi(w,\bar{w})) = \frac{\bar{h}}{(\bar{z}-\bar{w})^2}\Phi + \frac{1}{\bar{z}-\bar{w}}\partial_{\bar{w}} \Phi + ... \tag{2.46}$$

Equations (2.45) and (2.46) contain the conformal transformations properties of the field Φ, and can thus be used as a definition of a primary field[5] for Φ with conformal weight $(h, \bar{h})$. We observe that the above products are given by the expansion of poles plus regular terms, which we can omit.

[5] The formal definition of the primary field is: Φ is primary of conformal weight $(h, \bar{h})$ if it satisfies the transformation law $\Phi(z, \bar{z}) \to (\frac{\partial f}{\partial z})^h (\frac{\partial \bar{f}}{\partial \bar{z}})^{\bar{h}} \Phi(f(z), \bar{f}(\bar{z}))$, where h and $\bar{h}$ are real values.

2.3.2 Free Bosons and Free Fermions

We start by considering a massless free boson, $X(z,\bar{z})$, where we can split the holomorphic and anti-holomorphic components into $X_L(z)$ and $X_R(\bar{z})$. For our purpose, it is sufficient to consider the holomorphic part only. The propagator of the left component corresponds to $\langle X_L(z)X_L(w)\rangle = -log(z-w)$, which means that it is not a conformal field, but its derivative, $\partial X_L(z)$ is a $(1,0)$ conformal field. This is shown by taking the OPE with the stress tensor, which is defined as $T = -\frac{1}{2} : \partial X_L^2 :$, and comparing with (2.45) and (2.46), we obtain

$$T(z)\partial X_L(w) \sim \frac{1}{(z-w)^2}\partial X_L(w) + \frac{1}{z-w}\partial^2 X_L(w) + ... \tag{2.47}$$

We now consider two Majorana-Weyl fermions, $\psi^i(z), i = 1,2$, where a change of basis rearranges the fermions into the complex form

$$\psi = \frac{1}{\sqrt{2}}(\psi^1 + i\psi^2)\ ,\ \bar{\psi} = \frac{1}{\sqrt{2}}(\psi^1 - i\psi^2). \tag{2.48}$$

The theory contains a $U(1)$ current algebra[6] generated by the $(1,0)$ current $J(z) =: \psi\bar{\psi} :$. The OPE for $\psi\bar{\psi}$ and the holomorphic energy tensor are defined as

$$\psi(z)\bar{\psi}(w) = -\frac{1}{z-w}\ ,\ T(z) = \frac{1}{2} : \psi(z)\partial\psi(z) : . \tag{2.49}$$

If we calculate the product expansion $T(z)\phi(w)$ with the above definitions, we see that ψ is an affine primary field of conformal weight $(1/2, 0)$.

We first present the boson-fermion correspondence by showing that the same operator algebra is produced by two Majorana-Weyl fermions in one case and a chiral boson in the other case. In the fermionic case,

$$T(z) = \frac{1}{2} : J^2 :, \tag{2.50}$$

which says that the stress tensor has central charge $c = 1$. We can produce the same operator algebra by using a single chiral boson, $X(z)$, whose current is given by

$$J(z) = i\partial X(z), \tag{2.51}$$

[6] This will be discussed more in the following section, 2.4.

where the stress energy tensor is $T = -\frac{1}{2} : \partial X^2 :$, as at the beginning of the section. The definitions below thus contain explicitly the boson-fermion equivalence

$$\psi =: e^{iX(z)} : \ , \ \bar{\psi} =: e^{-iX(z)} : . \tag{2.52}$$

Alternately, we can consider the OPE of each of the fields with themselves to demonstrate the boson-fermion equivalence. If we consider the OPE of two bosonic fields, $X(z)$ and $X(w)$, we find

$$X(z)X(0) = -ln|z|^2 + \mathcal{O}(z), \tag{2.53}$$

where we have used the definition provided in the beginning of this section. Consider now the operators $e^{\pm iX(z)}$. Using the Campbell-Baker-Hausdorff formula,

$$e^{ipX} e^{iqX} = e^{ipX(z)+iqX(0)+\frac{1}{2}pq[X(z),X(0)]+\dots}, \tag{2.54}$$

we find that they have the OPEs

$$\begin{aligned} e^{iX(z)}e^{-iX(0)} &= \frac{1}{z} + \mathcal{O}(z), & (2.55) \\ e^{iX(z)}e^{iX(0)} &= \mathcal{O}(z), & (2.56) \\ e^{-iX(z)}e^{-iX(0)} &= \mathcal{O}(z). & (2.57) \end{aligned}$$

Similarly, consider now the OPE for two Majorana-Weyl fermions, as given in (2.48). Their OPEs are

$$\begin{aligned} \psi(z)\bar{\psi}(0) &= \frac{1}{z} + \mathcal{O}(z), & (2.58) \\ \psi(z)\psi(0) &= \mathcal{O}(z), & (2.59) \\ \bar{\psi}(z)\bar{\psi}(0) &= \mathcal{O}(z), & (2.60) \end{aligned}$$

which are equivalent to $(2.55 - 7)$.

Thus, we find the equivalence between bosons and fermions, and we can thus write

$$\psi(z) \cong e^{iX(z)} \ , \ \bar{\psi}(z) \cong e^{-iX(z)}, \tag{2.61}$$

which is the same result as (2.52) (24).[7]

2.4 The Heterotic String

The heterotic string was first constructed by (25). Employing both the bosonic string and the superstring, it came about after it was shown (26) that for consistency, an $N = 1$ supersymmetric string theory requires the presence of an $E_8 \times E_8$ or Spin(32) gauge symmetry. Ten dimensional supergravity with these gauge groups is free of gravitational and gauge anomalies. This observation fuelled an increase in activity in heterotic models. Before this discovery, the standard procedure to introduce gauge groups in string theory consisted of attaching the Chan-Paton charges at the endpoints of open strings (27). Such a prescription does not produce the $E_8 \times E_8$, a non-abelian GUT group which allows a more natural embedding of the Standard Model spectrum at low energy.

In this section, we describe the basics of the heterotic string, an orientable closed string theory in ten dimensions with $N = 1$ supersymmetry and gauge group $E_8 \times E_8$. Its low energy limit is supergravity coupled with Yang-Mills theory. This theory is a hybrid of the $D = 10$ fermionic string and the $D = 26$ bosonic string, and the resulting spectrum is supersymmetric, tachyon free, Lorentz invariant, and unitary. The absence of gauge and gravitational anomalies is obtained by compactification of the extra sixteen bosonic coordinates on a maximal torus of determined radius. All these properties make the heterotic string one of the most appealing candidates for a unified field theory.

[7] Note that here we use the $\cong$ symbol, whereas in (2.52) we use the : : symbol. The : : defines the standard normal ordering of operators in a quantum field theory (QFT) and is known as the regular part, while the = part of the $\cong$ should be interpreted as being valid primarily as a statement for the expectation values of the two fields. The two notations are synonomous and can be used interchangeably.

In heterotic models, the gauge symmetries are introduced by distributing symmetry charges on the closed strings. Such charges are not localized, so we obtain a continuous charge distribution throughout the string. A way to describe their currents is to introduce on the worldsheet fermions with internal quantum number, which are singlets under the Lorentz group. If we take n real Majorana fermions, $\lambda^a, a = 1, ..., n$, and we split them into right and left moving modes, $(\lambda^a_\pm)$, we can write the bosonic action on the worldsheet, including the new internal symmetries, as

$$S = -\frac{T}{2}\int d^2\sigma(\partial_\alpha X_\mu \partial^\alpha X^\mu - \lambda^a_- \partial_+ \lambda^a_- - \lambda^a_+ \partial_- \lambda^a_+). \tag{2.62}$$

The equivalence of bosons and fermions in two dimensions allows us to convert two Majorana fermions on the worldsheet into a real boson. We can then obtain $\frac{n}{2}$ bosons, ϕ^i, in the place of fermions, λ^a. With this substitution, the theory contains $D + n/2$ free bosons and has an $SO(D-1,1)$ Lorentz symmetry plus an internal $SO(n) \times SO(n)$ symmetry. Its consistency requires $D + n/2 = 26$, and in the case of a supersymmetric theory ($D = 10$), it means that $n = 32$. If we consider for our purposes only an $SO(n)_R$ symmetry, then the right moving fermion currents are given by

$$J^\alpha_+(\sigma) = \frac{1}{2\pi} T^\alpha_{ab} \lambda^a_+(\sigma)\lambda^b_+(\sigma). \tag{2.63}$$

The T^α generators satisfy the algebra $[T^\alpha, T^\beta] = i f^{\alpha\beta\gamma} T^\gamma$, and this relation fixes the commutation relation for the currents

$$[J^\alpha_+(\sigma), J^\beta_+(\sigma')] = i f^{\alpha\beta\gamma} J^\gamma_+(\sigma)\delta(\sigma - \sigma') + \frac{ik}{4\pi}\delta^{\alpha\beta}\delta'(\sigma - \sigma'). \tag{2.64}$$

The previous formula describes the affine Lie algebra $\hat{SO}(n)$ with central extension represented by the second term, or anomaly contribution. If this algebra is built up from n fermions in the fundamental representation of $SO(n)$, then $k = 1$. If the fermions are not in the fundamental representation, then we would obtain a different, or quantized, value of k.

We are now ready to describe the heterotic string as it was first formulated in (26). As we already mentioned, the left moving modes are described in a bosonic string theory with $D = 26$, while the right moving modes are supersymmetric with $D = 10$. Specific GSO projections ensure supersymmetry for our model. The gauge degrees of freedom are included in the left sector with an appropriate current algebra.

The general action of this theory is

$$S = -\frac{T}{2}\int d^2\sigma(\sum_{\mu}^{9}(\partial_\alpha X_\mu \partial^\alpha X^\mu - 2\psi_+^\mu \partial_- \psi_{+\mu}) - 2\sum_{a=1}^{n} \lambda_-^a \partial_+ \lambda_-^a - \lambda_+^a \partial_- \lambda_+^a). \quad (2.65)$$

We observe here that the spacetime fermions, ψ^μ have only right moving components, superpartners of X_R^μ. The content therefore differs from the type IIB, where supersymmetry is realized in both the left and right sectors. The left moving sector contains the spacetime fields X_L^μ and the internal Majorana fermions, λ_-^a.

If the boundary conditions for λ_-^a are all the same, we obtain the Spin (32) heterotic theory. Choosing different periodic/antiperiodic boundary conditions between two sets of 16 real internal fermions will provide the $E_8 \times E_8$ heterotic string. It can be shown that the two theories are continuously related (28). In fact, an equal number of states at every mass level appear in the two heterotic string theories.

In the MSSM we find an $N = 1$ spacetime supersymmetry and a $D = 4$ target space or space time. Again there has been considerable effort to reduce the number of dimensions and supersymmetries. Although many different ways have been employed, we note two that have been used extensively. Toroidal compactifications with twists are the most widely used compactification scheme, in particular orbifold compactifications. They are identified as $\mathbb{Z}_M$ and $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifolds. It has been shown that only a limited number of these types of orbifolds reduce the number of the supersymmetries to the number of the MSSM (29). Of particular interest for phenomenology are the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactifications. Since the MSSM is a chiral theory, heterotic string theory provides a convenient description in the search for phenomenological string theories.

CHAPTER THREE

Free Fermionic Models

In this chapter we describe the free fermionic formulation of the heterotic superstring and focus mainly on a subset of these models which are called semi-realistic free fermionic models. We provide an overview of the Nanopoulos-Antoniadis-Hagelin-Ellis (NAHE) set (30), before discussing our own models within this set in the next chapter. In this part of our discussion, we will describe the consistency rules necessary for the construction of the theory. The interested reader can find further details in the original papers (31; 32; 33; 34; 35).

The general procedure for the construction of free fermionic models is based on two steps. The first is the choice of boundary condition basis vectors for the class under consideration, and the second is the inclusion of additional basis vectors which reduce the number of generations in the model to three, while breaking the four dimensional gauge group. The presence of three Higgs doublets in the untwisted spectrum is a feature of semi-realistic free fermionic models, and the general procedure to reduce them to one pair is given by the analysis of the supersymmetric flat directions. This method consists of giving heavy masses to some of the Higgs doublets in the low energy field theory (36; 37). We will present some generalities on the analysis of flat directions and introduce the concept of stringent flat directions, as this allows the investigation of the low energy properties of free fermionic models. The flat direction analysis is needed because of an anomalous $U(1)$ which generally appears in this setup. Its presence gives rise to a Fayet-Illiopolous (FI) D-term which breaks supersymmetry. However, by looking at supersymmetric flat directions and imposing F and D flatness on the vacuum, supersymmetry can be restored. Analysis for specific models is presented in the following chapter.

3.1 The Free Fermionic Formulation

Free fermionic model building was developed simultaneously by two groups (33; 34). The quasi-realistic heterotic string models in the free fermionic formulation, which are related to $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactifications, are among the most realistic string models constructed to date. These models provide a wide variety of three generation models with an $SO(10)$ embedding of the Standard Model spectrum, including: the flipped $SU(5)$ models (38; 39), the standard-like models (40; 41; 42; 43; 44; 45; 46; 47; 48), the Pati-Salam models (49), and the Left-Right symmetric models (50). Within these models many of the issues pertaining to the phenomenology of the Standard Model and Grand Unification have been explored and investigated (51). Free fermionic models also produced the first known string models in which the matter content of the observable sector in the low energy effective quantum field theory consists solely of that of the Minimal Supersymmetric Standard Model (MSSM) (52; 45).

In light cone gauge, a free fermionic heterotic string model contains 64 real worldsheet fermions, $\psi^n, 1 \leq n \leq 20$, for left moving worldsheet fermions, and $21 \leq n \leq 64$, for right moving worldsheet fermions, in addition to the left and right moving worldsheet scalars, $X_{\mu=1,2}$, and $\bar{X}_{\bar{\mu}=1,2}$, which embed transverse coordinates of four dimensional spacetime. ψ^1 and ψ^2 are the worldsheet superpartners of the two left moving transverse scalars. The remaining 62 fermions are internal degrees of freedom, and some or all of these may be paired to form complex fermions, $\psi^{n,m} \equiv \psi^n + i\psi^m$. If m and n both denote left movers or right movers, then $\psi^{n,m}$ is a Weyl fermion, but if m denotes a right mover and n a left mover, or vice versa, then $\psi^{n,m}$ is a Majorana fermion. A specific model is defined by two factors:

(1) Sets of 64-component boundary vectors with components for complex fermions counted twice, which describe how the worldsheet fermions transform around non-comtractible loops on the worldsheet, and

(2) Sets of coefficients weighting contributions to the one loop partition function from fermions with specific boundary conditions.

In contrast with the ten dimensional superstrings, where the compactification of the extra dimensions is needed to reduce the spacetime to four dimensions, the free fermionic formulation directly provides a four dimensional theory with a certain number of internal degrees of freedom. An internal sector of two dimensional conformal field theories is required in order to fulfill the following:

- conformal invariance
- worldsheet supersymmetry
- modular invariance

In this approach, all internal degrees of freedom are fermionized, thus producing world-sheet fermions. Requiring anomaly cancellation fixes the number of fields in the left and right sector, retaining 18 left-moving Majorana fermions $\chi^a, (a = 1, ..., 18)$, and 44 right-moving Majorana fermions $\bar{\Phi}^I, (I = 1, ..., 44)$. The spacetime is described by the left-moving coordinates (X^μ, ψ^μ), and the right-moving bosons, $\bar{X}^\mu$. The heterotic string has $N = 1$ spacetime supersymmetry in D dimensions. This is realized non-linearly (32) among all the fields in the left sector, spacetime and internal ones, by the supercurrent

$$T_F = \psi^\mu \partial X^\mu + f_{abc}\chi^a\chi^b\chi^c, \tag{3.1}$$

where f_{abc} are the structure constants of a semi-simple Lie group G of dimension 18[1] . The χ^a transform in the adjoint representation of G. In (53) it is shown that $N = 1$ spacetime supersymmetry can be obtained in four dimensions when the Lie algebra $G = SU(2)^6$. In this case, it is convenient to group the χ^a into six triplets

[1] In general, the f_{abc} are the structure constants of a semi-simple Lie group, $\mathcal{L}$, of dimension $3(10 - D)$, which becomes dimesion 18 for $D = 4$.

$(\chi^i, y^i, w^i), (i = 1, ..., 6)$. Each triplet transforms as the adjoint representation of $SU(2)$. Thus far, we have ensured superconformal invariance of the theory. We still need to verify its modular invariance to get a consistent theory, which is acheived by investigating the properties of the partition function. Modular invariance exists if the one loop partition function is invariant under $S : \tau \to -1/\tau$ and $T : \tau \to \tau + R$ transformations of the complex worldsheet parameter τ defining the one loop worldsheet, a torus. In this case, a modular invariant partition function must be the sum over all different boundary conditions for the worldsheet fermions, with appropriate weights. For a genus-g worldsheet Σ_g, fermions moving around a non-trivial loop $\alpha \in \pi_1(\Sigma_g)$ transform as

$$\bar{\Phi}^I \to R_g(\alpha)^I_J \bar{\Phi}^J, \tag{3.2}$$

$$\psi^\mu \to -\delta_\alpha \psi^\mu, \tag{3.3}$$

$$\chi^a \to L_g(\alpha)^a_b \chi^b, \tag{3.4}$$

where the first transformation refers to the right-moving fields, $\bar{\Phi}^I$, $L^a_{ga'} L^b_{gb'} L^c_{gc'} f_{abc} = -\delta_\alpha f_{a'b'c'}$ and $\delta_\alpha = \pm 1$, and the second and third transformations refer to the left-moving fields, ψ^μ and χ^a. The spin structure of each fermion is a representation of the first homotrophy group $\pi_1(\Sigma_g)$ (54). The transformations $(2.2 - 4)$ ensure the invariance of the supercurrent. We need to require the orthogonality of $R_g(\alpha)$ to leave the energy tensor invariant in the right sector. In order to keep the theory tractable, commutativity of the boundary conditions has been assumed (31), implying that $L_g(\alpha)$ and $R_g(\alpha)$ have to be abelian matrix representations of $\pi_1(\Sigma_g)$. Note that commutativity is assumed between the boundary conditions on surfaces of different genus. The previous constraints allow the diagonalizations of the matrices $R(\alpha)$ and $L(\alpha)$, which are expressed purely as phase changes, and simplifying $(2.2 - 4)$ into

$$f \to -e^{i\pi\alpha(f)} f, \tag{3.5}$$

where f is any fermion $(\psi^\mu, \chi^a, \bar{\Phi}^I)$ and $\alpha(f)$ is the phase acquired by f when moving around the non-contractable loop α.

Then, the spin structure for a non-contractible loop can be expressed as a vector

$$\alpha = \{\alpha(f_1^r), ..., \alpha(f_k^r); \hat{\alpha}(f_1^c), ..., \hat{\alpha}(f_{k\prime}^c)\} \tag{3.6}$$

where $\alpha(f^r)$ is the phase for a real fermion, while $\hat{\alpha}(f^c)$ is the phase for a complex fermion. By convention, $\alpha(f) \in (-1, 1]$. Then, for the complex conjugate fermion $\alpha(f^*) \in [-1, 1)$. We set the notation

$$\delta_\alpha = \begin{cases} 1 & \text{if } \alpha(\psi^\mu) = 0 \\ -1 & \text{if } \alpha(\psi^\mu) = 1 \end{cases} \tag{3.7}$$

where, according to (2.5), the entry 1 represents a periodic (Ramond) boundary condition and 0 represents an anti-periodic (Neveu-Schwarz) boundary condition. Since there are $2g$ non-contractible loops for a genus g Reimann surface, we have to specify two sets of phases, $\alpha_1, ..., \alpha_g, \beta_1, ..., \beta_g$, to obtain the full partition function. In its general form, it can be written as a weighted sum over the individual partition functions, $Z \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ for specific pairs of boundary vectors

$$Z = \sum_{genus} \sum_{i,j=1}^{g} c \begin{pmatrix} \alpha_i \\ \beta_j \end{pmatrix} z \begin{pmatrix} \alpha_i \\ \beta_j \end{pmatrix}, \tag{3.8}$$

where $z \begin{pmatrix} \alpha_i \\ \beta_j \end{pmatrix}$ can be expressed in terms of θ-functions. The modular invariance imposes constraints onto the coefficients $c \begin{pmatrix} \alpha_i \\ \beta_j \end{pmatrix}$. It was shown (55) that modular invariance and unitarity imply that these coefficients for higher genus surfaces factorize into the form

$$c\begin{pmatrix}\alpha_1,...,\alpha_g\\ \beta_1,...,\beta_g\end{pmatrix} = c\begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} c\begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix} \dots c\begin{pmatrix}\alpha_g\\ \beta_g\end{pmatrix}. \tag{3.9}$$

For this reason, it is sufficient to consider only the one-loop coefficients.

3.1.1 Model Building Rules and Physical Spectrum

In the free fermionic framework, the construction of consistent string vacua in four dimensions is acheived by applying two sets of rules: the constraints for the boundary condition vectors, restricted to the case of rational spin structure (31), and the rules for one-loop phases.

A set of consistent boundary condition vectors form an additive group

$$\Xi \sim Z_{N_1} \otimes ... \otimes Z_{N_k}, \tag{3.10}$$

generated by the basis $B = \{b_1, ..., b_k\}$, where each b_i is in the form of (2.6). This basis must satisfy the following conditions:

- $\sum m_i b_i = 0 \Longleftrightarrow m_i = 0 \pmod{N_i}, \forall i,$
- $N_{ij} b_i \cdot b_j = 0 \bmod 4,$
- $N_i b_i \cdot b_i = \begin{cases} 0 \bmod 8, \; \mathrm{N}_i \text{ even} \\ 0 \bmod 4, \; \mathrm{N}_i \text{ odd} \end{cases}$
- $b_1 = 1$

where N_i is the smallest postive non-zero integer for which $N_i b_i = 0 \pmod 2$ and N_{ij} is the lowest common multiplier between N_i and N_j. The inner Lorentz product is defined by

$$b_i \cdot b_j = \left\{\frac{1}{2}\sum_{\text{real left}} + \sum_{\text{complex left}} - \frac{1}{2}\sum_{\text{real right}} - \sum_{\text{complex right}}\right\} b_i(f) b_j(f). \tag{3.11}$$

For a consistent basis B, there are several different modular invariant choices of phases, each one leading to a consistent string theory. The phases under consideration have to satisfy the requirements above, which provide the second group of constraints below

- $c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = \delta_{b_i} e^{\frac{2\pi i n_i}{N_j}} = \delta_{b_j} e^{\frac{2\pi i m_i}{N_i}} e^{\frac{i\pi b_i \cdot b_j}{2}}$,
- $c\begin{pmatrix} b_i \\ b_i \end{pmatrix} = -e^{\frac{i\pi b_i \cdot b_i}{4}} c\begin{pmatrix} b_i \\ 1 \end{pmatrix}$,
- $c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = e^{\frac{i\pi b_i \cdot b_j}{2}} c^* \begin{pmatrix} b_j \\ b_i \end{pmatrix}$,
- $c\begin{pmatrix} b_i \\ b_j + b_k \end{pmatrix} = \delta_{b_i} c\begin{pmatrix} b_i \\ b_j \end{pmatrix} c\begin{pmatrix} b_i \\ b_k \end{pmatrix}$,

where $1 < n_i < N_j$ and $1 < m_i < N_i$. In addition, there is some freedom for the phase $c\begin{pmatrix} b_1 \\ b_1 \end{pmatrix} = \pm e^{\frac{i\pi b_1 \cdot b_1}{4}}$, while by convention, $c\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1$ and $c\begin{pmatrix} \alpha \\ 0 \end{pmatrix} = \delta_\alpha$, which assures the presence of the graviton in the spectrum.

If we indicate [2] by α a generic sector in Ξ, the corresponding Hilbert space, H_α, contributes to the partition function of the model. We adopt the notation $\alpha = \{\alpha_L | \alpha_R\}$ to separate the left and right phases. The states in H_α have to satisfy the Virasoro conditions and the level matching condition, that, in our formulation, appear as

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + N_L = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R = M_R^2, \qquad (3.12)$$

[2] The notation can be confusing since we use α to indicate both a generic boundary condition vector and the generic sector in the Hilbert space. We assure from the context that it is always clear which quantity we are referring to.

where N_L and N_R are the total left and the total right oscillator number, respectively, acting on the vacuum, $|0>_\alpha$. The frequencies are given respectively for a fermion, f, and its conjugate, f^*, by

$$\nu_f = \frac{1+\alpha(f)}{2}, \text{ and } \nu_{f^*} = \frac{1-\alpha(f)}{2}. \tag{3.13}$$

The physical states contributing to the partition function are those satisfying the GSO conditions

$$e^{i\pi b_i \cdot F_\alpha}|s>_\alpha = \delta_\alpha c \begin{pmatrix} \alpha \\ b_i \end{pmatrix}^* |s>_\alpha, \tag{3.14}$$

where $|s>_\alpha$ is a generic state in the sector α, given by bosonic and fermionic oscillators acting on the vacuum. The operator $(b_i \cdot F_\alpha)$ is given by

$$b_i \cdot F_\alpha = \left\{ \sum_{left} - \sum_{right} \right\} b_i(f) F_\alpha(f), \tag{3.15}$$

where F is the fermion number operator with the following values

$$F(f) = \begin{cases} 1 & \text{for } f \\ -1 & \text{for } f^*. \end{cases} \tag{3.16}$$

If the sector α contains periodic fermions, the vacuum is degenerate and transforms in the representation of an $SO(2n)$ Clifford algebra. Hence, if f is such a periodic fermion, it will be indicated as $|\pm>$ and F assumes the value

$$F(f) = \begin{cases} 0 & \text{for } |+> \\ -1 & \text{for } |->. \end{cases} \tag{3.17}$$

The $U(1)$ charges for the physical states correspond to the currents $f^* f$ and are calculated by

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \tag{3.18}$$

hence the charge has possible values of $\{0, \pm 1\}$ for antiperiodic fermions, and $\{\pm\frac{1}{2}\}$ for periodic fermions.

