ABSTRACT

Automated Systematic Generation and Exploration of Flat Direction Phenomenology in Free Fermionic Heterotic String Theory



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Any good physical theory must resolve current experimental data as well as offer predictions for potential searches in the future. The Standard Model of particle physics, Grand Unified Theories, Minimal Supersymmetric Models and Supergravity are all attempts to provide such a framework. However, they all lack the ability to predict many of the parameters that each of the theories utilize. String theory may yield a solution to this naturalness (or self-predictiveness) problem as well as offer a unified theory of gravity. Studies in particle physics phenomenology based on perturbative low energy analysis of various string theories can help determine the candidacy of such models. After a review of principles and problems leading up to our current understanding of the universe, we will discuss some of the best particle physics model building techniques that have been developed using string theory. This will culminate in the introduction of a novel approach to a computational, systematic analysis of the various physical phenomena that arise from these string models. We focus on the necessary assumptions, complexity and open questions that arise while making a fully-automated flat direction analysis program.

Automated Systematic Generation and Exploration of Flat Direction Phenomenology in Free Fermionic Heterotic String Theory

by

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## ABBREVIATIONS

ABK/KLT Antoniadis-Bachus-Kounas/Kawai-Lewellen-Tye (p. 47)

B-L baryon minus lepton number (pp. 24, 34)

BV boundary vector (pp. 45, 47–49, 69)

CKM Cabibbo-Kobayashi-Maskawa (p. 2)

cm centimeter (p. 12)

CP charge-parity (pp. 2, 37)

EFT effective field theory (pp. 2, 13, 25, 27, 28, 36, 51, 57, 64, 90, 104)

EM electromagnetic (pp. 2, 8)

EW electroweak (pp. 3, 6, 9, 25–27, 29, 34, 36, 38, 40, 102)

FF Free Fermionic (p. 59)

FI Fayet-Illiopoulos (pp. 33, 35–38, 63, 66)

GeV giga-electron Volt (pp. 2, 3, 9, 10, 27, 31, 36)

GSDSW Green-Schwarz-Dine-Seiberg-Witten (pp. 33, 35)

GSO Gliozzi-Scherk-Olive (pp. 19, 20, 46, 48)

GUT grand unified theory (pp. 3, 7, 9, 22, 24–26, 32, 34, 44, 64, 66, 69, 70, 73, 75–77, 85)

IP Integer Programming (pp. 84, 104)

LAPACK Linear Algebra PACKage (p. 67)

LDS local discrete symmetry (pp. 35, 37)

LEEFT low energy effective field theory (pp. 23, 32, 45)

LM left mover/moving (pp. 16, 20, 42–44, 46, 47, 49, 50, 54, 55)

LP Linear Programming (pp. 84, 104)

LSP lightest supersymmetric particle (p. 8)

MSSM minimal supersymmetric standard model (pp. 5–7, 22, 23, 36, 37, 42)

NA non-Abelian (pp. 35, 39, 60, 63, 66, 69–74, 82, 85, 102, 103)

NAHE Nanopoulos-Antoniadis-Hagelin-Ellis (pp. 22, 38, 47–49)

NPHC Numerical Polynomial Homotopy Continuation (p. 101)

NS Neveu-Schwarz (pp. 20, 52–54)

QCD quantum chromodynamics (p. 2)

R Ramond (pp. 5, 12, 14, 20, 25, 37, 50, 52–54, 57, 61, 83, 84)

RG renormalization group (p. 3)

RM right mover/moving (pp. 16, 20, 42–44, 46–48, 54–56)

RNS Ramond Neveu-Schwarz (pp. iv, 18, 19, 49, 52, 53, 69, 70, 74)

SM standard model (pp. 2–6, 8, 9, 22, 24–26, 29, 32, 34, 46, 48, 64, 66, 69, 70, 73, 75–77, 85)

SUGRA supergravity (p. 13)

SUSY supersymmetry (pp. iv, 4–10, 18–21, 24, 25, 27–31, 34, 35, 39, 40, 44, 49, 50, 54, 57, 58, 61–63, 65, 75, 76, 81, 90, 91, 95, 96, 98–100, 102, 105)

SVD singular value decomposition (pp. v, 83–87, 103, 105)

TeV tera-electron Volt (pp. 6, 31, 35, 38, 40, 62)

UV ultraviolet (p. 7)

VEV vacuum expectation value (pp. 10, 25, 27–29, 31, 33, 35–39, 41, 57–59, 61–66, 68, 70, 71, 73–77, 80, 81, 83–85, 88, 89, 91, 95–99, 101–105)

WCFFH weakly-coupled free fermionic heterotic (pp. 44, 45, 47, 48, 100)

WCFFHS weakly-coupled free fermionic heterotic string (pp. 35, 42, 59, 105)

# CHAPTER ONE

## Introduction

Physics has long sought the ability to traverse pathways between the polar ends of theory and experiment. In this age of large hadron colliders and high energy cosmic ray experiments, it is necessary to understand various findings within the context of a theoretical framework. There are almost as many ideas as there are physicists and many, like string theory and quantum gravity, are beyond our experimental grasp. The work of string phenomenology is to provide a narrowing of the gap between high energy string phenomena and experimental limits. Specifically, there is a need to sift through the incredible wealth of string models to find those that may represent our universe during the early stages of its existence.

This work summarizes the need for deeper and more fundamental theories and offers a useful tool in filtering various string models. The computational algorithms presented later represent a highly extensible set of code meant to help determine many of the free parameters within a given string model. Building upon previous work, we show that it is possible to glean high energy phenomenology with a fully-automatic and systematic software package and present this as a starting point.

This code will be extended to perform more in-depth studies of both hidden and observable matter. It will also be implemented with current model generating software which will allow automatic analysis of potentially interesting models. Not limited to string theory, future revisions will allow for the consideration of generic particle physics models.

### *1.1 SMall GUT Problems*

The standard model (SM) is a beautifully descriptive theory but in terms of naturalness it is somewhat lacking in its self-predictiveness[1]. The theory requires the measurement of many parameters:

- 9 fermion masses (6 quarks + 3 charged leptons)
- 3 flavour-changing Cabibbo-Kobayashi-Maskawa (CKM) mixing angles and a phase
- $\alpha$, the electromagnetic (EM) coupling constant
- $\alpha_s$, the strong nuclear force coupling constant
- $g_W$, the weak nuclear force coupling constant
- $m_Z$, the mass of $Z^0$
- $m_{W^\pm}$, the mass of the $W^\pm$ bosons
- $m_H$, the mass of the Higgs boson.

There are other naturalness questions that arise such as why the gauge structure is: $G_{\text{SM}} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. In view of the scale of electroweak symmetry breaking, one may wonder why the $\approx 125$ giga-electron Volt (GeV) mass of the Higgs boson is not comparable to the Planck mass ($1.2209 \times 10^{19}$ GeV), which is warranted by quantum contributions from almost all detectable particles. Other problems, as yet unanswered, are the lack of charge-parity (CP) violations in quantum chromodynamics (QCD) and the cosmological constant problem. The former issue arises because of the potentially CP-violating terms that appear in the QCD Lagrangian. These terms seemed to be fine-tuned in such a way so to not break CP symmetry. The latter issue comes about when the quantum vacuum is presumed to be the source of the cosmological constant. When calculated in an effective field theory (EFT) framework the cosmological constant is predicted to be a coefficient of $\mathcal{O}(1)$ applied

[1]The question to ask is, how much of theory must be determined by experiment. For our purposes, a *natural* theory would be one whose constants and couplings are somehow determined within its own framework.

to the quartic power of the Planck mass. This is hugely in error to the observed coefficient value of $10^{-120}$.

These points illustrate that our understanding of the universe is far from complete. The patchwork of existing theories is not sufficient to be able to describe reality. This has motivated many researchers to investigate ideas that claim to be more fundamental and robust. This can be seen by the grand unified theory (GUT) approach to particle physics. Enlisting the idea of electroweak (electroweak (EW)) unification, GUT models are proposed as a higher energy representation of the lower energy $G_{\rm SM}$. As energy scales increase, electromagnetism and the weak nuclear force unify. The strong nuclear force merges with the EW at even higher energies ($\approx 10^{16}$ GeV) and it is hoped that gravity itself will eventually join the cadre near the Planck scale, which is thought to be the realm of quantum gravity. Independent of the form of this more-fundamental theory, one should expect a natural prediction for the finely-tuned values of the SM parameters.

Furthermore, as we scale up to higher energy physics, the following assumption can be tested: that the physical quantities that are measurable at one scale are connected, through mathematical extrapolation, to others which exist at higher energy scales. This need not be the case but, without plausible alternatives, unification schemes have persisted and become the natural methodology. The way to calculationally connect these different energy regimes is through the use of renormalization group (RG) equations which determine how some parameters evolve in order to keep phenomenologically important quantities constant. At this stage it is common to describe our physics using perturbative methods. This implies that our models are weakly coupled even to the highest theoretically-allowed energy scales. This also should make us pause to ask if there is not another approach. But, like RG flow, there does not yet seem to be another way.

## *1.2 SUSY*

Very high mass particles are predicted to be directly indiscernible in low energy physics except through self-energy diagrams of fundamental scalar particles. That would mean for particles, like the Higgs, large mass contributions could appear. This could be rectified through various renormalizations of the model's parameters, but doing so threatens the naturalness of our theory. We could be free from this problem if there were no fundamental scalars. This would require that those scalars be composites of yet smaller particles. This is also not favorable since it leaves us without any guidance on what form these constituents take. Instead, this so-called mass hierarchy problem can be solved by supersymmetry (SUSY), which is the famous idea requiring the existence of a symmetry that pairs fermions and bosons and extends the Poincaré Algebra. This creates fermionic (anticommuting) coordinates which are partners of the usual bosonic (commuting) spacetime coordinates. The new collection of coordinates is called the 'superspace'. The generators of that symmetry, which are Weyl spinors, are designed to transform superpartners into one another:

$$\begin{aligned} Q|Boson\rangle &= |Fermion\rangle \\ Q^{\dagger}|Fermion\rangle &= |Boson\rangle. \end{aligned} \tag{1.1}$$

The operators in Eq 1.1 are fermionic with spin-1/2 and must satisfy a host of commutation and anti-commutation relations because they will be operating on both bosons and chiral fermions. One side effect of this symmetry, in addition to the introduction of extra particle content with humorous 's'-prepended or 'ino'-appended names, is that superpartners must have the same mass as their original SM counterparts because they exist in the same supermultiplet. Spin is a quantum charge under the spacetime Poincaré group and SUSY partners (spartners) will have different spins. They will also live in the same gauge group representations, thereby exhibiting identical electrical, isospin and color charges since external spacetime symmetries commute with internals ones.

Of particular importance is the resolution of the Higgs mass hierarchy problem [1]. The Higgs boson couples to massive particles through self-energy diagrams which can create quadratic divergences. Controlling the contributions to this mass is of great importance. It turns out that the scalar field and fermion contributions to the Higgs mass quadratic divergence will exactly cancel if there exists a supersymmetry connecting fermions to bosons. Other observational effects of SUSY include the existence of Ramond (R)-parity. This discrete symmetry gives a charge of +1 to sparticles and 0 to SM particles. Since interaction vertices must have an R-charge of 0 *mod* 2, spartners must always appear in pairs. This restriction limits fast proton decay and requires that sparticles will not appear as tree-level intermediate states[2]. These weak-scale SUSY effects provide particle physics constraints and help maintain the SM-like qualities of SUSY.

### *1.2.1 Minimal Supersymmetric Standard Model*

The minimal supersymmetric standard model (MSSM) is the simplest extension of the SM to include SUSY. Due to the extra particle content, the number of parameters in our theory jumps beyond 30. Fermions are paired with spin-0 sfermions in chiral supermultiplets and gauge bosons are paired with Majorana spin-$\frac{1}{2}$ gauginos. The only required content beyond that which is directly related to the SM are two complex Higgs doublets (and their SUSY partners). The two lightest of these four particles becomes the SM Higgs ($h_{\mathrm{up}}$ and $h_{\mathrm{down}}$). Altogether, three become massive and decouple from the model as spin-0 modes of the Weak bosons: $W^+, W^-, Z^0$.[3]

When taken from low energy experimental data, the low-to-high energy gauge coupling unification of the MSSM seems to imply the 'big desert' scenario in which no new physi-

[2]In addition to R-parity, some mechanism for doublet-triplet splitting will be necessarily provided by a more-encompassing theory.

[3]These bosons when massless, like the photon, have two degrees of freedom. Upon gaining mass they receive a third in the form of the Higgs scalars.

cal states emerge between the EW and Planck-level unifications scales. Relatedly, one may wonder why no new content appears or how SM physics derives directly from Planck scale physics. Answering these questions requires an extension from the MSSM to a more fundamental theory. One could consider string or M-theory as the law in which the MSSM is embedded, but that requires both the string and unification scales to coincide during extrapolation into the strong coupling regime [2]. This does not seem to be the immediate case, but there has been work to try to bring the scales, which differ by a factor of 20, into agreement [3].

### *1.2.2 SUSY Problems*

With exception to quantum mechanical spin, supermultiplet elements must possess the same quantum number and masses. Therefore, SUSY partners should have been visible long ago. To date, there have been no observations of gluinos, photinos or selectrons, all of which would be readily seen in detectors since their SM partners have long been visible. This forces SUSY to be a symmetry that is broken at some energy scale above which is currently accessible. However, the dimensionless couplings that relate partners to superpartners must be unchanged by the SUSY breaking, otherwise the hierarchy problem would once again arise. That would make our supersymmetric extensions trivial. Therefore, SUSY must be broken 'softly' through additional effective Lagrangian terms. This allows the theory to be supersymmetric at its core but possess a vacuum which is not invariant under SUSY (softly-broken SUSY). It is the presence of these additional soft mass terms which allow for superpartners to gain additional mass thus making them currently unobservable. However, it is believed that the lightest superpartners will have mass just above that of the EW tera-electron Volt (TeV) scale.

