ABSTRACT

An Investigation on the Impact of Modified Mathematics Course Sequence
on Student's Retest Scores in Algebra I End-of-Course Exam

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Mathematics course sequence in the high school curriculum has been a source of continued debate in the United States due to the high failure rates in high school math courses and high failure rates in state-developed end-of-course exams (EOC). Identifying effective sequences of mathematics courses has the potential to offer high school students opportunities for advancing in mathematics as well as performing on increasingly rigorous exams like EOC. This researcher analyzed evidence that some mathematics course sequences may be more effective than traditional sequences.

In this study, the researcher investigated whether a group of high school students following a modified mathematics course sequence performed better on the state-administered EOC than a similar group following a traditional course sequence. Determination of a statistically significant difference was found through an analysis comparing the results of EOC scores of the two groups of students following different mathematics sequence.

The results indicated a significant difference in performance between the two groups of students. Specifically, the students following a modified mathematics course

sequence outscored students following a traditional mathematics sequence. A medium effect size for this study suggested the findings are relevant and of interest to public school practitioners and educational researchers.

An Investigation on the Impact of Modified Mathematics Course Sequence on Student's Retest Scores in Algebra I End-of-Course Exam

by

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A Dissertation

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DEDICATION

I dedicate this dissertation to

my loving wife, Kara

and my two sons, Hunter and Hayden

Their support through this process has been constant

and greatly appreciated

I also appreciate my mother and father for always believing in me

and helping me throughout my journey

CHAPTER ONE

Introduction

Numeracy has become an imperative prerequisite skill for aspiring students in a digital age. Educating students to become more math proficient has been a focused effort for many educators and education sytem leaders. Mathematics achievement became the forefront of American education after the Soviet launch of Sputnik in 1957 (Kruglak, 1970), and again reached critical awareness with the reports, A Nation at Risk (National Commission on Excellence in Education, 1983) and No Child Left Behind (NCLB) Act of 2001 (2002). As part of the focus on mathematics, Algebra I has been widely viewed as integral in advancing student achievement in mathematics (Achieve, 2009; Bitter & O'Day, 2010; Filer & Chang, 2008; Shoenfeld, 2004, Spielhagen, 2006; Zelkowski, 2010).

Policymakers have based their decision to offer Algebra I earlier in the middle school years to students based on numerous research studies that have shown students who take Algebra I in middle school have greater opportunities to enroll in advanced mathematics courses in high school (Bitter & O'Day, 2010; Ed Source, 2009; Loveless, 2009). The state of Texas has mandated all students take 4 years of mathematics, including Algebra I, Geometry, and Algebra II, to fulfill mathematics graduation requirements (Texas Education Agency, 2011). Additionally, there are aggressive efforts among administrators to encourage more and more students to take Calculus before graduating high school. This effort is predicated on studies that show greater success in college mathematics among those who take Calculus in high school and those students have greater opportunities than students who do not take high school Calculus (Schiller & Hunt, 2011).

The different areas of mathematics vary, but they are all important. Recently, much focus has been shifted towards the first course in Algebra, and requiring students to have some level of proficiency prior to graduation (U.S. Department of Education, 2008). This is especially true in the accountability climate of today where states use end-of-course (EOC) assessments in the context of the state evaluation system. These new state EOC assessments measure new student performance standards and scoring assessment systems. It was in 2012 that the state of Texas introduced a new performance accountability system assessing students on readiness and supporting college-ready standards. Furthermore, Texas is in a testing implementation transition that began in 2007 with Senate Bill 1031 (Texas Classroom Teachers Association, 2011). According to Texas Senate Bill 1031, the purpose of the EOC assessments is to measure students' academic performance in core high school courses and make them part of a student's graduation requirement and final course grade starting with the freshmen class of 2012. These new EOC assessments must include a series of special purpose questions that measure college readiness. This law required high school students in Texas to take EOC tests in order to graduate. The required EOC tests were World Geography, World History, U.S. History, Biology, Chemistry, Physics, Algebra I, Geometry, Algebra II, English I Reading, English I Writing, English II Reading, English II Writing, English III Reading, and English III Writing. When this bill was introduced, students who failed a single test could fail to graduate if that student did not retest and pass that EOC test. With the introduction of Texas House Bill 5, a default graduation program for all students entering ninth grade beginning with the 2014-2015 school year, the number of tests was reduced from 15 EOC tests to five EOC tests (Texas Education Agency, 2013a). The courses are English I, English II, Algebra I, Biology, and U.S. History. Therefore, it is critical for Texas high school students to be successful in Algebra I.

Yet, historically, failure rates in Algebra I have been among the highest for all courses, without acquiring the necessary skills to be considered college ready, is an urgent issue that has critical ramifications on a student's academic and economic future. Understanding Algebra I is a gateway toward higher levels of mathematic courses as well as the gatekeeper to college and career success for all students. For mathematics courses, the majority of Texas high schools have their students go through the mathematics sequence of Algebra I, Geometry, and Algebra II (Texas Education Agency, 2013a). Yet, this order of mathematics courses in the high school curriculum has been debated. Some say Algebra II should follow Algebra I. With the vast amount of data generated by the EOC testing program, it is feasible to study the question of course sequence for Texas schools. EOC testing may provide insight into the best order of mathematics courses.

Background of the Problem

The National Mathematics Advisory Panel stated algebraic proficiency is a gateway to later achievement, and that there is a strong correlation between the completion of Algebra I and Algebra II and the likelihood of college success and college graduation (U.S. Department of Education, 2008). Students who complete up to Algebra II are also more likely to have higher college GPAs, graduate from college, and attain higher earnings in later life (Gaertner, Kim, DesJardins, & McClarty, 2014).

Algebra proficiency is a requirement for employment in growing STEM fields like engineering, medicine, science, programming, and technology (Zeldin, Britner, & Pajares, 2008). There is a call from these and other professional communities for more students who are prepared to enter a career in the Science, Technology, Engineering and Mathematics (STEM) field (National Council of Teachers of Mathematics, 2014). For many students, the choice to pursue the types of college programs leading to STEM careers happens as early as middle school, and those choices are often a reflection of a student's learning experiences with science and math content, and not on their ability alone (Crisp, Nora, & Taggart, 2009; Hazari, Tai, & Sadler, 2007; Wai, Cacchio, Putallaz, & Makel, 2010).

Algebra proficiency is an expectation at the high school level as well. At the present time a number of states require students to take and pass at least one algebra course while in high school (O'Hara, 2012), making algebra a gateway to not just college but to high school graduation (Stein, Kaufman, Sherman, & Hillen, 2011). To meet this demand in Texas, mathematics student progress is measured through the State of Texas Assessments of Academic Readiness (STAAR; Texas Education Agency, 2012b). Moreover, STAAR has been designed to improve college and career readiness for the state's graduating high school students with the intention of making Texas students more competitive with other students both nationally and internationally. This assessment is said to measure the achievement of the goals set forth in the state-mandated curriculum guide known as the Texas Essential Knowledge and Skills (TEKS). The federal requirement established under NCLB (2002) required all school districts, campuses, and states to be accountable for student performance for mathematics for Grades 3-11.

The school district and the high school included in the current study did not meet the minimum standard for the percentage of students passing a mathematics exam in either 2011 or 2012. In addition, in 2012, TEA introduced a new testing accountability system assessing students on readiness and supporting college-ready standards. These new state EOC assessments placed greater emphasis on high school students performing at higher levels on more rigorous EOC assessments and introduced new student performance standards and scoring assessment systems. In 2011, 18% of the district's high school students did not score proficient on the Texas Assessment of Knowledge and Skills (TAKS) math exam (Texas Education Agency, 2012b). Texas schools whose students or subgroups of students fail to meet adequate yearly progress mandates 2 years in a row must respond with detailed improvement plans, or they risk losing federal funds and must develop a School Improvement Plan (SIP). If districts' or schools' SIPs do not result in improved student performance, they eventually risk either being taken over by their state educational agencies or becoming charter schools. All Texas schools must have 100% of their students pass math state exams or risk sanctions, such as losing Title I funds and becoming labeled as underachieving districts or schools (U.S. Department of Education, 2012).

Statement of the Research Problem

Since 1990, the year the Texas legislature first adopted a common curriculum framework for all schools in the state, school districts in Texas have aligned their course sequence of math classes to have Geometry follow Algebra I because of the value Geometry added to student knowledge as they prepared to take the TAKS test (Texas Education Agency, 2004). The TAKS test did not assess students on Algebra II content;

therefore, school districts only found it necessary to have students fully complete Algebra I and Geometry prior to the TAKS test. If students had not completed Algebra II prior to the TAKS test administration, students would not be at a disadvantage on the state assessment.

Schools have always had mathematics courses offered as one of the core courses required for students. The Common Core State Standards for Mathematics (CCSS) document, although not utilized in Texas, shows the traditional pathway that is typical in the United States in which students took Algebra I, Geometry, and Algebra II (Eddy et al., 2015). According to Eddy et al., performance differences are minimal with students who have taken Algebra I and Geometry but there are significant differences with students who have taken rigorous academic courses, such as Algebra II (Milewski & Sawtell, 2006). Further, Deng and Kobrin (2007) point to the fact that Algebra II has the greatest impact on SAT performance than either Algebra I or Geometry. Thus, the sequence of mathematics courses has the potential to offer high schools' students evidence that the mathematics course sequencing at the high school level should be more effective for passing the Algebra I EOC. The current study is a good first step to offer analytical evidence as to which course should be taken after Algebra I.

Given the nature of the Algebra I EOC test, the Texas public educational stakeholders have sought to redefine accountability of students and professionals, best practices, and learning environments. Political mandates, legislature, and high-stakes testing have established an educational environment in which teachers are subjected to accountability requirements and standards-based curriculum. Offering quality education to students remains the key goal of teachers, community members, administrators, and legislators. Nevertheless, reliance on a particular academic indicator often obliged educators to focus instruction on students who are, in most cases, not proficient in meeting a minimum pre-established proficiency standard. Consequently, many students are not getting the mathematics education they deserve. Hence, the current researcher investigated if re-sequencing of high school mathematics courses could be linked to an increased score to pass the Algebra I EOC retest. If so, then the study would suggest a re-sequencing of mathematics courses would benefit students acquiring a greater sense of numeracy prior to college admission.

Purpose of the Study

The purpose of this quantitative study was to investigate the impact of a modified mathematics course sequence to the students Algebra I EOC retest scores. Geometry is taken during the third year of a student's math course sequence. Students continue to have a second math course they are taking during their second year, which is math models. The plan of this study was to determine if there was a difference between the performance levels on the Algebra I EOC relative to groups taking different course sequences of mathematics.

If the findings of the study show a significant difference between two groups of students retest scores then educators will be encouraged to consider modifying the course sequence in mathematics for all students. Conversely, if the analysis shows there is no difference between group performances, then the modified sequence may not significantly influence the EOC exam as a standardized assessment.

Research Question

This study was anchored in the main research question:

Is there a statistically significant difference in test performance due to students taking different sequences of mathematics courses?

Significance of the Study

Many educators view Algebra I as the gateway course to college and as foundational to college preparatory coursework (Bitter & O'Day, 2010; Bracey, 2008; Filer & Chang, 2008). The traditional course sequence for students in high school mathematics, ignoring remedial mathematics and differentiated levels of courses, is: Algebra I, Geometry, Algebra II, Pre-Calculus, and Calculus (Schiller & Hunt, 2011). Because Algebra I students have an opportunity to take Algebra II in the next grade, any mathematics courses taken beyond Algebra II in 11th and 12th grades greatly improve students' likelihood of graduating from college (Adelman, 1999). Studies show that students who complete ninth grade Algebra I tend to take more accelerated mathematics courses in high school, such as Algebra II, Pre-Calculus, and Advanced Placement (AP) Calculus (Loveless, 2009; Ma, 2000). These students are also more likely to enroll in high school Calculus, apply to a 4-year college, and ultimately become better prepared for college than their peers (Bressoud, 2004; Schiller & Hunt, 2011).