The boundary vectors, β, contribute a set of GSO projections which act on the α sector states, projecting some of them out of the model. Which states survive is a function of the phase coefficients, $c\begin{pmatrix}\alpha_i \\ \beta_j\end{pmatrix}$. In a given α sector, a state is removed from the model unless it satisfies the GSO projection equation imposed by each basis vector

$$\mathbf{B}_j \cdot \mathbf{F}_\alpha = (\sum_i k_{j,i} a_i) + s_j - \frac{1}{2}\mathbf{B}_j \cdot \alpha \text{ (mod 2)}, \tag{3.19}$$

or, equivalently,

$$\mathbf{B}_j \cdot \mathbf{Q}_\alpha = (\sum_i k_{j,i} a_i) + s_j \text{ (mod 2)}. \tag{3.20}$$

3.1.2 Construction of Semi-Realistic Models

The construction of semi-realistic free fermionic models is related to a particular choice of boundary condition basis vectors, and the general procedure of the construction is based on two principal steps. For the particular class under investigation, the first stage is considering the Nanopoulos-Antoniadis-Hagelin-Ellis (NAHE) set (56; 57; 58) of boundary condition basis vectors $B = \{1, s, b_1, b_2, b_3\}$, which corresponds to $\mathbb{Z}_2 \times \mathbb{Z}_2$ compactification and the standard embedding of the gauge connection (**?** 59). The basis B is given explicitly below

$$1 = \{\psi^{1,2}, \chi^{1,\ldots,6}, y^{1,\ldots,6}, w^{1,\ldots,6} | \bar{y}^{3,\ldots,6}, \bar{w}^{1,\ldots,6}, \bar{\psi}^{1,\ldots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\ldots,8}\} \tag{3.21}$$

$$S = \{\psi^{1,2}, \chi^{1,\ldots,6}\} \tag{3.22}$$

$$b_1 = \{\psi^{1,2}, \chi^{1,2}, y^{3,\ldots,6} | \bar{y}^{3,\ldots,6}, \bar{\psi}^{1,\ldots,5}, \bar{\eta}^{1}\} \tag{3.23}$$

$$b_2 = \{\psi^{1,2}, \chi^{3,4}, y^{1,2}, w^{5,6} | \bar{y}^{1,2}, \bar{w}^{5,6}, \bar{\psi}^{1,\ldots,5}, \bar{\eta}^{2}\} \tag{3.24}$$

$$b_3 = \{\psi^{1,2}, \chi^{5,6}, w^{1,\ldots,4} | \bar{w}^{1,\ldots,4}, \bar{\psi}^{1,\ldots,5}, \bar{\eta}^{3}\}, \tag{3.25}$$

where the notation means that only periodic fermions are listed in the vectors. The left-moving internal coordinates are fermionized by the relation $e^{iX^i} = 1/\sqrt{2}(y^i + iw^i)$, as explained in Chapter 2, and a similar prescription holds for the right moving internal coordinates. The superpartners of the left moving bosons are indicated by

χ^i. The extra 16 degrees of freedom, $\bar{\psi}^{1,\ldots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\ldots,8}$, are complex fermions. The GSO one-loop phases for the NAHE set are

$$c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = -1,\ c\begin{pmatrix} 1 \\ S \end{pmatrix} = 1,\ c\begin{pmatrix} b_i \\ 1, S \end{pmatrix} = -1. \tag{3.26}$$

The gauge group introduced by the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ and $N = 1$ supersymmetry. The spacetime vector bosons generating the symmetry group arise in the Neveu-Schwarz (NS) sector, and in the sector $\xi_2 = 1 + b_1 + b_2 + b_3$. In particular, the $\bar{\psi}^{1,\ldots,5}$ are responsible for the $SO(10)$ symmetry, the $\bar{\phi}^{1,\ldots,8}$ generate the hidden E_8 and the internal fermions $\{\bar{y}^{3,\ldots,6}, \bar{\eta}^1\}, \{\bar{y}^1, \bar{y}^2, \bar{w}^5, \bar{w}^6, \bar{\eta}^2\}, \{\bar{w}^{1,\ldots,4}, \bar{\eta}^3\}$, generate the three horizontal $SO(6)$ symmetries. In the untwisted sector, we note the presence of states in the **10** vectorial representation of $SO(10)$, which represent the best candidates for the Higgs doublets. The three twisted sectors b_1, b_2, and b_3, produce 48 multiplets in the **16** representation of $SO(10)$, which carry $SO(6)^3$ charges, but are singlets under the hidden gauge group.

In the second stage of the construction, we consider additional basis vectors, generally indicated by α, β, γ, which reduce the number of generations to three and simultaneously break the four dimensional gauge group. This breaking is implemented by the assignment of boundary conditions, in the form of new basis vectors, which, through respective GSO projections, keep only the generators of the subgroup considered. For example, the breaking of $SO(10)$ is due to the boundary conditions of $\bar{\psi}^{1,\ldots,5}$ in α, β, γ, which can provide $SU(5) \times U(1)$ (38), $SO(6) \times SO(4)$ (49), or $SU(3) \times SU(2) \times U(1)^2$ gauge groups (40; 42; 57; 36). Further attempts in the construction of realistic models can be found in (50). The $SO(6)^3$ symmetries are also broken to flavor $U(1)$ symmetries. The worldsheet currents, $\eta^i \bar{\eta}^i, i = 1, 2, 3$, produce $U(1)$ charges in the visible sector, and further $U(1)^n$ symmetries arise by the pairing of real fermions among the right internal sector. If a left moving real fermion is paired with a right moving real fermion, the rank of the right gauge group is reduced by one. The pairing of the left and right movers is a key point in the phenomenology of free fermionic models, as it is strictly related to the reduction of the untwisted Higgs states, as we will discuss in detail later.

Once we extract the massless spectrum of a particular free fermionic model, the next step is the analysis of its superpotential. We postpone the explanation of this topic since it will be treated in the next sections. Further details concerning the construction of free fermionic models carried out step by step can be found in (47; 60).

3.2 Minimal Standard Heterotic String Models

After providing the main tools on the construction of model building, we would like to revisit some of the properties of semi-realistic Standard Model-like free fermionic models. One of their remarkable successes has been the fact that they can accomodate the top quark mass (61). These models offered an explanation as to why only the top quark mass is characterized by the electroweak scale, whereas the masses of the lighter quarks and leptons are supressed (42; 62). The reason is that only the top quark Yukawa coupling is obtained at the cubic level of the superpotential, whereas the Yukawa couplings of the lighter quarks and leptons are obtained from nonrenormalizable terms that are supressed relative to the leading order term. As explained earlier, the three generations arise from the three twisted sectors, whereas the Higgs doublets, to which they couple in leading order, arise from the untwisted (NS) sector. At leading order, each twisted generation couples to a separate pair of untwisted Higgs doublets. Analysis of supersymmetric flat directions implied that at low energies, only one pair of Higgs doublets can remain light, and the other Higgs doublets must obtain heavy mass from VEVs of Standard Model singlet fields. Thus, in the low energy effective field theory, only the coupling of the twisted generation that couples to the light Higgs remains at leading order. The consequence is that only the top quark mass is obtained at leading order, whereas the masses of the remaining quarks and leptons are obtained at subleading orders. Evolution of the calcuated Yukawa couplings from the string to the electroweak scale then gives a prediction for the top quark mass. The analysis of the top quark mass therefore relies on the analysis of supersymmetric flat directions and the decoupling of the additional untwisted electroweak Higgs doublets, that couple to the twisted generations at leading order.

3.3 D− and F− Flatness Constraints

The requirements for the preservation of spacetime supersymmetry, expressed in terms of the $D-$ and $F-$terms have been reviewed in (44; 52; 63; 45; 37). We will review them again here, with an emphasis on geometric interpretation of the non-Abelian VEVs.

Spacetime supersymmetry is broken in a model when the expectation value of the scalar potential,

$$V(\varphi) = \frac{1}{2}\sum_{\alpha} g_{\alpha}^{2}\big(\sum_{\alpha=1}^{dim(\mathcal{G}_{\alpha})} D_{a}^{\alpha} D_{a}^{\alpha}\big) + \sum_{i} |F_{\varphi_i}|^2, \tag{3.27}$$

becomes nonzero. The $D-$term contributions have the form,

$$D_{a}^{\alpha} \equiv \sum_{m} \varphi_{m}^{\dagger} T_{a}^{\alpha} \varphi_{m}, \tag{3.28}$$

where T_a^{α} is a matrix generator of the gauge group $\mathcal{G}_{\alpha}$ for the representation φ_m. The $F-$term contributions are

$$F_{\Phi_m} \equiv \frac{\partial W}{\partial \Phi_m}, \tag{3.29}$$

where W is the superpotential. The φ_m are spacetime scalar superpartners of the chiral spin-$\frac{1}{2}$ fermions, ψ_m, which together form a superfield, Φ_m. All of the D and F term contributions to (3.32) are positive semidefinite, so each must have a zero expectation value for supersymmetry to remain unbroken. In addition, because the

D and F terms are independent of one another, each must have zero expectation value separately.

For an Abelian gauge group, the $D-$term in (3.33) simplifies to

$$D^i \equiv \sum_m Q_m^{(i)} |\varphi_m|^2, \tag{3.30}$$

where $Q_m^{(i)}$ is the $U(1)_i$ charge of φ_m. When an Abelian symmetry is anomalous, meaning that the trace of its charge over the massless fields is nonzero,

$$\text{Tr } Q^{(A)} \neq 0, \tag{3.31}$$

the associated $D-$term acquires a Fayet-Iliopoulos (FI) term, given by

$$\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2} \text{Tr } Q^{(A)}, \tag{3.32}$$

where g_s is the string coupling and M_P is the reduced Planck mass, $M_P \equiv M_{Planck}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV. The $D-$term becomes

$$D^{(A)} \equiv \sum_{m'} Q_{m'}^{(A)} |\varphi_m|^2 + \epsilon. \tag{3.33}$$

The existence of the anomalous $U(1)$ symmetry is a common feature of free fermionic models (64). However, it is always possible to put the total anomaly into a single $U(1)$. The anomalous $U(1)_A$ is broken by the Green-Schwarz-Dine-Seiberg-Witten mechanism (65) in which a potentially large FI term is generated by the VEV of the dilaton. The FI term breaks supersymmetry near the string scale, $V \sim g_s^2 \epsilon^2$, unless it can be canceled by a set of scalar VEVs, $\{\langle\varphi_{m'}\rangle\}$, carrying anomalous charges $Q_{m'}^{(A)}$,

$$\langle D^{(A)} \rangle \equiv \sum_{m'} Q_{m'}^{(A)} |\langle\varphi_m\rangle|^2 + \epsilon = 0. \tag{3.34}$$

To maintain supersymmetry, a set of anomaly-canceling VEVs must simultaneously be $D-$flat for all additional Abelian and non-Abelian gauge groups,

$$\langle D^{i,\alpha} \rangle = 0. \tag{3.35}$$

A consistent solution to all constraints, $(3.32 - 40)$ specifies the overall VEV 'FI-scale', $\langle\alpha\rangle$ of the model. A typical FNY value is $\langle\alpha\rangle \approx 7 \times 10^{16}$ GeV.

In general, the FI $D-$term breaks supersymmetry unless there is a direction, $\hat{\phi} = \sum \alpha_i \phi_i$, in the scalar potential for which $\sum Q^i_A |\alpha_i|^2$ is of opposite sign to ϵ, and that is D-flat with respect to all the non-anomalous gauge symmetries, as well as F-flat. If such a direction exists, it will acquire a VEV, cancelling the FI term, restoring supersymmetry and stabilizing the vacuum. The solution to $(3.32 - 4)$, which corresponds to the choice of fields with non-vanishing VEVs, though non-trivial, is not unique. Therefore, in a typical model, there exists a moduli space of solutions to the F and D flatness constraints, which are supersymmetric and degenerate in energy (66). Much of the study of the superstring models phenomenology involves the analysis and classification of these flat directions. The methods for this analysis in string models have been systematized in (52; 67; 68; 37).

In general, it has been assumed in the past that in a given string model, there should exist a supersymmetric solution to the F and D flatness constraints. The simpler type of solutions utilize only fields that are singlets of all the non-Abelian groups in a given model. These are type I solutions. More involved solutions, type II solutions, that also use the non-Abelian fields, have also been considered (37). Non-Abelian fields have also been used in systematic methods of analysis (37). The general expectation that a given model admits a supersymmetric solution arises from analysis of supersymmetric point quantum field theories. In these cases, it is known that if supersymmetry is preserved at the classical level, there exist index theorems that forbid supersymmetry breaking at the perturbative quantum level (69). Therefore, in point quantum field theories, supersymmetry breaking may only be induced by non-perturbative effects (70).

3.3.1 Non-Abelian Flat Directions and Self-Cancellation

Past investigations have suggested that for several phenomenological reasons, including the production of viable three generation quark and lepton mass matrices and Higgs $h - \bar{h}$ mixing, non-Abelian fields must also acquire FI-scale VEVs (60).

In a number of these investigations, 'stringent' $F-$flatness is demanded, meaning that each superpotential term is forced to satisfy $F-$flatness by assigning no VEVs to at least two of the constituent fields. The absence of any nonzero terms from within $\langle F_{\Phi_m} \rangle$ and $\langle W \rangle$ is itself sufficient to guarantee $F-$flatness along a given $D-$flat direction, but such stringent demands are not necessary.

Complete absence of these terms can be relaxed, as long as they appear in collections which cancel among themselves in each $\langle F_{\Phi_m} \rangle$ and $\langle W \rangle$. It is desirable to examine the mechanisms of such cancellations, as they can allow additional flexibility for the tailoring of phenomenologically viable particle properties, while preserving supersymmetry (71). It should be noted, however, that success along these lines may be short-lived, with flatness retained in a given order only to be lost at a higher order.

Since Abelian $D-$flatness constraints limit only VEV magnitudes, we are left with the gauge freedom of each group with which to attempt a cancellation between terms, while retaining consistency with non-Abelian $D-$flatness. However, it can often be the case that only a single term from W becomes an offender in a given $\langle F_{\Phi_m} \rangle$ (37). If a contraction of non-Abelian fields bearing multiple components is present, it may be possible to effect a self-cancellation that is still 'stringently' flat in some sense. Even safe sectors of W, in particular with $\langle \Phi_m \rangle = 0$, may yield dangerous $\langle F_{\Phi_m} \rangle$ contributions. The individual $F-$terms may be separated into two classes based on whether or not Φ_m is Abelian.

CHAPTER FOUR

Semirealistic Models

In this chapter we present four semirealistic string models within the NAHE basis set. Before discussing the specifics of each model, we provide as motivation for investigating this specific class of models the first model within the free fermionic formulation with a reduced number of Higgs.

The first model is an $SO(10)$ model in the observable sector, while the last three models are $SU(5)$ models in the observable sector. The hidden sectors, however, are different in all four models Model 1 has an $SU(8) \times SU(2)$ hidden sector, while models 2 and 4 havel $SU(4) \times SO(10)$ hidden sectors. Model 5, however, has an $SU(2)^4 \times SU(4)$ hidden sector. In the analysis of the flat directions for each of the models, both Models 2 and 5 lack any singlet $D-$flat directions with negative anomalous charge. Thus, in these models, it was necessary to investigate $D-$flat directions which are non-Abelian.

4.1 Motivation

In (72), a semi-realistic free fermionic model was presented within the NAHE basis set which contains three chiral generations charged under the standard-like model subgroup of the underlying $SO(10)$ symmetry of the NAHE set. Fewer singlet particles were present from the untwisted sector. Analysis of flat directions showed that no stringent $F-$ and $D-$flat solutions appeared to exist to all order in the superpotential. In all previous investigations of semi-realistic free fermionic models, not only were such supersymmetric directions found, but all previous models yielded stringent flat directions which can be shown to be exact, or flat to all orders of nonrenormalizable terms. However, in this model, no physical $D-$flat direction that was generated kept $F-$flatness through sixth order. It was speculated that only stringent flat directions can be flat to all orders of renormalizable terms. If validated, it would indicate that the particular model under investigation appears to have no $D-$flat directions which can be shown to be $F-$flat to all orders, other than by order analysis.

Our aim, then, was to investigate a series of models within this class. The goal was to determine whether the flat direction analysis of these models yielded the same results as the model mentioned above. If that were the case, we could more certainly conclude that the non-existence of all order flat directions is a general property of this class of models. However, flat direction analysis of the four models presented herein shows this not to be the case, as all order flat directions were found in three of the four of the models.

4.2 Flat Direction Analysis

We begin with the flat direction analysis of each of the four models, beginning with the simpler models first, those with singlet flat directions. We then discuss the models with non-Abelian flat directions, and provide discussion of stringent flat directions.

4.2.1 Singlet Flat Directions

In this section, we investigate the simplest of our four models, those with singlet flat directions. We have called these models 1 and 4, and their basis vectors are given in Appendix A and C, respectively.

4.2.1.1 Model 1. Model 1 contains six $U(1)$s, five of which are anomalous,

$$TrQ_1 = 768, \quad (4.1)$$

$$TrQ_3 = TrQ_4 = -TrQ_5 = 192, \quad (4.2)$$

$$TrQ_6 = -2112. \quad (4.3)$$

The total anomaly can be rotated into a single $U(1)_A$, rescaling by a factor of 192, and the new basis reads

$$Q'_1 = 4Q_1 + Q_3 + Q_4 - Q_5 - 11Q_6, \quad (4.4)$$

$$Q'_2 = Q_2, \quad (4.5)$$

$$Q'_3 = Q_3 - Q_4, \quad (4.6)$$

$$Q'_4 = 11Q_1 + 4Q_6, \quad (4.7)$$

$$Q'_5 = Q_3 + Q_4 + 2Q_5, \quad (4.8)$$

$$Q_A = -12Q_1 + 137(Q_3 + Q_4 - Q_5) + 33Q_6. \quad (4.9)$$

From now on, we adopt the convention of calling $Q'_i, i = 1, ..., 5$, simply Q_i.

To search for flat directions, we employ the methodology in (73). We start by constructing a basis of $D-$flat directions under $Q_{1,...5}$, and then we investigate the existence of $D-$flat directions in the anomalous $U(1)_A$. Subsequently, we will have to impose $D-$flatness under the remaining gauge groups and $F-$flatness. As we will see for this model, however, it was only necessary to obtain a basis of $D-$flat directions under $Q_{1,...5}$. To generate the basis flat directions under $Q_{1,...5}$, we start by forming a basis of gauge invariant monomials under $U(1)_1$, which we then use to construct a basis of invariant monomials under $U(1)_2$, and so forth.

As a first step, we investigate the existence of flat directions involving vacuum expectation values for only the fields which are singlets under both the observable and hidden sector gauge groups. These fields are $\phi_{1,2,3}$, and $\psi_{1,...14}$, a total of 17 fields. Note that we don't include $\Phi_{1,2,3}$ because, though they are singlets under both the observable and hidden sector gauge groups, they carry no $U(1)$ charge. We have 5 constraints we need to impose, from the original number of $U(1)$s, six, minus one anomalous $U(1)_A$, so the basis set of flat directions should contain at most 12 elements. For a given model, finding less than the maximum amount of basis directions is possible and simply means that all the fields which comprise the directions are not completely independent of one another. For this model, all 17 fields can be incorporated into the flat directions, depicted in Table A.2 of Appendix A.

For any basis set of $D-$flat directions, the the basis directions can have positive, negative, or zero anomalous charge. In the maximally orthogonal basis used in the approach of (68; 37), each basis direction is uniquely identified with a particular VEV. In other words, although each basis direction generally contains many VEVs, each basis direction contains at least one particular VEV that appears solely in that direction. Such is the case for this model, as well. As can be seen in Table A.2 of Appendix A, the fields $\phi_{1,2,3}$ and $\psi_{1,...9}$ are uniquely associated with one basis direction.

In our notation, a physical flat direction may have a negative norm-square for a vector-like field. This denotes that it is the oppositely charged vector partner field which acquires the VEV, rather than the field itself. Basis directions themselves may have vector-like partner directions if all associated fields are vector-like. This is the case for model 1, as Table A.1 in Appendix A shows that all 17 fields which make up the $D-$flat basis directions for this model are vector-like. On the other hand, if, in particular, the field generating the VEV uniquely associated with a basis direction does not have a vector-like partner, that basis direction cannot have a vector-like partner direction.

Since the FI-term for this model is positive,[1] with $TrQ_A = 1344$, a physical flat direction must carry a negative anomalous charge in order to cancel the dangerous FI term, as previously discussed. From Table A.2 in Appendix A, we see that three of the twelve $D-$flat basis directions, $D_{1,2,3}$, carry negative anomalous charge. For this model, then, it is not necessary to continue to investigate a basis set of $D-$flat directions from non-Abelian fields which may take on VEVs, and we can proceed to search for $F-$flat directions.

In order to search for $F-$flat directions, we must first construct the superpotential, because according to (3.29), the $F-$terms are the derivatives of the superpotential with respect to the superfields of a given model. We generated the superpotential for third through sixth order,[2] and first tested for stringent $F-$flatness, in which at least two fields do not take on VEVs, described in more detail in section 4.2.2, from all third through sixth order superpotential terms. For those combinations in which two or more fields did not take on VEVs, we then investigated what combinations of $D-$flat basis fields provided F-flatness to what order in the superpotential. This analysis involved looking at which basis fields taking on VEVs came from which R/NS sector (1 to 3), summing the totals for each category, and matching them to the rules in (37). The total number of fields is given by n, and again for $3 \leq n \leq 5$, the combination was not further tested. For $n > 5$, the combination was tested. If the combination followed the rules in (37), it was dangerous, and broke supersymmetry at order n in the superpotential. However, if the combination did not follow the rules, supersymmetry was not broken, and we listed the direction as $F-$flat to all order.

[1] In general, there is no restriction on the sign of the FI-term, however models constructed from the NAHE set typically have an FI-term which is positive.

[2] Although we do provide the superpotential for this model here, and for three of the four models, we have produced it only for third through sixth order, we can generate the superpotential for any model to any arbitrary order. We have not yet generated the superpotential for model 5, even though, as discussed in section 4.2.2, we could not find any $D-$flat direction with negative anomalous charge.

For increasing number of basis fields taking on VEVs, the above analysis becomes time consuming to do by hand, and it became reasonable to implement a FORTRAN program to do the flat direction analysis. While implementation of the program was not necessary for this model, it was used for the flat direction analysis of the following two models, and can be used for any future model. An additional feature of the FORTRAN program is its ability to allow for the addition of coeffiecients to the fields present in the basis directions. Because we need supersymmetry to be broken at no less than 17th order in the superpotential to be consistent with the Standard Model, we ran coeffiecients from -17 to 17 for all fields. This ensures that a set of VEVed basis fields can be multiplied by any arbitrary coeffiecient and remain F-flat to sufficient order. Once a direction was found to be F-flat to at least 17th order in the superpotential, it was listed as F-flat to all order, as it is not important that supersymmetry breaks beyond 17th order to be consistent with the Standard Model.

We hoped to find at least four to six directions that are F-flat to all order for any given model, to be consistent with past investigations. For the parameter space of VEVed basis directions investigated for model 1 thus far, we found fifteen directions which were F-flat to all order. These are listed in Table A.3 of Appendix A. For model 1, we were able to find a basis set of F-flat directions to all order which included twelve of the seventeen basis fields making up the D-flat directions. At low orders, each individual superpotential term includes several stringent $F-$term constraints on the coefficients of physical flat directions. The set of constraints from the superpotential terms with only singlet fields translate into the requirement that two or more singlet fields in a given superpotential term cannot take on VEVs, as will be discussed in section 4.2.2. The five fields which do not appear in the all order flat directions, $\psi_{1,2,4,5,7}$, never took on VEVs in the superpotential, and were thus excluded from the all order flat directions.

For any given model, finding singlet flat directions to all order means that non-Abelian flat directions likely exist to all order for that model. Such is the case for model 1, as well as model 4, below. However, for our purposes, merely finding singlet directions which are flat to all order is sufficient. Thus, with the first model we investigated, we were able to show that the lack of F-flat directions to all order is not a general property of reduced Higgs models.

Once a flat direction basis is found for a given model, we can investigate which hidden sector fields take on mass, and at what scale they take on mass. In order to examine which hidden sector fields become massive, we generate the superpotential for each flat direction. In this analysis, only the superpotential terms which gain mass are generated. Unlike the observable sector fields, the hidden sector fields do not require the Higgs to become massive. Instead, we must examine at what order the hidden sector fields appear in the superpotential, and in what combinations with observable sector fields they occur. The order at which the hidden sector field appears in the superpotential corresponds to the order at which that field becomes massive. The higher the order at which the field appears, the lower the order at which the field takes on mass. In order for a hidden sector field to take on mass, at least two hidden sector fields and one observable sector field which takes on a VEV must appear in a superpotential term.