In appealing to naturalness, soft SUSY breaking requires many unknown parameters unless the specific cause of SUSY breaking is known. This causes researchers to work in a model-dependent framework. Therefore, a SUSY breaking scheme is sought, one that will allow for model-independence with calculable effects resulting from just a few parameters. This beckons the search for a theory that subsumes the MSSM.

#### *1.2.3 Types of SUSY Breaking*

There are many approaches to SUSY breaking:

1. Spontaneous
2. Soft (or explicit)
3. Dynamical
4. Fayet-Iliopoulos

In the case of spontaneous SUSY breaking, the theory's Lagrangian is manifestly supersymmetric but its vacuum state solution is not. This formalism allows the vacuum state to retain the positive benefits of the supersymmetric theory. The trigger for SUSY breaking in this scenario is non-zero vacuum energy: $V_0 = M_{\mathrm{SUSY}}^4$. When broken, the supersymmetric charge becomes global and a Goldstino is produced. For Supergravity theories, this Goldstino is consumed by the gravitino (making the symmetry local) thus creating the scale for SUSY breaking [4].

In the typical approach to soft SUSY breaking, the presence of additional low-energy superpotential terms represent our lack of knowledge concerning the overarching, ultraviolet (UV) complete theory. These terms appear at low energies in such a way so as to not dictate, too strongly, the form of the UV. The inherent ignorance as energies are scaled from low to high is parameterized by the soft terms.

Dynamical SUSY breaking was first introduced by Witten [5]. This form of spontaneous SUSY breaking requires some very small effect to, quite naturally, introduce the scale of SUSY breaking below some cutoff (such as the Planck or GUT scales). This scale can be introduced as an exponential: $N_{\text{SUSY}} = \Lambda_{\text{cutoff}} e^{-c/g\Lambda^2_{\text{cutoff}}}$ [4] which is much less than the cutoff scale. In this scenario, the hierarchical scales of the various gauge forces arise naturally.

Due to the higher energy scale at which superpartners would appear in interactions, the lightest supersymmetric particle (LSP) will be stable and will possess a large mass. This has spurred cosmological cold dark matter investigations since the LSP must be without electric and color charge. These characteristics are chief among the defining features of dark matter which is known to interact gravitationally with baryonic matter but remains unseen in the observable spectral ranges of EM. Dark matter may be charged under the weak nuclear force due to the incredibly short range of flavor interactions.

### *1.3 Supergravity*

As with the SM, SUSY parameters can not be precisely calculated from first principles. Additionally, there are many new parameters left undetermined[4]. This forces us to consider a theory that envelops and predicts SUSY models. As described in the last section, SUSY may be broken softly and to do so would require it to be a local symmetry. Additionally, SUSY as a global[5] symmetry leads to unrealistic phenomenology [6]. Local SUSY, however, is much more tenable and requires the presence of gravitational interactions. The inclusion of the spin-$\frac{3}{2}$ gravitino reformulates SUSY into a local supersymmetric theory. The gravitino is gauge mediator of these local SUSY interactions and gains mass when it absorbs a massless goldstino field. This hungry gravitino mechanism sets the scale for and drives the mass splitting between SM particle and sparticle pairs. This can sweep SUSY particles 'underneath the rug' by driving them to energies above current observational limits.

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[4]The SM has on the order of 20 parameters while SUSY has 100+. These include spartner versions of the SM couplings, masses and mixing angles.

[5]Globalness implies that symmetries remain constant throughout spacetime. They are described in terms of conservation laws but do not introduce a corresponding force. Local symmetries, which do vary from point to point in spacetime, also require conservation laws but are described in terms of boson gauge fields. These fields introduce a corresponding force to the theory.

Relatedly, radiative EW symmetry breaking is a natural result of supergravity theories wherein the Higgs mass-squared goes from positive to negative at lower energies. This is a signal for EW breaking and makes possible models with a sufficiently heavy top quark ($> 60$ GeV). It also keeps all mass-squared values for charged and color-charged particles positive.

This robust supergravity theory contains the parameters necessary to break SUSY: the Kähler and gauge kinetic functions. These functions determine the scaled spectrum of all SUSY particles. For example the gravitino mass, $m_{3/2}$, is related to the Kähler function, $G$, through:

$$m_{3/2} = e^{\langle G \rangle}, \tag{1.2}$$

where $G = K + \ln |W|^2$, $K$ is the Kähler potential and $W$ is the superpotential. The Kähler function then determines the super-scalar masses[6] through Eq 1.2 while the derivatives of the gauge kinetic function yield the gaugino masses. These functions, therefore, constrain the numerous SUSY parameters that are left undetermined in SUSY-only theories.

In addition to potentially solving SUSY breaking, supergravity may be able to provide a source for the cosmological constant, $\Lambda$, through the vacuum energy ($V_0 = \Lambda^4$). $V_0 \leq 10^{-31} M_{\text{Planck}} \approx 10^{-3}$ eV may indicate the presence of some approximate symmetry.[7] The so-called 'no-scale' supergravity models try to emulate this by investigating forms of the Kähler potential which cause the scalar potential, $V$, to yield $V_0 = \langle V \rangle = 0$. SUSY breaking is naturally included in these models through the gravitational mediation of SUSY breaking in a hidden sector to the observable (GUT or SM) sector.

[6]Mass sum rules equate the mass-squared of scalar and fermionic components of the superfields. In a supergravity context, these summations are affected by nonzero spacetime curvature as well as the inclusion of larger multiplet matter. In addition to these effects, the Kähler potential will also contribute to the partner/spartner mass deviation.

[7]This also happens to be the scale of the lightest neutrino. This may be an interesting coincidence or the signal for some deeper relationship. Note also that the experimental value of the cosmological constant is used to determine $V_0$ in the above relationship. However the quartic identification is not the only one possible. It is just the most widely accepted

Eq 1.2 also illustrates that SUSY breaking must occur[8] through the superpotential since $e^{\langle K \rangle} \neq 0$. It is likely for SUSY breaking to come from the hidden sector. Therefore, strongly-interacting hidden gauge forces are likely to exist. This means that, as mass scales are lowered to 'condensation' scales, gauge couplings increase dramatically as:

$$\lambda = M_{\text{Planck}} e^{-\frac{8\pi^2}{|\beta(g)| g^2}}, \tag{1.3}$$

where $\beta(g)$ is the usual gauge-beta function and $g$ is the gauge coupling at the Planck scale. This condensation scale usually forces gauginos to coalesce and take a nonzero vacuum expectation value (VEV) as well as form Higgs 'mesons'. These then contribute to SUSY breaking around $M_{\text{SUSY}}^{hidden} \approx 10^{10 \text{ to } 12}$ GeV. This translates to observable sector as [7]:

$$M_{\text{SUSY}}^{obs} \lesssim M_{\text{SUSY}}^{hidden} \mathcal{O}\left(\frac{1}{M_{\text{Planck}}}\right) \approx M_Z \text{ (91.19 GeV)}. \tag{1.4}$$

Due to its inability to access scales above the Planck mass, supergravity is an incomplete, effective theory. Additionally, the Kähler[9] and gauge kinetic functions and the superpotential are all without form and must be specified by some other theory. A further difficulty with the Kähler function is its susceptiblity to non-perturbative effects. This means that any over-archng theory that gives rise to Supergravity can control the form of the Kähler potential. This makes the issue of gravitino-mediated SUSY breaking theory-dependent. However, this is not the case with the superpotential which is relatively stable against non-perturbative effects [8]. The case of superpotential-mediated SUSY breaking is also pertintent for perturbative analyses. In fact, there exist non-renormalization theorems which show that if, at tree-level, SUSY is not broken then it will remain so for all perturbative orders.

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[8]Contained in the kinetic part of the Kähler potential are, what is known as, D-terms. These terms can lead to SUSY breaking but at the level of the Planck scale. This is not as desireable as lower-scale SUSY breaking but to fully analyze SUSY breaking, knowledge of the Kähler potential is needed.

[9]The Kähler poential usually consists of various moduli fields, such as the Axion. Due to the difficulty in working with these potentials, simple models are often considered and mostly within the context of cosmology.

## 1.4 String Theory

String theory offers a possible unification theory having great promise to describe our universe. As the next level "up" from supergravity, fundamental constants and parameters are all related to combinations of the Planck mass and the string scale. In fact, the only fundamental constants of perturbative string theory are the Regge slope parameter, $\alpha$', and the string coupling constant, $g_s$.[10] This is certainly a step in the right direction. However it has been shown that there exists at minimum somewhere between $10^{500}$ to $10^{1000}$ string models, collectively called the string landscape [10]. We arrive at this number based on taking the number of Calabi-Yau manifolds (approximately 100 trillion), the manifolds on which the 6 extra dimensions are compactified [11], and then allowing fluxes to live on the compactification manifold (where there are anywhere from $10^{500}$ to $10^{1000}$ flux combinations per manifold) [12]. Each of these string models corresponds to a different universe, all possessing their own physics and corresponding gauge groups. In order to test string theory as a quantum theory of gravity it is necessary to find out if any of these string models contain our universe as a solution or, more realistically, show that there is a high probablility that it is a solution. This implies that we perform a systematic check of these models (or at least regions of the string landscape).

The current state of strings research involves primarily the study of string theory as a first-quantized perturbative description of the two-dimensional worldsheet dynamics. Here expansions are performed on the two dimensional worldsheets of varying genus (*e.g.* sphere, torus, double torus, etc... ). This can be seen as a way of calculating S-matrix elements for finite massless and infinite massive particles [13]. Simple when compared to typical field theory, string perturbation theory requires only a single Feynman-like diagram at each order. For a brief introduction to string theory (within the context of phenomenology) many reviews

[10]In fact, $M_{\text{Planck}}$ and gauge couplings are found by calculating the vertex amplitude for three interacting gravitons. This then relates those fundamental quantities to $g_s$ and $\alpha$' [9].

Table 1.1: The five perturbative string theories

| Type | $N_{\text{SUSY}}$ | Stringy Objects |
|---|---|---|
| Heterotic$_{E_8\times E_8}$ | 1 | oriented and closed |
| Heterotic$_{SO(32)}$ | 1 | oriented and closed |
| I$_{SO(32)}$ | 1 | unoriented closed and open |
| IIA | 2 | oriented closed and open, D-branes |
| IIB | 2 | oriented closed and open, D-branes |

and books have been authored [13, 14]. Here we present a summary only to familiarize the reader with a few key ideas that will relate directly to our phenomenological interests. As is the usual course, bosonic strings will be highlighted and then will be followed by superstring formalism which allow for fermionic degrees of freedom. Both of these cases are required because the heterotic string theories are described by both prescriptions.

Figure 1.1 and Table 1.1 illustrate the five 10-dimensional "string theories" some of which are direct low energy approximations to 11-dimensional M-theory whose non-perturbative formulation is still unknown. As can be seen, Type IIA and Heterotic $E_8 \times E_8$ are directly related back to M-theory through $S_1/\mathbb{Z}_2$ (where $S_1$ is 1-sphere or circle) compactification of a single large spacetime direction. The Heterotic $SO(32)$ models are generated solely by the $S_1$ compactification with no integer modding. The various string theories are also related to one another through duality transformations:

- "T-duality" - This duality relates the size of the compactified dimension between two theories under the transformation of $\mathrm{R} \leftrightarrow \frac{1}{\mathrm{R}}$. This transformation places a minimal distinguishable length scale for physical stringy interactions. This smallest length is $\mathrm{R} = 1$ in string units ($\approx$ Planck length $= 10^{-33}$ centimeter (cm)).
- "S-duality" - Since this duality contrasts the string coupling strength, $g_s \leftrightarrow \frac{1}{g_s}$, this transformation allows comparisons to be made between non-perturbative and perturbative effects of the dual theories. In other words, this transformation allows strongly-

coupled dynamics in one string theory to be identified with weakly-coupled dynamics in another string theory.

As can also be seen in Figure 1.1, 11-dimensional supergravity (SUGRA) is related directly to M-theory, however, it is not a 'string theory' as it lacks the same stringy matter content. This SUGRA theory allows for gravitational interaction between extended, higher dimension membranes (branes). Branes also make appearances in some of the string theories but SUGRA does not allow for any strings to be present. Because of this, 11-D SUGRA is not a useful tool for particle investigations. We mention it here only for completeness and to illustrate that the general field of string theory encompasses gravity research as well as EFTs.

### *1.4.1 Bosonic Strings*

Strings are one dimensional objects that move through spacetime forming what is known as a *worldsheet*. Open- and closed-ended strings form two dimensional worldsheet surfaces in the shape of tubes or sheets, respectively (see Figure 1.2). As a fundamental object in string theory, the worldsheet, which lives in spacetime, can be seen as a mapping from a two dimensional rectangular (or parallelogrammic)[11] parameter space measured by the coordinates, $\tau$ and $\sigma$. The parameter space to spacetime mapping is represented as a set of bosonic degrees of freedom, $X^{\mu}(\tau, \sigma)$. In other words, these fields are a set of mappings that take the $\tau$ (worldsheet proper time) and $\sigma$ (string coordinate) parameters to a spacetime target space. $\mu$ then ranges over the dimensionality of spacetime, $[0, 1, ..., D-1]$. Based upon the reparametrization invariance of the $\tau, \sigma$ space, a metric is induced, $\gamma_{\alpha\beta}$ which is related to the target spacetime metric:

$$\gamma_{\alpha\beta} = \partial_{\alpha} X^{\mu} \partial_{\beta} X_{\nu} \eta_{\mu\nu}, \tag{1.5}$$

[11]For non-interacting strings, the worldsheet parameter space is the entirety of the complex plane. The parallegram is used, by identifying the edges, to represent a one-loop string interaction.