Thus, the grade level at which students enroll in high school mathematics courses depends on which grade level students first take Algebra I. If students take Algebra I in seventh or eighth grade, then they will be able to take Pre-Calculus by 10th in a high advanced sequence or 11th grade in an advanced sequence. The typical sequence of courses for most students begins Algebra I in ninth grade, with an opportunity to enroll in Pre-Calculus by 12th grade as shown in Table 1.

Table 1

High School Mathematics Course Sequences

	Course Sequence		
Grade Level	High advanced	Advanced	Traditional
07	Algebra I		
08	Geometry	Algebra I	
09	Algebra II	Geometry	Algebra I
10	Pre-Calculus	Algebra II	Geometry
11	AP Calculus	Pre-Calculus	Algebra II
12	AP Calculus BC	AP Calculus	Pre-Calculus

Note. Algebra I and Geometry in seventh and eighth grade are typically Pre-AP (advanced), but Algebra I, Geometry, Algebra II, and Pre-Calculus in high school can be taken as regular or Pre-AP.

In Texas, graduation requirements are set by the Texas Education Agency. Students may fulfill partial graduation course requirements prior to ninth grade. While the minimum number of credits required for graduation is determined by the Texas Education Agency, the number of credits required to be taken while in ninth through 12th grades is determined by individual school districts. The additional courses students may take to complete the third or fourth credit include: Mathematics Models and Applications (MMA), if completed before Algebra II; Advanced Placement (AP) Statistics, Advanced Mathematics Decision Making (AMDM) renamed in fall 2011 to Advanced Quantitative Reasoning (AQR); Pre-Calculus; and AP Calculus.

The current study results have the potential to offer public high schools throughout the United States and especially in the State of Texas evidence that might reveal the modified sequence as an intervention on how the mathematics alignment at the high school level should be, such as Algebra I, Algebra II, and then Geometry. In today's high stakes testing environment in public schools, any strategy that schools could build into their school day that would provide students a more effective learning opportunity would be beneficial. Currently, a majority of the schools are aligned with Geometry taken after Algebra I but before Algebra II. With little significant research addressing the course sequence of high school math courses, this study could be a good first step to offer concrete evidence as to which course should be taken after Algebra I. The current study is organized to focus solely on the sequence of mathematics classes and will not examine any outside factors that could play a part in the student's life that might cause the student to be successful or not successful with the Algebra I EOC.

Definition of Terms

Academic Excellence Indicator System (AEIS)−AEIS puts together a broad range of information on the performance of each student in each school and district in Texas every year (Texas Education Agency, 2015).

Adaptive reasoning−Adaptive reasoning is the "capacity for logical thought, reflection, explanation, and justification" (Kilpatrick, Swafford, & Findell, 2001, p. 5).

Algebra I End-of-Course (EOC)−The Algebra I EOC is a state-wide assessment that is used to determine the level of mastery of course content. The Algebra I EOC assesses a student's knowledge on: functional relationships, properties and attributes of functions, linear functions, Linear equations and inequalities, and quadratics and other nonlinear functions.

Common Core State Standards for Mathematics−The knowledge and skills students require to be ready for mathematics in college, career, and life through the mathematics standards (Kilpatrick et al., 2001).

Common Core State Standards−A set of objectives that students are required to learn to be able to do partitioned by grade levels, content strands and subject area (Kilpatrick et al., 2001).

Complexity Level−The level of thinking necessary to involve with and answer an item correctly: moderate complexity, low complexity, and high complexity.

Conceptual Understanding−The "comprehension of mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p. 5).

Content Strand−A group of interrelated concepts (e.g., number sense, number, and operations).

End-of-course (EOC) Assessments−In the spring of 2012, the EOC assessments replaced the TAKS for the high school courses of English I, English II, Algebra I, Biology, and U.S. History (Texas Education Agency, 2015).

Graduation−The Mandatory Assessment for Diploma: One of Texas comprehensive high school exit exams. As the name suggests, it is compulsory to pass this test to be eligible for graduation.

High School Advanced Mathematics−high school advanced mathematics would include pre-calculus, calculus, AP mathematics, and dual credit mathematics (Zelkowski, 2011).

High School Lower Level Mathematics−The three basic mathematics courses—Algebra I, Geometry, Algebra II—are required to graduate high school (Zelkowski, 2011). Lower level mathematics choices would be math models and advanced quantitative reasoning.

Mathematical Proficiency−Mathematical proficiency necessitates a conceptual understanding, a productive disposition, strategic competence, procedural fluency, and adaptive reasoning across a broad range of popular mathematical concepts and performances (Kilpatrick et al., 2001).

Procedural Fluency−The “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately (Kilpatrick et al., 2001, p. 5).

Retesting−A scenario in an educational setting in which a student is given an opportunity to retake any summative assessment.

State of Texas Assessments of Academic Readiness (STAAR)−In the spring of 2012, the STAAR replaced the Texas Assessment of Knowledge and Skills (TAKS). The STAAR program includes annual assessments for (a) reading and mathematics in Grades 3 through 8, (b) writing at Grades 4 and 7, (c) science in Grades 5 and 8, and (d) social studies in Grade 8. The program of assessments is tied to the TEKS curriculum standards (Texas Education Agency, 2018a).

Summative Assessment−Also known as the end of course, is an evaluation exam that is given to students at the end of the learning process, such as after the learning target has been reached or at the end of a unit of study. Summative tests can include written, performance, forced response, oral, and project-based assessments.

Texas Assessment of Knowledge and Skills (TAKS)−These are referenced evaluations, which assess student academic performance in Texas in Grade 3 all the way to Grade 11. Students are examined in two or more of the following subjects each year: mathematics, science, English language arts, and social studies (Texas Education Agency, 2018b).

Texas Essential Knowledge and Skills (TEKS): The authorized K-12 curriculum in the state of Texas which contains the details of the requirements for every course. State required standardized tests focus on testing if students have acquired the specific skills outcomes and knowledge in this curriculum (Texas Education Agency, 2018c).

Assumptions

According to Leedy and Ormrod (2013), assumptions are so basic that, without them, the project itself could not exist. A major assumption for this study was that all the data collected from the Texas Education Agency is valid and reliable, including the complexity level and content strand addressed by items. As EOC exams are kept secure, it is not feasible to acquire copies of administered exams for content analysis. Another key assumption for this study was that each of the exit exams is a valid and reliable measure of students' mathematical achievement. The Texas Algebra I EOC has been psychometrically determined to be a valid measure (Texas Education Agency, 2013a).

Limitations

The limitations of this study included: Data is based on students who did not pass the EOC Algebra I test during 2012 or the first administration of that test for the student.

Not all students who took the Algebra I EOC test will remain a student of the school district so only those who remained in the district were participants of the current study.

Delimitations

This study was confined to one district in Texas that implemented the Algebra I EOC for over 3 years. The study's target population was focused specifically on the students that did not meet the minimum standard for passing on previously administered Algebra I EOC tests in the last 3 years. Consequently, this study was further delimited to students' academic performance in this subject.

All students that failed the Algebra I EOC test are required to take two math courses during the 2012-2013 school years with one of the courses being math models and the other course being Geometry. Students were placed in the math models class with the expectations that many of the math models concepts would help support the student with the Algebra I EOC retest. The Geometry class would help the students as they prepare for the Geometry EOC test in the spring. The Geometry curriculum does not reinforce areas from the Algebra I EOC on which students were not successful. The fact that the math models course and Geometry class did not help the students on the Algebra I EOC retest was evident in the fact that only 20% of the students were able to pass the Algebra I EOC. Hence, an altered sequence of Mathematics courses: Algebra I, Algebra II, and Geometry respectively, is a possible help for a student to pass the Algebra I EOC Test and thus, the school`s passing rate for the Algebra I EOC would increase.

This study's plan focused on students who took and failed the Algebra I EOC test during the May 2012, May 2013, and May 2014 administrations. Additionally, data for each subsequent testing administration of the Algebra I EOC was collected. The students who were tested with the Algebra I EOC in May 2012 were enrolled in Geometry in the fall. The students who were tested with the Algebra I EOC in May 2013 and May 2014 were enrolled in Algebra II in the fall. The goal of this study was to determine whether gains observed on the Algebra I EOC retest scale scores are due to course sequence other than the traditional sequence.

Summary

This chapter provided an introduction of mathematics courses offered by the schools in Texas, the concept of mathematics course sequence, and established the need for further research on educational reforms linked to student test performance. A brief history of standards-based education is also presented which offers the necessary background to understanding the context for the study. Chapter Two is an outline of a body of the literature which is pertinent to the current study, as well sets a base for mathematics course sequence effects to retest examinations. Chapter Three includes a description of the participants involved in the current study and the methodology to be employed to guide the research. Chapter Four includes a presentation of the statistical results of the measures used and Chapter Five is a summary of those findings with interpretations and implications for future studies.

CHAPTER TWO

Literature Review

There is a prolific history of research on the effect course sequence has on student learning that continues through the present day. Additionally, there is a growing number of research articles that are devoted toward the topics of educational reform and educational testing in the current era of accountability. The goal of this literature review is to summarize the history of the study of course effect on student learning, support the notion of a trend in research to study attempts at educational reform, and provide a summary of the evolving landscape of high-stakes testing in the present era of accountability.

The review leads the reader down a path that gives insight into the curricular decisions and issues confronting public school officials as school administrators work on improving the performances of students on legislatively adopted measurement standards and instruments. This chapter is organized into five sections that represent the confluence of concepts that underlie this study. The first section concentrates on the literature related to the accepted notion that course sequence represents a curriculum offering that has important effects on the quality of student learning, in particular on algebraic understanding. The second section illustrates the growing interest in curriculum reform in mathematics education. The third section is an examination of the literature on the history of testing and accountability in Texas, which includes graduation requirements for students and House Bill 5 passed in Texas in January 2014. The fourth section is a discussion of the practical challenges facing decision-makers who are charged with alignment of mathematics courses. The final section is a summarization of the literature review's main concepts and how they form the basis for the context of this study's purpose and findings.

The Significance of Curriculum on Student Learning

There is a large body of evidence within research literature that points to course sequence as a significant contributor to student learning evidenced by achievement (Oakes, 2012). Since U.S. schools typically follow a common, traditional path of courses from middle school through high school, it is understandable why researchers focus on the effect of course of curriculum on student learning. Katz (2007) supports this reasoning when drawing the conclusion that curriculum design has an important impact on student attitude and effort toward learning the content. When curriculum is designed to bring forth prior knowledge, challenge misconceptions, and push for deeper understanding of new content, students become engaged and efforts are increased. This increased effort will, naturally, lead students to think more critically about the subject and how it may impact their life and the lives around them.

Polya (1981) contributed to the discussion by studying the kinds of habits that are exhibited by a mathematician while doing mathematics. His research revealed the notion that well-constructed learning tasks develop student thinking skills that correlate to the habits required in advanced classes and future careers (Polya, 1981). Research in mathematics education, like Polya's, demonstrated the idea that mathematics as an academic subject can be viewed as a network of correlated objectives that are developed over time as students participate in a sequence of advanced courses or develop within a specialized career path. For this reason, Polya goes on to argue that educators must develop lessons that relate to real world situations, develop critical thinking skills, and look toward the next step in their educational careers.

The National Council of Teachers of Mathematics (2018) has adopted the position that mathematics courses should be defined by their content that directs student's attention "to conceptual understanding, procedural fluency, problem solving, and mathematical reasoning and critical thinking practices" (p. 37). Huntley, Rasmussen, Villarubi, Sangton, and Fey (2000) argued that a particular sequence of learning has a significant effect on learning outcomes. Their research suggested there is a relationship between how rigor escalates in a sequence and the development of advanced growth by the students. When correlating these findings to mathematics, it delves into the idea of course sequence, specifically regarding Algebra. Participating in an algebra class, as opposed to a regular mathematics course, the curriculum forces higher order or critical thinking skills that will benefit the student both during their high school years and throughout their lifetime. Curriculum design has an important impact on student attitude and effort toward learning the content (Katz, 2007). The ability to derive new ideas from the procedural process, while adhering to the sequenced order is where imagination can combine with critical thinking to open new worlds in mathematics.