Preliminary investigation shows a total of twelve of the hidden sector fields appearing in the superpotential across all fifteen of the $F-$flat directions given for this model. The twelve fields are $H_{1,2,3,5,6,9,10}$ and $\bar{H}_{1,2,3,12}$, and most first appear at third order in the superpotential, meaning that they gain mass at a scale of approximately one tenth of the string scale, taking 17th order in the superpotential to be of the electroweak scale. If a field occurs at a higher order in the superpotential, or if a field appears twice at a given order in the superpotential, that mass is reduced by a factor of ten.

The superpotenitial for the first flat direction of model 1 is given at the end of Appendix A. Again, we begin at third order in the superpotential, which means that no mass terms exist for first or second order. The reason for this is that we need at least two non-Abelian fields to occur in a term before it can gain mass. After the superpotential is generated, we look at which fields in a given superpotential term take on VEVs. For example, in the superpotential at the end of Appendix A, the following term occurs,

$$H_1\bar{H}_2\bar{\phi}_3, \tag{4.10}$$

where H_1 and $\bar{H}_2$ are the non-Abelian hidden sector fields, and $\bar{\phi}_3$ is a singlet field. If $\bar{\phi}_3$ takes on a VEV, H_1 and $\bar{H}_2$ will become massive at one tenth the string scale. However, if $\bar{\phi}_3$ does not take on a VEV, H_1 and $\bar{H}_2$ will not become massive; instead the term represents an interaction term. If a fourth order term appeared in the superpotential of the form,

$$H_1\bar{H}_2\bar{\phi}_3\bar{\phi}_4, \tag{4.11}$$

then H_1 and $\bar{H}_2$ would only become massive if both $\bar{\phi}_3$ and $\bar{\phi}_4$ took on VEVs. If only one of the two takes on a VEV, the term will again be an interaction term. At any order, all additional singlet fields in a given superpotential term must take on VEVs for the hidden sector fields to become massive. Other non-singlet fields may be present in a term, but they are not required to take on VEVs in order for the hidden sector fields to become massive. Its the requirement that there be at least two non-Abelian fields and one singlet field in a given superpotential before the non-Abelian fields can become massive that requires us to look at no lower than third order in the superpotential.

In addition to any hidden sector fields which can become massive, additional observable sector matter can be generated in this manner, provided that a given superpotential term contains a Higgs. In the case of this superpotential, we have the following terms with Higgs,

$$\bar{h}_2\bar{h}_2\bar{\phi}_3,\ \ \bar{h}_1\bar{h}_1\bar{\phi}_3\overline{\psi}_3\overline{\psi}_3, \tag{4.12}$$

which means that both $\bar{\phi}_3$ and $\overline{\psi}_3$ become massive. The scales at which any additional observable sector fields become massive are the same as those at which the hidden sector fields become massive. In this case, just as with a hidden sector field, we would discard the massive term $\bar{\phi}_3$ because it appears at both third and fifth order in the superpotential, lessening the mass it gains. We still consider the mass from $\overline{\psi}_3$, because the first place it gains mass is at fifth order, or one thousandth the string scale.

We then consider a generic $SU(N_c)$ gauge group containing N_f flavors of matter states. When $N_f < N_c$, the gauge coupling, g_s, though weak at the string scale M_{str}, becomes strong at a condensation scale

$$\Lambda = M_P e^{8\pi^2/\beta g_s^2}, \tag{4.13}$$

where $\beta = -3N_c + N_f$. The N_f flavors counted are only those which ultimately receive masses $m << \Lambda$. This results in an expectation value of

$$W \sim N_c \Lambda^3 (\frac{m}{\Lambda})^{N_f/N_c} \tag{4.14}$$

for the nonperturbative superpotential. The scale of $\langle W \rangle$ corresponds to the scale at which hidden sector matter condensates into observable sector matter. In this case, the hidden sector gauge groups are $SU(8)$ and $SU(2)$, and we can therefore calculate the scale at which the hidden sector matter condensates under each gauge group for this model. Work on the hidden sector matter states for all the models presented herein will be presented in (74; 75).

4.2.1.2 Model 4. Model 4 contains eight $U(1)$s, seven of which are anomalous:

$$TrQ_1 = TrQ_3 = 120, \quad (4.15)$$

$$TrQ_2 = -TrQ_4 = -96, \quad (4.16)$$

$$TrQ_5 = 48, \quad (4.17)$$

$$TrQ_6 = -TrQ_7 = 24. \quad (4.18)$$

Again, we rotate the total anomaly into a single $U(1)_A$, rescaling by a factor of 24, and the new basis reads

$$Q'_1 = Q_1, \quad (4.19)$$

$$Q'_2 = Q_2 - Q_4, \quad (4.20)$$

$$Q'_3 = Q_7 + Q_8, \quad (4.21)$$

$$Q'_4 = Q_3 + Q_5, \quad (4.22)$$

$$Q'_5 = Q_6 - Q_7 + Q_8, \quad (4.23)$$

$$Q'_6 = 4(Q_2 + Q_4) + 5(Q_3 - Q_5), \quad (4.24)$$

$$Q_A = Q_2 + 5(Q_2 + Q_4) + 4(Q_5 - Q_3) + 2Q_6 + Q_7 - Q_8. \quad (4.25)$$

Again, we begin our search for flat directions as we did for model 1, by first constructing a set of basis vectors under $Q_{1,\dots 6}$. We again include in this analysis only the fields with vanishing hypercharge and which are singlets under the Standard Model gauge group.

Beginning again with the investigation of the existence of flat directions involving VEVs for only the fields which are singlets under both the observable and hidden sector gauge groups, we are left with the fields $E^c_{1,2,3}$, $\bar{E}^c_1$, ϕ_1, and $\psi_{1,\dots,24}$, where again we have excluded $\Phi_{1,2,3}$ because they carry no $U(1)$ charge. This leaves us with 28 fields and seven constraints, so the basis set of $D-$flat directions for this model should contain 21 elements, as depicted in Table C.2 in Appendix C.

As we can see from Table C.2, each basis direction again is uniquely identified with a particular field VEV. Unlike model 1, however, not all the fields comprising the basis set of $D-$flat directions for this model are vector-like. In fact, the only basis direction for this model which is vector-like is D_5, because it uniquely contains a vector-like field, ϕ_1. The other three vector-like fields comprising the $D-$flat basis, $\psi_{1,2,3}$ are associated with at least twelve basis directions.

Since the FI-term for this model is positive, with $TrQ_A = 2112$, a physical flat direction must carry a negative anomalous charge to cancel the FI-term, as mentioned before. From Table C.2, we see that five of the 21 $D-$flat basis directions, $D_{1,\ldots,5}$, carry negative anomalous charge. Once again, for this model, it is not necessary to continue to investigate a basis set of $D-$flat directions from non-Abelian fields which may take on VEVs, and we proceed to search for $F-$flat directions.

The search again begins with the construction of the superpotential, which we again generated from third through sixth order. In the $F-$flat direction analysis for this model, we employed the previously mentioned FORTRAN program because the basis set of fields for the $D-$flat directions was so large, containing 28 fields. The first iteration of the program tested for terms in the superpotential in which two or more fields do not take on VEVs, and the second iteration of the program tested the rules provided in (37) to assess at what order $F-$flatness was broken. This iteration included the tests for coefficients on the basis directions. For this model, however, a sufficient number of directions were found for which at least two fields in the superpotential did not take on VEVs that it was not necessary to run the second iteration of the program. This could, however, be done in future to see how many additional directions may be obtained which are $F-$flat to all order. Table C.3 in Appendix C lists the first fifteen $F-$flat directions found to all order for this model. The first iteration of the program produced approximately 100 $F-$flat directions to all order, however, we have listed only the first fifteen, as this search is meant to be representative, not exhaustive. This is also the reason we did not continue with the second iteration of the program. Again, not all fields in the $D-$flat basis appear in the $F-$flat basis. In this case, only 16 of 28 original fields appear in the $F-$flat basis due to the constraints on the coefficients of physical $D-$flat directions. This makes sense because more basis directions will have more constraints, which will eliminate more fields from the $F-$flat basis.

4.2.2 Non-Abelian Flat Directions

In general, systematic analysis of simultaneously $D-$ and $F-$flat directions in anomalous models is a complicated, nonlinear process.[3] In weakly coupled heterotic string (WCHS) model building, as mentioned earlier, $F-$flatness of a specific VEV direction in the low energy effective field theory may be proven to a given order by cancellation of $F-$term components, only to be lost at higher order at which cancellation is not found. An exception is directions with stringent $F-$flatness (50; 52; 77). Instead of allowing cancellation between two or more components in an $F-$term, stringent $F-$flatness requires that each possible component in an $F-$term have zero vacuum expectation value.

When only non-Abelian singlet fields acquire VEVs, stringent flatness implies that two or more fields in a given $F-$term cannot take on VEVs. For example, for the $\overline{\psi}_4$ term in the third order superpotential for model 2, given at the end of Appendix B, the components of the F-term are:

$$F_{\overline{\psi}_4} = \psi_5\psi_9 + \overline{\psi}_8\overline{\psi}_{12}. \tag{4.26}$$

For stringent $F-$flatness, we require not just that $\langle F_{\overline{\psi}_4}\rangle = 0$, but that each component within is zero, i.e.,

$$\langle\psi_5\psi_9\rangle = 0, \;\langle\overline{\psi}_8\overline{\psi}_{12}\rangle = 0. \tag{4.27}$$

[3] In (76) it is argued that, in addition to flat directions, isolated special points generically exist in the VEV parameter space which are not located along flat directions, but for which all $D-$ and $F-$terms are indeed zero. The interested reader can find additional information in (76; 72).

Thus, by not allowing cancellation between components in a given $F-$term, stringent $F-$flatness imposes stronger constraints than generic $F-$flatness, but requires significantly less fine-tuning between the VEVs of the fields.

The net effect of all stringent $F-$flatness constraints on a given superpotential term is that at least two fields in the term must not take on VEVs. This condition can be relaxed when non-Abelian fields acquire VEVs. Self-cancellation of a single component in a given $F-$term is possible between various VEVs within a given non-Abelian representation. Self-cancellation was discussed in (52) for $SU(2)$ and $SO(2n)$ states.

A given set of stringent $F-$flatness constraints are not independent and solutions to a set can be expressed in the language of Boolean algebra and applied as constraints to linear combinations of $D-$flat basis directions. Such a language makes it clear that the effect of stringent $F-$flat constraints is strongest for low order superpotential terms and lessens with increasing order. In particular, for the two models discussed in this section, stringent $F-$flatness is extremely constraining on the VEVs of the reduced number of untwisted singlet fields appearing in the third through fifth order superpotential, even to the point of excluding any stringent $F-$flat directions for model 5. This is in comparison to the larger number of singlets in the two models in the previous section, as well as in the model of (46).

Though it is possible to imagine that stringent $F-$flatness constraints require order-by-order testing of the superpotential terms, this is, in fact, not necessary. All-order stringent $F-$flatness can be proven or disproven by examining only a small set of possible dangerous superpotential terms, or terms which break $F-$flatness. Through various processes (78), a finite set of superpotential terms can be constructed which generates all possible dangerous superpotential terms for a given $D-$flat direction. The basis of gauge-invariants can always be formed with particular attributes:

(1) Each basis element term contains at most one unVEVed field, because to threaten $F-$flatness, a gauge-invariant term, which is necessarily without anomalous charge, can contain no more than one unVEVed field;

(2) There is at most one basis term for each unVEVed field in the model; and

(3) when an unVEVed field appears in a basis term, it appears only to the first power.

To appear in a string-based superpotential, a gauge invariant term must also follow R-NS worldsheet charge conservation rules. For free fermionic models, these rules have been generalized from finite order (79; 80) to all-order (37). The generic all-order rules can be applied to systematically determine if any product of $F-$flatness threatening superpotential basis elements generated via (78) survive in the corresponding string-generated superpotential. If none survive, then $F-$flatness proven to all finite order. This technique has been used to prove $F-$flatness to all finite order various directions in various models (50; 52; 37; 81; 82). Alternately, if any terms do survive, the lowest order at which stringent $F-$flatness is broken is determined.

All-order stringent flat directions contain a minimum number of VEVs and appear in models as the roots of more fine-tuned, and generally finite order, flat directions which require specific cancellations between $F-$term components. General flat directions, however, may involve cancellations between sets of components of different orders in the superpotential.

All-order stringent flat directions have indeed been discovered to be such roots in all prior free fermionic heterotic models for which systematic flat direction analysis and classification has been performed. Based on the results of the previous two models presented herein, we can conclude that the lack of all order stringent flat directions, as in the model presented in (72), is not a general property of low Higgs models, and we would expect to find all order stringent flat directions in future investigations into models in free fermionic heterotic models.

4.2.2.1 Model 2. Model 2 contains seven $U(1)$s, six of which are anomalous:

$$TrQ_1 = TrQ_2 = -TrQ_3 = -96, \tag{4.28}$$

$$TrQ_4 = TrQ_5 = TrQ_6 = -48. \tag{4.29}$$

Rotating into a single anomalous $U(1)_A$ and rescaling by a factor of 48 gives:

$$Q'_1 = Q_1 - Q_2, \tag{4.30}$$

$$Q'_2 = Q_5 - Q_6, \tag{4.31}$$

$$Q'_3 = Q_3 + 2Q_4, \tag{4.32}$$

$$Q'_4 = Q_1 + Q_2 - 2(Q_5 + Q_6), \tag{4.33}$$

$$Q'_5 = 2(Q_1 + Q_2 + 2Q_3 - Q_4) + Q_5 + Q_6, \tag{4.34}$$

$$Q'_6 = Q_7, \tag{4.35}$$

$$Q_A = -2(Q_1 + Q_2 + Q_3) - (Q_4 + Q_5 + Q_6). \tag{4.36}$$

Investigating only fields which are singlets under the observable and hidden sectors leaves us with the fields $E^c_{1,2,3}$, $\Phi_{1,2,3}$, $\phi_{1,\ldots,6}$, and $\psi_{1,\ldots,12}$. Again, simple analysis excludes $E^c_{1,2,3}$ from the basis directions, so we are left with 21 fields and six constraints, which gives us the 15 basis directions that can be seen in Table B.2 of Appendix B.

We can see that again each basis direction contains a unique field VEV, just as in the previous two models. In addition, all the fields which make up the set of basis $D-$flat directions are vector-like, which means that all 15 of the $D-$flat basis directions formed from singlet fields only for this model are vector-like, just as those in model 1 are. Again, the FI-term for this model is positive, with $TrQ_A = 720$, so again we need a physical $D-$flat direction with negative anomalous charge to cancel the dangerous FI term. Since all the $D-$flat directions in Table B.2 are vector like, a direction could be physical if it had postive anomalous charge, because its vector partner would have negative anomalous charge. However, we see from Table B.2

that no such $D-$flat directions exist for this model, since no $D-$flat direction listed in Table B.2 has any anomalous charge. Therefore, we needed to expand our $D-$flat basis direction search to include non-Abelian fields. This result is not altogether unexpected, as non-Abelian VEVs have been required for physical all order flat directions in other semi-realistic free fermionic heterotic models in the past (50).

The $D-$flat basis directions for non-Abelian fields only and for a mix of non-Abelian and singlet fields are shown in Tables B.3 and B.4 of Appendix B, respectively. Again, all $D-$flat basis directions for the both the non-Abelian and mixed fields contain one unique field VEV. However, none of the unique field VEVs for the non-Abelian basis directions are vector-like, and thus, none of the ten non-Abelian basis directions depicted are vector-like. Only one of the non-Abelian directions, D_9, has negative anomalous charge, $Q_A = -30$, which is sufficent to cancel the dangerous positive FI-term, even though none of the other basis directions are vector-like and thus would have vector partners which took on negative charge, opposite to their positive charge. Two of the unique fields for the mixed basis vectors are vector like, and thus the directions containing them, D_6 and D_7, are vector like. However, because they have no anomalous charge, they cannot cancel the positive FI-term.

With one physical $D-$flat direction found to cancel the dangerous FI-term, we proceeded to search for directions which were $F-$flat to all order, starting again with generating the superpotential for third through sixth order. The third order superpotential for model 2 is given at the end of Appendix B. We again implemented the first iteration of the FORTRAN program, whereby we looked for stringent $F-$flatness by requiring that at least two fields in the superpotential not take on VEVs. Results provided over 60 such directions, and listed in Table B.5 of Appendix B are the first ten of these. Again because the initial search provided so many directions which were $F-$flat to all order, we did not proceed to the second iteration of the program for the purposes of this thesis. We also note that only twelve of the original set of 45 fields are present in the final $F-$flat basis, which is to be expected for a large number of fields with a large number of constraints. Additionally, because our final flat direction basis includes non-Abelian fields, we have shown that the hidden sector breaks supersymmetry in this model, as these directions were involved in investigating which directions were flat to what order.

4.2.2.2 Model 5. Model 5 contains eight $U(1)$s, six of which have anomalous charge:

$$TrQ_3 = TrQ_4 = -TrQ_5 = TrQ_7 = -96 \quad (4.37)$$

$$TrQ_6 = TrQ_8 = 48. \quad (4.38)$$

Rotating into a single anomalous $U(1)_A$ and rescaling by a factor of 48 gives:

$$Q'_1 = Q_1 + Q_2, \quad (4.39)$$

$$Q'_2 = Q_3 - Q_4, \quad (4.40)$$

$$Q'_3 = Q_6 - Q_8, \quad (4.41)$$

$$Q'_4 = Q_5 + Q_7, \quad (4.42)$$

$$Q'_5 = Q_3 + Q_4 + 2(Q_6 + Q_8), \quad (4.43)$$

$$Q'_6 = Q_3 + Q_4 + Q_5 + Q_7 + 2(Q_6 + Q_8), \quad (4.44)$$

$$Q_A = -2(Q_3 + Q_4 - Q_5 + Q_7) + Q_6 + Q_8. \quad (4.45)$$

Once again, we proceed with searching for singlet flat directions. The fields which are singlets in both the observable and hidden sectors are the following: $E^c_{1,2,3}$, $\phi_{1,\ldots,6}$, and $\psi_{1,\ldots,6}$, where we do not include $\Phi_{1,\ldots,4}$ in the basis of $D-$flat directions since they do not carry any $U(1)$ charge. We are left with 16 fields and eight constraints from the $U(1)$ charges for a total of eight $D-$flat basis directions, shown in Table D.2 of Appendix D.

We notice that again, each basis direction has a unique field VEV associated with it, but the only direction with a unique field VEV from a vector-like field is D_5, which has no anomalous charge. For $D_{1,2,3}$ the total anomalous charge is positive, and the remaining basis directions have no anomalous charge. We therefore have no $D-$flat direction which can cancel the positive FI-term for this model, with $TrQ_A = 864$. We must then proceed to search for non-Abelian $D-$flat directions, the results of which will appear in (74).

4.3 Gauge Groups

In this section, we discuss the details of each of the models. Discussion of matter content and representations is also given. Further details can be found in Appendices A-D.

Each model has a reduced number of Higgs particles, less than the standard three, one for each generation. They are denoted by h_i. All Higgs particles are paired with an anti-Higgs particle, $\bar{h}_i$. While models 1,4, and 5 all have two pairs of Higgs under the 10 representation of $SO(10)$ and the 5 and $\overline{5}$ representation of $SU(5)$, respectively, model 2 lacks any Higgs particles at all, making it incompatible with Standard Model predictions. Additionally, model 4 contains six additional unpaired Higgs-like particles, denoted by $h_{3,..,8}$, which are exotic Higgs. The first three are 5 representations under $SU(5)$, while the last three are $\overline{5}$ representations of $SU(5)$. Exotic Higgs come from the twisted sector of a given model, while the standard Higgs come from the untwisted sector.

A general property of models with low number of Higgs is that they have fewer singlet particles from the untwisted sector. This is indeed the case for all of our models, as we see for each model that only the ϕ_i fields come from the untwisted sector. All other singlet fields come from a combination of one or more of the twisted sectors, which include the sectors from the additional basis vectors for each model.

Model 1 is the only model with four additional basis vectors; all other models presented herein have three additional basis vectors. The basis vectors of a given model determine the gauge groups of that model. Model 1 is our only model with an $SO(10)$ observable sector gauge group; the three other models all have $SU(5)$ observable sector gauge groups. Each break to the Standard Model gauge group, $SU(3)\times SU(2)\times U(1)$, and they are both equally valid groups from which to obtain semi-realistic Standard-like GUT Models. The $U(1)^{1,2,3}$ gauge groups are standard symmetries present in all three generation free fermionic models which employ the NAHE set. Additional horizontal $U(1)^n$, where $n > 4$, symmetries arise by pairing two real fermions from the sets $\{\bar{y}^{3,\ldots,6}\}$, $\{\bar{y}^{1,2},\bar{w}^{5,6}\}$, and $\{\bar{w}^{1,\ldots,4}\}$. The final gauge group depends on such pairings, which can be seen in the (d) tables of Appendices A-D, which contain the additional basis vectors for each model. The number of pairings corresponds to the number of additional $U(1)$s, which is two for model 1, three for model 2, and four for models 4 and 5.

The existence of these additional $U(1)$ gauge groups is correlated with the assignment of asymmetric boundary conditions with respect to the set of internal world-sheet fermions, $\{y,w|\bar{y},\bar{w}\}^{1,\ldots,6}$, in the basis vectors that extend the NAHE for a given model. This assignment of asymmetric boundary conditions in the basis vector that breaks the $SO(10)$ symmetry to $SO(6)\times SO(4)$ results in the projection of the untwisted Higgs color-triplet fields and preservation of the corresponding electroweak doublet Higgs representations (83).

In model 1, we find the states which correspond to the three generations, denoted by $G_{1,2,3}$, are the 16 representations of $SO(10)$, while in the remaining models, the states corresponding to the three generations are denoted by $F_{1,2,3}$, which are the 10 representations of $SU(5)$, and $\bar{F}_{1,2,3}$, which are the $\overline{5}$ representations of $SU(5)$. All other states in each model are singlets under the observable sector gauge groups. The non-Abelian fields, denoted by H_i and $\bar{H}_i$ are the only states which are not singlets under the hidden sector gauge groups. In model 1, the non-Abelian representations are the 8 of $SU(8)$, which only the last three fields contain, and the

2 of $SU(2)$, which the remaining non-Abelian fields contain. In model 2, $H_{10,11,12}$ are 6 representations of $SU(4)$, $H_{7,8,9}$ are the 4 representations of $SU(4)$, and $H_{1,..6}$ are the $\overline{4}$ representations of $SU(4)$. The remaining non-Abelian fields of model 2 are 10 representations of $SO(10)$. Model 4 contains only an $SU(4)$ hidden sector gauge group, with all the non-Abelian particles in the 8 representation. Finally, model 5 contains four $SU(2)$ gauge groups and one $SU(4)$ gauge group. $H_{4,6,32}$ are the 6 representations of $SU(4)$ and are singlets under all other gauge groups, while H_{47} is also a 6 representation of $SU(4)$ with 2 representations of $SU(2)^1$ and $SU(2)^3$ and contains no $U(1)$s. The particles which are 4 representations of $SU(4)$ are $H_{37,38}$, and those which are $\overline{4}$ representations of $SU(4)$ are $H_{40,\ldots,45}$, all of which are also singlets under all other gauge groups. All other non-Abelian fields for model 5 have at least one 2 representation of $SU(2)^{1,\ldots,4}$.

CHAPTER FIVE

The NAHE Variation With a Geometric Twist

In this chapter, we present a variation of the NAHE basis set for free fermionic heterotic string models. By rotating some of the boundary conditions of the NAHE periodic/anti-periodic fermions, $\{y^m, \bar{y}^m, w^m, \bar{w}^m\}$, for $m = 1$ to 6, associated with the six compact dimensions of a bosonic lattice/orbifold model, we show an additional method for enhancing the standard NAHE gauge group of $SO(10)$ back to E_6. This rotation transforms $(SO(10) \otimes SO(6)^3)_{rmobs} \otimes (E_8)_{hid}$ into $(E_6 \otimes U(1)^5)_{obs} \otimes SO(22)_{hid}$. When $SO(10)$ is enhaced to E_6 in this manner, the i^{th} Minimal Supersymmetric Standard Model (MSSM) matter generation in the $SO(10)$ $\mathbf{16}_i$ representation, originating in twisted basis vector $\mathbf{b}_i$, recombines with both its associated untwisted MSSM Higgs in a $\mathbf{10}_i$ representation and an untwisted non-Abelian singlet, ϕ_i, to form a $\mathbf{27}_i$ representation of E_6. Beginning instead with the E_6 model, the inverse transformation of the fermion boundary conditions corresponds to partial Grand Unified Theory (GUT) breaking via boundary rotation.

Correspondence between free fermionic models with $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist, especially of the NAHE class, and orbifold models with a similar twist has received further attention recently. The NAHE variation discussed here also involves a $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist and offers additional understanding regarding the free fermion/orbifold correspondence. Further, models based on this NAHE variation offer some different phenomenological features compared with NAHE based models. In particular, the more compact $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist of the NAHE variation offers a range of mirror models not possible from NAHE based models.