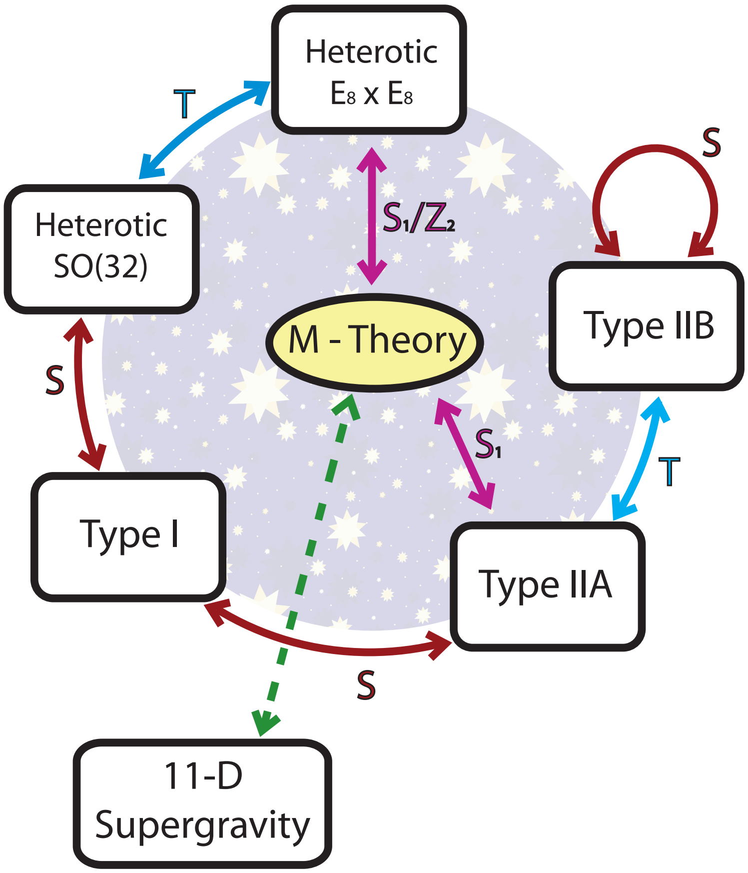


Figure 1.1: The five perturbative string theories as they relate to each other and to M-theory. 11-D supergravity is a low energy, non-stringy description of gravity in M-theory. The T- and S-dualities relate $\mathrm{R} \leftrightarrow \frac{1}{\mathrm{R}}$ and $g_s \leftrightarrow \frac{1}{g_s}$, respectively. These transformations map models in one string theory description to models in a different string theory.

with $\alpha, \beta \in [\tau, \sigma]$. This, coupled with the Lorentz invariant 'proper area' of the worldsheet, yields the famous Nambu-Goto action:

$$S_{NG} = -T_0 \int d\tau d\sigma \sqrt{-\gamma}, \tag{1.6}$$

where $T_0$ is the string tension, $T_0 = \frac{1}{2\pi\alpha'}$. The proper area of the worldsheet is the one quantity that can be agreed upon by all Lorentz observers and is analogous to the measurement of proper time in classical relativity. In addition to the two-dimensional coordinate invariance, the Nambu-Goto action is also invariant under a Weyl (conformal) symmetry. This

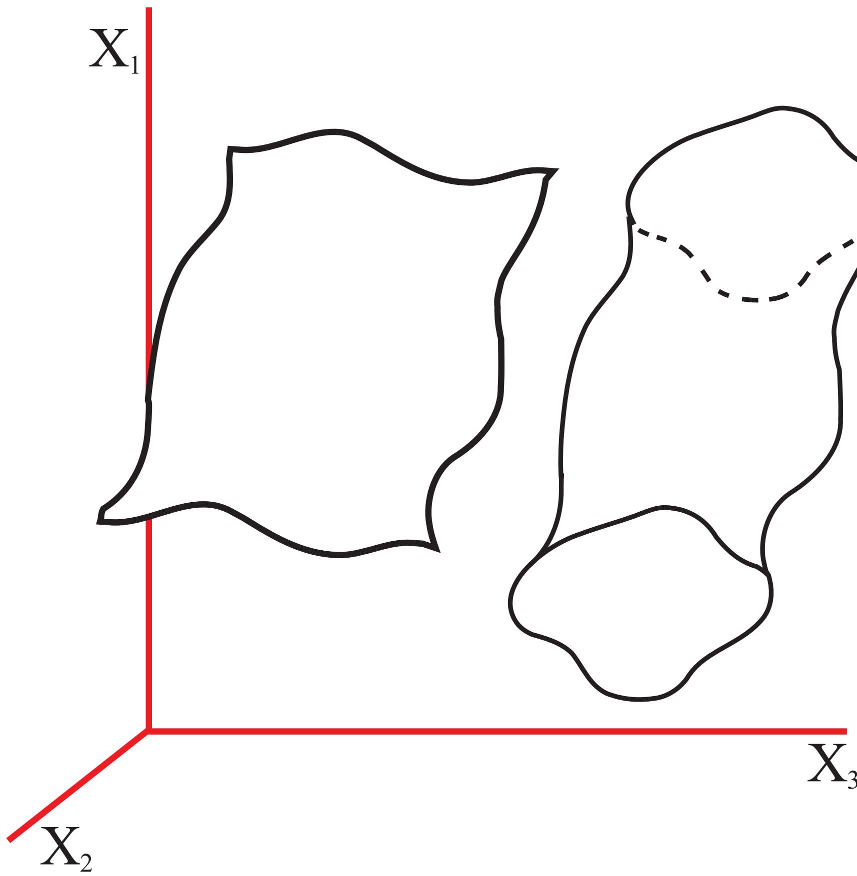


Figure 1.2: Open(Closed) strings propagating through spacetime trace out sheets(tubes).

Weyl invariance can be used to fully specify the induced metric. The above action can also be expressed as the Polyakov action through the introduction of another worldsheet metric, $g_{\alpha\beta}$:

$$S_P = -\frac{T}{2}\int d\tau d\sigma \sqrt{-g} g^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu. \tag{1.7}$$

Here the mappings are explicitly shown. The choice of $g_{\alpha\beta}$ conformally flat fixes the Weyl and reparametrization invariances inherent in the Nambu-Goto action. Complexifying the worldsheet coordinates to $z, \bar{z}$ allows one to define a Hilbert space of states using the normal mode operators of the mapping fields:

$$\begin{aligned}
\partial_z X^\mu &\approx \sum_n z^{-n-1}\alpha_n^\mu \\
\partial_{\bar{z}} X^\mu &\approx \sum_n \bar{z}^{-n-1}\bar{\alpha}_n^\mu.
\end{aligned} \tag{1.8}$$

The generic string solution from which Eq 1.8 is derived depends on the choice of closed versus open strings, as well as the boundary conditions applied to open strings. Closed strings are periodic: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$. Open strings can fall under Neumann or Dirichlet boundary conditions for any of the spatial dimensions. The choice of Dirichlet (fixed end) boundary conditions indicates the presence of *Dp-branes*, where $p$ is the number of spatial dimensions which are affixed to the membrane. The study of these objects becomes important in some of the various string theories. Neumann conditions represent dimensions along which the string is freely propagating. For our purposes, the closed string is the natural choice to consider. In that case, the equations of motion yield solutions that can be split into left mover/moving (LM) and right mover/moving (RM) waves which represent transverse vibrational modes that propagate (counter) clockwise around the string.

$$\begin{aligned}
X^\mu_{Left} &= \frac{1}{2}x^\mu + \frac{1}{2}l_s^2 p^\mu(\tau-\sigma) + \frac{i}{2}l_s\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-2in(\tau-\sigma)} \\
X^\mu_{Right} &= \frac{1}{2}x^\mu + \frac{1}{2}l_s^2 p^\mu(\tau+\sigma) + \frac{i}{2}l_s\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)}
\end{aligned} \tag{1.9}$$

In this language, $X^\mu$ is defined by: the spacetime position ($x^\mu$), the center of mass momentum ($p^\mu$) and lastly the vibrational modes. The string length, $l_s$, is related to the Regge slope parameter which was found in the string tension and is $\alpha' = \frac{1}{2}l_s^2$.

At this point it is necessary to quantize the theory in order to represent anything physical. After varying the action with respect to the dynamical fields, the energy-momentum tensor (there are only three unique possibilities due to Weyl Invariance), $T_{\alpha\beta}$, can be calculated. It turns out that the mixed terms vanish, so only the diagonal operators exist:

$$\begin{aligned} T_{zz} &= -\frac{1}{2}(\partial_z X)^2 \text{ with } z = \tau + \sigma \\ \bar{T}_{\bar{z}\bar{z}} &= -\frac{1}{2}(\partial_{\bar{z}} X)^2 \text{ with } \bar{z} = \tau - \sigma \end{aligned} \tag{1.10}$$

Expanding Eq 1.10 with Eq 1.8 yields the Virasoro operators, $L_n$ which are comprised of the normal mode operators, $\alpha_n^\mu, \tilde{\alpha}_n^\mu$ and which constitute the Fourier mode expansion of the stress-energy tensor. These Virasoro operators lead to an algebra whose commutator gives rise to a central charge, $D$, the dimension of the target spacetime:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{12}m(m^2-1)\delta_{m+n}. \tag{1.11}$$

The Virasosor operators, and subsequent algebra, are physically necessary. The mass operator is created by the action of the Virasoro operators.

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N} - \frac{D-2}{12}), \tag{1.12}$$

Note that the $N$'s are number operators and, like in quantum mechanics, are proportional to the sum of raising and lowering operators. In this case, the raising/lowering correspond to stringy wave modes.

As the Virasoro operators are applied to states it becomes apparent that not all values of $D$ are allowed. For $D \neq 26$, the theory will lack unitarity due to the presence of negative norm states. This is a result of the Weyl symmetry not being able to decouple the Liouville mode in the quantum action. It can also be seen from the quantization of the Virasoro operators. After quantization, the vibrational modes become creation and annihilation operators and the normal ordering requirement introduces an extra constant for $L_0$ which is not present in the classical case. Since the quantum and classical actions do not obey the same set of symmetries, a conformal anomaly appears in the theory. In order to remove this anomaly, the value of the normal ordering constant is fixed by invoking Lorentz invariance. This process is intimately connected to also fixing $D = 26$. The critical dimension of 26 removes the anomaly and allows the theory to keep Lorentz invariance at the quantum level in the spacetime.

When the massless states are investigated, one will find among them a tachyon, a dilaton, a graviton and an axion. The presence of tachyonic particle states indicates that the incorrect vacuum has been chosen. However, while the inclusion of faster-than-light particles is problematic, the free existence of a graviton is miraculous. In order to rid the theory of a tachyon, fermionic fields must be introduced.

### *1.4.2 Superstrings*

To rid our bosonic theory of the offending tachyon, spacetime SUSY must be introduced. This would force into existence a tachyonic fermion field to partner with the bosonic culprit. Since the Dirac equation is free of faster-than-light solutions this fermion could not exist and therefore its bosonic partner (who would also possess imaginary mass) would not be allowed in the supersymmetric spectrum. Using the conversion:

$$X^\mu \to X^\mu, \psi^\mu \tag{1.13}$$

and appropriate gamma matrices, the worldsheet action can be altered to accommodate worldsheet SUSY. This is called the Ramond Neveu-Schwarz (RNS) formalism. Each new fermion is really a Majorana spinor possessing two components: $\psi^\mu = (\psi^\mu_+, \psi^\mu_-)$. The new action for our theory would become:

$$S = -\frac{T_0}{2} \int d^2\sigma \left( \partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right)$$

where the two dimensional Dirac matrices are:

$$\rho^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

It can be shown that the action, Eq 1.14, is invariant under the explicitly supersymmetric infinitesimal transformations:

$$\delta X^\mu = \bar{\epsilon}\psi^\mu, \quad \delta\psi^\mu = \rho^\alpha \partial_\alpha X^\mu \epsilon,$$

where the $\epsilon$ is an infinitesimal spinor: $(\epsilon_-, \epsilon_+)$. This makes the worldsheet SUSY a global symmetry.

The anticommutation relations of fermionic fields implies the existence of Grassmann coordinates on the worldsheet. This requires that our worldsheet parameter space become superspace: $\tau, \sigma \longrightarrow \tau, \sigma, \theta_A$ with $A \in \{+, -\}$. This affects the measure of integration, but it also gives us the option to rewrite our fields as superfields:

$$Y^\mu(\tau, \sigma, \theta_A) = X^\mu(\tau, \sigma) + \bar{\theta}\psi^\mu(\tau, \sigma) + \frac{1}{2}\bar{\theta}\theta B^\mu(\tau, \sigma). \tag{1.14}$$

$B^\mu$ is an auxiliary field whose only requirement is to not affect the physical states of the theory, but which is needed to make SUSY manifest.

As in the bosonic case, varying the action leads to the equations of motion. These, in turn, yield the various string modes and the physical spectrum. The bosonic string's conformal invariance allowed the Virasoro algebra to remove states with negative norm and to fix the spacetime. In the RNS string there is a superconformal symmetry which performs the same operation. The effect of adding the fermion partners is to change the central charge of the Virasoro commutations. This changes the critical dimension of the theory to $D = 10$. When the central charge is shifted the physical states will be affected and it is possible that the tachyonic field will be resurrected. The way to counteract this is to introduce the Gliozzi-Scherk-Olive (GSO) projection.