Researchers have also focused on learning how course expectations contribute to building student self-efficacy in mathematics, specifically higher mathematics while others do not. In their study on parent expectations of their children's education, Reis and Graham (2005) affirmed the connection between learning expectations defined in a curriculum and the academic development of students progressing through the curriculum. Using data from the longitudinal study, Reis and Graham (2005) found using a well-developed course sequencing, even after controlling for SES, parent educational level, and other student background characteristics, students do not only build their conceptual knowledge from one course to the next, but increase their interest to attempt advanced courses.

Evidence resides within research on educational testing that there is a significant curricular effect on the development of student critical thinking. Deng and Kobrin (2007) suggested a positive correlation between engagement in higher level math courses and performance on national standardized tests. Other researchers argued the same effect specifically for College Board tests like PSAT (Milewski & Sawtell, 2006, Schiller, Schmidt, Muller, & Houang, 2010; Schoen & Hirsch, 2003).

Researchers continue to expand on the notion of the importance of course sequencing on student learning. “Course enrollments as opportunities to learn” is how Schiller et al. (2010, p. 605) looked at this concept. They discussed math curriculum that students may take during their high school career as, “while not directly addressed in these analyses, mathematics track incorporates a third dimension of a school curriculum (subject matter content) that contributes to inequity” (Schiller et al., 2010, p. 606). They were describing the exposure students will have to deeper, richer curriculum if they take Algebra II rather than just Geometry. Algebra II seems to be a good gauge for many studies to determine if a student has the critical thinking skills necessary to attend college. It is clear that research has also created evidence that course curriculum affects the educational opportunities for students to the point of being a significant single contributor to inequalities among student educational experiences.

Critical Thinking as a Goal for Education Reform

Education reform can drive decisions within schools and set expectations for students. These reforms, partnered with critical thinking, can drive goals for improvement in educational outcomes. If one takes the emphasis of research studies in this area as an indicator, there appears to be momentum in educational reform to emphasize the development of higher-order thinking skills among students through participation in advanced mathematics courses. For example, there have been studies that focus on the learning of higher-order thinking skills by improving mathematics course content that allow students to practice computation techniques (Hirschhorn, 1993), calculator usage (Huntley et al., 2000), and core mathematical concepts (D. R. Thompson & Senk, 2001).

D. R. Thompson and Senk (2001) noted educators have always been revising their courses to incorporate more efficient approaches to delivering content in an understandable and a palatable manner. However, they also noted in their research that the effort at reform has been limited in scope to promote more efficient ways to reinforce individual student perceptions of mathematics and practice mathematical procedures.

Furthermore, proponents of educational reform have pressed the need for identifying specific habits of thinking and processing in mathematics education to promote advancements in student reasoning skills (Asiala, Brown, Devries, Mathews, & Thomas, 1996). Advocates of this issue have identified several benefits in the study of advanced mathematics in high school courses. Foremost, it exercises the mind and makes the student learn and understand challenging concepts. Mathematics significantly increases the reasoning and understanding skills of people (National Council of Teachers of Mathematics, 2018). There is a body of research that identifies the importance of identifying student traits that exemplify a properly educated student which includes thinking skills and soft skills (Hardy & Snow, 1992). Among the traits identified by Hardy and Snow was intellectual curiosity, which leads directly back to critical thinking. Educators must keep this in mind when deducing which sequence of mathematics courses a student should take. The course sequence must be directly tied to increasing the intellectual curiosity of students in order to encourage more students to achieve success in higher level courses.

Hiebert et al. (2003) has raised the concern of educational administrators focusing on less important issues like maintenance costs and accountability outcomes rather than focusing on improving learning in mathematics. Sequencing mathematics courses to maximize retention, extend learning, increase rigor, and demonstrate critical thinking skills is paramount. By reworking the sequence, this can all be accomplished, but administrators' focus must shift.

By calling out administrators, the focus can be redirected. There is historic precedence for researchers being concerned with the best way to improve mathematics learning within the United States. Sleeman (1984) emphasized the development of concepts in mathematics; Kilpatrick (2003) brought attention to the learning environment and the importance of context. In 1991, the National Council of Teachers of Mathematics advocated for a shift from traditional models of teaching and since then, researchers have focused on studying the effect the various pathways' pedagogy and curriculum have on student outcomes.

Reform in Developing Attitudes Toward Mathematics Based on Race/Ethnicity, Gender, and SES

The research on educational improvement has also focused on the benefit of closing gaps of learning among different ethnicities, genders, and other demographic categories. For example, the literature on the effect of race on math attitudes has corrected the conventional misperception that Black students do not like mathematics has resulted from a faulty interpretation of traditionally low achievement in the subject relative to other races (Matthews, 2004). In contrast, Matthews found that while Black students are less likely than White students to view math as useful to their everyday lives, careers, or college education, they "like mathematics, find it interesting, have little math anxiety, and want to take more mathematics [courses]" (p. 90). Peng and Wright (2005) found eighth grade Black and Asian students were more likely than White, Latino/a, and Native American students to express that they liked mathematics. Additionally, Catsambis (2004) found Black and Latino/a students are more likely than White students to look forward to math class. These findings suggest a paradox that researchers have discovered in other educational areas: that Black students overwhelmingly buy into the importance of education, but do not perform at high levels matching their enthusiasm (Solórzano & Yosso, 2002).

In addition to race, education reform has made strides in helping one better understand the effect of gender on mathematics education. The research on girls' attitudes towards mathematics suggests on average, girls tend to view mathematics as a male subject that will not be useful to them (Fennema, 2000) and have less positive attitudes towards math than boys (Reis & Graham, 2005). Similarly, Catsambis (2004) found that within all ethnic groups, eighth grade girls are less likely than their male counterparts to look forward to math class. Yet, although White and Latina girls are more likely than White and Latino boys to be afraid to ask questions in math class, there are no such gender differences among Black students. However, any attitudinal advantage of Black girls has dissipated by 10th grade, when girls of all ethnic groups are less likely to agree than their male counterparts that math is one of their best subjects (Catsambis, 2004). These findings suggest several important phenomena: first, that girls in eighth grade may be affected by a perception that mathematics is a male domain; second, in the early years of high school White and Latina girls may be more affected than Black girls by gender stereotyping; and third, and perhaps most troubling, by 10th grade, girls, regardless of ethnic background, either legitimately feel that math is not their best subject or are hesitant to report that they are good in mathematics.

Knowing that students of different ethnic and gender backgrounds do not feel as though they excel in a mathematics classroom, brings forth the notion that something should be done to combat this thinking. Recent efforts at curricular reform can be observed in the adoption of the CCSS (2018) for Mathematics by most states in the country. CCSS outline two different approaches to teaching high school mathematics (Eddy et al., 2015). The first method would be considered the "traditional" sequence, which is made up of Algebra I, Geometry, and Algebra II. The second approach offers schools the opportunity to have an integrated series of mathematic courses where the curriculum is blended through 3 years of school rather than broken apart during 3 different years (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2014). States such as Florida, Georgia, Indiana, North Carolina, and Tennessee have all moved to the integrated approach of teaching math classes by having all three mixed classes taught within their states (Reys, Dingman, Nevels, & Teuscher, 2007). Integrated curriculum from Glencoe, known as Core-Plus Math, showed that students scored higher on Algebra EOC (Schoen & Hirsch, 2003). The study results implied that continuing to take courses that build Algebra skills would improve the students' performance.

Even research on methodology has concerned itself with the need for developing new techniques for examining curriculum improvements (Hirschhorn, 1993). The notable models pinpoint areas in which both traditional curricula and reform need to be enhanced if they are to reach extensively agreed-upon goals. Nevertheless, they also have to leave open the important questions about what understanding and skill in algebra do students need for them to acquire necessary mathematics experience from their school.

History of Testing and Accountability in Texas Public Schools

Each state in the U.S. has experienced its own evolution of change in using high stakes measurements as evidence of learning and as a basis for holding their schools accountable for improving student learning. Texas has a rich, well-documented history of testing and accountability that began in the late 1970s. In Texas, standardized testing was first instituted in 1979 when the 66th Legislature enacted a law requiring basic skills competencies in mathematics, reading, and writing for Grades 3, 5, and 9 (Texas Education Agency, 2013b). In 1980, the Texas Assessment of Basic Skills (TABS) was introduced, which assessed math, reading, and writing. In 1986, the Texas Educational Assessment of Minimum Skills (TEAMS) was required for students to be eligible for a high school diploma, and in 1990, the Texas Assessment of Academic Skills (TAAS) was introduced. TAAS shifted the focus of assessment from minimum skills to academic skills (Texas Education Agency, 2013c). In 2003, TAAS was replaced by the Texas Assessment of Knowledge and Skills (TAKS), which also measured state mandated curriculum. Students were required to pass TAKS exams in English language arts, mathematics, science, and social studies to earn their high school diploma (Texas Education Agency, 2013c). Then in 2010, to comply with NCLB (2002) requirements, the State of Texas created and implemented the State of Texas Assessment of Academic Readiness End-of-Course Assessments (STAAR EOCs) which replaced TAKS and was implemented beginning school year 2011-2012 (Texas Education Agency, 2014).

The trend is for the state legislature to adopt versions of test instruments that increasingly measure more rigor and complex thinking as evidenced in the most recent EOC exams (Texas Education Agency, 2010). Elected officials and TEA alike tout the high caliber and quality of these newly created STAAR EOC mandated tests. The rigor of items has been increased by assessing skills at a greater depth and level of cognitive complexity. In this way, according to TEA (2013b), the tests will be able to measure a greater range of student achievement and establish stronger links to postsecondary readiness.

School Accountability

As testing history has been evolving in Texas so has the high-stakes environment of accountability. Reich and Bally (2010) noted educators have been dealing with high-stakes testing due to the accountability systems for the nation and at the state levels. Hong and Youngs (2008) explained school accountability has forced educators to narrow curriculum in order to spend time preparing at-risk students on test preparation. The consequence was spending less time on real world skills and engaging instruction that was relevant and valued by students, universities, and employers.

Moore et al. (2010) noted that since 1991, Texas legislators have required public schools to report college readiness indicators in addition to the state exam results within the school and district accountability systems.

Hoffman, Assaf, and Paris (2001) noted Texas transitioned to a criterion-referenced test, known as the Texas Assessment of Academic Skills (TAAS) test, implemented in 1990. It was during the TAAS era when Texas required EOC exams for Algebra I, Biology, English II, and U.S. History. The transition from the TAAS test to the Texas Assessment of Knowledge and Skills (TAKS) test occurred in 2002 because Texas implemented a major curriculum revision with the approval of a new curriculum framework (Texas Education Agency, 2014). At the time of this study, Texas' newest test, called the State of Texas Assessments of Academic Readiness (STAAR) was administered for the first time during the 2011-2012 academic year.

Under House Bill 5, the 83rd Texas Legislature approved five assessments required for high school graduation (Texas Education Agency, 2014). Those exams were as follows: Algebra I, English I and English II (which combined reading and writing), biology, and U.S. History.

Graduation Requirements, House Bill 5, and Senate Bill 149

As reported in the side-by-side graduation requirements (Texas Education Agency, 2014), Texas approved House Bill 5, which changed the graduation requirements for students who entered Grade 9 in 2014-2015. The Foundation High School Plan (FHSP) came into effect at this time. FHSP requires students to earn four credits in English Language Arts, three credits in math, three credits in science, three in social studies, one credit in physical education, two credits within the same language other than English, and five electives credits for 22 credits. Speech was removed as a required one-half credit, however a student must demonstrate proficiency by being able to deliver clear verbal messages, utilize effective nonverbal behaviors, have proficient listening skills, apply critical thinking and problem solving skills, and implement and evaluate communication skills in professional and social settings. In addition to the credits listed, the FHSP allows students to earn an endorsement in science, technology, engineering, and math (STEM); business and industry; public services; arts and humanities; or multidisciplinary studies. These endorsements require four additional credits for a total of 26 high school course credits.