5.1 A Geometric Twist

The parameter space of the weakly coupled free fermionic heterotic string (WCFFHS) (33; 31; 34; 35; 53) region of the string landscape has been shown to be rich in semi-realistic models containing the MSSM or its extensions. The WCFFHS region has produced a vast range of semi-realistic Near-MSSM-like models (40; 41; 42; 67; 52; 45; 84; 77; 85; 44) semi-GUT models (50; 49; 86; 87), and GUT models (38), etc. The majority of these models are constructed as extensions of the NAHE set (56), with the five basis vectors of the NAHE set as their common core. Within the five basis vectors of the NAHE set, the twelve real free fermions representing the six compactified bosonic directions have boundary condition vectors equivalent to a $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ orbifold twist. While basis vector extensions to the NAHE set may or may not break $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry, the semi-realistic models consistently follow the latter scheme.

Of current focus is the correspndence between free fermionic and orbifold models (88; 89; 90). In (88) a complete classification was obtained for orbifolds of the form X/G, with X the product of three elliptic curves and G an Abelian extension of a group of $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twists acting on X. This includes $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ orbifolds. Each such orbifold was shown to correspond to a free fermionic model with geometric interpretation. The NAHE basis and certain model extensions were shown to have geometric interpretation and thus, to have orbifold equivalences. However, the general class of semi-realistic models with a NAHE basis were shown not to have geometric interpretation; specifically, their Hodge numbers were not reproducible by any orbifold X/G. In other words, the beyond-NAHE basis vectors necessary to yield a semi-realistic model, by reducing the number of copies of each generation from 16 to 1 and breaking $SO(10)$ to a viable sub-group,[1] consistently break the $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry in a manner that also eliminates geometric interpretation.

[1] $SO(10)$ must be broken via Wilson loop effects of basis vectors rather than by GUT Higgs, since adjoint or higher dimension scalars are not possible in Kač-Moody rank one models.

The non-geometric feature of the semi-realistic WCFFHS models inspired us to investigate variations of the NAHE set that might allow for semi-realistic models with a geometric interpretation, particularly with geometric $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ interpretation. In the next section, we construct a NAHE variation of this form by rotating (interchanging) the boundary conditions of a subset of the twelve real fermions in two of the twisted sectors. We conclude by considering some of the phenomenological aspects of this new model class, especially in comparison to those of the NAHE class.

5.2 Construction and Phenomenology of the NAHE Variation

As mentioned earlier, the NAHE basis set consists of five basis vectors: The all-periodic sector, **1**, which is present in all fermionic models, the supersymmetry generating sector, **S**, and the three generation sectors, $\mathbf{b}_{i=1,2,3}$. The NAHE set was given in (3.21 − 5), where the $(y, w)^m$, for $m = 1$ to 6, are the six pairs of real fermions that replace the right-moving bosonic scalar fields, χ_m for the six compactified directions, and the corresponding $(\bar{y}, \bar{w})^m$ are the six pairs of real fermions that replace the left-moving $\bar{\chi}_m$.

Again, the gauge group of the NAHE set is $SO(10) \otimes SO(6)^3 \otimes E_8$ with $N = 1$ spacetime supersymmetry. The matter content is 48 spinorial **16** representations of $SO(10)$ matter states, coming from sixteen copies from each sector, $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$. The sixteen copies in each sector are composed of two copies of $(\mathbf{16}, \mathbf{4}_i)$ representations and two copies of $(\bar{\mathbf{16}}, \bar{\mathbf{4}}_i)$ representations of $SO(10) \otimes SO(6)_i$. The untwisted sector contains six copies of a pair of Higgs for each generation in the form of $(\mathbf{10}, \mathbf{6}_i)$ representations of $SO(10) \otimes SO(6)_i$, in addition to a single $(\mathbf{6}_i, \mathbf{6}_j)$ representation of $SO(6)_i \otimes SO(6)_j$, for each case of $i, j \in \{1, 2, 3\}$ and $i \neq j$. In a real basis of the $\bar{y}$ and $\bar{w}$, the generators of $SO(6)_1$ are $(\bar{\eta}^1, \bar{y}^1, \bar{y}^2, \bar{w}^5, \bar{w}^6)$; of $SO(6)_2$ are $(\bar{\eta}^2, \bar{y}^3, \bar{y}^4, \bar{y}^5, \bar{y}^6)$; of $SO(6)_3$ are $(\bar{\eta}^3, \bar{w}^1, \bar{w}^2, \bar{w}^3, \bar{w}^4)$.

The three sectors, $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$, are the three unique twisted twisted sectors of the corresponding $\mathbb{Z}_2^a \otimes \mathbb{Z}_2^b$ orbifold compactification. The $\mathbb{Z}_2^a \otimes \mathbb{Z}_2^b$ acts on the $(y, w)_i$ and $(\bar{y}, \bar{w})_i$ in the $\mathbf{b}_i$ according to

$$\mathbb{Z}_2^a: \quad (y, \bar{y})^{m=3,\ldots,6} \to (y+1, \bar{y}+1)^m \text{ (mod 2)} \tag{5.1}$$

$$\mathbb{Z}_2^b: \quad (y, \bar{y})^{m=1,2}; (w, \bar{w})^{n=5,6} \to (y+1, \bar{y}+1)^m; (w+1, \bar{w}+1)^n \text{ (mod 2)}. \tag{5.2}$$

Thus, $\mathbf{b}_1$ is a $\mathbb{Z}_2^a$ twisted sector, $\mathbf{b}_2$ is a $\mathbb{Z}_2^b$ twisted sector, and $\mathbf{b}_3 + \mathbf{1}$ is a $\mathbb{Z}_2^a \otimes \mathbb{Z}_2^b$ twisted sector. The $\mathbb{Z}_2^a \times \mathbb{Z}_2^b$ NAHE orbifold is special precisely because of the existence of three twisted sectors, one per generation, with a permutation symmetry with respect to the horizontal $SO(6)^3$ symmetries. This symmetry also enables $\mathbf{b}_1+\mathbf{b}_2+\mathbf{b}_3+\mathbf{1}$ to generate the massless sector that produces the spinor components of the hidden sector E_8 gauge group.

The NAHE set is common to a large class of three generation free fermionic models. As previously discussed, model building proceeds by adding three or four additional boundary condition basis vectors to the NAHE set, which simultaneously break $SO(10)$ to one of its subgroups, $SU(5) \otimes U(1), SO(6) \otimes SO(4)$, or $SU(3) \otimes SU(2) \otimes U(1)^2$, and reduce the number of generations to three chiral, one from each of the sectors, $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$. The various three generation models differ in their detailed phenomenological properties based on the specific assignment of boundary condition basis vectors for the internal world sheet fermions, $\{y, w|\bar{y}, \bar{w}\}^{1,\ldots,6}$. This is one reason for our interest in examining the properties of a new class of models based on a NAHE variation for which some of the boundary conditions of the $\{y, w|\bar{y}, \bar{w}\}^{1,\ldots,6}$ are exchanged.

$$\mathbf{b}_1 = \{y^{3,\ldots,6}|\bar{y}^{3,\ldots,6}\} \tag{5.3}$$

$$\mathbf{b}_2 = \{y^{1,2,5,6}|\bar{y}^{1,2,5,6}\} \tag{5.4}$$

$$\mathbf{b}_3 = \{y^{1,\ldots,4}|\bar{y}^{1,\ldots,4}\} \tag{5.5}$$

The NAHE variation under discussion is produced by exchanging some of the periodic and antiperiodic boundary conditions in the second and third generation sectors, as shown above. In $\mathbf{b}_2$, the boundary conditions of $(y,\bar{y})^{m=5,6}$ and $(w,\bar{w})^{m=5,6}$ are interchanged. In $\mathbf{b}_3$, the boundary conditions of $(y,\bar{y})^{m=1,2,3,4}$ and $(w,\bar{w})^{m=1,2,3,4}$ are interchanged. Under this exchange, both $\mathbb{Z}^a$ and $\mathbb{Z}^b$ now induce twists solely among the $(y,\bar{y})^m$ and no longer among the $(w,\bar{w})^m$. In addition, $\mathbb{Z}^a \otimes \mathbb{Z}^b$ now corresponds exactly to $\mathbf{b}_3$, rather than to $\mathbf{b}_3 + \mathbf{1}$. This effect of the exchanged boundary conditions for the $\mathbb{Z}^a$ and $\mathbb{Z}^b$ twists is very non-trivial.

The observable gauge group is enhanced to $E_6 \otimes U(1)^5$, and the hidden sector gauge group transforms into $SO(22)$. The change in gauge group occurs because now it is the combination of $\mathbf{S} + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$, rather than of $\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 + \mathbf{1}$, that forms a mass spinor gauge group sector. Thus, in the NAHE variation, there is a massless spinor sector involving the five complex $\bar{\psi}$ and the three complex $\bar{\eta}$ observable sector fermions rather than the eight complex $\bar{\phi}$ hidden sector fermions. This massless spinor sector enhances the $SO(10)$ symmetry generated into E_6. The enhancement is into E_6 rather than E_8 because of the GSO constraints the $\mathbf{b}_i$ basis vectors place on the $\bar{\eta}^i$ spinors.

The trace component of the three complex $\bar{\eta}$ fermions is also absorbed into the E_6, leaving $\bar{\eta}^1 - \bar{\eta}^2$ and $\bar{\eta}^1 + \bar{\eta}^2 - 2\bar{\eta}^3$ as generating 2 extra $U(1)$ charges, along with the 3 extra $U(1)$'s generated by the complex $\bar{y}^I = \bar{y}^1 + i\bar{y}^2$, $\bar{y}^{II} = \bar{y}^3 + i\bar{y}^4$, and $\bar{y}^{III} = \bar{y}^5 + i\bar{y}^6$.

Instead of producing 8 copies of non-chiral generations of $SO(10)$ 16 representations in each $\mathbf{b}_i$ sector, this model produces one non-chiral generation of E_6 27 representations in each $\{\mathbf{1}, \mathbf{b}_i\}$ sector combination and an additional 4 non-chiral generations in each of the three $\{\mathbf{S} + \mathbf{b}_i + \mathbf{b}_j\}, i \neq j$ sectors. Thus, this model corresponds to $h^{1,1} = h^{2,1} = 15$. Thus, this model has the Hodge numbers and twisted sector matter distributions of the orbifold models $(1-2)$ and $(1-8)$of (88) and may be the free fermionic equivalent of one of these.

The NAHE variation also contains 45 pairs of vector-like non-Abelian matter singlets carrying $U(1)$ charges, with 9 pairs coming from the untwisted sector and 12 pairs from each of the three $\mathbf{b}_i + \mathbf{b}_j$ sectors. The untwisted sector contains 6 copies of **22** representations of the hidden sector $SO(22)$, while each $\mathbf{S} + \mathbf{b}_i + \mathbf{b}_j$ sector produces an additional 8 copies of **22** representations of $SO(22)$. The third order components of the model's superpotential are given in Appendix E. The next lowest order terms are fifth order; there are no fourth order terms.

We note finally that this NAHE variation has connection with another variation discussed in (86) that is formed from six basis vectors. In that model, the sector formed by the sum of the three $\mathbf{b}_i$ in our above variation was denoted as X and was added to the NAHE group. In the latter, the observable sector GUT gauge group is also raised to E_6, with the same $U(1)$ enhancing $SO(10)$ to E_6. The total gauge group becomes $E_6 \otimes U(1)^2 \otimes SO(4)^3 \otimes E_8$, in contrast to our $E_6 \otimes U(1)^5 \otimes SO(22)$.

5.3 Discussion

In (91) we introduced a general algorithm for systematic generation of the complete set of WCFFHS gauge group models up to a chosen number of basis vectors, L, and order N, the lowest common multiple of the orders N_i of the respective basis vectors[2] $\mathbf{V}_i$, whereby N_i is the smallest positive integer such that $N_i\mathbf{V}_i = \vec{0}$ (mod 2). The algorithm of systematic generation of models containing twisted matter sectors has been generalized, and we have begun a systematic investigation of $SO(10)$ NAHE based models (74). Now, with the construction of the E_6 NAHE variation presented in this chapter, we are also initiating a parallel systematic investigation of models with the NAHE variation as their core. The general phenomenology of this new

[2] By gauge basis vectors, we mean those with all anti-periodic left-moving boundary conditions.

class of models and the particular characteristics of subclasses of models defined by their observable gauge group will be presented in an upcoming series of papers.

One aspect of the NAHE variation class of models that we will pursue are mirror models. These models contain matching observable and hidden sector gauge groups and matter states. The possibility of NAHE based mirror models was explored in (92), in which it is shown that since the charges of observable sector states in NAHE based models are spread out beyond half (22) of the total number of right-moving complex fermions, GSO constraints imposed by the observable sector on the charges of the hidden sector states significantly hinder realization of mirror models. In fact, it was shown that in a large class, perhaps all, of NAHE based models with mirror basis vectors, these GSO constraints enforce spontaneous breaking of an initial mirror symmetry of gauge groups (92).

However, our variation on the NAHE set appears more condusive to mirror model construction, since the $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist in the NAHE variation allows observable sector states to carry charges within just the first 11 of the 22 right-moving complex fermions, allowing the additional 11 charges to be reserved for hidden sector states. Specifically, an additional three sectors denoted $\mathbf{b}'_{i=1,2,3}$ mirroring $\mathbf{b}_{i=1,2,3}$ in the hidden sector might be added to our NAHE variation to generate an $(E_6 \otimes U(1)^5)_{obs} \otimes (E_6 \otimes U(1)^5)_{hid}$ model with matching matter states.[3]

It should be noted that, nevertheless, the GSO projections between observable and hidden massless matter sectors can never be totally independent, since the observable and hidden matter sectors will always have a periodic complex spacetime fermion in common. Modular invariance contraints require that any pair of order-2 mirror matter sectors have at least one more non-zero complex fermion boundary condition in common, albeit the complex fermion can be either left-moving or right-moving. Hence, for order-2 the modular invariant rules cannot be satisfied by simply

[3] Nevertheless, singlet states carrying both observable $U(1)^5_{obs}$ and hidden $U(1)^5_{hid}$ charges are likely to exist, and therefore mix the observable and hidden sectors.

adding an additional set of hidden sector mirror matter sectors, $\mathbf{b}'_{i=1,2,3}$, with real right moving components defined by $(\mathbf{b}'_i)^n = (\mathbf{b}_i)^{44-n}$. In this case, while $\mathbf{b}'_i \cdot \mathbf{b}_i$ satisfy modular invariance requirements, $\mathbf{b}'_i \cdot \mathbf{b}_{j\neq i}$ do not. As we will show in (93), for higher order basis vectors, this requirement is lifted; mirror observable/hidden matter sectors with either only a periodic spacetime boundary condition in common or else only a periodic spacetime and left-moving complex fermion x boundary condition in common are consistent with modular invariance.

Results of our full exploration of gauge and mirror models based on our NAHE variation will appear in (93). Rather than discuss the range now, we close the chapter instead with an interesting NAHE variation-based example of a gauge (but not matter) mirror model that satisfies modular invariance requirements. The observable and hidden sector matter basis vectors are not completely mirrors among the $\{\bar{\eta}^{(\prime)}, \bar{y}^{(\prime)}, \bar{w}^{(\prime)}\}$. Hence observable and hidden sector matter are not mirror images. The gauge group is $(E_6)_{obs} \otimes U(1)^7 \otimes SU(4) \otimes (E_6)_{hid}$. The model is chiral with 21 **27** representations and 3 $\bar{\mathbf{27}}$ representations of $(E_6)_{obs}$. The untwisted sector provides 3 27 and 3 $\bar{\mathbf{27}}$ representations; the 18 net chiral representations are all from the twisted sectors. The model also contains 12 **4** and 12 $\bar{\mathbf{4}}$ representations, not in vector-like pairs, of $SU(4)$ and 48 $U(1)^5$ charged non-Abelian singlets. There are neither **27** nor $\bar{\mathbf{27}}$ representations of $(E_6)_{hid}$. A net $\mathbb{Z}_6$ twist from additional sectors is needed to (1) simultaneously reduce $(E_6)_{obs}$ to a (semi-)GUT that does not require adjoint or higher scalar representations to induce a spontaneous symmetry breaking to the MSSM at low energy and (2) reduce the number of copies of each matter generation from 6 to 1. The basis vectors and GSO projection matrix are given in Tables E.1 and E.2 in Appendix E.

CHAPTER SIX

Conclusions

In this thesis, we focus our study on heterotic superstring theories and their applications to particle physics. In particular, we are interested in the search of semi-realistic four dimensional superstring vacua which can reproduce the Standard Model physics at low energy. A highly successful approach is given by free fermionic models, which give rise to the most realistic three generation string models to date. Their phenomenology is studied in the effective low energy field theory by the analysis of supersymmetric flat directions. Before discussing each of the four models presented herein, we mention a model which consists of MSSM states in the observable Standard Model sector, and which was the first model found with a reduced number of Higgs content at the string scale. This result came about through the application of a new general mechanism which involved a choice of asymmetric boundary conditions for the internal fermions of the theory. An additional result for minimal Higgs spectrum models is the fact that the supersymmetric moduli space is reduced as well, which increases the predictive power of the theory.

A common feature of free fermionic models is the presence of an anomalous $U(1)$ which gives rise to a Fayet-Iliopoulos $D-$terms which breaks supersymmetry at the one loop level in string perturbation theory. Supersymmetry is restored by imposing $D-$ and $F-$ flatness on the vacuum. Generally, it has been assumed that in a given string model, there should exist a supersymmetric solution to $D-$ and $F-$flatness constraints. However, in the model mentioned at the beginning of Chapter 4, no flat solutions were found after employing the standard analysis for flat directions. Such a result lead to the investigation of similar free fermionic models, with low Higgs content, to assess whether the lack of such supersymmetric flat directions was a general property of this class of models.

In Chapter 4, we presented four semi-realistic free fermionic models, all with reduced Higgs content. Standard flat direction analysis showed that supersymmetric $D-$ and $F-$flatness existed for all four models. Additionally, two of the models exhibited non-Abelian singlet flat directions, as had been expected from past free fermionic heterotic models. While the results herein are not exhaustive, i.e. not all flat directions are presented for all four models, we required only that they be sufficient to make a general statement about this class of models. Therefore, the tables of flat directions listed do not reflect a complete flat direction characterization for a given model. For the purposes of this work, we are concerned only with whether or not these flat directions exist, rather than providing an exhaustive list of each flat direction for each model. In this case, it was sufficient to find just one flat direction from each model to assess whether the lack thereof was a general property of these models.

In Chapter 5, we presented a variation on the NAHE set from which the first four models were derived. Such a variation comes about through the rotation of some of the boundary conditions of the NAHE periodic/antiperiodic fermions associated with the six compact dimensions of a bosonic lattice/orbifold model, specifically those for a subset of the twelve real fermions in two of the twisted sectors, $\mathbf{b}_2$ and $\mathbf{b}_3$. Through this rotation, the observable sector gauge group is enhanced from $SO(10) \otimes SO(6)^3$ to $E_6 \otimes U(1)^5$, and the hidden sector gauge group transforms from E_8 into $SO(22)$. The significance of beginning with an E_6 model is that the inverse transformation of the fermion boundary conditions correspond to partial GUT breaking through the boundary rotation. In addition, investigation of models within this variation on the NAHE set offers the possibility of a range of mirror models, or models which contain matching observable and hidden sector gauge groups and matter states. To date, investigation into specific models within this variation, besides the example presented herein, has yet to be done, and is an obvious place to proceed with further research.

The main results of this thesis allow for a significant amount of future work, in addition to that just mentioned. As stated earlier, the search for flat directions was merely representative, not exhaustive. In this case, further work can be done to fully classify the flat directions for each of the models. Also, brief work was done to assess the scale at which hidden sector fields become massive for one of the models presented in chapter 4. The next step is to investigate at what scale the hidden sector fields condensate into observable sector fields for a given flat direction, which causes supersymmetry breaking along that direction. This analysis can obviously be done for all the four models presented in chapter 4, to more completely characterize these models.

Though we presented only four semi-realistic models herein, the goal of string phenomenology is to obtain a semi-realistic string model which accurately describes the Standard Model in the observable sector, and perhaps gives new insight into dark matter via the hidden sector. The string models in the free fermionic formulation give rise to a large class of semi-realistic models which produce solely the MSSM spectrum in the observable Standard Model charged sector of the effective low energy field theory. As such, free fermionic models provide an arena to study how string theory may be related to observed particle data. In turn, the properties of the models which make them attractive from the point of view of the phenomenological data may be instrumental in uncovering unexpected properties of string theory.

APPENDICES

APPENDIX A

Gauge Charges and Flat Directions of Model 1

Table A.1: Gauge Charges of Model 1. The names of the states appear in the first column, with the states' various charges appearing in the other columns. All $U(1)$ charges are multiplied by a factor of 4 (and similarly for all other models).

state	$SO(10)$	U_A	U'_1	U'_2	U'_3	U'_4	U'_5	$SU(8)$	$SU(2)$
G_1	16	10	0	-2	-4	0	20	1	1
G_2	16	6	2	0	2	0	-30	1	1
G_3	16	12	-2	2	2	0	10	1	1
h_1	10	-12	-2	-2	-2	0	-10	1	1
$\bar{h}_1$	10	12	2	2	2	0	10	1	1
h_2	10	-12	-2	2	-2	0	-10	1	1
$\bar{h}_2$	10	12	2	-2	2	0	10	1	1
Φ_1	1	0	0	0	0	0	0	1	1
Φ_2	1	0	0	0	0	0	0	1	1
Φ_3	1	0	0	0	0	0	0	1	1
$\phi_1(\bar{\phi}_1)$	1	0	0	-8	0	0	0	1	1
$\phi_2(\bar{\phi}_2)$	1	24	4	4	4	0	20	1	1
$\phi_3(\bar{\phi}_3)$	1	24	4	-4	4	0	20	1	1
$\psi_1(\overline{\psi}_1)$	1	12	-6	2	2	0	10	1	1
$\psi_2(\overline{\psi}_2)$	1	0	2	-2	2	0	-70	1	1
$\psi_3(\overline{\psi}_3)$	1	8	-2	2	-10	0	30	1	1
$\psi_4(\overline{\psi}_4)$	1	0	0	4	0	0	0	1	1
$\psi_5(\overline{\psi}_5)$	1	0	0	4	0	0	0	1	1

Table A.1: Gauge Charges of Model 1, Continued.

state	$SO(10)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	$SU(8)$	$SU(2)$
$\psi_6(\overline{\psi}_6)$	1	24	4	0	4	0	20	1	1
$\psi_7(\overline{\psi}_7)$	1	12	-6	-2	2	0	10	1	1
$\psi_8(\overline{\psi}_8)$	1	0	2	2	2	0	-70	1	1
$\psi_9(\overline{\psi}_9)$	1	8	-2	-2	-10	0	30	1	1
$\psi_{10}(\overline{\psi}_{10})$	1	-20	2	-2	2	16	-54	1	1
$\psi_{11}(\overline{\psi}_{11})$	1	-20	0	0	0	16	16	1	1
$\psi_{12}(\overline{\psi}_{12})$	1	12	0	0	8	-16	24	1	1
$\psi_{13}(\overline{\psi}_{13})$	1	-8	-4	0	4	16	-44	1	1
$\psi_{14}(\overline{\psi}_{14})$	1	-20	2	2	2	16	-54	1	1
$H_1(\bar{H}_1)$	1	24	0	0	0	8	48	1	2
$H_2(\bar{H}_2)$	1	0	-4	4	-4	8	28	1	2
$H_3(\bar{H}_3)$	1	0	-4	-4	-4	8	28	1	2
$H_4(\bar{H}_4)$	1	-12	2	2	-6	8	18	1	2
$H_5(\bar{H}_5)$	1	0	-2	2	-2	8	-42	1	2
$H_6(\bar{H}_6)$	1	-8	-2	2	6	8	-2	1	2
$H_7(\bar{H}_7)$	1	0	-4	0	-4	8	28	1	2
$H_8(\bar{H}_8)$	1	-12	2	-2	-6	8	18	1	2
$H_9(\bar{H}_9)$	1	0	-2	-2	-2	8	-42	1	2
$H_{10}(\bar{H}_{10})$	1	-8	-2	-2	6	8	-2	1	2
$H_{11}(\bar{H}_{11})$	1	0	0	-2	-4	-8	-28	8	1
$H_{12}(\bar{H}_{12})$	1	2	-2	2	2	-8	-38	8	1
$H_{13}(\bar{H}_{13})$	1	10	0	0	0	8	48	8	1

Table A.2: Basis set of U(1) D-flat directions for Model 1. Column 1 denotes the D-flat direction label. Column 2 is the anomalous charge. The remaining columns specify the norm squared VEV's of the respective non-Abelian singlet fields. All other "2" tables are structured in a like manner.

FD	Q_A	ϕ_1	ϕ_2	ϕ_3	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}	ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}
D_1	-1	0	0	0	0	0	0	0	0	-3	0	0	0	1	2	2	-2	1
D_2	-1	0	-3	0	0	0	0	0	0	0	0	0	0	-2	2	2	-2	4
D_3	-1	0	0	-3	0	0	0	0	0	0	0	0	0	4	2	2	-2	-2
D_4	0	1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	2
D_5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	1
D_6	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	-1
D_7	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	-1	0
D_8	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1
D_9	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	-1
D_{10}	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	-1
D_{11}	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	1	0	0	0
D_{12}	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0

Table A.3: Basis set of F-flat directions to all order for Model 1. Column 1 denotes the flat direction number. Column 2 is the anomalous charge. The remaining columns specify the norm squared VEVs of the respective non-Abelian singlet fields. All other "3" tables are structured in a like manner.