The GSO projection is meant to ensure the locality of all physical vertex operator products. The operators are the set of conformal fields which contain the quantum number of a given state under the symmetries of the model [15]. In order to create on-shell physical states, vertex operators must be calculated on various two dimensional manifolds of varying genus (e.g. spheres, tori, double tori, etc.). These Riemann surfaces are parametrized and all of them must be integrated over as part of Green functions that make up the vertex operators. Modular invariance is a result of general coordinate invariance on the set of parameters which are used to gauge the worldsheet metrics.

Before the GSO projection is applied, the tachyon-graviton vertex is local as well as the graviton-gravitino vertex. The gravitino-tachyon vertex is not local and that is precisely what the GSO projection is meant to remove. As a result, one of the states must disappear. The gravitino (as the SUSY generator) is chosen to remain. Another benefit to the GSO projection is the potential removal of massless states from the spectrum which could result in rapid proton decay. This is the stringy doublet-triplet splitting mechanism [16]. Unfortunately, the GSO projection cannot completely resolve the issue as nonrenormalizable, higher order terms that can cause proton decay will need to be removed by some other mechanism (*e.g.* discrete symmetries).

As bosons, the $X^\mu$ have periodic boundary conditions. The fermions, $\psi^\mu$ can have either periodic (Ramond) or antiperiodic (Neveu-Schwarz). R and Neveu-Schwarz (NS) boundary conditions will be seen to offer additional constraints on the existence of terms in a model's superpotential. What is important here is that since the LM and RM can each have their own boundary conditions, closed strings can have up to four sectors: $\mathrm{R}_L \times \mathrm{R}_R$, $\mathrm{NS}_L \times \mathrm{NS}_R$, $\mathrm{R}_L \times \mathrm{NS}_R$, $\mathrm{NS}_L \times \mathrm{R}_R$. Spacetime states which arise from the first two are bosons while the latter two form fermions.

## *1.5 String Phenomenology*

String phenomenology is the process of modeling specific string theories and investigating them group theoretically. The five string theories cover overlapping regions of the string landscape and are able to collectively represent the breadth of group theoretic possibilities (both physically meaningful and non-sensical). Depending on the string theory in use, matter and gauge content can arise from various geometrically disparate mechanisms. For type I and II string theories, the presence of D-branes allow for charges to live at brane-brane or brane-string intersections that are formed after the compactification of six out of the ten dimensions. Heterotic strings offer a very different approach as it is a theory of closed strings where waves moving counter-clockwise differ from those that run clockwise. One set of waves describe spacetime SUSY while the other yield the gauge symmetries and charges. In all cases, the issue at hand is the removal of extra large spacetime dimensions through compactification. It is through the compactification process that gauge symmetries are obtained. Various manifold structures, around which the shrinking spatial dimensions are wrapped, give rise to different gauge group isometries. These internal gauge symmetries are expressed by zero-modes of the ten dimensional graviton, which interacts with the six-dimensional compactified space and then effect those gauge symmetries in the noncompact dimensions. This is the very heart of Kaluza-Klein compactification which began as an investigation of a five dimensional gravity theory which, when compactified to four dimensions, produced electromagnetic-like effects. Gauge groups such as $U(N)$ and $SO(N)$ can be produced by compactifying on N-tori or (N-1)-spheres, respectively.

There are many methods used to describe the compactification process. Some of the most popular are: Calabi-Yau, orbifolds and free fermions. Each of these constructions can cover different sub-regions of the landscape and provide, in some cases, a geometric interpretation for compactification processes. There are benefits to the different languages of string model constructions.

- Calabi-Yau: Being the most geometrical approach, it imparts a sense of physical-ness to string modeling. This method gives a nice sampling of the output space yielding the power to tune the input space parameters into producing nice phenomenological properties. The $SO(10)$ and $SU(5)$ gauge groups are readily found. Unfortunately, this approach is often fraught with mathematical complexity and requires much in the way of analytics.
- Orbifolds: Compactification comes on flat tori and endows the worldsheet with free bosons. These orbifolds are twisted giving rise to discrete symmetries. A $\mathbb{Z}_3$ twisting can yield a SM$\times U(1)^n$ gauge group. A downside is that the weak-hypercharge does not have a GUT embedding and often is in disagreement with the low-energy values of the weak-mixing angle and strong nuclear force coupling.
- Free fermions: Related to the orbifold compactification on $\mathbb{Z}_2 \times \mathbb{Z}_2$, this method has produced some of the most realistic models to date. Three generation models are found in free fermionic models roughly due to splitting of the compactified six dimensional space into three sectors of $\mathbb{Z}_2 \times \mathbb{Z}_2$. Here the weak-hypercharge can come from an $SO(10)$ embedding[12] unlike the general orbifolding and Calabi-Yau approaches. This language lies at the other end of the spectrum: they are easy to compute but give a many-to-one mapping of the string vacua. This makes statistical random sampling difficult due to floating correlations between input and output spaces. Therefore it is difficult to pick, *a priori*, the models that yield MSSM properties and to compute their probabilistic likelihoods. However, the constructions lend themselves well to computational and systematic build approaches.

Later in this work we will detail the construction process but it is helpful to have an intuitive picture of the process. The string is fundamental to understanding the symmetries

[12]This is true for the Nanopoulos-Antoniadis-Hagelin-Ellis (NAHE)-based models which are used by our research group. They will be described in more detail later.

existing in a stringy space. Since models are constructed perturbatively, there is a need to specify input space parameters which are seen as a description of worldsheet dynamics and properties. These input parameters are also heavily constrained making it more difficult to intuitively relate properties of the worldsheet to symmetries in spacetime. Once the parameters are specified, a model will possess some non-Abelian gauge group, massless representations of matter and additional $U(1)$ gauge factors. At this point, vertex operators, describing all the gauge degrees of freedom, represent each of the states. These vertex operators are essential in discovering the low energy effective field theory (LEEFT), which is the evaluation of S-matrix elements, and allow for the calculation of correlators. The correlators of the fermion and scalar vertex components yield cubic and higher superpotential terms [15],

$$\langle \cdots V_i^f V_j^b \cdots \rangle, \tag{1.15}$$

which are then used to calculate gauge and Yukawa coupling as well as D- and F-terms. Beyond the usual gauge theoretic quantum numbers, these states also are constrained by extra stringy quantum numbers that allow for complete specification of the super- and Kähler potentials.

The main goal is to be able to find stringy realizations that possess at least some of the characteristics of the MSSM. It is hoped that results from this work will be two fold: 1. give a predictive, experimentally accessible road for verifying string theoretic approaches and 2. to gain a greater understanding of the non-perturbative, model-independent aspect of string/M-theory. According to Lykken: "all phenomenological properties of the effective low energy theory are related by world sheet properties. To understand superstring physics we need to understand these relationships." [13]

Many groups are not trying to find the an exact MSSM from the get-go. Instead, 'semi-realistic' or 'pseudo-realistic' models are sought where the minimal desirable characteristics are:

- six compactified dimensions,
- $N = 1$ SUSY,
- three particle generations and
- the SM or GUT gauge group and an extended Higgs-mechanism.

The last item is of particular interest since larger Lie groups are prevalent in string constructions. Gauge coupling unification is a common property of string theoretic models. In the worst case, unification is guaranteed at the string scale. Often, though, it is achieved at much lower scales: $M_{\text{GUT}} << M_{\text{Planck}}$. This relaxes model building approaches to allow for GUT-like gauge groups and bolsters string theory's naturalness as a 'theory of everything'.

A problem of phenomenological import that is testable is possible proton decay. Stringy models tend to be parity-breaking due to the presence of Baryon-Lepton (baryon minus lepton number (B-L)) violating terms in the third and fourth order effective superpotential terms. These can lead to, respectively, instantaneous or rapid proton decay [13]. For any serious sampling of string models, searching for models which gauge the $U(1)_{\text{B-L}}$ group or which do not allow the offending superpotential terms will be a necessary task.

As can be seen by the above discussion, string phenomenology and model building are studies that seek to relate various physical phenomena together in meaningful and mathematical ways. The string vacua that are investigated may not yield the 'correct' results of being an exact model of our own universe. There may be many that seem close to reality. Because we are dealing with effective field theories all we can hope to do is to approximately find our universe in the string landscape. It will take the work of non-perturbative analysis to definitively discover whether string theory is a veritable theory of everything. It is hoped that 'M'- or 'F'-theory will provide a reason why specific vacua are chosen. At this point all we can hope to do is to uncover possible relationships which will guide future studies in both perturbative and non-perturbative regimes [16]. The patterns and properties found in the meantime are anticipated to illustrate the phenomenology of the 'true' string vacuum.

Another reason for considering GUT groups within the context of stringy models is that these string GUTs can also reveal the source of R and discrete symmetries[13]. Additionally, they can be the source of extra $U(1)$'s [17]. All of these things help to constraint stringy EFTs, as well as provide mass hierarchies. Other properties of the SM have arisen from the free fermionic approach to string model construction. The existence of three generations, the nature of the fermion mass spectrum, proton stability and the mass source of the top quark have all been seen naturally in the context of the free fermion description. These may provide some answers to the naturalness problems found in the SM.

One GUT group is particularly promising is flipped $SU(5)$, denoted $fSU(5)$. This group is superior to the other usual GUT groups (*i.e.* $E_6$, $SO(10)$, Pati-Salam and $SU(5)$) in that it does not require an adjoint Higgs scalar in order to break to the SM [18]. In fact, it does not need any larger Higgs either. This fact is important since the presence of adjoint (or higher), massless, scalar multiplets is inconsistent with $N = 0, 1$ SUSY string model with a Kač-Moody level-1 algebra. The $fSU(5)$ matter content for any given generation is distributed differently than its $SU(5)$ counterpart. The **1**, $\bar{\mathbf{5}}$, **10** representations are:

$$\begin{aligned} \mathbf{1}_i &= e_i^c, \\ \bar{\mathbf{5}}_i &= \{u_i^c, L_i\} \text{ and} \\ \mathbf{10}_i &= \{Q_i, d_i^c, N_i^c\}. \end{aligned} \tag{1.16}$$

The SM breaking occurs in this way: the Higgs decuplets, **10** and $\bar{\mathbf{10}}$, allow for an EW singlet which, when taking on a VEV, breaks the $SU(5)$ down to the SM [18]. This fact

[13]The source of R-symmetries is dependent on which string theory is used. For our purposes (the Heterotic sting), R-symmetries appear from what are are called left-moving sectors. This means that (since gauge groups are created by the right-moving sector) the gauge structure of a given model does not affect the form and presence of R-symmetries. In other string theories (*e.g.* Type-II) there is a direct relationship between gauge groups and R-symmetries.

is important because clearer phenomenology occurs at lower Kač-Moody level since fewer massless states will be allowed to exist in a given string model. The standard $SU(5)$ is problematic since at level 1 (the simplest possible) the adjoint Higgs GUT-breaking mechanism is not supported. If we want models to be simple and allow for breaking to the SM, flipped $SU(5)$ is a strong candidate. Other benefits include a lack of strong proton decay operators and a natural neutrino mass hierarchy. The $fSU(5)$ right-handed neutrino also can help generate the baryon asymmetry in the universe.

There are many ways in which to approach the GUT and GUT-like construction of models [16].

- Intermediate GUTs: This option yields the SM only after breaking from a GUT (in general, some non-Abelian) gauge group, like $SO(10)$ or $SU(5)$, at an intermediate energy between the string and EW scales. A requirement here is that this intermediate group will be of some higher Kač-Moody level (this does not apply to the flipped $SU(5)$).
- Semi-simple GUTs: Again at an intermediate energy scale the SM is realized as coming from some simpler-than-GUT group(s). Examples here include: $SU(3)^3$, $fSU(5)$, $SU(4) \times SU(2)^2$ (called Pati-Salam)[14], or $SU(3) \times SU(2)_L \times SU(2)_R$ (called Left-Right Symmetric).
- String scale: Instead of some intermediate energy regime where the SM is realized, this option anticipates generating the non-Abelian parts of the SM directly at the string scale. This might be one of the best methods for limiting fast proton decay.

Unlike the first option, the latter two can be realized with a Kač-Moody level of unity. In general, intermediate energy states may be a desirable aspect for experimental observation. Perturbative string scale unification may give rise to exotic color triplets and EW doublets

[14]Pati-Salam is sometimes written as: $SO(6) \times SO(4)$. Mathematically speaking, $SU(4)$ is isomorphic to $SO(6)$ and $SU(2) \times SU(2)$ is a double cover of $SO(4)$.

that persist at scales above the EW unification scale [16]. This provides an oasis in the desert and may be observational in future higher energy detectors.

### *1.7 SUSY's All Tied Up In Strings*

The presence of SUSY-breaking mechanisms among string models is an important criterion to generating semi-realistic models. With the abundance of 'hidden sector' matter that exists in stringy models, one approach is to gravitationally couple any SUSY breaking effects in the hidden sector to the observable matter. This will naturally allow for hidden sector SUSY breaking as high as $10^{14}$ GeV [13]. Alternatively, in string constructions the hidden sector will not be in complete isolation from the visible. Some of the model's $U(1)$'s will give nontrivial charges to both hidden and observable matter. The fields that fall under this category are called the 'shadow sector'. Their related particles are constrained to have mass significantly above the SUSY scale [7].