The new graduation requirements implemented by TEA (2014) also allow students to earn a Distinguished Level of Achievement along with their FHSP. This is achieved by students earning four credits in math, where one must be in Algebra II and by earning four credits in science. Additionally, seniors should complete the curriculum requirements for one endorsement. Performance acknowledgements can also be achieved for outstanding performance in dual credit courses; success in bilingualism and biliteracy; success on AP or IB tests; high scores on PSAT, ACT-Plan, SAT, or ACT; and for earning a nationally or internationally recognized business or industry certification or license. These performance acknowledgements must be documented on each individual printed diploma. This, plus passing five EOC exams are required for graduation.

However, TEA Commissioner Williams (2015) stated in a letter to Texas school administrators, emergency rules to amend the assessment graduation requirements were to take effect as of May 11, 2015. The amendment to the assessment requirements for students in 11th or 12th grades during the 2014-2015, 2015-2016, or 2016-2017 school years included:

- Consider the student's grade in each course for which the student did not meet standard on EOC.
- Meeting required standards on Texas Success Initiative (TSI) college readiness exams.

Challenges with High School Mathematic Course Sequence

There are numerous challenges when trying to decipher the alignment of mathematic courses at the high school level. The experts have not weighed in on any particular model that works best to serve the needs of students. At best, many mathematic experts have only weighed in on this topic to acknowledge that the typical sequence of courses is Algebra I–Geometry–Algebra II. Through The Mathematics Education in the United States 2008, Dossey et al. (2008) focused on the appropriate age level that a student should be to take algebra. The researchers also focused on student success after Algebra I if the student was unsuccessful the first time. In the Mathematics Education in the United States 2008 study, the researchers recognized "at the high school level, the mainstream curriculum currently found in the U.S. secondary school classrooms was built around a sequence of three full-year courses, Algebra I−Geometry−Algebra II or Algebra I−Algebra II−Geometry" (Dossey et al. (2008, p. 18). The current Geometry classes taught around the nation align with demonstrative, plane, and solid geometry as described in *The Reorganization of Mathematics in Secondary Education* (National Committee on Mathematical Requirements, 1923). The current Algebra II standards align with what was taught within the years 10, 11, and 12 from this study. However, the most important element from the current study was the order was of far less importance than whether or not a student completed all of the course offerings (National Committee on Mathematical Requirements, 1923). In 1985, the Secondary School Mathematics Curriculum highlighted what is still true today and that fact is for the past 30 years, a majority of schools are in the same alignment of high school courses. "The standard college preparatory mathematics program in U.S. high schools is probably some modest variation on the following basic scheme: Algebra I, Euclidean geometry, Algebra II, trigonometry, analytic geometry" (Hirsch & Zweng, 1985, p. 125).

Research articles are in abundance that supported the idea of needing a different approach working with students as they advance in their educational experience in mathematics. The National Research Council advocated in 1989 for the creation of a new national curriculum and new approaches to math challenges, but they failed to alter the need to address the current alignment of courses (Kilpatrick et al., 2001). Overall, most articles which referenced the arrangement of classes left the decisions to the determination of what is in the school and the school district's best interest.

> Fewer graduates who had beginner algebra I or geometry courses went on to complete advanced mathematics courses. About 60 percent of graduates who completed beginner algebra I courses went on to finish algebra II course or higher as their highest level math course. (J. Brown et al., 2013, p. 6)

Mauch and Shi (2005) pointed out in math and computer education that "sequence offers a fascinating example that can be used in teaching mathematics subjects at various levels" (p. 199). Alper (1997) shared the idea of having a focus on bringing the community to the schools to help write the curriculum for the school. However, within the Designing a High School Mathematics Curriculum for All Students study, Alper failed to look at the concept of the alignment of courses. Kuhn and Dempsey (2011) in their “Leading & Learning with Technology” article, pointed out that some schools look the same as they did in “1999, 1989, or even 1959” (p. 19). They discussed the differences in innovation, not alignment, “the algebra-geometry-trigonometry-calculus sequence” (p. 20).

Focus on Retakers

Student failure of a mathematics EOC exam necessitates a multitude of local policy decisions regarding instruction and student learning (Fong, Jacquet, & Finkelstein, 2014; Liang, Heckman, & Abedi, 2012). Should a student be held back and repeat the course attached to the failed EOC exam? Should the student move forward in the secondary mathematics sequence, despite exam failure? What instructional supports do students need after EOC exam failure, and how are they best supported to pass it upon retake? The uncertainty surrounding students who fail a mathematics EOC exam, and must subsequently retake the exam and possibly the course, prompted a study to focus on these students, and how teachers respond to them in the classroom. The institutional response is discussed in Chapter 5; this section focuses specifically on individual instructional decisions and actions for students who were retakers. In this section, retakers refers to students who must retake a failed mathematics EOC exam, and students who must retake a mathematics course and the exam due to EOC exam failure. Some parts focus specifically on students who are retaking a course as a result of exam failure. Teachers discussed their observations of students who had previously failed a mathematics EOC exam, as well as observations of students who ended up retaking a course as a result of exam failure. These observations factored into the instructional decisions they made over the course of the year, and came to influence their views of the EOC testing system itself.

An example of this can be illustrated in Fong et al.'s (2014) study by looking at students who had failed a mathematics EOC exam, either while at Springhare High School (SHS) or at their middle school. For these students, course placement became a pressing issue. As noted in discussion of the SHS retaker response, some students were retained as a result of failing a mathematics EOC exam (Fong et al., 2014). Students who had not successfully completed EOC testing requirements were automatically subject to retake the exam, but there was still the question of whether to have students retake a course, either during the academic year or during summer school, or move on to the next course in the secondary mathematics sequence. Decisions regarding course placement were made by SHS mathematics teachers, who shared their insights and observations of the students. Minerva, a SHS mathematics teacher, made one observation (Fong et al., 2014). At SHS, students who retake a mathematics course as a result of EOC exam failure were placed in heterogeneous classes; students were not placed in classes exclusively with other students who had failed an EOC exam. Minerva explained the institutional rationale behind having some students retake the course was to provide students a second pass at course material (Fong et al., 2014).

Fong et al. (2014) also found that for some Algebra I students, repeating the course bolstered their grades upon subsequent retake, though they found that positive effects were most frequent for students who had struggled during their first attempt at the course, as opposed to students who had performed well the first time taking the course. Minerva noted that with a year of cognitive maturation and personal development, students might then be prepared to improve their course performance and pass the corresponding EOC exam (Fong et al., 2014). In some cases, students might also retake a course with a different teacher, exposing students to a different instructional perspective. Having students retake a course after struggling or failure presumes that repeating a course provides students with an opportunity to address deficiencies and be prepared to move forward (Roderick & Nagaoka, 2005). At SHS, the same line of thinking applied and had been an option for addressing student learning after EOC failure (Fong et al., 2014).

At the same time, prior research has drawn attention to potential adverse effects in the practice of having students retake courses (Fong et al., 2014; Liang et al., 2012), some of which Minerva and Sam observed in their students. Minerva found that the impact of having a student retake a course was contingent on the student's level of engagement. While some students benefitted from revisiting curricular content, other students reduced their class participation. Minerva believed this might be the case because these students had seen the exact same curricular content the year prior and felt they already understood these concepts. Other students had poor learning experiences that discouraged their participation and academic effort. Of her students who had retaken either an Algebra I or Geometry course, Minerva observed these students were often the most disinterested. Related to this, Minerva expressed that retaking a course would be effective only for students who recognized their learning issues and deficiencies from the prior year and were actively working to address them. For course retake to be effective, she felt students had to be aware that recognition of a concept was not the same as conceptual understanding. Students had to distinguish learning gaps and errors, and have the intention to correct them during the second time through. Consequently, Minerva was concerned that, in and of itself, retaking a course might not be an effective way to address all the issues that contributed to students failing a mathematics EOC exam.

Specific Context of the Present Study

Due to the number of students failing the Algebra I EOC test in May, coupled with the high stakes ramifications attached to a proficient performance on the Algebra I EOC, the school district administration was faced with a challenging problem to minimize, if not eliminate Algebra I EOC failures. Therefore, the decision to adjust the course sequence was a challenging decision. The campus principal, district math coordinator, campus math specialist, math department chair, and the campus associate principal formed a committee with the purpose of brainstorming how to help students be successful after they failed the Algebra I EOC test. Students in the sample failed the first administration in May of their freshman year. Students would have an opportunity to retake the EOC in July. After that failure, students in the study were obligated to enroll in a math course before taking the retest of the Algebra I EOC in December and May of the following year. If a student persisted to fail the exam throughout their senior year, the student would not be allowed to graduate.

After an unsuccessful attempt at finding published resources that applied research to their problem, the committee researched course sequence to see what the logic was behind the current sequence. They also spoke to school districts within Texas to see what other high schools had as their math course sequence. After conversing with other school districts, the committee, school principal, and school superintendent decided to make two major recommendations that would impact the high school teachers and students. The first decision was to look at the course sequence and adjust the sequence from Algebra I, Geometry, and Algebra II, to Algebra I, Algebra II, and then Geometry. The committee felt like a continuation of the curricular ideas partnered with the critical thinking of Algebra I and Algebra II that the group would have a great chance for success. Additionally, they were unable to locate research that led them to a decision of which math course was best to follow Algebra I. The second recommendation was to move from a traditional 7 period day to an 8 period day. The logic, from the committee, was that an additional period allowed for students to have a 45-minute intervention built into their school day. These recommendations were made to the superintendent and the School Board, who supported them, and the changes were implemented during the 2012-2013 school year.

With the adjustment of the mathematics courses, it allowed a group of students to be researchers and see what impact a course that followed a failed EOC could have. The study would be focused on just students who failed the Algebra I EOC. The first year, students who failed the Algebra I EOC would be taking Geometry. The second and third year of the study, students who failed the Algebra I EOC would be taking Algebra II.

Summary

More research is needed to understand the role course sequence has in determining student understanding of mathematical concepts. Such research has the potential for synthesizing parallel research efforts in curricular effects on student learning, educational reform trends, affecting the purpose of high stakes testing, and resolving issues involving barriers to improving mathematics curriculum sequencing.

The goal of this study was to investigate whether a modified course sequence that differs substantially from the traditional sequence had a greater effect on student achievement on the Texas state end of course exam. In the next chapter, the researcher discusses the method used for studying this question.

CHAPTER THREE

Methodology

The purpose of this quantitative study was to investigate the impact of a modified mathematics course sequence to the students Algebra I EOC retest scores. The intent of this investigation was to analyze whether a group of students who received instruction through a modified mathematics course sequence experienced greater gains in performance on the Algebra I EOC exam than a group receiving instruction through a traditional course sequence.

In this chapter, the application of appropriate statistical tests are discussed in-depth as well as the dependability of the instruments used and process followed. The research design, research questions and hypothesis, sampling, data collection and analysis, and ethical issues are also primary components of this chapter.

Restatement of the Research Question and Hypothesis

This study was conducted to build a theory to answer the following research question:

Do students receiving instruction through a modified mathematics course sequence perform better on Algebra I EOC retests than students receving instruction through a traditional sequence?

To address this question scientifically, statistical analyses were applied to the following null and alternative hypotheses:

H_0: There is no statistical difference in the mean difference in Algebra I EOC test and retest scores for students receiving instruction in a modified mathematics course (Group 2) than for students receiving instruction in a traditional mathematics course sequence (Group 1).

H_A: There is a statistical difference in the mean difference in Algebra I EOC test and retest scores for students receiving instruction in a modified mathematics course (Group 2) than for students receiving instruction in a traditional mathematics course sequence (Group 1).

Research Paradigm and Design

Scientific research to examine what works in education has been applied since the formation of formal education (Lodico, Spaulding, & Voegtle, 2010). Scientific research is systematic investigation that leads to generalizable knowledge following the acceptable guidelines within an area of expertise (U.S. Department of Health and Human Services, 2010). Scientific research in education "is defined as the application of systematic methods and techniques to help researchers and practitioners understand and enhance the teaching and learning process" (Lodico et al., 2010, p. 10).