FD	Q_A	ϕ_1	ϕ_2	ϕ_3	ψ_3	ψ_6	ψ_8	ψ_9	ψ_{10}	ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}
1	-49	0	-3	-144	-98	0	0	0	92	98	0	-98	-92
2	-1	0	-1	-2	-2	0	0	0	0	2	0	-2	0
3	-1	0	0	-3	-4	0	0	0	0	2	-2	-2	-2
4	-1	0	-1	0	0	-2	0	-2	0	2	0	-2	0
5	-2	0	-3	-1	0	-2	0	-4	0	4	0	-4	0
6	-1	0	0	1	0	-4	0	-2	0	2	0	-2	0
7	-1	0	0	-1	-2	-2	0	0	0	2	0	-2	0
8	-2	0	-1	-3	-4	-2	0	0	0	4	0	-4	0
9	-1	0	1	0	-2	-4	0	0	0	2	0	-2	0
10	-2	1	0	0	0	-6	0	-4	0	4	0	-4	0
11	-5	1	0	3	0	-18	0	-10	0	10	0	-10	0
12	-2	-1	0	0	-4	-6	0	0	0	4	0	-4	0
13	-5	-1	0	-3	-10	-12	0	0	0	10	0	-10	0
14	-1	0	0	0	0	-3	-2	-3	1	0	-1	-2	0
15	-1	0	0	1	0	-4	-2	-4	0	0	-2	-2	0

Superpotential, W_{FD1}, for $F-$flat Direction 1, for Model 1.

$$
\begin{array}{lll}
\phi_1\ \phi_2\ \bar{\phi}_3 & +\ \bar{\phi}_1\ \phi_3\ \bar{\phi}_2 & +\ \psi_1\ \overline{\psi}_{10}\ \psi_{11} \\
+\ \psi_9\ \Phi_1\ \overline{\psi}_3 & +\ \overline{\psi}_2\ \overline{\psi}_2\ \bar{\phi}_2 & +\ \psi_3\ \psi_3\ \bar{\phi}_2 \\
+\ \psi_3\ \psi_{12}\ \psi_{10} & +\ \psi_5\ \overline{\psi}_{11}\ \psi_{10} & +\ \overline{\psi}_4\ \overline{\psi}_{10}\ \overline{\psi}_{13} \\
+\ \overline{\psi}_6\ \overline{\psi}_{10}\ \overline{\psi}_{14} & +\ \overline{\psi}_8\ \psi_{14}\ \psi_{11} & +\ \overline{\psi}_9\ \psi_{12}\ \overline{\psi}_{14} \\
+\ \overline{\psi}_{10}\ \Phi_2\ \psi_{10} & +\ \psi_{12}\ \Phi_3\ \psi_{11} & +\ \psi_{13}\ \Phi_1\ \overline{\psi}_{14} \\
+\ \psi_{14}\ \Phi_1\ \overline{\psi}_{13} & +\ H_1\ \bar{H}_3\ \bar{\phi}_2 & +\ H_1\ \bar{H}_2\ \bar{\phi}_3 \\
+\ H_2\ \bar{H}_6\ \overline{\psi}_3 & +\ \bar{h}_2\ \bar{h}_2\ \bar{\phi}_3 & +\ \bar{H}_{10}\ \bar{H}_5\ \overline{\psi}_{13} \\
+\ \psi_3\ \psi_{12}\ \bar{\phi}_3\ \overline{\psi}_3\ \psi_{11} & +\ \psi_5\ \overline{\psi}_{11}\ \bar{\phi}_3\ \overline{\psi}_3\ \psi_{11} & +\ \overline{\psi}_{10}\ \overline{\psi}_{10}\ \bar{\phi}_2\ \overline{\psi}_{11}\ \overline{\psi}_{11} \\
+\ \bar{h}_1\ \bar{h}_1\ \bar{\phi}_3\ \overline{\psi}_3\ \overline{\psi}_3 & +\ \overline{\psi}_2\ \overline{\psi}_{10}\ \bar{\phi}_3\ \bar{\phi}_3\ \overline{\psi}_3\ \overline{\psi}_3\ \psi_{11} &
\end{array}
$$

Table A.4: Additional Basis Vectors of Model 1.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,\ldots,5}$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,\ldots,8}$
α_1	0	0	0	0	1,...,1	1	0	0	0,0,0,0,0,0,0,0
β_1	0	0	0	0	1,...,1	0	1	0	0,0,0,0,0,0,0,0
γ_1	0	0	0	0	1,...,1	1	1	0	0,0,0,1,1,0,0,0
δ_1	0	0	0	0	1,...,1	1	1	1	1,1,1,1,1,1,1,1

Table A.4: Additional Basis Vectors of Model 1, Continued.

	$y^{3,\ldots,6}$	$\overline{y}^{3,\ldots,6}$	$y^{1,2}, w^{5,6}$	$\overline{y}^{1,2}, \overline{w}^{5,6}$	$w^{1,\ldots,4}$	$\overline{w}^{1,\ldots,4}$
α_1	1,0,0,1	1,0,0,1	0,0,0,1	1,0,1,1	0,0,1,0	0,1,1,1
β_1	0,0,1,0	1,0,1,1	1,0,1,0	1,0,1,0	1,0,0,0	1,1,0,1
γ_1	0,1,0,0	0,1,0,0	0,1,0,0	0,1,0,0	0,1,0,1	0,0,0,0
δ_1	0,0,0,0	0,0,0,0	0,0,0,0	0,0,0,0	0,0,0,0	0,0,0,0

Table A.5: GSO Projection Matrix for Model 1.

$k_{i,j}$	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	α_1	β_1	γ_1	δ_1
$\mathbf{1}$	0	0	0	1	1	0	0	0	0
$\mathbf{S}$	0	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	0	1	0	1	1	0	0	1	1
$\mathbf{b}_2$	1	1	1	1	1	0	1	1	1
$\mathbf{b}_3$	1	1	1	1	1	1	0	1	1
α_1	0	0	1	1	0	0	1	1	0
β_1	0	0	1	0	1	1	0	0	1
γ_1	0	0	0	0	1	0	1	0	1
δ_1	0	0	0	0	0	1	0	1	1

APPENDIX B

Gauge Charges and Flat Directions of Model 2

Table B.1: Gauge Charges of Model 2.

state	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	$SU(4)$	$SO(10)$
F_1	10	6	-2	-2	0	2	-6	2	1	1
F_2	10	6	2	0	-4	-2	0	2	1	1
F_3	10	6	0	2	2	4	6	2	1	1
$\bar{F}_1$	-5	2	-2	2	0	-6	-2	-6	1	1
$\bar{F}_2$	-5	2	2	0	4	-2	-8	-6	1	1
$\bar{F}_3$	-5	2	0	-2	2	-4	10	-6	1	1
E_1^c	1	2	-2	2	0	-6	-2	10	1	1
E_2^c	1	2	2	0	4	-2	-8	10	1	1
E_3^c	1	2	0	-2	2	-4	10	10	1	1
Φ_1	1	0	0	0	0	0	0	0	1	1
Φ_2	1	0	0	0	0	0	0	0	1	1
Φ_3	1	0	0	0	0	0	0	0	1	1
$\phi_1(\bar{\phi}_1)$	1	8	0	0	0	16	-8	0	1	1
$\phi_2(\bar{\phi}_2)$	1	0	0	-8	0	0	0	0	1	1
$\phi_3(\bar{\phi}_3)$	1	8	0	4	-8	8	4	0	1	1
$\phi_4(\bar{\phi}_4)$	1	0	0	-4	-8	-8	12	0	1	1
$\phi_5(\bar{\phi}_5)$	1	8	0	-4	-8	-8	4	0	1	1
$\phi_6(\bar{\phi}_6)$	1	0	0	4	-8	-8	12	0	1	1

Table B.1: Gauge Charges of Model 2, Continued.

state	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	$SU(4)$	$SO(10)$
$\psi_1(\overline{\psi}_1)$	1	-4	0	-2	4	-4	-2	0	1	1
$\psi_2(\overline{\psi}_2)$	1	-4	0	-2	4	-4	-2	0	1	1
$\psi_3(\overline{\psi}_3)$	1	-4	0	-2	4	-4	-2	0	1	1
$\psi_4(\overline{\psi}_4)$	1	-4	0	6	4	-4	-2	0	1	1
$\psi_5(\overline{\psi}_5)$	1	-4	0	0	0	-8	4	0	1	1
$\psi_6(\overline{\psi}_6)$	1	-4	0	0	0	-8	4	0	1	1
$\psi_7(\overline{\psi}_7)$	1	4	0	4	-8	0	8	0	1	1
$\psi_8(\overline{\psi}_8)$	1	4	0	-4	-8	0	8	0	1	1
$\psi_9(\overline{\psi}_9)$	1	0	0	6	4	4	-6	0	1	1
$\psi_{10}(\overline{\psi}_{10})$	1	-8	0	-2	4	-12	2	0	1	1
$\psi_{11}(\overline{\psi}_{11})$	1	0	0	-2	4	4	-6	0	1	1
$\psi_{12}(\overline{\psi}_{12})$	1	0	0	-2	4	4	-6	0	1	1

Table B.1: Gauge Charges of Model 2, Continued.

state	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	$SU(4)$	$SO(10)$
$H_1(\bar{H}_1)$	1	-8	0	-2	-1	-2	2	-5	-4	1
$H_2(\bar{H}_2)$	1	-4	0	0	-5	2	4	-5	-4	1
$H_3(\bar{H}_3)$	1	-4	0	-2	-1	6	-2	-5	-4	1
H_4	1	2	0	-2	-3	6	10	5	-4	1
H_5	1	2	2	0	-1	8	-8	5	-4	1
H_6	1	2	-2	2	-5	4	-2	5	-4	1
H_7	1	6	0	-2	-5	2	-6	-5	4	1
H_8	1	6	-2	0	1	8	0	-5	4	1
H_9	1	6	2	2	-3	4	6	-5	4	1
H_{10}	1	6	2	2	2	-6	6	0	6	1
H_{11}	1	6	-2	0	6	-2	0	0	6	1
H_{12}	1	6	0	-2	0	-8	-6	0	6	1
H_{13}	1	10	-2	0	-2	-2	8	0	1	10
H_{14}	1	10	0	2	0	0	-10	0	1	10
H_{15}	1	10	2	-2	2	2	2	0	1	10

Table B.2: Basis set of non-anomalous U(1) Singlet D-flat directions for Model 2.

FD	Q_A	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}	ψ_{11}	ψ_{12}
D_1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	-1	0	-1
D_2	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	1	0	-3
D_3	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
D_4	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	-1	0	1
D_5	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	3
D_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1
D_7	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1
D_8	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	-1	0	1
D_9	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	-2	-1	0	1
D_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	0	1
D_{11}	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	-1	0	-1
D_{12}	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0
D_{13}	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
D_{14}	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1
D_{15}	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1

Table B.3: Basis set of non-anomalous U(1) non-Abelian Singlet D-flat directions for Model 2.

FD	Q_A	H_1	H_2	H_3	F_1	$\bar{F}_1$	F_2	$\bar{F}_2$	F_3	$\bar{F}_3$	H_4	H_{14}	H_{13}	H_{15}	H_7	H_8	H_9
D_1	60	3	-3	-3	0	0	0	0	5	0	0	0	0	0	5	0	0
D_2	60	3	-3	-3	5	0	0	0	0	0	0	0	0	0	0	0	0
D_3	60	3	-3	-3	0	0	5	0	0	0	0	0	0	0	0	5	0
D_4	30	0	0	-2	0	0	0	0	0	0	0	0	0	1	1	1	0
D_5	30	0	-2	0	0	0	0	0	0	0	0	0	1	0	1	0	1
D_6	60	2	-4	-4	0	0	0	0	0	0	1	0	0	0	3	2	2
D_7	120	-9	-1	-1	0	0	0	0	0	5	0	0	0	0	5	0	0
D_8	180	-9	19	9	0	0	0	5	0	0	0	0	0	0	-10	-5	-10
D_9	-30	0	2	2	0	0	0	0	0	0	0	1	0	0	-2	-1	-1
D_{10}	180	-9	9	19	0	5	0	0	0	0	0	0	0	0	-10	-10	-5

Table B.4: Basis set of non-anomalous U(1) Mixed Singlet D-flat directions for Model 2.

FD	Q_A	ψ_{10}	ψ_{11}	ψ_{12}	E_1^c	E_2^c	E_3^c	H_1	H_2	H_3	H_5	H_6	H_{10}	H_{11}	H_{14}	H_{12}	H_{13}	H_{15}	H_7	H_8	H_9
D_1	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	1	0	0	-1	0	0
D_2	0	0	0	0	0	0	0	-1	1	1	0	0	0	1	0	0	0	0	0	-1	0
D_3	0	0	0	0	0	0	0	-1	1	1	0	0	1	0	0	0	0	0	0	0	-1
D_4	0	0	0	0	0	0	0	2	0	-2	1	0	0	0	0	0	0	0	0	1	0
D_5	0	0	0	0	0	0	0	2	-2	0	0	1	0	0	0	0	0	0	0	0	1
D_6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_7	0	-1	0	1	0	0	0	2	0	-2	0	0	0	0	0	0	0	0	0	0	0
D_8	0	0	0	1	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0
D_9	30	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	3	0	0	0
D_{10}	30	0	0	0	-1	-1	2	0	0	0	0	0	0	0	3	0	0	0	0	0	0
D_{11}	30	0	0	0	-1	2	-1	0	0	0	0	0	0	0	0	0	3	0	0	0	0

Table B.5: Basis set of F-flat directions to all order for Model 2.

FD	Q_A	ψ_9	ψ_{10}	ψ_{11}	$\bar{H}_1$	$\bar{H}_2$	$\bar{H}_3$	H_4	H_{10}	H_{12}	H_7	H_8	H_9
1	0	-5	6	-9	0	-66	-48	18	0	24	40	46	46
2	-1	-2	6	0	0	-42	-42	15	0	18	31	34	34
3	-14	0	-3	-3	-1	-13	-7	7	0	21	0	14	14
4	-15	0	-3	-3	-1	-15	-9	8	0	23	1	16	16
5	-16	1	-3	0	0	-12	-12	9	0	24	1	16	16
6	-1	0	33	-33	-44	-68	-2	23	0	24	45	46	46
7	-1	0	30	-30	-42	-60	0	21	0	24	39	42	42
8	-1	0	9	-9	0	-56	-38	19	0	20	37	38	38
9	-14	0	3	-3	-13	-7	-1	7	0	21	0	14	14
10	-14	-2	6	0	-23	-1	-1	6	1	22	0	16	15

Table B.6: Additional Basis Vectors of Model 2.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,\ldots,5}$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,\ldots,8}$
α_2	0	0	0	0	1,...,1	1	0	0	0,0,0,0,0,0,0,0
β_2	0	0	0	0	1,...,1	0	1	0	0,0,0,0,0,0,0,0
γ_2	0	0	0	0	1,...,1	1	1	1	0,0,0,0,1,1,1,1

Table B.6: Additional Basis Vectors of Model 2, Continued.

	$y^{3,\ldots,6}$	$\overline{y}^{3,\ldots,6}$	$y^{1,2}, w^{5,6}$	$\overline{y}^{1,2}, \overline{w}^{5,6}$	$w^{1,\ldots,4}$	$\overline{w}^{1,\ldots,4}$
α_2	1,0,0,1	1,0,0,1	0,0,0,1	1,0,1,1	0,0,1,0	0,1,1,1
β_2	0,0,1,0	1,0,1,1	1,0,1,0	1,0,1,0	1,0,0,0	1,1,0,1
γ_2	0,2,0,0	0,2,0,0	0,2,0,0	0,2,0,0	0,2,0,2	0,0,0,0

Table B.7: GSO Projection Matrix for Model 2.

$k_{i,j}$	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	α_2	β_2	γ_2
$\mathbf{1}$	0	0	1	1	1	0	0	$-\frac{1}{2}$
$\mathbf{S}$	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	1	1	1	1	1	0	0	$-\frac{1}{2}$
$\mathbf{b}_2$	1	1	1	1	1	0	1	$\frac{1}{2}$
$\mathbf{b}_3$	1	1	1	1	1	1	0	1
α_2	0	0	1	1	0	0	1	$\frac{1}{2}$
β_2	0	0	1	0	1	1	0	$\frac{1}{2}$
γ_2	0	0	1	0	0	0	0	$-\frac{1}{2}$

Third Order Superpotential, W_3, for Model 2.

$$\phi_1\ \phi_4\ \bar{\phi}_5 \quad + \phi_1\ \phi_6\ \bar{\phi}_3 \quad + \phi_1\ \psi_5\ \psi_5 \quad + \phi_1\ \psi_6\ \psi_6 \quad + \phi_2\ \phi_3\ \bar{\phi}_5$$

$$+ \phi_2\ \phi_6\ \bar{\phi}_4 \quad + \phi_2\ \psi_7\ \overline{\psi}_8 \quad + \bar{\phi}_2\ \phi_4\ \bar{\phi}_6 \quad + \bar{\phi}_2\ \phi_5\ \bar{\phi}_3 \quad + \bar{\phi}_2\ \psi_8\ \overline{\psi}_7$$

$$+ \bar{\phi}_1\ \phi_3\ \bar{\phi}_6 \quad + \bar{\phi}_1\ \phi_5\ \bar{\phi}_4 \quad + \bar{\phi}_1\ \overline{\psi}_5\ \overline{\psi}_5 \quad + \bar{\phi}_1\ \overline{\psi}_6\ \overline{\psi}_6 \quad + \bar{\phi}_1\ H_5\ H_{14}$$

$$+ \phi_3\ \psi_1\ \psi_1 \quad + \phi_3\ \psi_2\ \psi_2 \quad + \phi_4\ \psi_3\ \psi_4 \quad + \phi_5\ \psi_9\ \psi_{10} \quad + \phi_6\ \psi_{11}\ \psi_{11}$$

$$+ \phi_6\ \psi_{12}\ \psi_{12} \quad + \bar{\phi}_4\ \overline{\psi}_4\ \overline{\psi}_3 \quad + \bar{\phi}_3\ \overline{\psi}_1\ \overline{\psi}_1 \quad + \bar{\phi}_3\ \overline{\psi}_2\ \overline{\psi}_2 \quad + \bar{\phi}_3\ H_6\ H_{15}$$

$$+ \bar{\phi}_6\ \overline{\psi}_{11}\ \overline{\psi}_{11} \quad + \bar{\phi}_6\ \overline{\psi}_{12}\ \overline{\psi}_{12} \quad + \bar{\phi}_5\ \overline{\psi}_{10}\ \overline{\psi}_9 \quad + \bar{\phi}_5\ H_4\ H_{13} \quad + \psi_1\ \overline{\psi}_1\ \Phi_1$$

$$+ \psi_1\ \psi_5\ \overline{\psi}_{10} \quad + \psi_1\ \overline{\psi}_5\ \overline{\psi}_{11} \quad + \psi_1\ \overline{\psi}_6\ \overline{\psi}_{12} \quad + \psi_1\ \psi_7\ \psi_{12} \quad + \psi_1\ \psi_8\ \psi_9$$

$$+ \psi_2\ \overline{\psi}_2\ \Phi_1 \quad + \psi_2\ \psi_6\ \overline{\psi}_{10} \quad + \psi_2\ \overline{\psi}_5\ \overline{\psi}_{12} \quad + \psi_2\ \overline{\psi}_6\ \overline{\psi}_{11} \quad + \psi_2\ \psi_7\ \psi_{11}$$

$$+ \psi_2\ H_2\ \bar{H}_1 \quad + \psi_3\ \overline{\psi}_3\ \Phi_1 \quad + \psi_3\ \psi_5\ \overline{\psi}_{11} \quad + \psi_3\ \psi_6\ \overline{\psi}_{12} \quad + \psi_4\ \overline{\psi}_4\ \Phi_1$$

$$+ \psi_4\ \overline{\psi}_5\ \overline{\psi}_9 \quad + \psi_4\ \psi_8\ \psi_{11} \quad + \overline{\psi}_1\ \psi_5\ \psi_{11} \quad + \overline{\psi}_1\ \psi_6\ \psi_{12} \quad + \overline{\psi}_1\ \overline{\psi}_5\ \psi_{10}$$

$$+ \overline{\psi}_1 \overline{\psi}_7\ \overline{\psi}_{11} \quad + \overline{\psi}_1\ H_1\ \bar{H}_2 \quad + \overline{\psi}_2\ \psi_5\ \psi_{12} \quad + \overline{\psi}_2\ \psi_6\ \psi_{11} \quad + \overline{\psi}_2\ \overline{\psi}_6\ \psi_{10}$$

$$+ \overline{\psi}_2\ \overline{\psi}_8\ \overline{\psi}_9 \quad + \overline{\psi}_2\ \overline{\psi}_7\ \overline{\psi}_{12} \quad + \overline{\psi}_4\ \psi_5\ \psi_9 \quad + \overline{\psi}_4\ \overline{\psi}_8\ \overline{\psi}_{12} \quad + \overline{\psi}_3\ \overline{\psi}_5\ \psi_{11}$$

$$+ \overline{\psi}_3\ \overline{\psi}_6\ \psi_{12} \quad + \overline{\psi}_3\ \overline{\psi}_7\ \overline{\psi}_{10} \quad + \psi_5\ \overline{\psi}_6\ \Phi_3 \quad + \psi_6\ \overline{\psi}_5\ \Phi_3 \quad + \psi_6\ H_3\ \bar{H}_1$$

$$+ \overline{\psi}_6\ H_1\ \bar{H}_3 \quad + \psi_7\ \overline{\psi}_7\ \Phi_3 \quad + \psi_8\ \overline{\psi}_8\ \Phi_3 \quad + \psi_9\ \overline{\psi}_9\ \Phi_2 \quad + \psi_{10}\ \overline{\psi}_{10}\ \Phi_2$$

$$+ \psi_{11}\ \overline{\psi}_{11}\ \Phi_2 \quad + \psi_{11}\ H_2\ \bar{H}_3 \quad + \psi_{12}\ \overline{\psi}_{12}\ \Phi_2 \quad + \overline{\psi}_{12}\ H_3\ \bar{H}_2 \quad + \Phi_1\ H_3\ \bar{H}_3$$

$$+ \Phi_2\ H_1\ \bar{H}_1 \quad + \Phi_3\ H_2\ \bar{H}_2$$

APPENDIX C

Gauge Charges and Flat Directions of Model 4

Table C.1: Gauge Charges of Model 4.

state	$SU(5)$	U_A	U'_1	U'_2	U'_3	U'_4	U'_5	U'_6	U'_7	$SU(4)$
F_1	10	20	2	-2	2	-2	-2	-28	0	1
F_2	10	24	0	0	0	2	16	-104	0	1
F_3	10	20	2	2	-2	-2	-2	-28	0	1
$\bar{F}_1$	-5	-20	-6	-2	2	-2	-34	-148	0	1
$\bar{F}_2$	-5	-24	-8	0	0	-2	-16	104	0	1
$\bar{F}_3$	-5	-24	-6	2	2	2	-34	16	0	1
E^c_1	1	60	10	-2	2	-2	30	92	0	1
$\bar{E}^c_1$	1	-60	-10	2	-2	2	-30	-92	0	1
E^c_2	1	56	8	0	0	-2	48	344	0	1
E^c_3	1	56	10	2	2	2	30	256	0	1
$h_1(\bar{h}_1)$	5	-4	-4	4	0	0	-36	-12	0	1
$h_2(\bar{h}_2)$	5	12	0	-2	-2	-2	8	-52	-8	1
h_3	5	20	6	2	-2	2	34	148	0	1
h_4	5	8	0	2	-2	2	8	112	8	1
h_5	5	8	0	-2	2	2	8	112	-8	1
h_6	-5	-24	-6	-2	-2	2	-34	16	0	1
h_7	-5	8	4	0	0	2	36	-152	0	1
h_8	-5	32	4	0	0	-2	-4	272	0	1

Table C.1: Gauge Charges of Model 4, Continued.

state	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(4)$
Φ_1	1	0	0	0	0	0	0	0	0	1
Φ_2	1	0	0	0	0	0	0	0	0	1
Φ_3	1	0	0	0	0	0	0	0	0	1
$\phi_1(\bar{\phi}_1)$	1	36	-4	-4	0	0	-4	108	0	1
$\psi_1(\overline{\psi}_1)$	1	44	4	2	2	2	4	220	-8	1
$\psi_2(\overline{\psi}_2)$	1	4	0	4	0	2	0	-164	0	1
$\psi_3(\overline{\psi}_3)$	1	-4	0	4	0	-2	0	164	0	1
ψ_4	1	56	10	-2	-2	2	30	256	0	1
ψ_5	1	12	8	2	2	-2	8	-52	8	1
ψ_6	1	12	8	-2	-2	-2	8	-52	-8	1
ψ_7	1	16	6	0	4	2	26	-128	8	1
ψ_8	1	20	6	0	0	-2	26	-292	8	1
ψ_9	1	12	6	0	0	-6	26	36	-8	1
ψ_{10}	1	8	6	0	-4	-2	26	200	-8	1
ψ_{11}	1	8	0	4	-4	-2	0	-328	0	1
ψ_{12}	1	0	0	4	4	6	0	0	0	1
ψ_{13}	1	0	-4	0	0	-2	36	176	0	1
ψ_{14}	1	0	0	-4	-4	6	0	0	0	1
ψ_{15}	1	8	0	-4	4	-2	0	-328	0	1

Table C.1: Gauge Charges of Model 4, Continued.

state	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(4)$
ψ_{16}	1	40	-4	0	0	2	-4	-56	0	1
ψ_{17}	1	-12	-4	2	2	-2	-44	-124	-8	1
ψ_{18}	1	-48	-4	2	-2	2	-4	-56	-8	1
ψ_{19}	1	-12	-4	-2	-2	-2	-44	-124	8	1
ψ_{20}	1	-8	-6	0	4	2	-26	-200	-8	1
ψ_{21}	1	-4	-6	0	0	-2	-26	-364	-8	1
ψ_{22}	1	-48	-4	-2	2	2	-4	-56	8	1
ψ_{23}	1	-12	-6	0	0	-6	-26	-36	8	1
ψ_{24}	1	-16	-6	0	-4	-2	-26	128	8	1
H_1	1	32	4	2	2	-2	24	8	-4	8
H_2	1	32	6	0	0	2	6	-80	-4	8
H_3	1	16	-2	-2	2	2	-2	136	4	8
H_4	1	20	0	0	0	2	-20	-116	4	8
H_5	1	16	-2	2	-2	2	-2	136	4	8
H_6	1	20	-2	2	2	-2	-2	-28	4	8
H_7	1	-28	-4	2	-2	-2	-24	-172	-4	8

Table C.2: Basis set of non-anomalous U(1) D-flat directions for Model 4.