Two avenues for investigation imply the source for SUSY breaking to be either field theoretic or stringy by nature. The latter is not naturally favored as the energy scale would be extremely high. The typical EFT approach is to find the scale of SUSY-breaking by some moduli or field taking on a VEV and thus giving rise to a VEVed (*i.e.* non-zero) D- or F-term. Each of these solutions is degenerate in energy so they must be distinguished in terms of other phenomenological factors. This analysis is the course that we will pursue for viable SUSY phenomenology (though it is used in non-supersymmetric theories as well). In dealing with stringy-EFTs, the assumption can be made that the usual index theorems apply: if a models allows SUSY at the tree (classical) level then SUSY must be kept at the perturbative quantum level. This requires that point-like QFTs have SUSY broken non-perturbatively. However, stringy effects may offer an exception. For example, a model was found [19] to offer no stringent flat directions[15] with which to cancel the Fayet-Iliopoulos terms which

[15] This particular kind of flat direction will be detailed later. It suffices to know that stringent flat

breaks SUSY perturbatively at the one-loop level. This model did possess three generations and a cosmological constant that was null to one-loop calculations. This illustrates that the non-perturbative character of SUSY in strings may yield new unknown features.

### *1.8 SUSY Breaking Mechanisms*

The various sources of EFT-like SUSY breaking are

- hidden sector condensates,
- Gravity-mediation
- dilaton superfields and
- anomalous $U(1)$'s.

String models can contain large non-Abelian gauge groups that are distinct from the observable ones. These are labeled 'hidden' as they are usually minimally interacting with the observable sector. In these scenarios, hidden gaugino condensates can form: $\langle \lambda\lambda \rangle$ which may contribute to the gravitino mass thus breaking SUSY. Secondly, dilaton-mediated effects arise through the Kähler and gauge kinetic functions which in turn predict some soft SUSY breaking couplings.

When SUSY is broken in the hidden sector, the effect can be transferred purely gravitationally to the observable matter where it will break SUSY at some lower energy scale. This SUSY breaking mechanism is popular in supergravity theories and hence it survives in string-based models. One issue that arises with gravity-mediation is that squark masses are not degenerate which leads to large flavor-changing neutral currents at low energy. This scenario also has cosmological implications. In the early universe, flat directions produced by scalar particles (with $\mathcal{O}\left(m_{3/2}\right)$ mass and $\mathcal{O}\left(M_{\text{Planck}}\right)$ VEVs) introduce soft mass terms proportional to the Hubble parameter squared. This can induce oscillations about the present-day vacuum which will overclose the universe. This is called the Polonyi problem [20].

directions characterize the general phenomenological properties of a given model.

The first three SUSY breaking options are generally disfavored due to lack of results with hidden condensates and for the increase in flavor-changing neutral currents for the middle two. While covered in more detail later, it is important to highlight anomalous $U(1)$'s as a method of SUSY breaking[16]. The usual anomaly cancellation method[17] employed requires that both hidden and observable sectors fields be non-trivially charged under this $U(1)$ thereby allowing it to be a SUSY breaking messenger. Soft mass hierarchies are allowed due to the extra D-term contributions of the anomalous $U(1)$ [20]. This suppresses the SUSY flavor-changing neutral currents. In general, the anomalous $U(1)$ SUSY breaking is the most transparent and easily calculated. Therefore, we will primarily focus on flat directions with anomalous $U(1)$'s. The first option is the most desirable since SUSY breaking scales and mechanisms can more easily tuned. Note that it is possible to consider combinations of anomalous $U(1)$ and gravity-mediated SUSY breaking [20] or $U(1)$ and dynamical SUSY breaking via gaugino condensation [21].

### *1.8.1 Gaugino Condensation*

This topic is seen most often in supergravity extensions of strong and EW, SM-like models and is meant to mediate SUSY breaking in the hidden sector [22]. The condition necessary for gaugino-mediated SUSY breaking is that the gauge kinetic function is a nontrivial function of the dilaton. Otherwise, dilaton-mediation or flat directions must break SUSY. This is an important effect because hidden SUSY breaking can be carried to the observable sector through gravitational mediation or anomalous $U(1)$ interactions. There are two ways for gaugino condensates to be manifest in a given theory [22]:

1. F-term symmetries

[16]This method is a particular case of the familiar Fayet-Iliopoulos method which utilizes an auxilliary field of the gauge supermultiplet of $U(1)$. Its inclusion in the Lagrangian signifies the opportunity for it to receive a VEV. This VEV shifts the scalar masses in the theory while leaving the fermion partners untouched, thus spontaneously breaking SUSY [1].

[17]The Green-Schwarz-Dine-Seiberg-Witten method.

2. Effective superpotential terms

In the first case, gaugino bilinear terms arise indirectly in the scalar potential when they become renormalization group scales. It is these scales which are explicit in the scalar potential. The second option has the gaugino bilinears appear directly in the superpotential.

For generic string models, multiple hidden sector gauge groups are present. This will likely introduce multiple gaugino condensates. Since these condensates may form at different energy scales, one may wonder at what scale then will SUSY be broken. It has been shown [8] that the condensate possessing the largest beta function coefficient is also the one which most strongly affects the scalar potential.

Irrespective of how they present themselves, the gauginos will condense at a scale:

$$\langle \lambda\lambda \rangle = \Lambda^3, \tag{1.17}$$

where $\Lambda = \mu \exp(-1/b_0 g^2(\mu))$ and $b_0$ is from the one-loop $\beta$ function for asymptotically free hidden sector gauge groups. This is similar to the quantum chomodynamics fermion-fermion condensation. This method of condensation will be unaffected so long as $\Lambda$, the one-loop renormalization group invariant, is small compared to the Planck mass. Otherwise, gravitational effects may disturb the condensation. The condensation scale is transmitted to SUSY breaking effects via the direct relationship with the gravitino mass [8]:

$$m_{3/2} \approx \sqrt{\langle \lambda\lambda \rangle}. \tag{1.18}$$

In general, the condensate can be represented by a nonlinear function of the dilaton and the non-perturbative corrections to the Kähler potential (which is also a function of the dilaton) [8]. The dilaton-gaugino relationship causes, when the condensation occurs, a dilatonic potential to be generated which in turn stabilizes the dilaton field. Additionally, in the right formalism [8], the dilaton dominated SUSY breaking occurs because of the gaugino condensates.

## 1.9 Flat Directions

In particle physics, the VEV of a field, denoted as $\langle\Phi\rangle$, is a parameter of the model[18]. As VEVs are specified, a surface or point in the parameter space governed by the model's scalar potential is chosen. This then dictates the physics (particle masses, interaction types, many of the force strengths, *etc* ...) of the model.

Flat directions are those collections of field VEVs which leave the model's scalar potential flat down to the anticipated observable sector SUSY breaking scale of 100 GeV to 1 TeV (see Eq 1.19). This is essential for phenomenological viability and is the beginning of the further analyses. If the scalar potential, $V(\phi)$, gains a positive VEV then any global spacetime SUSY is broken (*i.e.* the mass difference between partner and spartner becomes non-zero),

$$V(\phi) = \frac{1}{2}\sum_{\alpha} g_{\alpha}^{2}\left(\sum_{\alpha=1}^{dim(\mathcal{G}_{\alpha})} D_{a}^{\alpha} D_{a}^{\alpha}\right) + \sum_{i} |F_{\phi_i}|^{2}\,. \tag{1.19}$$

Here $\alpha$ enumerates the gauge groups, $\mathcal{G}_{\alpha}$, and $a$ runs from 1 to the dimensions of the gauge group matrix representations.

In order to have a flat scalar potential, the D-terms and F-terms (both will be defined later) must be zero individually since they are both positive semi-definite. Note that D-terms originate in a given model's kinetic Lagrangian, while the F-terms are derived from the superpotential. Since D-terms rely on the gauge group representation we break them into two types. For non-Abelian gauge groups the fields can be either: Type I (singlets) or Type II (multiplets other than singlet). The type II D-terms will clearly be more complicated and therefore a singlet search is, more often than not, the first and only search performed for flat direction investigations.

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[18] In spacetimes with 4 large dimensions, the VEV has units of mass(energy).

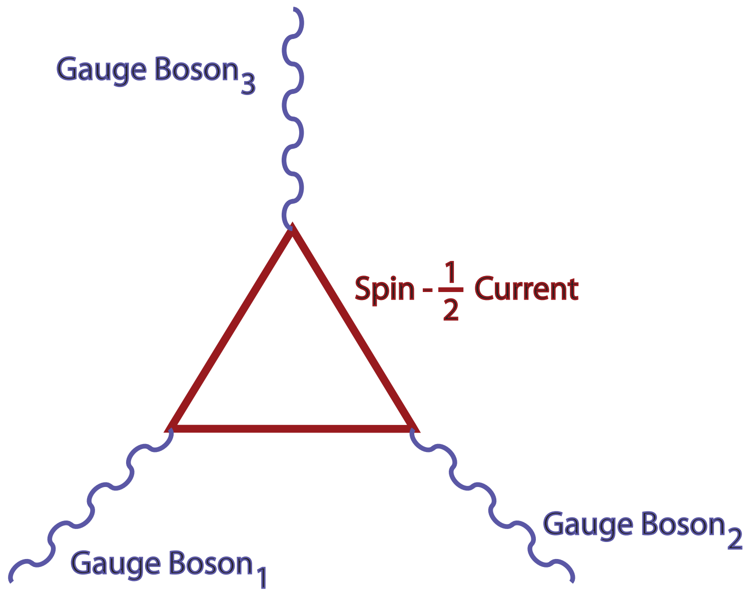


Figure 1.3: The one-loop Feynman diagram that can give rise to anomalies in four dimensions. In order for the anomaly to appear, an odd number of the incoming bosons ('legs') must be the gauge particle of an anomalous $U(1)$. These bosons interact via a fermionic current (the loop). Note also that these anomalies only appear in an even number of large spacetime dimensions, $D$, with the required number of bosons, $n$, being given by $n = 1+D/2$.

## *1.10 D-flatness*

During the compactification process in which spacetime dimensions become periodic, $U(1)$'s that are orthogonal to the desired observable gauge group (such as SM) are generated. Additionally, $U(1)$ charges can be freed from GUT groups due to gauge symmetry breaking. In the full ten dimensions, the local $U(1)$'s are embedded in $E_8 \times E_8$ or $SO(32)$, but once freed (via compactification down to four dimensions), they can be anomalous. Compactification schemes such as bosonic lattices, orbifolds, and free fermions have been shown to generically produce these anomalous U(1)s. These U(1)s possess a charge trace over massless states which is non-zero. In particular, the anomalous $U(1)$ is observed in a low energy effective field theory (LEEFT) through a 'triangle diagram'. This one-loop Feynman diagram consists of three gauge fields interacting in a triangle pattern through a virtual spin-1/2 fermion current and where one or all three fields are gauge bosons of the candidate anomalous $U(1)$

(see Figure 1.3). If the charge associated with this loop is found to not be conserved, then the $U(1)$ is anomalous:

$$\mathrm{Tr}Q^{(A)} \neq 0. \tag{1.20}$$

If multiple $U(1)$'s are anomalous, the anomaly can always be rotated into a single $U(1)$:

$$U(1)^{(A)} = c_A \sum_i \left(\mathrm{Tr}Q^{(i)}\right) U(1)_i \tag{1.21}$$

(here $c_A$ is a renormalization constant) and the remaining orthogonal $U(1)$'s all become non-anomalous ($i.e$ traceless) [23]. The existence of an anomalous $U(1)$, denoted: $U(1)_A$ implies an inconsistent theory where the classical action possess symmetries which do not appear in the related quantum actions. This is a problem unless $U(1)_A$ can be made Planck/string scale massive. The Green-Schwarz-Dine-Seiberg-Witten (GSDSW) cancellation mechanism is the standard anomaly cancellation method where counter-diagrams are generated by forcing[19] a non-zero dilaton VEV [18]. This in turn sets the string-loop coupling constant and creates a Fayet-Iliopoulos term. This action lifts the degeneracy of the vacuum by creating a Fayet-Illiopoulos (FI) D-term from the now-modified Kähler potential. The effect of this post-compactification, unbroken anomalous $U(1)$ is canceled by this FI term [24,25]. The FI term introduces a shift in the classical vacuum of the string but it, fortunately, still corresponds to a consistent perturbative quantum string vacuum, therefore no other anomaly is introduced.

As stated, not all $U(1)$'s will be anomalous and after the appropriate rotation, only one will remain anomalous. The remaining non-anomalous $U(1)$'s do not constrain the flat direction moduli space beyond what is demanded by F-flatness[20]. This is because both F-terms and non-anomalous $U(1)$ (contributing to Abelian or type I D-terms) require gauge invariant monomials of fields. The $U(1)$'s are not without merit though. Local $U(1)$'s can be

[19]This is allowable through an effective degree of freedom afforded by the dilaton-axion coupling [21].

[20]That is, for a given F-flat solution, a rotation may always be performed in which D-flatness is recoverable [26].

used to prevent various nonrenormalizable operators from rapid proton decay. This decay can occur through baryon minus lepton number (B-L) violating terms. Additionally, the conventional neutrino seesaw mass-gaining mechanism requires an intermediate scale where B-L symmetries are broken. In the standard seesaw mechanism, extra neutrinos are added to the model. These new neutrinos are paired with the SM neutrinos and take on masses at some large, intermediate mass scale (usually a GUT scale) forcing the SM neutrinos to remain light when the $U(1)_{\text{B-L}}$ symmetry is broken. These extra stringy $U(1)$'s can be used to restabilize proton lifetimes as well as remove the intermediate scale dependence[21] in seesaw models [7].

Hidden sector $U(1)$'s also can play a role in SUSY breaking. In both supergravity and GUT theories, SUSY breaking can occur through kinetic mixing, where an appropriate kinetic mixing term appears in the gauge Lagrangian:

$$\mathcal{L}_{\text{gauge}} \approx \chi F^{\mu\nu}_{(i)} F^{(j)}_{\mu\nu} \tag{1.22}$$

where $i$ represents $U(1)$'s in the observable sector and $j$ is for the hidden. The effects of breaking a hidden $U(1)$ can shift the kinetic mixing term causing any of the following:

- observable states gain hidden sector charges and couplings,
- hidden gauginos couple to observable states,
- hidden sector D-flatness is shifted and/or
- hidden D-terms contribute to soft scalar masses.