The design of the current quantitative study was an ex-post facto or causal-comparative design. Quantitative research is a discipline focused on theory testing by examining relationships among variables (Creswell & Clark, 2008). Ex post facto causal comparative research design that examines relationships among variables is common in education because of the difficulty in manipulating an independent variable experimentally with randomized groups; instead, experiences that have already occurred are examined (Lodico et al., 2010). Kerlinger explained that ex post facto design is research in which the treatment or independent variable occurred in the past, so it is not a controlled treatment. Instead, the researcher examines, in retrospect, the effects of something that occurred (independent variable) and the ensuing outcome (dependent variable) in an effort to establish a causal link (Lodico et al., 2010). As such, ex post facto design is akin to experimental design in retrospect. Like experimental design, the purpose of ex post facto causal comparative design is to seek causal explanations through controlled experimentation and is best conceived as theory testing (Creswell & Clark, 2008). The distinguishing feature between ex post facto causal comparative research and experimental research is the non-randomization of treatment and control groups due to convenience sampling (L. Cohen, Manion, & Morrison, 2007; Levin & Calcagno, 2008). Another difference is the location and timing of treatment; ex post facto causal comparative research takes place in a natural setting and/or uses archival data (Lodico et al., 2010).

The current study design is ex post facto causal comparative using archival data. Data for the current study was obtained by using the scores of two groups of students. Group 1 was students who failed the Algebra I EOC in May 2012 and received instruction in Geometry and math models during the 2012-2013 school years. Group 2 was students who failed the Algebra I EOC in May 2013 and May 2014 and received instruction in Algebra II and math models during the 2013-2014 and 2014-2015 school years respectively. The data focused solely on students who did not pass the Algebra I EOC because this event was the education system's prerequisite for being eligible for an alternative course sequence other than the traditional sequence. Specifically, Group 1 students received instruction in Geometry and math models −traditional mathematics course sequence and Group 2 students received instruction in Algebra II and math models −the modified mathematics course sequence. The differential between test and retest scores on the Algebra I EOC, taken by both groups after instruction, were compared.

The currently evolving paradigm shift in public education from student access to educational opportunity to student success on measured achievement coupled with the advancement of research on what works for student's mathematics success are rich environments for a quantitative examination of how mathematics courses can be designed and sequenced to increase the probability of student success as measured by an EOC exam. The validity of the data produced by the EOC examination instrument and the effort and variance in mathematics course sequence of student subjects producing these data aligns very well with a quasi-experimental design to analyze differences in mean exam scores. Moreover, it is important to investigate if there is an impact on students' success after a modified mathematics course sequence on their EOC retest scores. Therefore, ex post facto causal comparative research design was employed to answer the research questions by testing the hypotheses listed.

Sample and Setting

The researcher selected participants in the current study through purposive sampling. Purposive sampling produced a sample logically assumed to be representative of the high school student population requiring an improved mathematics education experience. The sampling was based on the student attribute of having failed the EOC Algebra I exam and received instruction in a mathematics course−traditional or modified−prior to retesting on the EOC exam.

Before conducting the experiment, a power analysis was performed using G*Power (Faul, Erdfelder, Lang, & Buchner, 2007) in order to check on the minimal sample size necessary to answer the research question. Power is defined as the probability that, when there is an effect, the statistical test indeed rejects null hypothesis. The power computation using G*Power showed that, for the independent samples *t*-test with an alpha level set to 0.05 a sample size of at least 85 students would be required for finding small effect sizes ($d = 0.3$) with at least 80% power.

The study setting is a real high school in a large school district located in Central Texas. The selected school had a total enrollment of 2,229 students during the years of this study (2012-2014). During that same span of time, 1,629 first-year students were admitted to the school.

Notably, this study examined the data of students who had failed the Algebra I EOC exam during the second, third, and fourth year of the study (2012, 2013, 2014) with an assumption that these students had taken a full schedule of courses in other studies. The data gathered is in Appendix A, Appendix B, and Appendix C. Another assumption considered in the current study was that the students who failed to pass the Algebra I EOC exams were allowed to retest for their exams the immediate summer after the results were released. Table 2 displays the total enrollment figures for the current study's selected school during the years of the study.

The sample for the current study was 103 who first enrolled in the school in 2010-2012 and had failed the Algebra I EOC exam that same spring. Each of the 103 students belonged in one of two groups, differentiated by the course taken after the first failure to pass the Algebra I EOC exam.

Table 2

Population of Students in the School Each Year of the Study

Year entering	Year graduating	Enrollment	% of total enrollment
2010	2015	544	24.4
2011	2016	519	23.3
2012	2017	566	25.4
2013	2018	600	26.9
Total		2,229	100.0

Group 1, 35 students, received instruction in Geometry which is the traditional course to take after Algebra I. Group 2, 68 students, received instruction in Algebra II, a non-traditional course to take right after Algebra I. The study eligibility criteria for participants was constituted by the students who were enrolled in the ninth grade for the first time and who were admitted for a year-long Algebra I mathematics class but failed to attain the required pass mark on the Algebra I EOC exam.

Research Instrumentation

The EOC test in Algebra I was the instrument used for the post-test assessment in the current inquiry. The state's EOC assessment was preferred as it is a test aligned to instructional standards and is used to assess course-specific skills in algebra. Every student in the study who had failed to meet the Algebra I EOC was required to retake the test on a different form. The participants were retested, and their score was used to determine whether they met or failed to meet standards expectations. The scores were then analyzed. An independent *t*-test was used to determine whether the researcher could accept or reject the null hypothesis that there is a difference in the two groups' mean difference scores where the difference is between the first and second exams for each student. The researcher rejected or accepted the null hypothesis. To assure a high level of confidence in the study's findings, the significance testing level was set at 95% where alpha = .05. Lastly, the researcher analyzed additional student groups by disaggregating the sample population into subgroups based on gender, age, grade level, pass/fail Algebra I class, first-time test taker, and ethnicity categories.

Data Collection Procedures

Existing archival data from the Algebra I state EOC assessment was used for this quasi-experimental study. The researcher collected these data systematically through a formal request to the district's testing coordinator. The data set was downloaded through the district's secure database transactional tool. The data included these variables: student ID; course enrolled, Geometry or Algebra II; admitted into special education, yes or no; economically disadvantaged status, yes or no; limited English proficiency (LEP), yes or no; gender; ethnicity; initial EOC scale score; and the retest EOC scale score.

All data were kept confidential and secure. Students remained anonymous by not downloading any student names and designating a record with a unique, mock student ID number.

Reliability and Validity

The quantitative scale score converted from the raw score correct answers on the Texas EOC Algebra I exam has been documented as reliable (Texas Education Agency, 2012a). Since the researcher in the current study analyzed student performance across years, the use of the scale score was desirable because the scale score is a conversion of the raw score onto a scale common to all test forms. Scale scores allow direct comparisons of student performance between specific sets of test questions from different test forms (Thissen & Wainer, 2001). The exam had been administered previously four times (2012, 2013, 2014) to measure student proficiency based on legislatively adopted state standards for Algebra I (Texas Education Agency, 2012a). Two exams were utilized as the dependent variables for this study, the State EOC test, and a re-test of the same test but different form. Exam forms have been examined by content specialists and equated by psychometricians contracted by the education agency publishing the exam (Texas Education Agency, 2012a, 2013b).

Data Analysis

After receipt of the secured electronic data from the school district's testing director, the researcher prepared a spreadsheet that organized the following variables for each student: ethnicity, gender, EOC exam score first EOC Algebra I scale score, second−re-test−EOC Algebra I scale score, and course-level of the course taken prior to the second Algebra I EOC exam. The researcher imported the data into IBM SPSS Version 20 software for further conditioning of the data set and comparing two group means using Student's *t*-test. The researcher imported the data into IBM SPSS statistics software for further conditioning of the data set. Once the data resided in the software, the difference between the first and second test scores was computed for each student.

The independent variables course level−categorical and the dependent variables−test scores were used to calculate independent sample means *t*-test to examine whether differences in the mean score gains are significant between two independent groups on their mean difference of first and second test scores.

While *t*-tests are quite robust, it is imperative to evaluate the statistical test's assumptions to assess the quality of the results. In this study, the researcher looked at the independent samples *t*-test's four main assumptions: (a) the dependent variable must be continuous (interval/ratio), (b) the observations are independent of one another, (c) the dependent variable should be approximately normally distributed, and (4d) the independent variable should be similarly distributed for each group.

The first assumptions investigated were variable type (continuous) and independence. The dependent variable (mean scale score difference) is a calculation of the difference between two scale scores (from the first Algebra I EOC test and the second Algebra I EOC test). Since the score reported for the Algebra I EOC is a raw score (interval) converted to a scale score (interval) using Rasch methodology (Texas Education Agency, 2012a), the result of subtracting the two scores remains an interval (Wright & Stone, 1979). Therefore, the assumption for a continuous dependent variable was satisfied.

The assumption for independence of the independent variable (course sequence) was met through observing that the assignment of a student to a course sequence was not dependent upon enrollment in the other course sequence. Additionally, the only criteria applied to selecting the student scores for this ex-post facto study were from students who failed the first Algebra I EOC exam, not by which course sequence in which they enrolled. This ensured minimal risk of collecting a biased sample.

To test the assumption of normality, a variety of methods were available, but many researchers recommend the Shapiro-Wilk test as the best choice for testing the normality of data (Razali, Wah, & Alam, 2011; Thode, 2002). Thodes (2002) described the test as robust with respect to the assumption of normality arguing that some deviation away from normality does not have a large influence on Type I error rates.

Using the Shapiro-Wilk test, the researcher analyzed the normality of *scale score mean difference* on individuals within each course sequence group. The Shapiro-Wilks test is a statistical test of the hypothesis that sample data have been drawn from a normally distributed population (Thodes, 2002). Using an alpha level of .05 to test this hypothesis is common and acceptable for educational testing data (McCluskey, 2007). Given that $p = .627$ for Group 1 and $p = .341$ for Group 2, the researcher was able to accept the null hypothesis for the Shapiro-Wilk test which is each of the levels of the independent variable (course sequence group) was sufficiently normally distributed to continue with a *t*-test.

The independent samples *t*-test also assumes the variances of the two groups are equal in the population. If the variances are unequal, this can affect the Type I error rate (M. B. Brown & Forsythe, 1974). The assumption of homogeneity of variance can be tested using Levene's test of equality of variances, which is produced in SPSS statistics when running the independent samples *t*-test procedure. This test for homogeneity of variance provides an F-statistic and a significance value (*p*-value). We were primarily concerned with whether the *p*-value for the proportional comparison of the two groups was greater than 0.05.

To determine whether the group means are statistically significant, a confidence level of $\alpha = 0.05$ was used for the independent samples *t*-test. Any outcome that is obtained with a *p*-value of less than .05 can be regarded as a significant finding and considered important to educational studies (Klippel, 1984).

Limitations of the Research Design

It is recognized that inferences of causality in quantitative research with ex post facto causal comparative research design are limited. Though experimental design would be superior for inferring causality, the ethical considerations supersede the ability to manipulate experimentally with students in this research. It is recognized that there are limitations of this research design. The threats to internal validity are discussed below.

Internal Validity

There are two threats to internal validity. The first is that a score on a high stakes exam is a reliable measure of student understanding of mathematical concepts. Dependence on the reliability of a test score can threaten the reliability of this study's findings. However, historically and in most reporting mechanisms, i.e., National Center for Education Statistics, The Integrated Postsecondary Education Data System (2009), validated test scores are generally accepted as a measure of student success. Further, using passing scores as a measure of student success is an acceptable performance indicator (Dougherty, Reid, & Nienhusser, 2006; Dowd & Tong, 2007).

The second limitation of internal validity is due to self-selection in the sample over time. That is, because students elect to enroll in the Algebra I course each year, the sample population was not randomly added to each year of the study. Creswell (2009) has explained history is an internal threat because as "time passes during an experiment; events can occur that unduly influence the outcome beyond the experimental treatment" (p. 163). To control for this threat, the researcher can have all groups experience the same events (Creswell, 2009). The choice of participants who failed during their first-take of Algebra I EOC based on concurrent enrollment in academic, from the study school, will control for this threat.