	Q_A	ϕ_1	E_1^c	ψ_1	ψ_2	ψ_3	E_2^c	E_3^c	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}
		ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}	ψ_{15}	ψ_{16}	ψ_{17}	ψ_{18}	ψ_{19}	ψ_{20}	ψ_{21}	ψ_{22}	ψ_{23}	ψ_{24}
D_1	-1	0	-5	-3	-3	-2	1	7	-4	0	3	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_2	-1	0	-2	-3	-3	-2	-2	7	-4	0	0	0	0	0	3
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_3	-1	0	-5	-3	-3	-2	-2	7	-1	0	0	0	0	3	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_4	-1	0	1	-3	-3	1	-2	4	-1	0	0	0	0	0	0
		0	0	0	0	0	0	0	3	0	0	0	0	0	0
D_5	-1	-3	-2	0	-3	-2	1	1	-1	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_6	1	0	2	0	0	2	-1	-1	1	0	0	0	0	0	0
		0	0	0	0	0	3	0	0	0	0	0	0	0	0
D_7	1	0	2	3	3	-4	2	-1	-2	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	3

Table C.2: Basis set of non-anomalous U(1) D-flat directions for Model 4, Continued.

	Q_A	ϕ_1	E^c_1	ψ_1	ψ_2	ψ_3	E^c_2	E^c_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}
		ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}	ψ_{15}	ψ_{16}	ψ_{17}	ψ_{18}	ψ_{19}	ψ_{20}	ψ_{21}	ψ_{22}	ψ_{23}	ψ_{24}
D_8	1	0	-1	3	3	-4	2	-1	1	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	3	0
D_9	1	0	-1	3	3	-4	5	-1	-2	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	3	0	0	0	0	0
D_{10}	0	0	1	0	0	1	-2	0	1	0	0	0	0	0	0
		0	0	1	0	0	0	0	0	0	0	0	0	0	0
D_{11}	0	0	-1	-1	0	-1	1	1	0	0	0	0	0	0	0
		0	0	0	0	0	0	1	0	0	0	0	0	0	0
D_{12}	0	0	-1	0	0	1	0	0	1	0	0	0	0	0	0
		0	0	0	0	1	0	0	0	0	0	0	0	0	0
D_{13}	0	0	1	0	0	1	0	0	-1	0	0	0	0	0	0
		0	0	0	1	0	0	0	0	0	0	0	0	0	0
D_{14}	0	0	1	1	1	1	0	-2	1	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	1	0	0

Table C.2: Basis set of non-anomalous U(1) D-flat directions for Model 4, Continued.

	Q_A	ϕ_1	E^c_1	ψ_1	ψ_2	ψ_3	E^c_2	E^c_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}
		ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}	ψ_{15}	ψ_{16}	ψ_{17}	ψ_{18}	ψ_{19}	ψ_{20}	ψ_{21}	ψ_{22}	ψ_{23}	ψ_{24}
D_{15}	0	0	-1	1	0	-1	1	-1	0	1	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{16}	0	0	0	1	0	1	0	-2	1	0	0	1	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{17}	0	0	-1	1	-1	0	0	0	0	0	0	0	1	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{18}	0	0	-1	-1	-1	0	0	2	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	1	0	0	0
D_{19}	0	0	-1	0	-2	-1	0	2	-1	0	0	0	0	0	0
		1	0	0	0	0	0	0	0	0	0	0	0	0	0
D_{20}	0	0	0	-1	0	1	0	0	1	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	1	0	0	0	0
D_{21}	0	0	1	0	0	1	0	-2	1	0	0	0	0	0	0
		0	1	0	0	0	0	0	0	0	0	0	0	0	0

Table C.3: Basis set of F-flat directions to all order for Model 4. The entries in all columns, beginning with Q_A have been divided by a factor of three.

FD	Q_A	ϕ_1	E^c_1	ψ_1	ψ_2	ψ_3	E^c_2	E^c_3	ψ_4	ψ_6	ψ_7	ψ_9	ψ_{10}	ψ_{15}	ψ_{19}	ψ_{20}	ψ_{22}
1	-3	0	-4	1	1	4	0	1	0	6	0	2	1	0	0	0	10
2	-3	-6	-4	1	-5	-2	0	1	0	0	0	2	1	0	0	0	4
3	-1	0	-8	1	-3	-2	0	3	0	0	4	4	1	0	2	0	0
4	-2	0	-17	5	-6	-1	0	0	4	0	11	9	1	0	4	0	0
5	-1	0	-8	-7	-3	-2	0	11	0	0	0	4	1	0	2	4	0
6	-2	-3	-10	-3	-6	0	0	8	0	1	0	1	1	4	0	0	0
7	-4	-7	-4	2	-5	-1	0	0	0	1	0	2	2	0	0	0	7
8	-2	-2	-2	1	-1	1	0	0	0	2	0	1	1	0	0	0	5
9	-3	-4	-4	1	-3	0	0	1	0	2	0	2	1	0	0	0	6
10	-4	-6	-5	1	-4	0	0	0	1	2	0	3	1	0	0	0	8
11	-3	-1	-4	1	0	3	0	1	0	5	0	2	1	0	0	0	9
12	-3	0	-4	1	1	4	0	1	0	6	0	2	1	0	0	0	10
13	-4	-2	-5	2	0	4	1	0	0	7	0	2	1	0	0	0	12
14	-4	-1	-4	2	1	5	0	0	0	7	0	2	2	0	0	0	13
15	-4	-7	-11	2	-12	-1	0	0	0	1	7	2	2	0	0	0	0

Table C.4: Additional Basis Vectors of Model 4.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,...,5}$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,...,8}$
α_4	1	0	0	0	1,...,1	0	1	0	0,0,0,0,0,0,0,0
β_4	1	0	1	0	1,...,1	1	0	0	0,0,0,0,0,0,0,0
γ_4	2	0	0	2	1,...,1	1	1	1	1,1,1,1,1,1,1,1

Table C.4: Additional Basis Vectors of Model 4, Continued.

	$y^{3,...,6}$	$\overline{y}^{3,...,6}$	$y^{1,2}, w^{5,6}$	$\overline{y}^{1,2}, \overline{w}^{5,6}$	$w^{1,...,4}$	$\overline{w}^{1,...,4}$
α_4	1,0,0,1	0,0,0,0	0,0,1,0	1,0,1,1	0,0,0,1	0,0,0,1
β_4	0,0,0,0	1,0,0,1	1,0,1,1	0,0,1,0	0,1,0,0	0,1,0,0
γ_4	0,2,0,0	2,2,0,2	0,2,0,0	0,2,0,0	2,0,2,0	0,0,0,0

Table C.5: GSO Projection Matrix for Model 4.

$k_{i,j}$	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	α_4	β_4	γ_4
$\mathbf{1}$	0	0	1	1	1	1	1	1
$\mathbf{S}$	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	1	1	1	1	1	0	0	$\frac{1}{2}$
$\mathbf{b}_2$	1	1	1	1	1	0	0	1
$\mathbf{b}_3$	1	1	1	1	1	0	1	0
α_4	1	1	1	1	0	1	1	1
β_4	1	1	1	1	1	1	1	$-\frac{1}{2}$
γ_4	0	1	0	0	0	0	1	$\frac{1}{2}$

APPENDIX D

Gauge Charges and Flat Directions of Model 5

Table D.1: Gauge Charges of Model 5.

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
F_1	10	8	0	2	-2	0	-2	-4	-2	(1, 1, 1, 1, 1)
F_2	10	6	0	2	2	4	0	2	-6	(1, 1, 1, 1, 1)
F_3	10	6	0	2	0	-2	4	2	12	(1, 1, 1, 1, 1)
$\bar{F}_1$	-5	8	0	-6	-2	0	-2	-4	-2	(1, 1, 1, 1, 1)
$\bar{F}_2$	-5	2	0	-6	2	-4	0	-2	-14	(1, 1, 1, 1, 1)
$\bar{F}_3$	-5	2	0	-6	0	-2	-4	6	4	(1, 1, 1, 1, 1)
E_1^c	1	8	0	10	-2	0	-2	-4	-2	(1, 1, 1, 1, 1)
E_2^c	1	2	0	10	2	-4	0	- 2	-14	(1, 1, 1, 1, 1)
E_3^c	1	2	0	10	0	-2	-4	6	4	(1, 1, 1, 1, 1)
$h_1(\bar{h}_1)$	-5	-8	0	4	-4	0	0	0	20	(1, 1, 1, 1, 1)
$h_2(\bar{h}_2)$	-5	-8	0	4	4	0	0	0	20	(1, 1, 1, 1, 1)
Φ_1	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1,1)
Φ_2	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1, 1)
Φ_3	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1, 1)
Φ_4	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1, 1)
$\phi_1(\bar{\phi}_1)$	1	4	0	0	0	0	-12	-4	8	(1, 1, 1, 1, 1)
$\phi_2(\bar{\phi}_2)$	1	12	0	0	0	0	4	-12	24	(1, 1, 1, 1, 1)
$\phi_3(\bar{\phi}_3)$	1	-8	0	0	0	-8	-8	0	-16	(1, 1, 1, 1, 1)
$\phi_4(\bar{\phi}_4)$	1	0	0	0	0	-8	8	-8	0	(1, 1, 1, 1, 1)

Table D.1: Gauge Charges of Model 5, Continued.

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
$\phi_5(\bar{\phi}_5)$	1	0	0	0	4	4	0	-8	-36	(1, 1, 1, 1, 1)
$\phi_6(\bar{\phi}_6)$	1	0	0	0	-4	4	0	-8	-36	(1, 1, 1, 1, 1)
ψ_1	1	4	8	0	2	-2	2	8	-10	(1, 1, 1, 1, 1)
ψ_2	1	4	-8	0	2	-2	2	8	-10	(1, 1, 1, 1, 1)
ψ_3	1	6	8	0	-2	-6	0	2	-6	(1, 1, 1, 1, 1)
ψ_4	1	6	-8	0	-2	-6	0	2	-6	(1, 1, 1, 1, 1)
ψ_5	1	6	8	0	0	0	-4	2	-24	(1, 1, 1, 1, 1)
ψ_6	1	6	-8	0	0	0	-4	2	-24	(1, 1, 1, 1, 1)
$H_1(\bar{H}_1)$	1	0	8	0	0	0	0	0	0	(1, 2, 2, 1, 1)
H_2	1	-2	4	5	-2	5	-4	2	-4	(1, 1, 2, 1, 1)
H_3	1	2	-4	-5	2	-5	4	-2	4	(1, 1, 1, 2, 1)
H_4	1	4	0	0	2	-2	2	8	-10	(1, 1, 1, 1, 6)
H_5	1	-4	0	0	-2	2	-2	-8	10	(1, 2, 1, 2, 1)
H_6	1	10	0	0	-2	2	0	6	2	(1, 1, 1, 1, 6)
H_7	1	-10	0	0	2	-2	0	-6	-2	(2, 1, 2, 1, 1)
H_8	1	-4	4	5	0	3	2	8	10	(1, 1, 2, 1, 1)
H_9	1	4	4	5	0	3	-2	0	26	(1, 1, 2, 1, 1)
H_{10}	1	2	4	5	-2	5	4	-2	4	(1, 1, 2, 1, 1)
H_{11}	1	2	-4	5	-2	5	4	-2	4	(2, 1, 1, 1, 1)
H_{12}	1	-4	-4	5	2	1	6	0	-8	(2, 1, 1, 1, 1)
H_{13}	1	4	4	5	2	1	2	-8	8	(1, 1, 1, 2, 1)
H_{14}	1	-2	4	5	-2	-3	4	-6	-4	(1, 1, 1, 2, 1)
H_{15}	1	-2	4	5	4	-1	0	2	14	(1, 1, 1, 2, 1)

Table D.1: Gauge Charges of Model 5, Continued.

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
H_{16}	1	-2	-4	5	-4	-1	0	2	14	(2, 1, 1, 1, 1)
H_{17}	1	-4	-4	5	2	1	6	0	-8	(1, 2, 1, 1, 1)
H_{18}	1	0	-4	5	2	1	-6	-4	0	(1, 2, 1, 1, 1)
H_{19}	1	0	-4	-5	0	5	2	4	-18	(1, 1, 2, 1, 1)
H_{20}	1	8	-4	-5	0	5	-2	-4	-2	(1, 1, 2, 1, 1)
H_{21}	1	6	4	-5	2	3	4	2	12	(1, 2, 1, 1, 1)
H_{22}	1	2	4	-5	2	3	-4	6	4	(1, 2, 1, 1, 1)
H_{23}	1	6	4	-5	2	3	4	2	12	(2, 1, 1, 1, 1)
H_{24}	1	8	4	-5	-2	-1	2	-4	16	(2, 1, 1, 1, 1)
H_{25}	1	0	-4	-5	-2	-1	6	4	0	(1, 1, 1, 2, 1)
H_{26}	1	2	-4	-5	4	1	0	-2	-14	(1, 1, 1, 2, 1)
H_{27}	1	2	4	-5	-4	1	0	-2	-14	(2, 1, 1, 1, 1)
H_{28}	1	0	-4	-5	-2	-1	6	4	0	(1, 1, 2, 1, 1)
H_{29}	1	4	-4	-5	-2	-1	-6	0	8	(1, 1, 2, 1, 1)
H_{30}	1	10	0	0	0	0	4	-2	-16	(1, 2, 2, 1, 1)
H_{31}	1	6	0	0	0	0	-4	2	-24	(2, 1, 1, 2, 1)
H_{32}	1	10	0	0	0	0	4	-2	-16	(1, 1, 1, 1, 6)
H_{33}	1	10	0	0	-2	2	0	6	2	(2, 1, 1, 2, 1)
H_{34}	1	6	0	0	-2	-6	0	2	-6	(1, 2, 2, 1, 1)
H_{35}	1	12	0	0	2	-2	-2	0	6	(2, 1, 1, 2, 1)
H_{36}	1	12	0	0	2	-2	-2	0	6	(1, 2, 2, 1, 1)
H_{37}	1	2	-4	0	2	2	0	6	22	(1, 1, 1, 1, 4)
H_{38}	1	-2	-4	0	-2	-2	0	-6	-22	(1, 1, 1, 1, 4)
H_{39}	1	10	0	0	-2	2	0	6	2	(1, 2, 1, 2, 1)

Table D.1: Gauge Charges of Model 5, Continued.

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
H_{40}	1	4	4	0	-2	-2	-2	0	26	(1, 1, 1, 1, -4)
H_{41}	1	4	4	0	2	2	-2	-8	-10	(1, 1, 1, 1, -4)
H_{42}	1	8	-4	0	0	4	2	-4	16	(1, 1, 1, 1, 4)
H_{43}	1	4	4	0	0	4	-6	0	8	(1, 1, 1, 1, -4)
H_{44}	1	4	-4	0	0	-4	2	-8	8	(1, 1, 1, 1, 4)
H_{45}	1	0	4	0	0	-4	-6	-4	0	(1, 1, 1, 1, -4)
H_{46}	1	12	0	0	2	-2	-2	0	6	(2, 1, 2, 1, 1)
H_{47}	1	0	0	0	0	0	0	0	0	(2, 1, 1, 2, 6)

Table D.2: Basis set of non-anomalous U(1) D-flat directions for Model 5.

FD	Q_A	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	E_1^c	E_2^c	E_3^c	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6
D_1	144	0	7	0	-2	-2	1	0	0	0	6	0	0	0	0	6
D_2	144	0	4	-3	-2	1	-2	0	0	0	0	0	6	0	0	6
D_3	144	0	4	-3	1	-2	-2	0	0	0	0	0	0	0	6	6
D_4	0	0	0	0	-1	1	0	0	0	0	0	0	0	2	0	-2
D_5	0	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0
D_6	0	0	-1	0	0	0	-1	2	0	-2	0	0	0	0	0	0
D_7	0	0	1	1	-1	0	1	0	0	0	0	2	0	0	0	-2
D_8	0	0	0	0	-1	-1	0	0	2	-2	0	0	0	0	0	0

Table D.3: Additional Basis Vectors of Model 5.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,...,5}$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,...,8}$
α_5	1	1	0	0	1,...,1	0	1	0	0,0,0,0,0,0,0,0
β_5	1	0	1	0	1,...,1	1	0	0	0,0,0,0,0,0,0,0
γ_5	2	0	0	2	1,...,1	1	1	1	1,1,1,1,2,2,0,0

Table D.3: Additional Basis Vectors of Model 5, Continued.

	$y^{3,...,6}$	$\overline{y}^{3,...,6}$	$y^{1,2}, w^{5,6}$	$\overline{y}^{1,2}, \overline{w}^{5,6}$	$w^{1,...,4}$	$\overline{w}^{1,...,4}$
α_5	1,0,0,1	0,0,0,0	0,0,1,0	1,0,1,1	0,0,0,1	0,0,0,1
β_5	0,0,0,0	1,0,0,1	1,0,1,1	0,0,1,0	0,1,0,0	0,1,0,0
γ_5	0,2,0,0	2,2,0,2	0,0,0,0	2,0,0,2	2,2,2,0	0,2,0,0

Table D.4: GSO Projection Matrix for Model 5.

$k_{i,j}$	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	α_5	β_5	γ_5
$\mathbf{1}$	0	0	1	1	1	0	0	$-\frac{1}{2}$
$\mathbf{S}$	0	0	0	0	0	1	1	0
$\mathbf{b}_1$	1	1	1	1	1	1	1	$-\frac{1}{2}$
$\mathbf{b}_2$	1	1	1	1	1	1	1	$\frac{1}{2}$
$\mathbf{b}_3$	1	1	1	1	1	1	1	0
α_5	0	0	0	0	1	0	1	$-\frac{1}{2}$
β_5	0	0	0	0	1	1	0	$-\frac{1}{2}$
γ_5	1	1	1	1	0	1	1	1

APPENDIX E

Basis Vectors and GSO Projection Matrix for Mirror Models Based on a NAHE Variation

Table E.1: Basis Vectors for Mirror Gauge Group Model Based on NAHE Variation.

	ψ^{μ}	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,...,5}$	$\overline{\eta}^{1}$	$\overline{\eta}^{2}$	$\overline{\eta}^{3}$	$\overline{\eta'}^{1}$	$\overline{\eta'}^{2}$	$\overline{\eta'}^{3}$	$\overline{\psi'}^{1,...,5}$
$\mathbf{1}$	1	1	1	1	1,...,1	1	1	1	1	1	1	1,...,1
$\mathbf{S}$	1	1	1	1	0,...,0	0	0	0	0	0	0	0,...,0
$\mathbf{b}_1$	1	1	0	0	1,...,1	1	0	0	0	0	0	0,...,0
$\mathbf{b}_2$	1	0	1	0	1,...,1	0	1	0	0	0	0	0,...,0
$\mathbf{b}_3$	1	0	0	1	1,...,1	0	0	1	0	0	0	0,...,0
$\mathbf{b}_1'$	1	0	1	1	1,...,1	0	1	1	1	0	0	0,...,0
$\mathbf{b}_2'$	1	0	1	0	1,...,1	1	0	1	0	1	0	0,...,0
$\mathbf{b}_3'$	1	0	0	1	1,...,1	1	1	0	0	0	1	0,...,0

	$y^{1,2}$	$\overline{y}^{1,2}$	$y^{3,4}$	$\overline{y}^{3,4}$	$y^{5,6}$	$\overline{y}^{5,6}$	$w^{1,2}$	$\overline{w}^{1,2}$	$w^{3,4}$	$\overline{w}^{3,4}$	$w^{5,6}$	$\overline{w}^{5,6}$
$\mathbf{1}$	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
$\mathbf{S}$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}_1$	0,0	0,0	1,1	1,1	1,1	1,1	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}_2$	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}_3$	1,1	1,1	1,1	1,1	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}'_1$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}'_2$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}'_3$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

Table E.2: GSO Projection Matrix for Mirror Gauge Group Model Based on a NAHE Variation.

$k_{i,j}$	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	$\mathbf{b}_1^{'}$	$\mathbf{b}_2^{'}$	$\mathbf{b}_3^{'}$
$\mathbf{1}$	0	0	1	1	1	1	1	1
$\mathbf{S}$	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	1	1	1	1	1	1	0	0
$\mathbf{b}_2$	1	1	1	1	1	0	1	0
$\mathbf{b}_3$	1	1	1	1	1	0	0	1
$\mathbf{b}_1^{'}$	1	1	0	0	0	1	0	0
$\mathbf{b}_2^{'}$	1	1	0	0	0	0	1	0
$\mathbf{b}_3^{'}$	1	1	0	0	0	0	0	1

APPENDIX F

Gauge Charges and Superpotential Terms of a NAHE Variation

Table F.1: $E_6 \otimes U(1)^5 \otimes SO(22)$ States. Note: all $U(1)$ charges below have been multiplied by a factor or 4 to eliminate fractions.

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(22)$
1	G_1	**27**	0	8	0	0	0	**1**
	G_2	**27**	4	-4	0	0	0	**1**
	G_3	**27**	-4	-4	0	0	0	**1**
	$\overline{G}_1$	**27**	0	-8	0	0	0	**1**
	$\overline{G}_2$	**27**	-4	4	0	0	0	**1**
	$\overline{G}_3$	**27**	4	4	0	0	0	**1**
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2$	G_4	**27**	0	-4	-2	-2	0	**1**
	G_5	**27**	0	-4	-2	2	0	**1**
	G_6	**27**	0	-4	2	-2	0	**1**
	G_7	**27**	0	-4	2	2	0	**1**
	$\overline{G}_4$	**27**	0	4	2	2	0	**1**
	$\overline{G}_5$	**27**	0	4	2	-2	0	**1**
	$\overline{G}_6$	**27**	0	4	-2	2	0	**1**
	$\overline{G}_7$	**27**	0	4	-2	-2	0	**1**
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_3$	G_8	**27**	-2	2	-2	0	-2	**1**
	G_9	**27**	-2	2	-2	0	2	**1**
	G_{10}	**27**	-2	2	2	0	-2	**1**
	G_{11}	**27**	-2	2	2	0	2	**1**

Table F.1: $E_6 \otimes U(1)^5 \otimes SO(22)$ States, Continued.

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(22)$
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_3$	$\overline{G}_8$	**27**	2	-2	2	0	2	**1**
	$\overline{G}_9$	**27**	2	-2	2	0	-2	**1**
	$\overline{G}_{10}$	**27**	2	-2	-2	0	2	**1**
	$\overline{G}_{11}$	**27**	2	-2	-2	0	-2	**1**
$\mathbf{S}+\mathbf{b}_2+\mathbf{b}_3$	G_{12}	**27**	2	2	0	-2	-2	**1**
	G_{13}	**27**	2	2	0	-2	2	**1**
	G_{14}	**27**	2	2	0	2	-2	**1**
	G_{15}	**27**	2	2	0	2	2	**1**
	$\overline{G}_{12}$	**27**	-2	-2	0	2	2	**1**
	$\overline{G}_{13}$	**27**	-2	-2	0	2	-2	**1**
	$\overline{G}_{14}$	**27**	-2	-2	0	-2	2	**1**
	$\overline{G}_{15}$	**27**	-2	-2	0	-2	-2	**1**
1	ϕ_1 $(\overline{\phi}_1)$	**1**	0	0	0	-4	-4	**1**
	ϕ_2 $(\overline{\phi}_2)$	**1**	0	0	0	-4	4	**1**
	ϕ_3 $(\overline{\phi}_3)$	**1**	0	0	-4	0	-4	**1**
	ϕ_4 $(\overline{\phi}_4)$	**1**	0	0	-4	0	4	**1**
	ϕ_5 $(\overline{\phi}_5)$	**1**	0	0	-4	-4	0	**1**
	ϕ_6 $(\overline{\phi}_6)$	**1**	0	0	-4	4	0	**1**
	ϕ_7 $(\overline{\phi}_7)$	**1**	4	-12	0	0	0	**1**
	ϕ_8 $(\overline{\phi}_8)$	**1**	4	12	0	0	0	**1**
	ϕ_9 $(\overline{\phi}_9)$	**1**	-8	0	0	0	0	**1**

Table F.1: $E_6 \otimes U(1)^5 \otimes SO(22)$ States, Continued.