These hidden $U(1)$'s threaten to affect observable gauge hierarchies if there exists some scalar field other than the Higgs which takes on a high mass: $M_{\text{scalar}} >> M_Z$. It could also raise the EW scale to that of the hidden sector SUSY breaking scale. Some of these effects could be too strong, therefore it is necessary to control the strength of kinetic mixing [7].

[21] Of particular interest here are the Pati-Salam models. This GUT scenario naturally includes a $U(1)_{\text{B-L}}$ which remains unbroken. Therefore, this intermediate scale is not required to create a see-saw mechanism.

Local discrete symmetry (LDS)s could be used to regulate these effects by disallowing various strong kinetically mixing states (*e.g.* $\mathbb{Z}_2$ *pairing symmetry*). Weakly-coupled free fermionic heterotic string (WCFFHS) models can be populated by LDSs which are residual from the breaking of non-Aelian gauge groups. These are also able to limit flavor-changing neutral currents and to mitigate proton decay by forbidding operators that would, normally, appear in supersymmetric models [27].

### *1.10.1 Anomalous Benefits*

If not for the induced VEVs, the GSDSW mechanism itself would lead to SUSY breaking near the string/Planck scale through the appearance of the FI term. This occurs at one-loop in the string expansion. Since evidence for SUSY is anticipated to be seen at the TeV scale, it will still be necessary to break SUSY at an energy scale far below the point where the GSDSW mechanism could. To prevent SUSY breaking until desired, the FI term must then be canceled by other contributions to the anomalous D-term as fields take on VEVs. In this process, the D-terms for the other $U(1)$s and non-Abelian (NA) groups must also remain flat in VEV space. This process is called vacuum restabilization. This restabilization has two side-effects. Firstly, since the now-VEVed scalars can be charged under both anomalous and non-anomalous $U(1)$s, a generalized Higgs effect causes the gauge generators of the $U(1)$s to become string scale massive [23]. This means that non-anomalous $U(1)$s can be broken along with the anomalous. Secondly, nonrenormalizable terms in the superpotential of order $n > 3$ can lead to effective mass and Yukawa terms. Terms in the form of

$$\begin{aligned} \text{Mass:} \quad & \langle \Phi_1 \dots \Phi_{n-2} \rangle \Phi_{n-1} \Phi_n, \\ \text{Yukawa:} \quad & \langle \Phi_1 \dots \Phi_{n-3} \rangle \Phi_{n-2} \Phi_{n-1} \Phi_n \end{aligned} \tag{1.23}$$

are responsible for these effects when $n-2$ (mass contributions) and $n-3$ (Yukawa interactions) fields take on a VEV.

The appearance and cancellation of the FI term can have positive phenomenological effects on constraining a given stringy model. When investigating MSSM models, one will find that exotic, fractionally-charged states readily appear [28, 29]. These states, in general are massless down to the EW scale and are phenomenologically undesirable. In fact, the problem is worsened due to selection rules required by string theory [15]. This is due, in part, to the restricted forms of superpotential terms which could impart large mass contributions to these exotic states. Flat direction VEVs could otherwise contribute, through terms as in Eq 1.23, to the mass of the exotics pushing them outside of the range of observation [30].

The lightest fractionally charged field in a given model will be stable and can contribute strongly to cosmological and astronomical phenomena if allowed to persist. In fact, the energy density of the universe can be overtaken completely by this light exotic if its mass is of order 100 GeV [13]. Lastly, these fractionally charged particles can contribute to the renormalization group flow equations of the gauge couplings and can affect the unification of the forces. The FI term can help mitigate the effects of these problematic exotics by requiring flat directions to generate near string mass scales through unsuppressed renormalizable terms in the superpotential, thereby killing off many of the undesired fields [23]. This effect can push the masses of the exotics to MSSM unification scale magnitudes, $\approx 10^{16}$ GeV.

Exotics are not necessarily a completely negative aspect to string models. Arising from non-Abelian symmetries broken at the string scale (as opposed to EFT scales), these exotic states may be able to give rise to observational signatures that would be direct evidence for stringy effects [16]. Understanding the allowed exotic mass scales near our experimental upper limits may be able to offer predictive analysis.

In general, for stringy models with no anomalous $U(1)$, the couplings of the effective superpotential are order unity for third (and possibly fourth) order superpotential terms. Due to the vacuum shift of the FI-term, additional Yukawa couplings from higher order terms are generated by the non-trivial VEVs of scalars or fermion condensates. These

Yukawa terms create nonrenormalizable VEV-to-quark/lepton interactions in the form of effective lower dimension Yukawa operators. Not shown in Eq 1.23 are the inverse powers of the string scale, $M_S^{-n+3}$, which work to suppress those nonrenormalizable effective Yukawa terms [9].

Analysis of these nonrenormalizable contributions to the Higgs doublet and fermion mass matrices will show that the first two generations will be less massive and the top quark Yukawa will remain at order unity[22]. Frequenting string model MSSM phenomenology is the presence of four (or more) Higgs which is problematic. Fortunately, at least two of these fields will receive FI-scale masses and be pushed out of the low energy spectrum. A third can be cast out if the hidden sector condensation scale is sufficiently high [18]. The remaining Higgs then becomes the electroweak doublet. Since FI-term generation is a generic feature this effect occurs with minimal tuning [31] . The anomalous $U(1)$ is a powerful feature of string models.

Other benefits of the existence of $U(1)_A$ are the rank reduction and breaking of the four-dimensional gauge group, possible neutrino mixing and correlations between anomaly cancellation and coupling normalization of the weak mixing angle [31]. Additionally, there have been connections with the strong CP problem. Another benefit arises in a possible connection between the LDS, R-parity, and the presence of the $U(1)_A$. In general, LDSs arise when a local $U(1)_{Z'}$, with charge $q$, is broken by a state whose VEV is equal to $Nq$. This forces an extra symmetry constraint on superpotential terms in the form of: $\sum_i Q_i^{Z'} = 0$ mod $N$. These additional constraints can survive the removal of the anomalous $U(1)$ and can affect flat direction geometry. This may signal a relationship to R-symmetries [7].

One issue that is noteworthy here is that since $U(1)$'s lie outside of the universal gauge group(s) of a model (*e.g.* $SO(10)$), the $U(1)$ symmetries will typically be family-dependent.

[22]VEVs generated by the breaking of the anomalous $U(1)$ give rise to the nonrenormalizable terms with the largest couplings. Hidden sector condensates yield the next largest contributions. Lastly, if there exists a $U(1)_{Z'}$ from the $SO(10)$ embedding, its breaking will contribute next to the nonrenormalizable terms.

Be it collections of generations, fermions, bosons or leptons, the lack of universality can pose a danger as certain flat directions can give rise to various currents. For example, an anomalous D-flat direction on the order of the EW scale can produce flavor-changing neutral currents through non-uniform squark masses. This problem can be resolved through extensions of the specific kind of string constructions called the NAHE set. This will be discussed in more detail below. These extensions can preserve the universality of the three chiral generations if the underlying $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold symmetry of the NAHE set is preserved after the addition of extra constraints. This helps further constrain the range of quasi-realistic free fermionic models to investigate on the Landscape.

### *1.10.2 D-flatness, mathematically-speaking*

For the case of Type-I ($U(1)$-charged) fields, D-flatness requires: $\langle D_A \rangle = \langle D_\alpha \rangle = 0$ where:

$$D^A = \sum_k Q_k^{(A)} |\langle \Phi_k \rangle|^2 + \xi \ ; \tag{1.24}$$

$$D^\alpha = \sum_k Q_k^{(\alpha)} |\langle \Phi_k \rangle|^2 \ , \ \alpha \neq A \ ; \tag{1.25}$$

$$\xi = \frac{g^2 \mathrm{Tr}(Q_A)}{192\pi^2} M_{\mathrm{Pl}}^2. \tag{1.26}$$

The $\Phi_k$ are thesuperfields for which the scalar components of $\phi_k$ acquire VEVs of order $\sqrt{\xi}$ and so their VEVs must reside within a hyper-sphere defined by the FI-term [9]. The $Q_k^{(A)}$ and $Q_k^\alpha$ denote the anomalous and non-anomalous charges of the $k$-th field and $M_{\mathrm{Pl}} \approx 2\times 10^{15}$ TeV denotes the reduced Planck mass. It is clear that the Eq 1.24 represents the anomalous $U(1)$ shifting the classical vacuum of the theory.

For Type-II fields, the corresponding equation is:

$$D_a^\alpha = \sum_k \Phi_k^\dagger T_a^\alpha \Phi_k \tag{1.27}$$

where $\alpha$ and $a$ are as defined in Eq 1.19 and $T_a^\alpha$ is a matrix generator of the gauge group,

$(G)_\alpha$. Non-Abelian fields consist of multiple components and each can take on a distinct VEV of differing phase, or no VEV at all. At this point, D-flatness can be reduced to a group space geometric problem. In the work that has been performed, it was shown that SUSY-keeping D-term constraints were mapped to a closed VEV vector sum in the groups' adjoint space. This was shown explicitly for $SU(2)$ and $SO(2n)$ [32]. This non-Abelian approach will increase the VEV moduli space and could increase the overall number of solutions and help focus in on more realistic VEV vacua.

As stated earlier, it is the usual practice to consider first the NA singlet case, due to computational simplicity. Unfortunately, singlet flat directions have so far not been able to yield any realistic phenomenology. For the case of heterotically-derived $fSU(5)$ models, the root space of Type-I flat directions has been completely reviewed [18]. The case of full NA D-flat searches has yet to be fully implemented but the ground work is laid and a hybrid scenario is currently in place.

## *1.11 F-flatness*

The superpotential is a gauge invariant, holomorphic function of chiral superfields which are considered to be complex variables. Each chiral superfield is a linear combination whose real part is a scalar while the imaginary piece is a spin-1/2 fermion. Taking derivatives of the superpotential with respect to a model's superfields will yield the respective F-terms. General superpotential monomial terms begin at order one and range through infinity. The $n$-th order term of W is a sum of all possible terms whose powers add up to $n$:

$$\begin{aligned} W_n &= \sum \Phi_1^{r_1} \dots \Phi_M^{r_M}, \\ n &= \sum_{i=1}^{M} r_i. \end{aligned}$$

In stringy potentials, first and second orders do not exist due to extra worldsheet, supersymmetric currents[23]. Additionally, the order at which the superpotential fails to be F-flat (i.e. does not satisfy Eq 1.28-1.29) corresponds to the scale at which SUSY is broken. The requirements for F-flatness[24] are:

$$\langle F_i \equiv \frac{\partial W}{\partial \Phi_i} \rangle = 0; \tag{1.28}$$

$$\langle W \rangle = 0. \tag{1.29}$$

At $3^{rd}$ through $4^{th}$ order SUSY is broken at about the Planck scale: $10^{15}$ TeV and for each superpotential order increase, the energy scale decreases by a factor of 10 [9]. SUSY breaking at 1 TeV then corresponds to $n = 17$, though this is only in the weak-coupling limit. As the coupling strength increases, so will the superpotential order that describes the EW scale [18]. $17^{\text{th}}$ order SUSY breaking is a desirable condition. Having the superpotential flat to all orders (or at least to $17^{\text{th}}$ order) is also acceptable since there are other mechanisms that allow for SUSY breaking such as gaugino condensation.

### *1.11.1 Stringent Flat Directions*

F-flat directions are dependent on the order of the superpotential calculated. A brute-force flat direction analysis (Eqs 1.28,1.29) would entail generating the superpotential to a given order and testing the above constraints. However, flatness at a given order does not insure that a higher order will also be flat. As mentioned before, the model must be flat to $16^{\text{th}}$ or $17^{\text{th}}$ order to be exactly consistent with the standard model. However this is a lower limit and if we can find a direction that is flat to all-order then there is no need to check for flatness on an order-by-order basis. Therefore, it is very useful to keep F-flatness to all-order and it has been observed that for most models, all-order flatness is tenable when stringent

[23] These are ghost picture-changing rules [33] and will be detailed later.

[24] Eq. 1.29 is a requirement from unbroken supergravity models with a vacuum energy that goes to zero [20].

flat directions are imposed [19]. This stringency requirement forces individual components in an F-term to be zero rather than allowing for cancellation between them. By looking for stringent all-order flatness, one removes the risk of finding F-flatness at one level and then losing it at the next higher order. In analyses comparing stringent flat directions to flat directions prepared by brute force, it has been seen that the stringent directions often are main roots from which other flat directions branch (see figure 1.4). This method then significantly simplifies calculations while preserving the general patterns inherent in these models. We feel justified in pursuing this course of analysis until more robust computational methods can be utilized. The net effect of stringent flat directions is realized by having in each superpotential term, at minimum, two unique non-VEVed fields or one non-VEVed field whose power is greater than one.