Ethical Issues

The use of archival, masked data protects participants from harm while reducing the need for informed consent. Archival data also contradicts the need for assurance of volunteerism and issues related to the potential negative risks for participants. The right to privacy of the archival data is protected through Family Educational Rights and Privacy Act. To protect the students *right to privacy*, student identifiers were removed. The researcher signed the appropriate confidentiality agreements required. Additionally, the data was stored in a locked safe in the researcher's office when not being used for data input into IBM SPSS for analysis. Only the researcher had access to the locked safe. As explained in the Publication Manual of the American Psychological Association (2010), the data will still be made available to the committee members and appropriate personnel at Baylor University at any time during this process in the event questions arise regarding the accuracy of the archival reports being used for raw data. The raw data will remain locked in the safe for 5 years following publication of this study per APA guidelines.

Summary

The quantitative research method employing ex post facto causal comparative research design has been explained, detailing the target population, sampling method, participants, and related procedures; data collection procedures; data analysis procedures including organizing, managing, preparing and processing the raw data for analysis; limitations of the research design; internal validity and controlling for these threats; and ethical issues. The research questions have been restated with the hypotheses of the study.

The goal of this chapter was to outline the research method used to answer the research question in the context of an ex post facto causal comparative research design. A discussion of the sampling method, data collection and data analysis procedures, limitations of the research design, internal validity and controlling for threats, and ethical issues outlined the specifics of how the study was conducted.

CHAPTER FOUR

Results and Findings

The purpose of this chapter is to summarize the analysis of the research question using the quantitative calculations of the statistical tests applied to the data collected in the study. Specifically, the analysis investigated the impact of a modified mathematics course sequence on student Algebra I EOC retest scores by comparing these data to equated Algebra I EOC retest scores obtained by students in a traditional mathematics course sequence.

In this chapter, the analysis begins with a restatement of the research hypotheses that guided the analysis followed by a detailed description of the sample in terms of demographic characteristics. The analysis continues with a discussion about the statistically significant result of the main test statistic: the independent samples *t*-test. The analysis concludes with an ANOVA extension of the t-test comparison of group means.

Results Obtained for Research Hypotheses

In order to address this question, the researcher tested the following null and alternative hypotheses, respectively:

> H_0: There is no statistically significant difference in the mean score difference of Algebra I EOC test and re-test scores for students in the modified mathematics course sequence (Group 2) than the the mean score difference of Algebra I EOC test and re-test scores for students in the traditional mathematics course sequence (Group 1).

H_A: There is a statistically significant difference in the mean score difference of Algebra I EOC test and re-test scores for students in the modified mathematics course sequence (Group 2) than the the mean score difference of Algebra I EOC test and re-test scores for students in the traditional mathematics course sequence (Group 1).

Demographic Descriptive Statistics

The target school recorded 1,629 first year students receiving instuction in Algebra I and taking the Algebra I EOC exam from 2012-2014. The current researcher was interested in the students who did not meet passing standards on the exam the first time it was taken. The number of students qualifying under this criterion was 178 from 2012-2014. Table 3 shows the population of eligible students.

Table 3

Population of Eligible Students Each Year of the Study

Year	Eligible students	Included in study	% included
2012	48	28	58.3
2013	70	47	67.1
2014	60	28	46.6
Total	178	103	57.8

Note. Figures are counts of students who did not meet the passing standard for the Texas Algebra I EOC exam.

Sample of Study

While 178 students were eligible for the study due to their not meeting the passing standard of the Algebra I EOC exam, the sample size for the current study was 103

(57.8%) due to not taking the retest after instruction. Students meeting the first assumption were excluded from the sample if (a) they withdrew from the course treatment, (b) they were exempt for school reasons from taking the Algebra I EOC retest, (c) they were absent from the day of testing and elected to not make-up the exam at a later date. The sample was further disaggregated according to the course taken between the first administration of the Algebra I EOC exam and the retest. Those students receiving instruction in the traditional course sequence were placed in Group 1. Those students who received instruction in the modified course sequence were placed in Group 2. The sample size of both groups are in Table 4.

Table 4

Sample Size and Group Assignment of Study

	Group 1		Group 2		
Year	Frequency	% of Total Sample	Frequency	% of Total Sample	Total
2012	28	27.2	0	0	28
2013	3	2.9	44	42.7	47
2014	4	3.8	24	23.3	28
Total	35	33.9	68	66.1	103

Note: Students assigned to Group 1 received instruction in a traditional course sequence. Students assigned to Group 2 received instruction in a modified course sequence. Students in both groups did not meet the passing standard for their first attempt on the Algebra I EOC exam but took the retest after instruction.

As seen in Table 4, no students received instruction in the modified course sequence as it had not yet been offered in 2012. However, in 2013-2014, the modified course sequence was offered to students who failed as an attempt by the school to improve assessment results.

Retest and Posttest Students Performance Statistical Analysis

The researcher conducted independent samples *t*-tests to compare the performances on the Texas Algebra I EOC exam by students belonging to two different course sequences. The null hypothesis stated that there is no statistically significant difference in the performance scores of the two groups of students taking the first exam and its subsequent retest. Viewing the descriptive statistics in Table 5 shows a difference between the means of each group.

Observing Group 2's mean differences in test scores were higher than Group 1's mean differences warranted investigation whether the difference was statistically significant. The means were calculated as a group mean of each child's increase or decrease in scale score on the retest as compared to the first test.

Table 5

Descriptive Statistics of Independent Groups of Students in Different Course Sequences

Group	*N*	*M*	*SD*	Std. Error
1	35	77.83	189.95	32.10
2	68	173.47	182.54	22.14

Note. Mean is the group's mean of the differences in two test scores (first test and re-test).

The intepretation is that, on average, the students in Group 2 had greater gains on the second test than did the students in Group 1. Group 1−Geometry students received instruction in a traditional course sequence than did Group 2−Algebra II. While both groups increased the value of their scale score, students in Group 2, overall, appeared to have increased their initial exam score more than the gains calculated in Group 1.

To determine whether the higher gains in Group 2 was due to an effect other than sampling bias, the researcher tested the null hypothesis with an independent samples *t*-test at an alpha level equal to .05.

The *t*-test for this study was interpreted against the null and alternative hypotheses.

> H_0: There is no statistically significant difference between the two groups of students on the mean differences in the Algebra I EOC test and re-test scale scores.
>
> H_A: There is a statistically significant difference between the two groups of students on the mean differences in the Algebra I EOC test and re-test scale scores.

For the main analysis, first a test on homogeneity of variance was calculated. An independent samples *t*-test was performed comparing the mean difference for first and re-test scale scores of students on the Algbera 1 EOC exam.

Given there was no violation of Levene's test for homogeneity of variances, $F(1,101)$ =.008, p = .928, a independent means *t*-test assuming equal variances was calculated. The results of the *t*-test indicated that there was a significant difference in mean gains for test scores between the two groups, $t(101)$ = -2.48, p = .015; d = .56. The effect size for this analysis, *d* = .56, was found to exceed J. Cohen's (1988) convention for a medium effect, *d* = .50. These results suggested that students in the modified course sequence group, M = 173.47; SD = 182.54, had greater gains in scale score than did the students who in the traditional course sequence group, M = 77.83; SD = 189.95.

Analysis of Categorical Characteristics of the Population

The sample population could be described in terms of demographic infromation collected during the study. That is, members of the sample could be grouped by gender, ethnicity, qualifying for special education services, or socioeconomic status.

Since much of educational research is concerned with the impact that demographic identities have on educational outcomes (VanderStel, 2014), the study extended the primary finding of a statistically significant difference in two groups identified by which course sequence they followed.

Two-way analyses of variance (ANOVAs) were performed on the data to detect the presence of statistically significant differences between Group 1 and Group 2 by specific demographic characteristics. By using a two-way ANOVA one mitigates the risk of a Type I error by performing all analyses within the same model (Albers, Anja, & Meijer, 2015). For each of the four variables characterizing the sample, a research question was posed parallel to the primary research of this study. That is, for each demographic grouping, the question was asked, "Are there differences in the mean scale score change from test to retest by [demographic attribute]?" Additionally, the two-way ANOVA enables one to study the question, "Is there an interaction effect of course sequence with a particular demographic variable on mean scale score change."

Gender

Table 6 shows the proportional representation of membership by gender within the sample.

Table 6

Gender Distribution of Students in Sample

Gender	Group 1	%	Group 2	%	Total	%
Female	18	51.4	29	42.6	47	45.6
Male	17	59.6	39	57.4	56	54.4
Total	35	100.0	68	100.0	103	100.0

Note. Percentages are within group percentages.

Since course sequence contributes to the performance of students on the Algebra I EOC, the researcher looked at whether that effect may differ across gender. A two-way analysis of variance tested the mean difference of scale scores between course sequences by levels of gender. Group 2 (modified course sequence) showed a significantly higher mean difference in scale scores from test to retest than Group 1 (traditional course sequence), $F(1, 99) = 6.128$, $p = .015$; however, gender itself was found not to be a significant factor in the determining the mean difference of test scores on the Algebra I EOC as shown in Table 7. While the 47 females in the sample had a slightly higher mean scale score gain of 145.8 ($SD = 170.4$) over the 56 males who had a gain of 136.9 ($SD = 205.9$), there was insufficient evidence to reject the interaction effect null hypothesis between course sequence and gender $F(1,99) = 4.013$, $p = .068$, partial $\eta^2 = .039$, observed power = .510.

Table 7

Two-Way Analysis of Variance of Mean Scale Score Difference by Course and Gender

Source	*df*	SS	MS	*F*	*p*	Partial η^2	Observed Power
Group (course seq)	1	205368	2023	6.128	.015	.058	.688
Gender	1	37455	36322	1.118	.293	.011	.182
Group x Gender	1	134485	134485	4.013	.068	.039	.510
Error	99	33512	33512				

Note. Significant at $p < .05$.

Ethnicity

Table 8 shows the proportional representation of membership by ethnicity within the sample.

Table 8

Ethnicity Distribution of Students in Sample

Ethnicity	Group 1	%	Group 2	%	Total	%
Black/Afr Amer	5	14.3	15	22.1	20	19.4
Hispanic	9	25.7	23	33.8	32	31.1
Multiple	0	0.0	3	4.4	3	2.9
White	21	60.0	27	39.7	48	46.6
Total	35	100.0	68	100.0	103	100.0

Note. Percentages are within group percentages. The Multiple category indicates students who were reported as having more than one ethnicity.

Since course sequence contributes to the performance of students on the Algebra I EOC, the researcher looked at whether that effect may differ across ethnicity. A two-way analysis of variance tested the mean difference of scale scores between course sequences by levels of gender. Group 2 (modified course sequence) showed a significantly higher mean difference in scale scores from test to retest than Group 1 (traditional course sequence), $F(1, 96) = 3.169$, $p = .048$; however, ethnicity itself was found not to be a significant factor in the determining the mean difference of test scores on the Algebra I EOC as shown in Table 9. While the 32 Hispanic students in the sample had the highest mean scale score gain with 162.06 scale score points ($SD = 212.1$), followed by Black/African American at 148.9 ($SD = 206.8$), followed by students with multiple ethnicities at 123.3 ($SD = 175.5$), and White students at 124.7 ($SD = 170.1$), there was insufficient evidence to reject the interaction effect null hypothesis between course sequence and ethnicity $F(2,96) = .368$, $p = .068$, partial $\eta^2 = .008$, observed power = .108.

Table 9

Two-Way Analysis of Variance of Mean Scale Score Difference by Course and Ethnicity

Source	*df*	SS	MS	*F*	*p*	Partial η^2	Observed Power
Group (course seq)	1	112757	112757	3.169	.048	.032	.422
Ethnicity	3	18766	6255	.176	.293	.005	.082
Group x Ethnicity	2	26192	13096	.368	.068	.008	.108
Error	96	3415509	35578				

Note. Significant at p < .05.

Special Education

Table 10 shows the proportional representation of membership by Special Education status within the sample.

Table 10

Students in the Sample by Special Education Status

Special Education	Group 1	%	Group 2	%	Total	%
No	32	91.4	56	82.4	88	85.4
Yes	3	8.6	12	18.6	15	14.6
Total	35	100.0	68	100.0	103	100.0

Note. Percentages are within group percentages.