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(22)$
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2$	ψ_1 $(\overline{\psi}_1)$	**1**	0	12	2	2	0	**1**
	ψ_2 $(\overline{\psi}_2)$	**1**	0	12	2	-2	0	**1**
	ψ_3 $(\overline{\psi}_3)$	**1**	0	12	-2	2	0	**1**
	ψ_4 $(\overline{\psi}_4)$	**1**	0	12	-2	-2	0	**1**
	ψ_5 $(\overline{\psi}_5)$	**1**	4	0	2	2	-4	**1**
	ψ_6 $(\overline{\psi}_6)$	**1**	4	0	2	2	4	**1**
	ψ_7 $(\overline{\psi}_7)$	**1**	4	0	2	-2	-4	**1**
	ψ_8 $(\overline{\psi}_8)$	**1**	4	0	2	-2	4	**1**
	ψ_9 $(\overline{\psi}_9)$	**1**	4	0	-2	2	-4	**1**
	ψ_{10} $(\overline{\psi}_{10})$	**1**	4	0	-2	2	4	**1**
	ψ_{11} $(\overline{\psi}_{11})$	**1**	4	0	-2	-2	-4	**1**
	ψ_{12} $(\overline{\psi}_{12})$	**1**	4	0	-2	-2	4	**1**
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_3$	ψ_{13} $(\overline{\psi}_{13})$	**1**	2	6	2	-4	2	**1**
	ψ_{14} $(\overline{\psi}_{14})$	**1**	2	6	2	-4	-2	**1**
	ψ_{15} $(\overline{\psi}_{15})$	**1**	2	6	2	4	2	**1**
	ψ_{16} $(\overline{\psi}_{16})$	**1**	2	6	2	4	-2	**1**
	ψ_{17} $(\overline{\psi}_{17})$	**1**	2	6	-2	-4	2	**1**
	ψ_{18} $(\overline{\psi}_{18})$	**1**	2	6	-2	-4	-2	**1**
	ψ_{19} $(\overline{\psi}_{19})$	**1**	2	6	-2	4	2	**1**
	ψ_{20} $(\overline{\psi}_{20})$	**1**	2	6	-2	4	-2	**1**
	ψ_{21} $(\overline{\psi}_{21})$	**1**	6	-6	2	0	2	**1**
	ψ_{22} $(\overline{\psi}_{22})$	**1**	6	-6	2	0	-2	**1**
	ψ_{23} $(\overline{\psi}_{23})$	**1**	6	-6	-2	0	2	**1**
	ψ_{24} $(\overline{\psi}_{24})$	**1**	6	-6	-2	0	-2	**1**

Table F.1: $E_6 \otimes U(1)^5 \otimes SO(22)$ States, Continued.

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(22)$
$\mathbf{S}+\mathbf{b}_2+\mathbf{b}_3$	ψ_{25} $(\overline{\psi}_{25})$	**1**	-2	6	-4	2	2	**1**
	ψ_{26} $(\overline{\psi}_{26})$	**1**	-2	6	-4	2	-2	**1**
	ψ_{27} $(\overline{\psi}_{27})$	**1**	-2	6	-4	-2	2	**1**
	ψ_{28} $(\overline{\psi}_{28})$	**1**	-2	6	-4	-2	-2	**1**
	ψ_{29} $(\overline{\psi}_{29})$	**1**	-2	6	4	2	2	**1**
	ψ_{30} $(\overline{\psi}_{30})$	**1**	-2	6	4	2	-2	**1**
	ψ_{31} $(\overline{\psi}_{31})$	**1**	-2	6	4	-2	2	**1**
	ψ_{32} $(\overline{\psi}_{32})$	**1**	-2	6	4	-2	-2	**1**
	ψ_{33} $(\overline{\psi}_{33})$	**1**	-6	-6	0	2	2	**1**
	ψ_{34} $(\overline{\psi}_{34})$	**1**	-6	-6	0	2	-2	**1**
	ψ_{35} $(\overline{\psi}_{35})$	**1**	-6	-6	0	-2	2	**1**
	ψ_{36} $(\overline{\psi}_{36})$	**1**	-6	-6	0	-2	-2	**1**
1	H_1	**1**	0	0	0	0	-4	**22**
	H_2	**1**	0	0	0	0	4	**22**
	H_3	**1**	0	0	0	-4	0	**22**
	H_4	**1**	0	0	0	4	0	**22**
	H_5	**1**	0	0	-4	0	0	**22**
	H_6	**1**	0	0	4	0	0	**22**

Table F.1: $E_6 \otimes U(1)^5 \otimes SO(22)$ States, Continued.

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(22)$
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2$	H_7	**1**	4	0	2	2	0	**22**
	H_8	**1**	4	0	2	-2	0	**22**
	H_9	**1**	4	0	-2	2	0	**22**
	H_{10}	**1**	4	0	-2	-2	0	**22**
	H_{11}	**1**	-4	0	2	2	0	**22**
	H_{12}	**1**	-4	0	2	-2	0	**22**
	H_{13}	**1**	-4	0	-2	2	0	**22**
	H_{14}	**1**	-4	0	-2	-2	0	**22**
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_3$	H_{15}	**1**	2	6	2	0	2	**22**
	H_{16}	**1**	2	6	2	0	-2	**22**
	H_{17}	**1**	2	6	-2	0	2	**22**
	H_{18}	**1**	2	6	-2	0	-2	**22**
	H_{19}	**1**	-2	-6	2	0	2	**22**
	H_{20}	**1**	-2	-6	2	0	-2	**22**
	H_{21}	**1**	-2	-6	-2	0	2	**22**
	H_{22}	**1**	-2	-6	-2	0	-2	**22**
$\mathbf{S}+\mathbf{b}_2+\mathbf{b}_3$	H_{23}	**1**	-2	6	0	2	2	**22**
	H_{24}	**1**	-2	6	0	2	-2	**22**
	H_{25}	**1**	-2	6	0	-2	2	**22**
	H_{26}	**1**	-2	6	0	-2	-2	**22**
	H_{27}	**1**	2	-6	0	2	2	**22**
	H_{28}	**1**	2	-6	0	2	-2	**22**
	H_{29}	**1**	2	-6	0	-2	2	**22**
	H_{30}	**1**	2	-6	0	-2	-2	**22**

$E_6 \otimes U(1)^5 \otimes SO(22)$ Third Order Superpotential (No Fourth Order Terms Exist)

$$\overline{G}_{1}\, G_{2}\, G_{3} + G_{1}\, \overline{G}_{2}\, \phi_{7} + G_{1}\, \overline{G}_{3}\, \overline{\phi}_{8} + G_{1}\, G_{4}\, G_{7} + G_{1}\, G_{5}\, G_{6}$$

$$+ G_{1}\, \overline{G}_{4}\, \overline{\psi}_{1} + G_{1}\, \overline{G}_{5}\, \overline{\psi}_{2} + G_{1}\, \overline{G}_{6}\, \overline{\psi}_{3} + G_{1}\, \overline{G}_{7}\, \overline{\psi}_{4} + G_{2}\, \overline{G}_{1}\, \overline{\phi}_{7}$$

$$+ G_{2}\, \overline{G}_{3}\, \phi_{9} + G_{2}\, G_{8}\, G_{11} + G_{2}\, G_{9}\, G_{10} + G_{2}\, \overline{G}_{8}\, \overline{\psi}_{21} + G_{2}\, \overline{G}_{9}\, \overline{\psi}_{22}$$

$$+ G_{2}\, \overline{G}_{10}\, \overline{\psi}_{23} + G_{2}\, \overline{G}_{11}\, \overline{\psi}_{24} + G_{3}\, \overline{G}_{1}\, \phi_{8} + G_{3}\, \overline{G}_{2}\, \overline{\phi}_{9} + G_{3}\, G_{12}\, G_{15}$$

$$+ G_{3}\, G_{13}\, G_{14} + G_{3}\, \overline{G}_{12}\, \overline{\psi}_{33} + G_{3}\, \overline{G}_{13}\, \overline{\psi}_{34} + G_{3}\, \overline{G}_{14}\, \overline{\psi}_{35} + G_{3}\, \overline{G}_{15}\, \overline{\psi}_{36}$$

$$+ \overline{G}_{1}\, \overline{G}_{2}\, \overline{G}_{3} + \overline{G}_{1}\, G_{4}\, \psi_{1} + \overline{G}_{1}\, G_{5}\, \psi_{2} + \overline{G}_{1}\, G_{6}\, \psi_{3} + \overline{G}_{1}\, G_{7}\, \psi_{4}$$

$$+ \overline{G}_{1}\, \overline{G}_{4}\, \overline{G}_{7} + \overline{G}_{1}\, \overline{G}_{5}\, \overline{G}_{6} + \overline{G}_{2}\, G_{8}\, \psi_{21} + \overline{G}_{2}\, G_{9}\, \psi_{22} + \overline{G}_{2}\, G_{10}\, \psi_{23}$$

$$+ \overline{G}_{2}\, G_{11}\, \psi_{24} + \overline{G}_{2}\, \overline{G}_{8}\, \overline{G}_{11} + \overline{G}_{2}\, \overline{G}_{9}\, \overline{G}_{10} + \overline{G}_{3}\, G_{12}\, \psi_{33} + \overline{G}_{3}\, G_{13}\, \psi_{34}$$

$$+ \overline{G}_{3}\, G_{14}\, \psi_{35} + \overline{G}_{3}\, G_{15}\, \psi_{36} + \overline{G}_{3}\, \overline{G}_{12}\, \overline{G}_{15} + \overline{G}_{3}\, \overline{G}_{13}\, \overline{G}_{14} + G_{4}\, G_{10}\, G_{15}$$

$$+ G_{4}\, G_{11}\, G_{14} + G_{4}\, \overline{G}_{7}\, \overline{\phi}_{5} + G_{4}\, \overline{G}_{10}\, \psi_{30} + G_{4}\, \overline{G}_{11}\, \psi_{29} + G_{4}\, \overline{G}_{14}\, \psi_{16}$$

$$+ G_{4}\, \overline{G}_{15}\, \psi_{15} + G_{5}\, G_{10}\, G_{13} + G_{5}\, G_{11}\, G_{12} + G_{5}\, \overline{G}_{6}\, \overline{\phi}_{6} + G_{5}\, \overline{G}_{10}\, \psi_{32}$$

$$+ G_{5}\, \overline{G}_{11}\, \psi_{31} + G_{5}\, \overline{G}_{12}\, \psi_{14} + G_{5}\, \overline{G}_{13}\, \psi_{13} + G_{6}\, G_{8}\, G_{15} + G_{6}\, G_{9}\, G_{14}$$

$$+ G_{6}\, \overline{G}_{5}\, \phi_{6} + G_{6}\, \overline{G}_{8}\, \psi_{26} + G_{6}\, \overline{G}_{9}\, \psi_{25} + G_{6}\, \overline{G}_{14}\, \psi_{20} + G_{6}\, \overline{G}_{15}\, \psi_{19}$$

$$+ G_{7}\, G_{8}\, G_{13} + G_{7}\, G_{9}\, G_{12} + G_{7}\, \overline{G}_{4}\, \phi_{5} + G_{7}\, \overline{G}_{8}\, \psi_{28} + G_{7}\, \overline{G}_{9}\, \psi_{27}$$

$$+ G_{7}\, \overline{G}_{12}\, \psi_{18} + G_{7}\, \overline{G}_{13}\, \psi_{17} + G_{8}\, \overline{G}_{6}\, \overline{\psi}_{28} + G_{8}\, \overline{G}_{7}\, \overline{\psi}_{26} + G_{8}\, \overline{G}_{11}\, \overline{\phi}_{3}$$

$$+ G_{8}\, \overline{G}_{13}\, \psi_{8} + G_{8}\, \overline{G}_{15}\, \psi_{6} + G_{9}\, \overline{G}_{6}\, \overline{\psi}_{27} + G_{9}\, \overline{G}_{7}\, \overline{\psi}_{25} + G_{9}\, \overline{G}_{10}\, \overline{\phi}_{4}$$

$$+ G_{9}\, \overline{G}_{12}\, \mathrm{s7} + G_{9}\, \overline{G}_{14}\, \mathrm{s5} + G_{10}\, \overline{G}_{4}\, \overline{\psi}_{32} + G_{10}\, \overline{G}_{5}\, \overline{\psi}_{30} + G_{10}\, \overline{G}_{9}\, \phi_{4}$$

$$+ G_{10}\, \overline{G}_{13}\, \psi_{12} + G_{10}\, \overline{G}_{15}\, \psi_{10} + G_{11}\, \overline{G}_{4}\, \overline{\psi}_{31} + G_{11}\, \overline{G}_{5}\, \overline{\psi}_{29} + G_{11}\, \overline{G}_{8}\, \phi_{3}$$

$$+ G_{11}\, \overline{G}_{12}\, \psi_{11} + G_{11}\, \overline{G}_{14}\, \psi_{9} + G_{12}\, \overline{G}_{5}\, \overline{\psi}_{14} + G_{12}\, \overline{G}_{7}\, \overline{\psi}_{18} + G_{12}\, \overline{G}_{9}\, \overline{\psi}_{7}$$

$$+ G_{12}\, \overline{G}_{11}\, \overline{\psi}_{11} + G_{12}\, \overline{G}_{15}\, \overline{\phi}_{1} + G_{13}\, \overline{G}_{5}\, \overline{\psi}_{13} + G_{13}\, \overline{G}_{7}\, \overline{\psi}_{17} + G_{13}\, \overline{G}_{8}\, \overline{\psi}_{8}$$

$$+ G_{13}\, \overline{G}_{10}\, \overline{\psi}_{12} + G_{13}\, \overline{G}_{14}\, \overline{\phi}_{2} + G_{14}\, \overline{G}_{4}\, \overline{\psi}_{16} + G_{14}\, \overline{G}_{6}\, \overline{\psi}_{20} + G_{14}\, \overline{G}_{9}\, \overline{\psi}_{5}$$

$$+ G_{14}\, \overline{G}_{11}\, \overline{\psi}_{9} + G_{14}\, \overline{G}_{13}\, \phi_{2} + G_{15}\, \overline{G}_{4}\, \overline{\psi}_{15} + G_{15}\, \overline{G}_{6}\, \overline{\psi}_{19} + G_{15}\, \overline{G}_{8}\, \overline{\psi}_{6}$$

$$+ G_{15}\, \overline{G}_{10}\, \overline{\psi}_{10} + G_{15}\, \overline{G}_{12}\, \phi_{1} + \overline{G}_{4}\, \overline{G}_{10}\, \overline{G}_{15} + \overline{G}_{4}\, \overline{G}_{11}\, \overline{G}_{14} + \overline{G}_{5}\, \overline{G}_{10}\, \overline{G}_{13}$$

$$+ \overline{G}_{5}\, \overline{G}_{11}\, \overline{G}_{12} + \overline{G}_{6}\, \overline{G}_{8}\, \overline{G}_{15} + \overline{G}_{6}\, \overline{G}_{9}\, \overline{G}_{14} + \overline{G}_{7}\, \overline{G}_{8}\, \overline{G}_{13} + \overline{G}_{7}\, \overline{G}_{9}\, \overline{G}_{12}$$

$$+ \phi_{1}\, \phi_{4}\, \overline{\phi}_{5} + \phi_{1}\, \phi_{6}\, \overline{\phi}_{3} + \phi_{1}\, \psi_{25}\, \overline{\psi}_{26} + \phi_{1}\, \psi_{29}\, \overline{\psi}_{30} + \phi_{1}\, \psi_{33}\, \overline{\psi}_{36}$$

$E_6 \otimes U(1)^5 \otimes SO(22)$ Third Order Superpotential, Continued.

$+\ \phi_1\ H_2\ H_4 \quad +\ \phi_1\ H_{23}\ H_{27} \quad +\ \phi_2\ \phi_3\ \overline{\phi}_5 \quad +\ \phi_2\ \phi_6\ \overline{\phi}_4 \quad +\ \phi_2\ \psi_{26}\ \overline{\psi}_{25}$

$+\ \phi_2\ \psi_{30}\ \overline{\psi}_{29} \quad +\ \phi_2\ \psi_{34}\ \overline{\psi}_{35} \quad +\ \phi_2\ H_1\ H_4 \quad +\ \phi_2\ H_{24}\ H_{28} \quad +\ \phi_3\ \overline{\phi}_1\ \overline{\phi}_6$

$+\ \phi_3\ \psi_{13}\ \overline{\psi}_{18} \quad +\ \phi_3\ \psi_{15}\ \overline{\psi}_{20} \quad +\ \phi_3\ \psi_{21}\ \overline{\psi}_{24} \quad +\ \phi_3\ H_2\ H_6 \quad +\ \phi_3\ H_{15}\ H_{19}$

$+\ \phi_4\ \overline{\phi}_2\ \overline{\phi}_6 \quad +\ \phi_4\ \psi_{14}\ \overline{\psi}_{17} \quad +\ \phi_4\ \psi_{16}\ \overline{\psi}_{19} \quad +\ \phi_4\ \psi_{22}\ \overline{\psi}_{23} \quad +\ \phi_4\ H_1\ H_6$

$+\ \phi_4\ H_{16}\ H_{20} \quad +\ \phi_5\ \overline{\phi}_1\ \overline{\phi}_4 \quad +\ \phi_5\ \overline{\phi}_2\ \overline{\phi}_3 \quad +\ \phi_5\ \psi_1\ \overline{\psi}_4 \quad +\ \phi_5\ \psi_5\ \overline{\psi}_{11}$

$+\ \phi_5\ \psi_6\ \overline{\psi}_{12} \quad +\ \phi_5\ H_4\ H_6 \quad +\ \phi_5\ H_7\ H_{11} \quad +\ \phi_6\ \psi_2\ \overline{\psi}_3 \quad +\ \phi_6\ \psi_7\ \overline{\psi}_9$

$+\ \phi_6\ \psi_8\ \overline{\psi}_{10} \quad +\ \phi_6\ H_3\ H_6 \quad +\ \phi_6\ H_8\ H_{12} \quad +\ \phi_7\ \phi_8\ \phi_9 \quad +\ \phi_7\ \psi_{25}\ \psi_{32}$

$+\ \phi_7\ \psi_{26}\ \psi_{31} \quad +\ \phi_7\ \psi_{27}\ \psi_{30} \quad +\ \phi_7\ \psi_{28}\ \psi_{29} \quad +\ \phi_7\ H_{23}\ H_{26} \quad +\ \phi_7\ H_{24}\ H_{25}$

$+\ \phi_8\ \overline{\psi}_{13}\ \overline{\psi}_{20} \quad +\ \phi_8\ \overline{\psi}_{14}\ \overline{\psi}_{19} \quad +\ \phi_8\ \overline{\psi}_{15}\ \overline{\psi}_{18} \quad +\ \phi_8\ \overline{\psi}_{16}\ \overline{\psi}_{17} \quad +\ \phi_8\ H_{19}\ H_{22}$

$+\ \phi_8\ H_{20}\ H_{21} \quad +\ \phi_9\ \psi_5\ \psi_{12} \quad +\ \phi_9\ \psi_6\ \psi_{11} \quad +\ \phi_9\ \psi_7\ \psi_{10} \quad +\ \phi_9\ \psi_8\ \psi_9$

$+\ \phi_9\ H_7\ H_{10} \quad +\ \phi_9\ H_8\ H_9 \quad +\ \overline{\phi}_1\ \psi_{28}\ \overline{\psi}_{27} \quad +\ \overline{\phi}_1\ \psi_{32}\ \overline{\psi}_{31} \quad +\ \overline{\phi}_1\ \psi_{36}\ \overline{\psi}_{33}$

$+\ \overline{\phi}_1\ H_1\ H_3 \quad +\ \overline{\phi}_1\ H_{26}\ H_{30} \quad +\ \overline{\phi}_2\ \psi_{27}\ \overline{\psi}_{28} \quad +\ \overline{\phi}_2\ \psi_{31}\ \overline{\psi}_{32} \quad +\ \overline{\phi}_2\ \psi_{35}\ \overline{\psi}_{34}$

$+\ \overline{\phi}_2\ H_2\ H_3 \quad +\ \overline{\phi}_2\ H_{25}\ H_{29} \quad +\ \overline{\phi}_3\ \psi_{18}\ \overline{\psi}_{13} \quad +\ \overline{\phi}_3\ \psi_{20}\ \overline{\psi}_{15} \quad +\ \overline{\phi}_3\ \psi_{24}\ \overline{\psi}_{21}$

$+\ \overline{\phi}_3\ H_1\ H_5 \quad +\ \overline{\phi}_3\ H_{18}\ H_{22} \quad +\ \overline{\phi}_4\ \psi_{17}\ \overline{\psi}_{14} \quad +\ \overline{\phi}_4\ \psi_{19}\ \overline{\psi}_{16} \quad +\ \overline{\phi}_4\ \psi_{23}\ \overline{\psi}_{22}$

$+\ \overline{\phi}_4\ H_2\ H_5 \quad +\ \overline{\phi}_4\ H_{17}\ H_{21} \quad +\ \overline{\phi}_5\ \psi_4\ \overline{\psi}_1 \quad +\ \overline{\phi}_5\ \psi_{11}\ \overline{\psi}_5 \quad +\ \overline{\phi}_5\ \psi_{12}\ \overline{\psi}_6$

$+\ \overline{\phi}_5\ H_3\ H_5 \quad +\ \overline{\phi}_5\ H_{10}\ H_{14} \quad +\ \overline{\phi}_6\ \psi_3\ \overline{\psi}_2 \quad +\ \overline{\phi}_6\ \psi_9\ \overline{\psi}_7 \quad +\ \overline{\phi}_6\ \psi_{10}\ \overline{\psi}_8$

$+\ \overline{\phi}_6\ H_4\ H_5 \quad +\ \overline{\phi}_6\ H_9\ H_{13} \quad +\ \overline{\phi}_7\ \overline{\phi}_8\ \overline{\phi}_9 \quad +\ \overline{\phi}_7\ \overline{\psi}_{25}\ \overline{\psi}_{32} \quad +\ \overline{\phi}_7\ \overline{\psi}_{26}\ \overline{\psi}_{31}$

$+\ \overline{\phi}_7\ \overline{\psi}_{27}\ \overline{\psi}_{30} \quad +\ \overline{\phi}_7\ \overline{\psi}_{28}\ \overline{\psi}_{29} \quad +\ \overline{\phi}_7\ H_{27}\ H_{30} \quad +\ \overline{\phi}_7\ H_{28}\ H_{29} \quad +\ \overline{\phi}_8\ \psi_{13}\ \psi_{20}$

$+\ \overline{\phi}_8\ \psi_{14}\ \psi_{19} \quad +\ \overline{\phi}_8\ \psi_{15}\ \psi_{18} \quad +\ \overline{\phi}_8\ \psi_{16}\ \psi_{17} \quad +\ \overline{\phi}_8\ H_{15}\ H_{18} \quad +\ \overline{\phi}_8\ H_{16}\ H_{17}$

$+\ \overline{\phi}_9\ \overline{\psi}_5\ \overline{\psi}_{12} \quad +\ \overline{\phi}_9\ \overline{\psi}_6\ \overline{\psi}_{11} \quad +\ \overline{\phi}_9\ \overline{\psi}_7\ \overline{\psi}_{10} \quad +\ \overline{\phi}_9\ \overline{\psi}_8\ \overline{\psi}_9 \quad +\ \overline{\phi}_9\ H_{11}\ H_{14}$

$+\ \overline{\phi}_9\ H_{12}\ H_{13} \quad +\ \psi_1\ \psi_{23}\ \psi_{36} \quad +\ \psi_1\ \psi_{24}\ \psi_{35} \quad +\ \psi_1\ \overline{\psi}_{19}\ \overline{\psi}_{30} \quad +\ \psi_1\ \overline{\psi}_{20}\ \overline{\psi}_{29}$

$+\ \psi_1\ H_{21}\ H_{30} \quad +\ \psi_1\ H_{22}\ H_{29} \quad +\ \psi_2\ \psi_{23}\ \psi_{34} \quad +\ \psi_2\ \psi_{24}\ \psi_{33} \quad +\ \psi_2\ \overline{\psi}_{17}\ \overline{\psi}_{32}$

$+\ \psi_2\ \overline{\psi}_{18}\ \overline{\psi}_{31} \quad +\ \psi_2\ H_{21}\ H_{28} \quad +\ \psi_2\ H_{22}\ H_{27} \quad +\ \psi_3\ \psi_{21}\ \psi_{36} \quad +\ \psi_3\ \psi_{22}\ \psi_{35}$

$+\ \psi_3\ \overline{\psi}_{15}\ \overline{\psi}_{26} \quad +\ \psi_3\ \overline{\psi}_{16}\ \overline{\psi}_{25} \quad +\ \psi_3\ H_{19}\ H_{30} \quad +\ \psi_3\ H_{20}\ H_{29} \quad +\ \psi_4\ \psi_{21}\ \psi_{34}$

$+\ \psi_4\ \psi_{22}\ \psi_{33} \quad +\ \psi_4\ \overline{\psi}_{13}\ \overline{\psi}_{28} \quad +\ \psi_4\ \overline{\psi}_{14}\ \overline{\psi}_{27} \quad +\ \psi_4\ H_{19}\ H_{28} \quad +\ \psi_4\ H_{20}\ H_{27}$

$+\ \psi_5\ \psi_{17}\ \psi_{33} \quad +\ \psi_5\ \psi_{25}\ \overline{\psi}_{20} \quad +\ \psi_5\ \overline{\psi}_{24}\ \overline{\psi}_{32} \quad +\ \psi_5\ H_2\ H_{14} \quad +\ \psi_5\ H_{21}\ H_{25}$

$E_6 \otimes U(1)^5 \otimes SO(22)$ Third Order Superpotential, Continued.