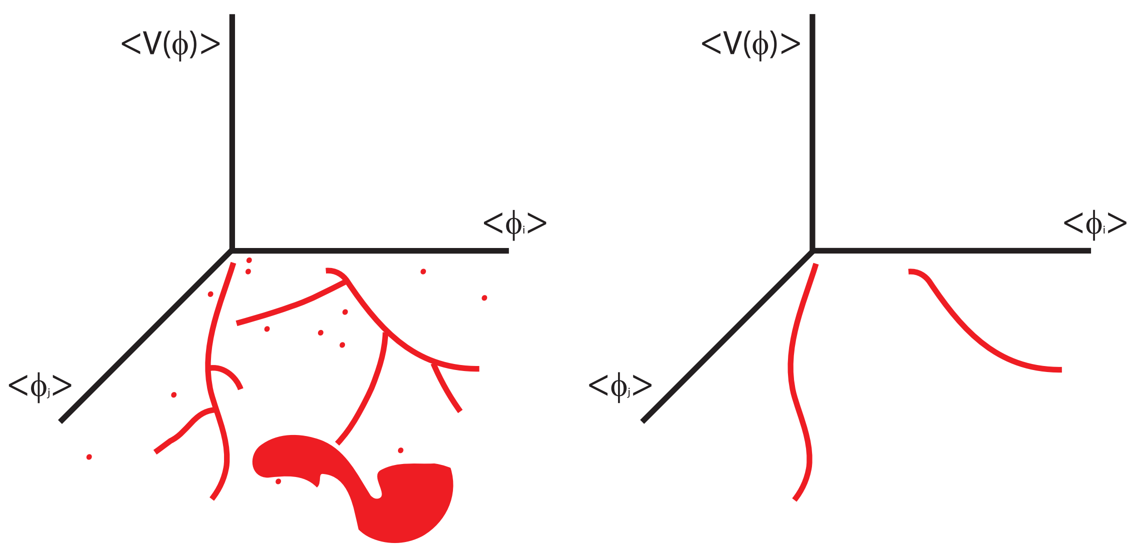


Figure 1.4: Investigating stringent flat directions (left), one will find general characteristics of a model. It seems that every all-order flat direction (right) is attached to a 'root' direction found via the stringency requirement.

# CHAPTER TWO

## Heterotic Flat Directions

Sifting through the various string models to search for the MSSM or its Grand Unified embedding necessitates choosing a particular low-energy string construction method. Since there has been shown to be dualities [34] between the 5 main string theories (heterotic $E_8 \times E_8$, heterotic $SO(32)$, IIa, IIb and I), we are free to choose the most convenient interpretation to present the most efficient modeling. The meaning of 'efficient' is subjective and for our purposes, we seek to quickly scan various models for semi-realistic phenomenological character. Thus we have chosen to use the free fermionic language of the heterotic $E_8 \times E_8$ approach.

In this chapter, we will briefly describe heterotic string theory. This will prepare us for the topic of interest: flat direction calculations within the framework of a specific subset of heterotic string models (WCFFHS). Therefore, we will focus on the data needed and salient calculations required to produce the results necessary for future phenomenological discussions of this portion of the heterotic string landscape.

### *2.1 Heterotic String Theory*

Heterotic strings come in two flavors: $E_{8_{obs}} \times E_{8_{hidden}}$ or $SO(32)$. In these theories, standing waves of the closed strings are represented by fields that move in either a clockwise or a counter-clockwise direction. We use the shorthand RM(LM) for right(left)-moving or right(left)-mover. The appropriate abbreviation shall be obvious based upon the context. Now the choice of which direction ones calls 'left' or 'right' is a matter of convention. In fact, we are chosing a particular description which is opposite to other sources [34]. What makes the heterotic theories unique is that the RM waves possess the same degrees of freedom of strings modes living in a 26 dimensional (*i.e.* purely bosonic) string theory. The LM waves

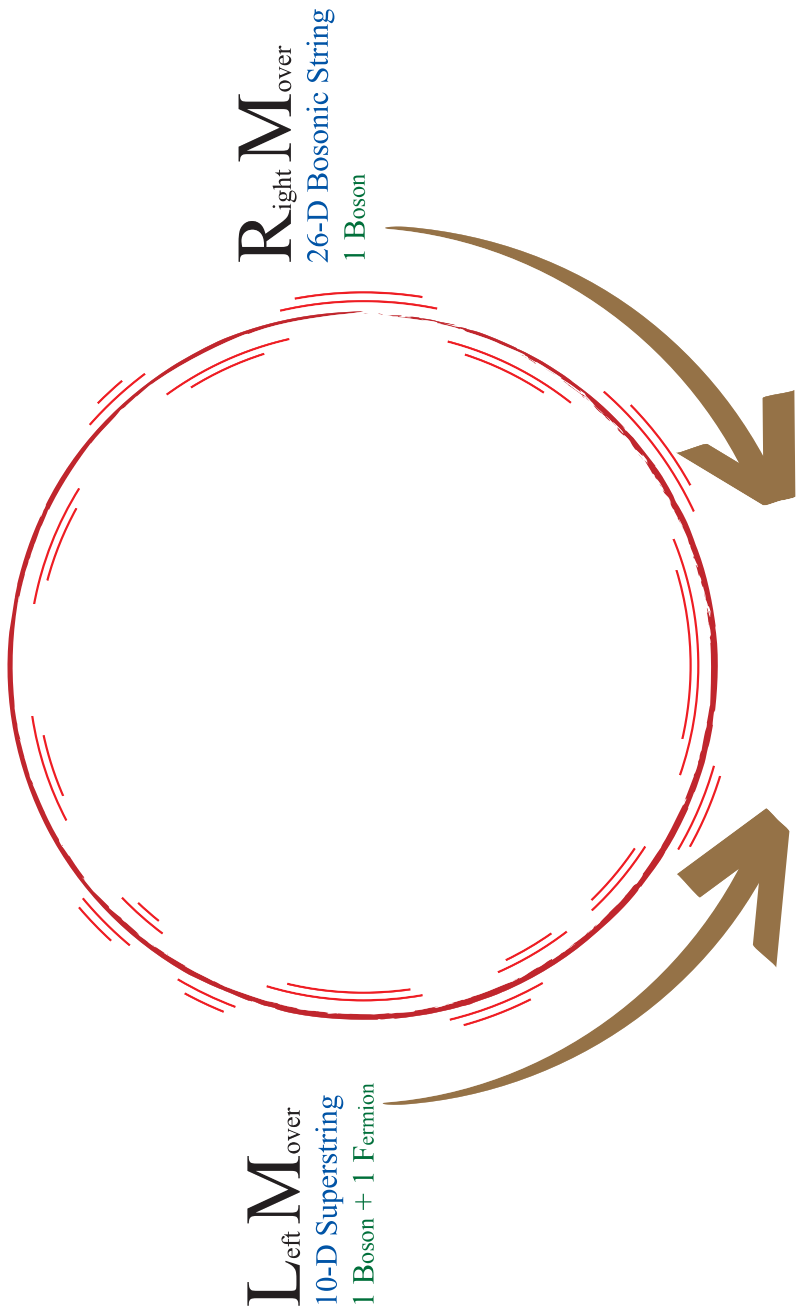


Figure 2.1: A heterotic closed string has both left-moving and right-moving waves. The LM live in a 10-D superstring theory while the RM live in a 26-D bosonic string theory. In a superstring theory there is one bosonic wave and one fermionic wave for each dimension while in bosonic string theory there is only a bosonic wave for each dimension.

maintain the usual 10-dimensional superstring degrees of freedom (*see figure 2.1*). In a null (light cone) gauge, the LMs have 8 transverse degrees of freedom. Since these are part of a 10-dimensional superstring theory, each dimension contributes one bosonic and one fermionic field. In a four dimensional theory, two of the light cone dimensions remain uncompactified and each dimension corresponds to a LM bosonic field, a LM fermionic field and a RM bosonic field. The two LM spacetime bosonic fields produce only Planck-scale massive modes and the two RM spacetime bosonic fields only occur for gravitons or gravitinos. In terms of an effective field theory, we will ignore the graviton since we are solely concerned with the gauge and particle content of each string model we investigate. The gravitino will resurface in our discussion SUSY breaking mediation.

Heterotic string theory received a lot of early attention due to its ease of generating GUT models [35]. Particular success was found among the free fermionic constructions since its naturally generated $SO(10)$ GUT group can be broken to a variety of phenomenologically realistic models [28, 36, 37, 37–45]. In fact, the $fSU(5)$ groups was the first semi-realistic model produced by fermionic investigations. It is estimated that approximately 100 trillion string models are of this type.

### *2.1.1 Free Fermions*

We pursue the course defined by the weakly-coupled free fermionic heterotic (WCFFH) string theory. In this approach, closed strings are endowed with the above-mentioned fermionic fields. It is these fields which give rise to the degrees of freedom with which the conformal anomaly is canceled. It is equally possible to describe the heterotic string in a bosonic framework but quantum field theories prefer neither worldsheet fermions nor bosons. Translating between bosonic and fermionic theories requirings pairing fermion modes to form bosons.

In the WCFFH approach and in four spacetime dimensions, we associate with these fermionic fields a 64-component vector, called a boundary vector (BV), $\vec{\alpha}$. A given collection of boundary vectors is the starting point to model building [40, 43]. These vectors, and any subsequent additions, represent the spin structure boundary conditions of fermions propagating around the string worldsheet. Since these fermionic fields represent waves we can characterize them by a phase factor as they progress around non-contractible loops (the worldsheet):

$$f \rightarrow e^{-i\pi\alpha(f)}f, \quad \alpha(f) \in (-1, 1]. \tag{2.1}$$

This gives us a range for the components of the BV, $-1 < \alpha_i \leq 1$ where $i = 1$ to 64 and the $\alpha_i$ are rational.

The vectors also represent the compactification of the extra large spacetime dimensions. This indicates that the geometry/topology of the compactified dimensions is directly connected to the allowed wave modes that precess the string worldsheet. It is not obvious, but one shortcoming to the WCFFH construction is the inability to map a given model, as generated by its boundary vectors, to a desired LEEFT. Thus, some groups prefer to utilize string theory constructions where it is easier to, *a priori*, generate a desirable model. This set-back is compensated by the ease in which the WCFFH landscape subregions can be computationally analyzed. It is in this vein that we proceed to discuss some of the advantages and difficulties of automating WCFFH string searches.

The action for the conformally invariant [15] free fermions, in the light cone gauge for ten large spacetime dimensions, is [34]:

$$S \sim \int d^2\sigma (2\partial_+ X_\mu \partial_- X^\mu + i\psi^\mu \partial_+ \psi_\mu + i\sum_{A=1}^{32} \lambda^A \partial_- \lambda^A). \tag{2.2}$$

This action is manifestly symmetric under $SO(32)$ when all of the $\lambda^A$ possess the same boundary conditions (Ramond or Neveu-Schwarz)[1]. Various GSO projections allow for mixing the periodicity of the fermions and doing so creates an $E_8 \times E_8$ gauge group instead. This is more desireable because $E_8$ has a natural embedding for the SM:

$$SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8. \tag{2.3}$$

The unique number of heterotic degrees of freedom, 64, is not an obvious result. String dynamics are usually written in terms of a light cone gauge which is an extension of the familiar idea used in black hole studies in relativity. The light cone gauge is important as it removes two longitudinal dimensions from our field counting. Compactification removes four bosons from each of the sectors (RM and LM). This leaves the (26-4 =) 22 RM bosons. Fermionizing, 44 RM fields are created. The LMs have (10-4=) 6 bosons after compactification. Adding to this the (10 - 2 transverse) = 8 fermions allows for 20 fermionic LMs after fermonization. Comparing our counting to Eq 2.2, we can see that the $\lambda^A$ are RMs. The remaining RMs come from bosonic $X^\mu$ which, after compactification, contribute 12 fermions. Noticing the factor of two in Eq 2.2, we can see that the other set of ten $X^\mu$ as well as the $\psi^\mu$ constitute the LMs.

After performing such a counting exercise, it is natural to ask whether heterotic strings are truly ten dimensional or some abstract mathematical description[2]. However, by observing the presence of ten bosons (for each sector) it is clear that this is, in fact, a ten dimensional theory. The remaining RM modes then constitute the internal gauge symmetries of our theory. On the LM side, sit 18 Majorana-Weyl fermionic LM fields , labelled: $x^I, y^I, w^I (I \in [1, 6])$, which transform under the adjoint of $SU(2)^6$. Two LM spacetime fermion remain and they are used for identifying a state's fermion/boson classification and its chirality.

[1] See the end of Chapter 1.

[2] This latter statements is often said of string theory as a whole.

During heterotic model construction, sets of BVs are generated but they are not freely specifiable and have been shown to be highly constrained [38, 39]. The first set of rules are the Antoniadis-Bachus-Kounas/Kawai-Lewellen-Tye (ABK/KLT) constraints:

$$
\begin{aligned}
N_{i,j}\vec{V_i}\cdot\vec{V_j} &= 0 \ (mod\,4), i \neq j && (2.4)\\
N_i\vec{V_i}\cdot\vec{V_i} &= 0 \begin{cases} (mod\,8) & N_i \text{ odd} \\ (mod\,4) & N_i \text{ even} \end{cases} && (2.5)\\
\vec{V_i}, \vec{V_j}, \vec{V_k} & \quad \text{must have an even number of simultaneous periodic real fermions.}
\end{aligned}
$$

for $\vec{V_i}$ the $i^{\text{th}}$ BV, $N_i$ is defined to be the lowest positive integer such that $N_i\vec{V_i} = \vec{0}\ (mod\,2)$ (called the order of $\vec{V_i}$) and $N_{i,j}$ is the lowest common multiple of $N_i$ and $N_j$. Also note that the dot products in equations (2.4) and (2.5) are actually Lorentz dot products where the RM products are subtracted from the LM products. The periodicity constraint means that there must be an even number of value 1 components shared between all three vectors. These constraints make the BV consistent with conformal/Lorentz invariance of the worldsheet/spacetime.