Since course sequence contributes to the performance of students on the Algebra I EOC, the researcher looked at whether that effect may differ across Special Education status. A two-way analysis of variance tested the mean difference of scale scores between course sequences by levels of Special Education status. Group 2 (modified course sequence) showed a significantly higher mean difference in scale scores from test to retest than Group 1 (traditional course sequence), $F(1, 99) = 3.509$, $p = .044$; however, Special Education status itself was found not to be a significant factor in the determining the mean difference of test scores on the Algebra I EOC as shown in Table 11.While the three Special Education students in the sample had slightly lower mean scale score gain of 117.5 (*SD* = 163.7) than did non-Special Education students at 145.0 (*SD* =194.3), there was insufficient evidence to reject the interaction effect null hypothesis between course sequence and gender $F(1,99) = .149$, p = .700, partial $\eta^2 = .002$, observed power = .067.

Table 11

Two-Way Analysis of Variance of Mean Scale Score Difference by Course and Special Education Status

Source	*df*	SS	MS	*F*	*p*	Partial η^2	Observed Power
Group (course seq)	1	121557	121557	3.509	.044	.034	.458
Special Education	1	28573	28573	.825	.366	.008	.147
Group x SpEd	1	5166	5166	.149	.700	.002	.067
Error	99	3429846	34644				

Note. Significant at $p < .05$*.*

Socioeconomic Status

Table 12 shows the proportional representation of membership by socioeconomic status within the sample.

Table 12

Socioeconomic Distribution of Students in Sample

Economically Disadvantage	Group 1	%	Group 2	%	Total	%
No	24	68.6	37	54.4	61	59.2
Yes	11	31.4	31	45.6	42	40.8
Total	35	100.0	68	100.0	103	100.0

Note. Percentages are within group percentages.

Since course sequence contributes to the performance of students on the Algebra I EOC, the researcher looked at whether that effect may differ across socioeconomic status.

A two-way analysis of variance tested the mean difference of scale scores between course sequences by levels of socioeconomic status. Group 2 (modified course sequence) showed a significantly higher mean difference in scale scores from test to retest than Group 1 (traditional course sequence), $F(1, 99) = 5.228$, $p = .024$; however, socioeconomic status itself was found not to be a significant factor in the determining the mean difference of test scores on the Algebra I EOC as shown in Table 13). While the 42 economically disadvantaged students in the sample had slightly higher mean scale score gain of 154.1 (*SD* = 201.5) than did non-disadvantaged students at 131.9 (*SD* =182.2), there was insufficient evidence to reject the interaction effect null hypothesis between course sequence and socioeconomic status, $F(1,99) = .004$, $p = .953$, partial $\eta^2 = .000$, observed power = .050.

Table 13

Two-Way Analysis of Variance of Mean Scale Score Difference by Course and Socioeconomic Status

Source	*df*	SS	MS	*F*	*p*	Partial η^2	Observed Power
Group (course seq)	1	182542	182542	5.228	.024	.050	.620
Econ Disadvantage	1	2381	2381	.068	.795	.001	.058
Group x Econ Dis	1	122	122	.004	.953	.000	.050
Error	99	3456797	34917				

Note. Significant at $p < .05$.

Summary

The purpose of this quantitative study was to investigate the impact of a modified mathematics course sequence to the students Algebra I EOC retest scores. A comparison was conducted between the difference of the test and retest scores on the Algebra I EOC taken by the two groups of students after instruction. The Group 1 students received instruction in Geometry and math models (the traditional mathematics course sequence) and the Group 2 students received instruction in Algebra II and math models (the modified mathematics course sequence). The results indicated there was a statistically significant difference in the performance scores among the two groups of students. The Group 2 students, or those who received instruction in Algebra II and math models, did better than the students who received instruction in Geometry and math models, Group 1. In conclusion, the null hypotheses, that there is no statistical difference in the mean difference in Algebra I EOC test and retest scores for students receiving instruction in a modified mathematics course (Group 2) than for students receiving instruction in a traditional mathematics course sequence (Group 1) was rejected. Therefore, the study results suggested the course sequence taken by students in Mathematics does contribute to the Algebra I EOC score; a result that is intriguing especially in light of the fact the course sequence was not a repeat of Algebra I itself.

CHAPTER FIVE

Summary of Findings, Conclusion, and Recommendations

In this study, the researcher looked at the effect of a modified high school mathematics course sequence on the improvement of Texas Algebra I EOC exam scale scores. The study involved first-year students who did not pass their first attempt on the exam. Students who did not pass the exam received instruction in one of two courses. Group 1 students followed a traditional mathematics course sequence which was Geometry and math models and Group 2 students followed a modified mathematics course sequence which was Algebra II and math models. The findings of this study postulate that the educational system schedules course sequences in mathematics that may work against students and frustrate them. Such an experience has the potential effect of discouraging students from future progress.

Little can be found to support the concept that the present study advocates which is to move Algebra II before Geometry. In one subtle article, W. I. Thompson (2005) works to present an argument that Geometry should come before Algebra I. W. I. Thompson's (2003) basis of this line of thought was aligned with an earlier study presented by Snow (2003), in the article "Charting the Domains of Human Thought." Snow shows a new idea of how the human thought is divided and aligned into different stages. W. I. Thompson (2003) drew inspiration from four steps–psychosocial, practical, abstract, and temporal–to show a similar division of cognitive stages that math students would go through and made the argument that Geometry should go before Algebra.

However, this article was a one-of-a-kind in that the researcher was unable to locate any additional items that apparently took a research-based stance on the alignment of one mathematic course before the other course. Nichols (1988), math teacher at Shawnee Mission Northwest High School, shared her opinion within an article that supported Geometry before Algebra II. She immediately acknowledged that this topic had been a source of teacher debate for years. While her stance is clearly aligned with Geometry coming before Algebra II, her research is not scientific and does not lend credibility to the alignment. The research, indeed, is utilized to fit around her personal teaching position of where the courses should align (Nichols, 1988).

L. C. Miller and Mittleman (2011) conducted a study to examine math and science curriculum. Their research findings did not dictate a particular course sequence but rather was focused on what math and science those students should take for the three courses. For mathematics, those three courses are Algebra I, Geometry, and Algebra II. L. C. Miller and Mittleman's research focused on the idea of a *pipeline* of mathematic courses to take each year within a school, which would help students not miss any opportunities such as college admission or scholarships. The overarching point that related to mathematics was it is not so much which order courses are taken, but that students are prepared for success if they get through Algebra II.

Summary of Findings

In a large Texas high school (2,229 students enrolled in Grades 9-12), 178 students failed the state's official exam—Algebra I EOC that measures student learning concluding a year of studying Algebra I—out of 1,629 enrolled in the course. As an attempt to help these students pass a scheduled retest of the Algebra I EOC, the school administration offered an alternative course sequence to these students to improve their understanding of mathematics. Students voluntarily chose which of the two course sequences they wanted to take. These two groups of students became the subjects of this ex post facto study. In the sample population, 35 students (labeled as Group 1 in the study) elected to take the traditional course sequence and 68 students (labeled as Group 2) elected to take the modified course sequence. At the conclusion of their respected courses, each group took a retest, equated version of the Algebra I EOC in an officially scheduled administration.

The current researcher analyzed the quantitative data produced by the results of the test scores and demographic identity of the students in the sample. The data was analyzed with the intent of determining whether there was a course sequence effect on the Texas Algebra I EOC exam scale score gain when comparing the first administration of the Algebra I EOC to the retest administration.

Therefore, the statistical procedure used to compare the two groups' performances on the exam was an independent samples *t*-test with a null hypothesis defined as no difference in mean gain scores between two groups of students distinguished by the course sequence taken by each group. The results of the *t*-test indicated there was a significant difference in mean gains for test scores between the two groups, $t(101) = -2.48$, $p = .015$; $d = .56$.

Therefore, the null hypothesis was rejected to accept the alternative hypothesis that students enrolled in the modified course sequence, $M = 173.47$; $SD = 182.54$, outscored students enrolled in a traditional sequence, $M = 77.83$; $SD = 189.95$. The effect size statistic ($d = .56$) indicated a medium effect size according to Cohen's (1988) criteria; therefore, the current study results should be of interest to educational researchers and practitioners within public schools.

Discussion

The discovery of a statistically significant finding in education warrants healthy discussion to leverage the value residing within the findings. One of the more important questions to contemplate is the question of why. Why did the group of students who took Algebra II outperform the students who took Geometry? A reasonable answer to this question would apply the arguments framed in Chapter 2 of this study. Earlier, the researcher described the current landscape of research in education as developing the notions of course content impacting student learning, reform leading to higher expectations in critical thinking among students, and the pressure applied by accountability measures for students to perform on more analytical high-stakes tests. Applying the confluence of these concepts to the question, the researcher postulated the students in the Algebra II group matured in their analytical thinking skills more than the students in the Geometry group. This indicated the expectations of the Algebra II course content itself, may likely provide more frequent and more substantial opportunities for students to engage in analytical thought than the content in the Geometry class. The researcher warns that the expectations of learning the course content should not be confused with the discipline of study in algebra or geometric reasoning. It may specifically be the course's content and assessment experiences that are different from one another.

The researcher will now discuss specific implications of this study's major findings for practicing educators relative to three prevalent issues present in today's schools. In particular, the researcher discusses the study's relevance to school administrators and teachers assisting students to overcome negative perceptions of mathematics once failure is experienced; then applications for instructional treatment is discussed, followed by a discussion on the pros and cons of retesting and its effect on student learning, and measurement interpretation. Afterward, the discussion transitions to implications the study has for research as well as addressing observable limitations of the study that naturally arise from a study using educational data that describes children.

Discussion on the Implications for Practice

The practical significance of this study is dependent on the subject matter and if the size of the effect is large enough to be relevant to the research questions being investigated. In the example of this study, the difference in mean scale score gain between the two groups was statistically significant. This observation can be paired with the observation that the effect is considered a medium effect (reference) which should make the study of important interest to researchers and practitioners.

The methodology in this study utilized tools and procedures that are very practical for local school systems to replicate. Software that conducts *t*-testing and ANOVAs are economically feasible for most school district's budget. Additionally, most school districts readily have the data available for replicating this type of study. Thus, school districts can readily investigate innovative course sequences to determine if there are resulting differences among those sequences on interval measures such as EOC tests in Science, English Language Arts, and History.

School systems are often rich in the amount of data they have stored in their data warehouses. However, many school administrators do not leverage these data to investigate innovative solutions to issues they are charged with resolving at the local school level.

Discussion on Overcoming Negative Perceptions of Mathematics

One issue often perplexing teachers and administrators is how to help students overcome a negative perceptions of mathematic and testing. Levels of anxiety, fear, and disdain are often an outcome of failing a high-stakes exam like the Texas Algebra I EOC. There is evidence indicating the importance of overcoming the initial failure of a high school course in order to advance toward higher courses later in school (Weiss, 2001). Therefore, it is important for school officials to consider a course sequence that gives a student an opportunity to mitigate the effects of failure by experiencing academic growth within the same subject of study (Weiss, 2001).

Math anxiety interrupts on-going activities of working memory, which makes mathematical performance more time consuming, and less accurate (Ashcraft, 2002). The effects are intrusive thoughts of meagerness and failure results in a cycle of math avoidance. Instructors can have a large hand in plummeting math anxiety in their classrooms.

The results of this study's suggest that practitioners may find a viable solution to resolving the negative effects of failing an exam by offering the student a different curriclum pathway than what the administration may feel is producing to high a number of failures. When EOC tests were first introduced, most Texas schools did not have a built-in remediation opportunity for students who failed the Algebra I EOC test offered during the school day. If accelerated instruction is not provided to high school students during the day, the challenge for school leaders would be to develop strategies to ensure students come before or after school to receive that instruction.

However, a modified course sequence should be supplemented with data from the failed test administration as well as contining to collect data from effective evaluation and grading practices so that the modified course experience is one that helps restore the learner to a level of confidence. That is, it is significant for students to appreciate that they must learn from their errors (Wormeli, 2006).