$+ \psi_6\ \psi_{18}\ \psi_{34}$	$+ \psi_6\ \psi_{26}\ \overline{\psi}_{19}$	$+ \psi_6\ \overline{\psi}_{23}\ \overline{\psi}_{31}$	$+ \psi_6\ H_1\ H_{14}$	$+ \psi_6\ H_{22}\ H_{26}$
$+ \psi_7\ \psi_{19}\ \psi_{35}$	$+ \psi_7\ \psi_{27}\ \overline{\psi}_{18}$	$+ \psi_7\ \overline{\psi}_{24}\ \overline{\psi}_{30}$	$+ \psi_7\ H_2\ H_{13}$	$+ \psi_7\ H_{21}\ H_{23}$
$+ \psi_8\ \psi_{20}\ \psi_{36}$	$+ \psi_8\ \psi_{28}\ \overline{\psi}_{17}$	$+ \psi_8\ \overline{\psi}_{23}\ \overline{\psi}_{29}$	$+ \psi_8\ H_1\ H_{13}$	$+ \psi_8\ H_{22}\ H_{24}$
$+ \psi_9\ \psi_{13}\ \psi_{33}$	$+ \psi_9\ \psi_{29}\ \overline{\psi}_{16}$	$+ \psi_9\ \overline{\psi}_{22}\ \overline{\psi}_{28}$	$+ \psi_9\ H_2\ H_{12}$	$+ \psi_9\ H_{19}\ H_{25}$
$+ \psi_{10}\ \psi_{14}\ \psi_{34}$	$+ \psi_{10}\ \psi_{30}\ \overline{\psi}_{15}$	$+ \psi_{10}\ \overline{\psi}_{21}\ \overline{\psi}_{27}$	$+ \psi_{10}\ H_1\ H_{12}$	$+ \psi_{10}\ H_{20}\ H_{26}$
$+ \psi_{11}\ \psi_{15}\ \psi_{35}$	$+ \psi_{11}\ \psi_{31}\ \overline{\psi}_{14}$	$+ \psi_{11}\ \overline{\psi}_{22}\ \overline{\psi}_{26}$	$+ \psi_{11}\ H_2\ H_{11}$	$+ \psi_{11}\ H_{19}\ H_{23}$
$+ \psi_{12}\ \psi_{16}\ \psi_{36}$	$+ \psi_{12}\ \psi_{32}\ \overline{\psi}_{13}$	$+ \psi_{12}\ \overline{\psi}_{21}\ \overline{\psi}_{25}$	$+ \psi_{12}\ H_1\ H_{11}$	$+ \psi_{12}\ H_{20}\ H_{24}$
$+ \psi_{13}\ \psi_{26}\ \overline{\psi}_4$	$+ \psi_{13}\ \overline{\psi}_{12}\ \overline{\psi}_{30}$	$+ \psi_{13}\ H_4\ H_{22}$	$+ \psi_{13}\ H_{13}\ H_{28}$	$+ \psi_{14}\ \psi_{25}\ \overline{\psi}_4$
$+ \psi_{14}\ \overline{\psi}_{11}\ \overline{\psi}_{29}$	$+ \psi_{14}\ H_4\ H_{21}$	$+ \psi_{14}\ H_{13}\ H_{27}$	$+ \psi_{15}\ \psi_{28}\ \overline{\psi}_3$	$+ \psi_{15}\ \overline{\psi}_{10}\ \overline{\psi}_{32}$
$+ \psi_{15}\ H_3\ H_{22}$	$+ \psi_{15}\ H_{14}\ H_{30}$	$+ \psi_{16}\ \psi_{27}\ \overline{\psi}_3$	$+ \psi_{16}\ \overline{\psi}_9\ \overline{\psi}_{31}$	$+ \psi_{16}\ H_3\ H_{21}$
$+ \psi_{16}\ H_{14}\ H_{29}$	$+ \psi_{17}\ \psi_{30}\ \overline{\psi}_2$	$+ \psi_{17}\ \overline{\psi}_8\ \overline{\psi}_{26}$	$+ \psi_{17}\ H_4\ H_{20}$	$+ \psi_{17}\ H_{11}\ H_{28}$
$+ \psi_{18}\ \psi_{29}\ \overline{\psi}_2$	$+ \psi_{18}\ \overline{\psi}_7\ \overline{\psi}_{25}$	$+ \psi_{18}\ H_4\ H_{19}$	$+ \psi_{18}\ H_{11}\ H_{27}$	$+ \psi_{19}\ \psi_{32}\ \overline{\psi}_1$
$+ \psi_{19}\ \overline{\psi}_6\ \overline{\psi}_{28}$	$+ \psi_{19}\ H_3\ H_{20}$	$+ \psi_{19}\ H_{12}\ H_{30}$	$+ \psi_{20}\ \psi_{31}\ \overline{\psi}_1$	$+ \psi_{20}\ \overline{\psi}_5\ \overline{\psi}_{27}$
$+ \psi_{20}\ H_3\ H_{19}$	$+ \psi_{20}\ H_{12}\ H_{29}$	$+ \psi_{21}\ \psi_{25}\ \overline{\psi}_{10}$	$+ \psi_{21}\ \psi_{27}\ \overline{\psi}_{12}$	$+ \psi_{21}\ H_{13}\ H_{26}$
$+ \psi_{21}\ H_{14}\ H_{24}$	$+ \psi_{22}\ \psi_{26}\ \overline{\psi}_9$	$+ \psi_{22}\ \psi_{28}\ \overline{\psi}_{11}$	$+ \psi_{22}\ H_{13}\ H_{25}$	$+ \psi_{22}\ H_{14}\ H_{23}$
$+ \psi_{23}\ \psi_{29}\ \overline{\psi}_6$	$+ \psi_{23}\ \psi_{31}\ \overline{\psi}_8$	$+ \psi_{23}\ H_{11}\ H_{26}$	$+ \psi_{23}\ H_{12}\ H_{24}$	$+ \psi_{24}\ \psi_{30}\ \overline{\psi}_5$
$+ \psi_{24}\ \psi_{32}\ \overline{\psi}_7$	$+ \psi_{24}\ H_{11}\ H_{25}$	$+ \psi_{24}\ H_{12}\ H_{23}$	$+ \psi_{25}\ H_6\ H_{30}$	$+ \psi_{25}\ H_8\ H_{20}$
$+ \psi_{26}\ H_6\ H_{29}$	$+ \psi_{26}\ H_8\ H_{19}$	$+ \psi_{27}\ H_6\ H_{28}$	$+ \psi_{27}\ H_7\ H_{20}$	$+ \psi_{28}\ H_6\ H_{27}$
$+ \psi_{28}\ H_7\ H_{19}$	$+ \psi_{29}\ H_5\ H_{30}$	$+ \psi_{29}\ H_{10}\ H_{22}$	$+ \psi_{30}\ H_5\ H_{29}$	$+ \psi_{30}\ H_{10}\ H_{21}$
$+ \psi_{31}\ H_5\ H_{28}$	$+ \psi_{31}\ H_9\ H_{22}$	$+ \psi_{32}\ H_5\ H_{27}$	$+ \psi_{32}\ H_9\ H_{21}$	$+ \psi_{33}\ H_8\ H_{18}$
$+ \psi_{33}\ H_{10}\ H_{16}$	$+ \psi_{34}\ H_8\ H_{17}$	$+ \psi_{34}\ H_{10}\ H_{15}$	$+ \psi_{35}\ H_7\ H_{18}$	$+ \psi_{35}\ H_9\ H_{16}$
$+ \psi_{36}\ H_7\ H_{17}$	$+ \psi_{36}\ H_9\ H_{15}$	$+ \overline{\psi}_1\ \overline{\psi}_{23}\ \overline{\psi}_{36}$	$+ \overline{\psi}_1\ \overline{\psi}_{24}\ \overline{\psi}_{35}$	$+ \overline{\psi}_1\ H_{15}\ H_{24}$
$+ \overline{\psi}_1\ H_{16}\ H_{23}$	$+ \overline{\psi}_2\ \overline{\psi}_{23}\ \overline{\psi}_{34}$	$+ \overline{\psi}_2\ \overline{\psi}_{24}\ \overline{\psi}_{33}$	$+ \overline{\psi}_2\ H_{15}\ H_{26}$	$+ \overline{\psi}_2\ H_{16}\ H_{25}$
$+ \overline{\psi}_3\ \overline{\psi}_{21}\ \overline{\psi}_{36}$	$+ \overline{\psi}_3\ \overline{\psi}_{22}\ \overline{\psi}_{35}$	$+ \overline{\psi}_3\ H_{17}\ H_{24}$	$+ \overline{\psi}_3\ H_{18}\ H_{23}$	$+ \overline{\psi}_4\ \overline{\psi}_{21}\ \overline{\psi}_{34}$
$+ \overline{\psi}_4\ \overline{\psi}_{22}\ \overline{\psi}_{33}$	$+ \overline{\psi}_4\ H_{17}\ H_{26}$	$+ \overline{\psi}_4\ H_{18}\ H_{25}$	$+ \overline{\psi}_5\ \overline{\psi}_{17}\ \overline{\psi}_{33}$	$+ \overline{\psi}_5\ H_1\ H_7$
$+ \overline{\psi}_5\ H_{16}\ H_{28}$	$+ \overline{\psi}_6\ \overline{\psi}_{18}\ \overline{\psi}_{34}$	$+ \overline{\psi}_6\ H_2\ H_7$	$+ \overline{\psi}_6\ H_{15}\ H_{27}$	$+ \overline{\psi}_7\ \overline{\psi}_{19}\ \overline{\psi}_{35}$

$E_6 \otimes U(1)^5 \otimes SO(22)$ Third Order Superpotential, Continued.

$$
\begin{array}{lllll}
+\,\overline{\psi}_7\ H_1\ H_8 & +\,\overline{\psi}_7\ H_{16}\ H_{30} & +\,\overline{\psi}_8\ \overline{\psi}_{20}\ \overline{\psi}_{36} & +\,\overline{\psi}_8\ H_2\ H_8 & +\,\overline{\psi}_8\ H_{15}\ H_{29} \\
+\,\overline{\psi}_9\ \overline{\psi}_{13}\ \overline{\psi}_{33} & +\,\overline{\psi}_9\ H_1\ H_9 & +\,\overline{\psi}_9\ H_{18}\ H_{28} & +\,\overline{\psi}_{10}\ \overline{\psi}_{14}\ \overline{\psi}_{34} & +\,\overline{\psi}_{10}\ H_2\ H_9 \\
+\,\overline{\psi}_{10}\ H_{17}\ H_{27} & +\,\overline{\psi}_{11}\ \overline{\psi}_{15}\ \overline{\psi}_{35} & +\,\overline{\psi}_{11}\ H_1\ H_{10} & +\,\overline{\psi}_{11}\ H_{18}\ H_{30} & +\,\overline{\psi}_{12}\ \overline{\psi}_{16}\ \overline{\psi}_{36} \\
+\,\overline{\psi}_{12}\ H_2\ H_{10} & +\,\overline{\psi}_{12}\ H_{17}\ H_{29} & +\,\overline{\psi}_{13}\ H_3\ H_{15} & +\,\overline{\psi}_{13}\ H_8\ H_{25} & +\,\overline{\psi}_{14}\ H_3\ H_{16} \\
+\,\overline{\psi}_{14}\ H_8\ H_{26} & +\,\overline{\psi}_{15}\ H_4\ H_{15} & +\,\overline{\psi}_{15}\ H_7\ H_{23} & +\,\overline{\psi}_{16}\ H_4\ H_{16} & +\,\overline{\psi}_{16}\ H_7\ H_{24} \\
+\,\overline{\psi}_{17}\ H_3\ H_{17} & +\,\overline{\psi}_{17}\ H_{10}\ H_{25} & +\,\overline{\psi}_{18}\ H_3\ H_{18} & +\,\overline{\psi}_{18}\ H_{10}\ H_{26} & +\,\overline{\psi}_{19}\ H_4\ H_{17} \\
+\,\overline{\psi}_{19}\ H_9\ H_{23} & +\,\overline{\psi}_{20}\ H_4\ H_{18} & +\,\overline{\psi}_{20}\ H_9\ H_{24} & +\,\overline{\psi}_{21}\ H_7\ H_{29} & +\,\overline{\psi}_{21}\ H_8\ H_{27} \\
+\,\overline{\psi}_{22}\ H_7\ H_{30} & +\,\overline{\psi}_{22}\ H_8\ H_{28} & +\,\overline{\psi}_{23}\ H_9\ H_{29} & +\,\overline{\psi}_{23}\ H_{10}\ H_{27} & +\,\overline{\psi}_{24}\ H_9\ H_{30} \\
+\,\overline{\psi}_{24}\ H_{10}\ H_{28} & +\,\overline{\psi}_{25}\ H_5\ H_{25} & +\,\overline{\psi}_{25}\ H_{14}\ H_{17} & +\,\overline{\psi}_{26}\ H_5\ H_{26} & +\,\overline{\psi}_{26}\ H_{14}\ H_{18} \\
+\,\overline{\psi}_{27}\ H_5\ H_{23} & +\,\overline{\psi}_{27}\ H_{13}\ H_{17} & +\,\overline{\psi}_{28}\ H_5\ H_{24} & +\,\overline{\psi}_{28}\ H_{13}\ H_{18} & +\,\overline{\psi}_{29}\ H_6\ H_{25} \\
+\,\overline{\psi}_{29}\ H_{12}\ H_{15} & +\,\overline{\psi}_{30}\ H_6\ H_{26} & +\,\overline{\psi}_{30}\ H_{12}\ H_{16} & +\,\overline{\psi}_{31}\ H_6\ H_{23} & +\,\overline{\psi}_{31}\ H_{11}\ H_{15} \\
+\,\overline{\psi}_{32}\ H_6\ H_{24} & +\,\overline{\psi}_{32}\ H_{11}\ H_{16} & +\,\overline{\psi}_{33}\ H_{11}\ H_{21} & +\,\overline{\psi}_{33}\ H_{13}\ H_{19} & +\,\overline{\psi}_{34}\ H_{11}\ H_{22} \\
+\,\overline{\psi}_{34}\ H_{13}\ H_{20} & +\,\overline{\psi}_{35}\ H_{12}\ H_{21} & +\,\overline{\psi}_{35}\ H_{14}\ H_{19} & +\,\overline{\psi}_{36}\ H_{12}\ H_{22} & +\,\overline{\psi}_{36}\ H_{14}\ H_{20}
\end{array}
$$

BIBLIOGRAPHY

[1] P. W. Higgs, Phys. Lett. **12**, 132 (1964).

[2] C. Kounnas, A. Masiero, D. Nanopoulos, and K. A. Olive, *Grand Unification With and Without Supersymmetry and Cosmological Implications*, World Scientific, Singapore, Singapore, 1984.

[3] P. Langacker, Phys. Rept. **72**, 185 (1981).

[4] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[5] H. Georgi, AIP Conf. Proc. **23**, 575 (1975).

[6] G. Veneziano, Nuovo. Cim. **A57**, 190 (1968).

[7] M. A. Virasoro, Phys. Rev. Lett. **22**, 37 (1969).

[8] A. Sen, arXiV:hep-th/9609176 **58**, 5 (1997).

[9] C. M. Hull and P. K. Townsend, arXiV:hep-th/9505073, Nucl. Phys. **B451**, 525 (1995).

[10] J. Scherk and J. H. Schwarz, Phys. Lett. **B82**, 60 (1979).

[11] E. Cremmer and J. Scherk, Phys. Lett. **B72**, 117 (1974).

[12] S.-T. Yau, Proc. Nat. Acad. Sci. **74**, 1798 (1977).

[13] A. M. Polyakov, Phys. Lett. **B103**, 207 (1981).

[14] A. M. Polyakov, Phys. Lett. **B103**, 211 (1981).

[15] M. B. Green, J. H. Schwarz, and E. Witten, *Cambridge Monographs on Mathematical Physics*, Univ. Pr., Cambridge, UK, 1987.

[16] L. Brink and H. B. Nielsen, Phys. Lett. **B45**, 332 (1973).

[17] A. Neveu and J. H. Schwarz, Nucl. Phys. **B31**, 86 (1971).

[18] P. Ramond, Phys. Rev. **D3**, 2415 (1971).

[19] F. Gliozzi, J. Scherk, and D. I. Olive, Phys. Lett. **B65**, 282 (1976).

[20] M. B. Green, J. H. Schwarz, and E. Witten, *Cambridge Monographs on Mathematical Physics*, Univ. Pr., Cambridge, UK, 1987.

[21] F. Gliozzi, J. Scherk, and D. I. Olive, Nucl. Phys. **B122**, 253 (1977).

[22] E. Kiritsis, Univ. Pr., Princeton, USA, 2007.

[23] E. Kiritsis, arXiV:hep-th/9709062 (1997).

[24] S. E. M. Nooij, arXiV:hep-th/0603035 .

[25] M. B. Green and J. H. Schwarz, Phys. Lett. **B149**, 117 (1984).

[26] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985).

[27] J. E. Paton and H.-M. Chan, Nucl. Phys. **B10**, 516 (1969).

[28] P. H. Ginsparg, Phys. Rev. **D35**, 648 (1987).

[29] F. Quevedo, arXiV:hep-th/9603074 (1996).

[30] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, Phys. Lett. **B194**, 231 (1987).

[31] I. Antoniadis and C. Bachas, Nucl. Phys. **B298**, 586 (1988).

[32] I. Antoniadis, C. Bachas, C. Kounnas, and P. Windey, Phys. Lett. **B171**, 51 (1986).

[33] I. Antoniadis, C. P. Bachas, and C. Kounnas, Nucl. Phys. **B298**, 87 (1987).

[34] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Nucl. Phys. **B288**, 1 (1987).

[35] H. Kawai, D. C. Lewellen, J. A. Schwartz, and S. H. H. Tye, Nucl. Phys. **B299**, 431 (1988).

[36] G. B. Cleaver, A. E. Faraggi, and D. Nanopoulos, arXiV:hep-ph/9811427, Phys. Lett. **B455**, 135 (1999).

[37] G. B. Cleaver, A. E. Faraggi, D. Nanopoulos, and J. W. Walker, arXiV:hep-ph/0104091, Nucl. Phys. **B620**, 259 (2002).

[38] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, Phys. Lett. **B231**, 65 (1989).

[39] J. L. Lopez, D. Nanopoulos, and K. Yuan, Nucl. Phys. **B399**, 3 (1993).

[40] A. E. Faraggi, D. Nanopoulos, and K. Yuan, Nucl. Phys. **B335**, 347 (1990).

[41] A. E. Faraggi, Phys. Rev. **D46**, 3204 (1992).

[42] A. E. Faraggi, Phys. Lett. **B278**, 131 (1992).

[43] A. E. Faraggi, Phys. Lett. **B274**, 47 (1992).

[44] G. B. Cleaver, arXiV:hep-ph/9901203 .

[45] G. B. Cleaver, A. E. Faraggi, D. Nanopoulos, and J. W. Walker, arXiV:hep-ph/0002060, Mod. Phys. Lett. **A15**, 1191 (2000).

[46] A. E. Faraggi, E. Manno, and C. Timirgaziu, Eur. Phys. Jour. **C50**, 701 (2007).

[47] G. B. Cleaver, Nucl. Phys. **B456**, 219 (1995).

[48] E. Manno, arXiV:hep-th/09083164 .

[49] I. Antoniadis, G. K. Leontaris, and J. Rizos, Phys. Lett. **B245**, 161 (1990).

[50] G. B. Cleaver, A. E. Faraggi, and C. Savage, Phys. Rev. **D63** (2001).

[51] A. E. Faraggi, arXiV:hep-ph/9707311, Int. J. Mod. Phys. **A19**, 5523 (2004).

[52] G. B. Cleaver, A. E. Faraggi, and D. Nanopoulos, arXiV:hep-ph/9904301, Int. J. Mod. Phys. **A16**, 425 (2001).

[53] H. K. Dreiner, J. L. Lopez, D. Nanopoulos, and D. B. Reiss, Nucl. Phys. **B320**, 401 (1989).

[54] L. Alvarez-Gaume, G. W. Moore, and C. Vafa, Commun. Math. Phys. **106**, 1 (1986).

[55] N. Seiberg and E. Witten, Nucl. Phys. **B276**, 272 (1986).

[56] A. E. Faraggi and D. Nanopoulos, Phys. Rev. **D48**, 3288 (1993).

[57] A. E. Faraggi, arXiV:hep-th/9208024, Nucl. Phys. **B387**, 239 (1992).

[58] S. Ferrara, L. Girardello, C. Kounnas, and M. Porrati, Phys. Lett. **B194**, 358 (1987).

[59] J. R. Ellis, A. E. Faraggi, and D. Nanopoulos, arXiV:hep-th/9709049, Phys. Lett. **B419**, 123 (1998).

[60] G. B. Cleaver, D. Nanopoulos, and J. T. Perkins, Int. J. Mod. Phys. **A23**, 3461 (2008).

[61] A. E. Faraggi, arXiV:hep-ph/9506388, Phys. Lett. **B377**, 43 (1996).

[62] A. E. Faraggi, Nucl. Phys. **B407**, 57 (1993).

[63] G. B. Cleaver, A. E. Faraggi, D. Nanopoulos, and J. W. Walker, arXiV:hep-ph/9910230, Nucl. Phys. **B593**, 471 (2001).

[64] G. B. Cleaver and A. E. Faraggi, arXiV:hep-th/9505073, Int. J. Mod. Phys. **A14**, 2335 (1999).

[65] M. Dine, N. Seiberg, and E. Witten, Nucl. Phys. **B289**, 585 (1987).

[66] A. Font, L. E. Ibanez, H. P. Nilles, and F. Quevedo, Nucl. Phys. **B307**, 109 (1988).

[67] G. B. Cleaver, M. Cvetic, J. Espinosa, L. Everett, and P. Langacker, Nucl. Phys. **B525**, 3 (1998).

[68] G. B. Cleaver, Mass hierarchy and flat directions in string models, page 101, 1997.

[69] E. Witten, Nucl. Phys. **B202**, 253 (1982).

[70] K. Intriligator and N. Seiberg, arXiV:hep-ph/0702069 .

[71] G. Dvali, R. Kallosh, and A. V. Proeyen, arXiV:hep-th/0312005, J. High Energy Phys. **01** (2004).

[72] G. B. Cleaver, A. E. Faraggi, E. Manno, and C. Timirgaziu, arXiv:hep-th/08020470, Phys. Rev. **D78**, 1103 (2008).

[73] G. B. Cleaver et al., arXiV:hep-ph/9807479, Phys. Rev. **D59** (1999).

[74] J. Greenwald, K. Pechan, T. Renner, M. Robinson, and G. Cleaver, Systematic Phenomenological Study of NAHE-Based Free Fermionic Heterotic Models, in preparation .

[75] G. B. Cleaver et al., Investigation of Low Higgs Models, in preparation .

[76] W. Buchmuller, K. Hamaguchi, O. Lebedev, and M. Ratz, arXiV:hep-th/0606187 .

[77] G. B. Cleaver, arXiV:hep-ph/0703027 .

[78] e. a. Press, Numerical recipes, 1989.

[79] S. Kalara, J. L. Lopez, and D. Nanopoulos, Nucl. Phys. **B353**, 650 (1991).

[80] J. Rizos and K. Tamvakis, Phys. Lett. **B262**, 227 (1991).

[81] G. B. Cleaver, J. R. Ellis, and D. Nanopoulos, arXiV:hep-ph/0009338, Nucl. Phys. **B600**, 315 (2001).

[82] G. B. Cleaver, A. E. Faraggi, and S. Nooij, arXiV:hep-ph/0301037, Nucl. Phys. **B672**, 64 (2003).

[83] A. E. Faraggi, Nucl. Phys. **B428**, 111 (1994).

[84] J. Perkins et al., arXiV:hep-ph/0310155 .

[85] G. B. Cleaver, Phenomenological survey of a minimal superstring standard model, in *7th International Symposium on Particles, Strings and Cosmology (PASCOS 99)*, page 108, 1999.

[86] G. B. Cleaver, D. Nanopoulos, A. E. Faraggi, and T. Veldhuis, arXiV:hep-ph/0002292, Int. J. Mod. Phys. **A16**, 3565 (2001).

[87] A. E. Faraggi and E. Manno, arXiV:hep-ph/09082034 .

[88] R. Donagi and K. Wendland, arXiV:hep-th/08090330 .

[89] O. Lebedev, H. P. Nilles, S. Ramos-Sanchez, M. Ratz, and P. K. S. Vaudrevange, arXiV:hep-th/0807438 .

[90] R. Valandro, arXiV:hep-th/08010584 .

[91] M. Robinson, G. B. Cleaver, and M. Hunziker, arXiV:hep-th/08095094, Mod. Phys. Lett. **A24**, 2703 (2009).

[92] B. Dundee, J. Perkins, and G. B. Cleaver, arXiV:hep-ph/0506183, Int. J. Mod. Phys. **A21**, 3367 (2006).

[93] J. Greenwald, D. Moore, K. Pechan, T. Renner, and G. Cleaver, Mirror Models from a NAHE Variation, in preparation .