Discussion on Applications for Instructional Treatment

Research on retesting isolates the opportunity to retest into educational backgrounds that presently is comprised of only post-secondary schools, and institutional settings, such as places of employment and state offices for licensing and certification. The retesting research investigates how practice (or retest) influences the performance on the retest, variances in retest time intervals, types of tests, demographic differences, types of retesting situations, and retest performance. Hausknecht, Halpert, DiPaolo, and Gerrard (2007) studied the reason why people chose to retest in a 2007 meta-analysis. The authors found if candidates presumed they would have a significant increase in opportunity for gaining admission or promotion to a college or program that they anticipated, they were more likely to retest. Eventually, students may also retest if they believe that a higher score may significantly increase their chances of an expected outcome. Finally, Hausknecht et al. also concluded if candidates are convinced they have the ability to score highly the second time around to realize a particular outcome, they might choose to retest.

Discussion on the Concept of Retesting

External difficulties, such as lack of parental involvement, SES, and obligations, that consume a significant time duration for students to finish their homework takes priority in the educational environment (McConnell & Doolittle, 2005). Educators can be involved in solving some of these challenges by creating multiple opportunities for student success. Permitting students to re-take the EOC assessment is a significant way of promoting learning. Learning can be sustained by inspiring students to reflect on an unsuccessful trial for an evaluation and to come up with a good strategy of action to do better the next time (McConnell & Doolittle, 2005).

Retesting in an educational background is described as having an opportunity to re-sit any summative assessment (McConnell & Doolittle, 2005). Renowned researchers describe summative assessment or EOC assessment as examinations that students are given after learning has taken place by their teachers or the examining body. A summative test can include performance, written, oral, forced response, and project-based assessments. Investigation on retesting separates the opportunity to retest into educational backgrounds, which presently includes only post-secondary institutions; and organizational settings such employment places and governmental offices for licensing and certification. In several studies, researchers have continued to examine how retesting influences the performance on the types of retesting situations, variances in retest time, retest assessments, intervals, demographic differences, types of tests, and retest performances (Catanzano & Wilson, 1977; Hausknecht et al., 2007; Juhler, Rech, From, & Grogan, 2010).

Hausknecht et al. (2007), while studying the reasons why people prefer to retest in a meta-analysis, concluded many people were willing to retake any test because of the increased prospects of gaining admission into a college or program of their choice or for promotion. Hausknecht et al. also revealed students were more likely to retest when they believed that a higher score would increase their chances of an anticipated outcome. Finally, the researchers found if the candidates thought there was the ability to earn a higher score the second time for a particular objective, they would choose to retest.

Juhler et al. (2010) studied the outcome of optional retesting on college students' success in an individualized algebra course. Mathematics need learners to master specific models that are sequential in nature and thus makes sense for a student to learn the concepts before moving on to the next concepts. These authors postulated that retesting would be an efficient strategy to inspire students to relearn information to a mastery level in case they had not understood the concepts well the first time they were taught. Juhler et al. created a learning setting that any student scoring a B or less on every test in the course would be obliged to retake a retest. They established that there was a substantial improvement between the initial test and the retest for the 91% of the students that were sampled for the exercise.

Catanzano and Wilson (1977) examined three different retesting scenarios while studying the effects of retesting on students' performances. The first sample that Catanzano and Wilson used was a case where the students did not have an option to retest. In the second scenario, the students were given an opportunity to retest, and the third situation obliged the students to retake a test if they scored less than 80% on the initial test. The study was run by an individual teacher who was teaching three different seventh-grade science classes. Catanzano and Wilson established that there were significant variations in achievement scores in the compulsory and optional retesting settings in the free-response questions and re-exam post-tests only. Nevertheless, the two scholars noted there were no significant differences in test anxiety amongst the three different testing likelihoods. Students who were in the optional retesting settings had considerably better attitudes than the compulsory testing scenario. The mandatory retesting students were expected to have high test anxiety, but eventually achieved better results academically than those students in the optional and no retesting settings.

Practice effects have been deemed by the researchers to play a significant role in the retesting environment (Catanzano & Wilson, 1977). Since the students in the obligatory and optional retesting scenarios had been subjected to more free-response questions, they did better on the post-tests. Catanzano and Wilson also concluded that voluntary retesting has a higher effect on a positive attitude than the obligatory situation. Optional retesting is also preferred since there is less test creation and tracking on the part of the teachers.

Permitting students to re-take a summative assessment is a superb way of promoting learning, as students get encouraged to reflect on a futile attempt at evaluation. Notably, educators come up with a plan of action that is aimed at enabling students to do better the next time (Butler & McMunn, 2006). Retesting in an educational background is used to refer to a scenario where a student is given an opportunity to retake any summative assessment. The summative evaluation, also known as the EOC test, is a test that is usually given to students in the learning process, such as after the learning target has been reached. Summative tests can include written, performance, forced response, oral, and project-based assessments.

When considering summative, campus-based assessments, schools may require additional standards for students to complete which may include tasks such as a student must correct errors in the original assessment, complete all missing assignments, and write a review of the student's original performance on the first evaluation, all before they take the retest. These criteria are in place to make students understand that it is essential to gain more knowledge when the expectation is to improve on a retest (Hausknecht et al., 2007). Lastly, teachers must edit the questions on the retest to avoid a situation where the same test is given for the second time. Otherwise, the effects from taking a retest are less likely to skew the assessment score.

A fruitful implementation of a modified mathematics course sequence on the state EOC exam after the high school math classes also necessitates a positive teacher mindset, professional advancement, and mentoring for approaches to be integrated successfully. Educators who differentiate learning should focus on various activities, permitting student choice, encouraging personal connections to the learning, and stimulating all students to achieve high expectations. Psychological findings, which focus on testing and educational policy, should enabled the education stakeholder to transform instructional principles and views of individuals towards the development of curriculum.

Conclusions

With national emphasis on college and career readiness, a significant number of states are enacting comprehensive strategies to make certain that high school graduates are equipped to take credit-bearing courses toward a postsecondary degree, to lessen the need for remedial education, and to improve on college completion rates. Equally, teachers in the current era are faced with an inclusive classroom, where all students are supposed to be challenged mathematically. Meeting this task is difficult but can be attained using a modified sequence in teaching mathematics that puts emphasis on building on prior mathematics concepts and learning. Together with state and federal student accountability, Texas runs a yearly Performance-Based Monitoring Analysis System to monitor the effectiveness of the program in all school districts in the state. Notably, individual students' performance is centered on state assessment scores.

Curricular change can be done through the creation of a positive campus climate dedicated to individual student achievement. Teachers should be obliged to assess their existing instructional practices, to analyze the students critically who are benefitting from current strategies, and to decide how mathematics course sequence can be modified to meet specific needs. For students to excel in Algebra and any other math related courses, there is a need for teachers to adopt a pedagogical approach to teaching which often necessitates veteran and greenhorn teachers changing their mindset toward structured learning. The curricular changes can be impactful. However, there is a need for overwhelming support from administrators and district personnel.

Similarly, continuing staff development is imperative for effective applications of differentiated instruction for the various mathematics courses since individual students are endowed with unique IQ levels, and students are influenced by external factors proportionately. Teachers should also get encouraged to challenge students to think critically, persistently assess their learning, and collaborate with parents and colleagues for success. Implementing different instructional practices usually necessitates productive, ongoing staff development. Trainin and Swanson (2005) emphasized, "Staff growth is reflective, up-to-date, connective, diagnostic, application-oriented, problem-focused, collaborative, quality-concerned, helpful, constant, and differentiated" (p. 271). Professional training is thus indispensable to empower teachers and offer them a pathway for successful implementation. Established support systems should also assist teachers to become productive, valued members of the educational setting.

Decisive implementation of differentiated instruction by teachers also will help instructors to allow students to make their choice correctly, promoting personal influences to the learning, and motivating all students to achieve high expectations. When teachers commit to ensuring quality instruction is offered and, at the same time, guaranteeing students are sufficiently prepared to meet a minimum state standard fruitfully, there is a higher probability that students will excel in all mathematics courses. Conclusively, all education stakeholders must be challenged to help students achieve their maximum excellence in academics even though, new obstacles will always present themselves in the school sector on a continuing basis. Discovering effective ways of enabling the 21st-century students to excel universally should remain a priority and a motivating force of today's education system.

Limitations and Suggestions for Future Research

In consideration of the significant finding in this study that course sequence contributes to the outcome of student learning in mathematics, more research is needed that applies this concept to other subject areas than mathematics. One of the challenges in creating modified course pathways for students to follow in high school is the paucity of literature on this specific question. Most of the current research relative to this topic is performed on traditional courses sequencing (i.e., Algebra I–Geometry–Algebra II).

This study also has implications for the research on assessment practices within education. Conventionally, it is believed students excel at higher levels when they are engaged in self-assessment exercises and it is also believed that a critical component of effective formative assessment includes the teacher's understanding of reaching students through the different learning abilities of the students. The current study revealed that simply tracking the performance of students in an EOC exam and their subsequent mathematics performance was not enough for high school students to gauge what they need to do to improve. Thus, additional research on the most significant ways for students to excel in Algebra courses and other related math courses would assist teachers to implement systemic assessment practices that would work for individuals within a classroom setting.

In light of a strong tradition within educational research to investigate demographic variables as probable effects on nearly all educational outcomes (e.g., achievement, graduation rates, self-efficacy, subject matter interests), the non-statistically significant finding of interaction effects of course sequence with gender, ethnicity, socioeconomic status, and Special Education status, respectively, should be of high interest to researchers and practitioners. This study may raise the question of whether gender gaps and other long-time population category gaps are narrowing. Additionally, this study raises the question of whether some course sequences have the effect of narrowing previously existing gaps among levels of population variables.

In this study's sample, it can be argued that the White population was over-represented since the ratio of White students to the other ethnicities was 3:1. The sampling within this study was bound by the enrollment choices of the student combined with their independent decision to retake the Algebra I EOC. Additionally, economically disadvantaged students, 43%, were slightly under-represented in the sample. Therefore, replication of this study within schools in different geographies may allow for a more substantive analysis of any interaction effects due to population demographics.

The current researcher concentrated on a single school in Texas. Consequently, the findings may not be generalized to all students within Texas or outside of Texas. Further research studies should include, but not limited to, a larger sample of Algebra I students from a variety of institutions and on other high school mathematics related courses and other subjects. Additionally, inquiry exploring course sequences other than the alternative sequence focused on in the current study would contribute significantly to the endeavor to give students the best opportunity possible for success.

Recommendations

The number of states that have put in place a plan to implement or have already applied compulsory exit exams is a potentially changing number. Three additional states−Connecticut, Pennsylvania, and Oregon−have considered implementing high school EOC tests; state officials in Oregon and Pennsylvania chose to permit the use of alternative assessments standards. The officials within Connecticut have not reached any final decisions in the past year. In the same regard, 26 states are currently suppressing or planning to withhold high school diplomas due to students' scores in state-mandated exams.

As the number of statewide EOC tests continues to increase, along with an expansion of their role, development, accountability, and implementation, decisions are more significant than ever. This process begins with stating how the assessment fits into a dependable theory of action that explains how all elements of the system could work together to promote the desired educational outcomes. Once explained, policymakers should be motivated to study the full range of policy, technical, and practical deliberations carefully that are related to the intended purposes and uses of EOC exams, many of which have been discussed in this document.

The recommendations of this researcher are that the instructional leaders should be directed to research and further explore the differences in performances involving retesting from males and females, as well as other sub-groups based on factors such as SES factors.

There is also a need for teachers and the related education stakeholders to develop the love of learning in every student, which will in turn drive decisions about how to improve schools. Not all outcomes of school improvement efforts can just be whittled down to numbers. School improvement efforts should include strategies that encourage the ability in students to reach the goals to improve school performance. In conclusion, a progressive monitoring and evaluation plan should go along with the implementation to support system improvement. As a result of that work, states have the potential to become positioned in leveraging the results of EOC tests and mitigate unintended consequences associated with high stakes testing.

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