An important set of consistent BVs is called the NAHE [40, 43] set. Since they are the starting point for much of the WCFFH model building, we list them here:

$$
\begin{aligned}
V_I &= (1_{64})\\
V_{II} &= (1_2, (1,0,0)_6 \| 0_{44})\\
V_{III} &= (1_2, (1,0,0)_2, (0,1,0)_4 \| 1_{12}, 0_6, 1_4, 0_{22})\\
V_{IV} &= (1_2, (0,1,0)_2, (1,0,0)_2, (0,0,1)_2 \| 1_{10}, 0_2, 1_2, 0_2, 1_2, 0_8, 1_2, 0_{16})\\
V_V &= (1_2, (0,0,1)_4, (1,0,0)_2 \| 1_{10}, 0_4, 1_2, 0_6, 1_4, 0_{18})\,, \qquad (2.6)
\end{aligned}
$$

where $X_n = \overbrace{X, X, ..., X, X}^{n}$. This set is of order two since the $\alpha_i \in \{0,1\}$. The form of the

NAHE set constrains the allowable wave phases of the created stringy states. These same constraints are then imparted to any extra BVs that are added to the set, as is the usual custom when performing any type of WCFFH string model building.

States of a generated model are created from symmetry considerations on linear combinations of the BVs. These combinations produce states of various charges which can be assembled into representations of various gauge groups. An important part of calculating the allowed internal gauge symmetries involves pairing the BV worldsheet phases. This can be done in two ways, each of which result in a different type of fermion:

1. Complex fermions - individual fermions are paired within their own sectors: left-left or right-right. Models containing only complex fermions possess a gauge group of rank 22. Each RM pair of fermions contributes a $U(1)$ subgroup which is a Kač-Moody current that generates the rank 22 gauge group.
2. Real fermions - through an orbifolding process, complex fermions are split and are re-paired between left and right movers. This pairing causes a rank reduction in the four dimensional gauge group of the theory since a RM complexifed fermion is broken. This breaks one of the $U(1)$ subgroups mentioned above. These fermions are dubbed "Ising" fermions in the literature [46].

In viewing a set of BVs (for example, the NAHE set), the RM must be divided up into pairs of real components with the exception of $\alpha_i$ with $i \in [37, 48]$ remaining possibly unpaired. With that in mind, the first 6 must be the same followed by the next 4 being the same to ultimately produce the standard model gauge group, $SU(3)_c \times SU(2)_L \times U(1)_Y$ for the most compact SM embedding. The remainder of the BV consists of a set of 6 paired and 12 possibly unpaired, which are followed by 16 paired components. The last 16 produce the $E_8$ hidden sector gauge group. The entire RM is freely specifiable within order $N$. Order $N$ means that the elements are composed of $\alpha_i \in \left\{0, \pm\frac{1}{\frac{N}{2}}, \pm\frac{2}{\frac{N}{2}}, , \pm\frac{\frac{N}{2}-1}{\frac{N}{2}}, 1\right\}$. These form the first set of constraints. After application of the GSO projections, the NAHE set yields a

Grand Unified Theory. In terms of gauge groups this is: $SO(10)\times SO(6)^3\times E_{8_{hidden}}$. Because of this result, many semi-realistic models are formed by breaking the $SO(10)$ symmetry of the NAHE set. This is done by adding more BVs. There has been work in consistently varying or extending the NAHE set en masse [47, 48].

By adding additional BVs, it is possible to produce a model with three chiral generations. This is done often with three additional BVs which break the four dimensional gauge group (in this case, $SO(10)$) to one of its subgroups. The $E_{8_{hidden}}$ is also broken as is the $SO(6)^3$, the latter being reduced to a set of flavor $U(1)$'s. These $U(1)_{flavor}$ groups are correlated to the number of generations so three of them are anticipated. Additional $U(1)$'s come from the (Real or Complex) pairing of worldsheet currents which may or may not reduce the rank of the to-be-formed observable gauge group. One last benefit that comes from the proper choice of BVs is the reduction of the Higgs doublet spectrum [19, 49]. This means that one does not have to rely on flat directions to remove unwanted Higgs doublets [49].

### *2.1.2 RNS Constraints*

The supercurrent (see [33]), as defined by the eighteen LM fields, is

$$T_F = \psi^\mu \partial X_\mu + i\sum_I x^I y^I w^I \tag{2.7}$$

where $\psi^\mu$ and $X^\mu$ are, respectively, spacetime fermions and bosons. The $x^I, y^I, w^I$ are Majorana-Weyl worldsheet fermions mentioned in the previous section and which satisfy the conformal anomaly and communicate the geometry of the compact dimensions. The $x^I$ can be collected into complex fermions and then bosonized via exponentiation:

$$\frac{1}{\sqrt{2}}\left(x^n + ix^{n+1}\right) = e^{iS_{n\,n+1}} \text{ for } n = 1, 3, 5. \tag{2.8}$$

An $N = 1$ spacetime superstring theory that is modular invariant requires two things: $N = 2$ worldsheet SUSY and LM local conformal invariance. The $N = 2$ worldsheet SUSY algebra generates a global $U(1)$ current, $J_1$, and two supercurrents: $T_F^1$ and $T_F^2$. The conformal invariance (which is really superconformal) results in the existence of a $U(1)_J$ ghost current generator living on the worldsheet. This LM current can be written in terms of the bosons:

$$J(z) = i\partial_z \left(S_{12} + S_{34} + S_{56}\right). \tag{2.9}$$

These bosons then splinter the $U(1)_J$ by each generating a unique global LM $U(1)$. These new $U(1)$'s are R-symmetries of the SUSY algebra since they appear in the gravitino vertex operator:

$$Q_\alpha^{\mathrm{SUSY}} = e^{-c/2} S_\alpha e^{i\beta_1 S_{12}} e^{i\beta_2 S_{34}} e^{i\beta_3 S_{56}}. \tag{2.10}$$

$S_\alpha$ is a spacetime spinor, $c$ is the ghost charge and the $S_{nn+1}$ come from Eq 2.8. Since our theory is superconformal, the corresponding algebra is closed for on-shell states due to the presence of the $U(1)$ bosonic generators.

For the sake of phenomenological investigations of our $N = 1$ SUSY models, it is necessary to consider order $n$ nonrenormalizable superpotential terms created from chiral superfields:

$$\int d^2\theta \Phi_1 \ldots \Phi_n. \tag{2.11}$$

These terms are proportional to $n$-point correlators:

$$\langle V_1^f V_2^f V_3^b \ldots V_n^b \rangle, \tag{2.12}$$

where $f(b)$ represent the fermion(boson) components of the superfields. As opposed to ten dimensional heterotic Calabi-Yau or left-right symmetric orbifold models, these terms can remain unsuppressed in four dimensional free fermionic models [50]. If let unchecked, these nonrenormalizable terms can have devastating effects on the hierarchical mass structure of a given model.

It has been shown that models composed entirely of complex fermions are safe from nonrenormalizable terms in perturbative analysis [50]. This is not the case for strict real fermion models or models where a combination of pairings are permitted. This latter scenario is more common in the age of systematic, large scale investigations of the free fermionic landscape and is the case with which we will be dealing. Fortunately, the pairing process can exclude various nonrenormalizable terms making stringy superpotentials much more constrained than their conventional EFT counterparts. This may be another hint of naturalness arising from string models.

Modular invariance of the correlator term, Eq (2.12), requires that the total ghost[3] charge of a tree-level amplitude be equal to $-2$. Since it is a quantum number, ghost charge is conserved at the vertices of the theory, although conformal invariance allows any given vertex to have different values of ghost charge. In other words, the superconformal invariance of our theory allows for “picture-changing” to redistribute the charge in order to yield the appropriate overall ghost number. It is the conservation of the three global $U(1)$'s that supply the picture-changing freedom. If there does not exist a transformation allowing a picture in which the -2 is realized, that term will be forbidden in a stringy superpotential. A given vertex fermion(boson) operator has ghost charge of $-1/2(-1)$. Since our construction produces spacetime fermions, the minimum superpotential will be order three: two fermions and one boson[4]. This is an example of string theory's potential naturalness manifestly controlling the type of allowed interactions. There are a few things to note at this point.

[3]These Fadeev-Popov ghosts are introduced in order to change the measure of integration in the action by breaking gauge symmetries. This procedure introduces extra non-physical particle content which is considered *virtual*. Unitarity is maintained through this process [51].

[4]It is natural to suppose that two bosons could combine to form an order superpotential term. These terms are disallowed for another reason: the worldsheet fermion charges: $y_i, w_i$ can only be conserved if there are at least three fields.

- Any given vertex needs to be invariant under the three $U(1)$'s generated by Eq (2.9), forcing the charged of the matter states in a superpotential term to sum up to zero (after picture-changing).
- Since we are only concerned with the vanishing of the correlator, the choice of fields '1' and '2' to be fermions is arbitrary (up to an ignorable phase). It is customary to keep the first two as such while bosonizing the remainder.
- Any of the bosons in a correlator can be picture changed, this is due to conformal invariance.

Table 2.1: Adapted from another paper (see [33]), this table lists the charges of the three $U(1)_i$ and indicates the various allowed picture-changing forms of the vertex operators.

| Field types | Charges: $(S_{12}, S_{34}, S_{56})$ |
|---|---|
| $\mathrm{NS}^b_{i(-1)}$ | $(1,0,0)$ + perms |
| $\mathrm{NS}^f_{i(-1/2)}$ | $\frac{1}{2}(1,-1,-1)$ + perms |
| $\mathrm{R}^b_{i(-1)}$ | $\frac{1}{2}(0,1,1)$ + perms |
| $\mathrm{R}^f_{i(-1/2)}$ | $\frac{1}{2}(-1,0,0)$ + perms |
| $\mathrm{NS}^b_{i(0)}$ | $(0,0,0)$ |
| $\mathrm{R}^b_{i(-1/2)}$ | $\frac{1}{2}(0,\pm 1,\mp 1)$ + perms |

Table 2.2: The categories of the fields can be defined according to which of the three $U(1)_i$; it is uniquely charged under. Also shown is the appropriate bosonized charge which is used in the full RNS picture-changing check. This table is adapted from another work ( [33] ).

| Field types | Category | Fermionic charges: $(S_{12}, S_{34}, S_{56})$ | Bosonized charged |
|---|---|---|---|
| | 1 | $(\pm\frac{1}{2},0,0)$ | $\frac{1}{2}(0,\mp 1,\mp 1)$ |
| R | 2 | $(0,\pm\frac{1}{2},0)$ | $\frac{1}{2}(\mp 1,0,\mp 1)$ |
| | 3 | $(0,0,\pm\frac{1}{2})$ | $\frac{1}{2}(\mp 1,\mp 1,0)$ |
| | 1 | $\frac{1}{2}(\pm 1,\mp 1,\mp 1)$ | $(\pm 1,0,0)$ |
| NS | 2 | $\frac{1}{2}(\mp 1,\pm 1,\mp 1)$ | $(0,\pm 1,0)$ |
| | 3 | $\frac{1}{2}(\mp 1,\mp 1,\pm 1)$ | $(0,0,\pm 1)$ |

There are many works [15, 33, 50] which delve into the explicit detail concerning the calculation of superpotential correlators and their stringy constraints. It would take us too far afield to consider it now. Therefore, we shall summarize the points necessary for numerical computation which allow for automated construction and investigation of semi-realistic free fermionic heterotic string model.

*2.1.2.1 RNS First Pass Constraints.* During the early investigations of Ramond/Neveu-Schwarz (RNS) picture-changing constraints, a set of simple boolean constraints on the form of the three global U(1) charges was found. This set of rules is necessary but not sufficient to forbid certain superpotential terms. These rules were summarized [33] and then generalized [2] and are included here because they offer a quick first pass test. Once a candidate superpotential term is found, each field is classified according to its RNS category by comparing the form of its $S_{ii+1}$ charges to the fermionic column in table 2.2. For each superpotential term, a two-by-six RNS category matrix can be generated by adding unity to the specified category column of the given R or NS row for each field within that term. An order $n$ superpotential term must pass the following rules:

1. All R categories must have the same value *mod* 2, and
2. For $n = 3$:
   (a) each R category must be 1, or
   (b) each NS category must be 1, or
   (c) an R category must be equal to 2 and the corresponding NS category must equal 1.
3. For $n > 3$:
   (a) A minimum of 4 R fields must be present, and
   (b) all R fields can not exist in the same category, and
   (c) For R = 4, only permutations of (R = 2, R = 2, NS = n-4) are allowed, or

(d) When R $> 4$, NS must be null for the largest R category (if there is a maximal R).

*2.1.2.2 Picture Changing (RNS Second Pass) Constraints.* The sufficiency test (which embodies the previous set of rules) is much more complicated but lends itself well to algorithmic, if not brute force, methodologies. Here are the steps to test the order $n$ term:

1. Bosonization: First we must bosonize the last $(n-2)$ fields. Technically speaking, this involves operating on each state with the zero-momentum gravitino vertex operator [15] which generates SUSY (Eq 2.10). This will shift the values of the paired $x_i$ worldsheet fermions and will most likely result in nonzero $U(1)_i$ traces of the three worldsheet currents. The absolute value sum of the $U(1)$ traces, $\Delta$, must be at most $(n-3)$ otherwise picture-changing will fail. To perform the shift, fermionic $(x_i, x_{i+1})$ charges are transformed into the related bosonic values as seen in Table 2.2. It is this $\Delta$ that must be shifted into other worldsheet fermions.

2. Other fermion pairs

   (a) Complex pairs - these $y$'s and $w$'s will be paired with other LMing fields. The pairing charge they possess must be summed for all the fields, *e.g.* $\sum_{i=1}^{n} Q_i^{\alpha_j \beta_k}$ for $\alpha, \beta \in y_j, w_k,\ j,k \in [1,6]$. A sum of nullity it required for these pairings.

   (b) Real fermions - due to the $\mathbb{Z}_2$ orbifolding symmetry of the compactified directions, the charges for the LM and RM worldsheet fermions that are paired are required to exist in equal numbers modulo 2. *i.e.* For all the fields within a given term, the (#1/2's, $\#-1/2$'s, #1's) = (0,0,0) or (1,1,1) *mod* 2 for any given left-right paired fermion.

3. Shifting the $\Delta$ charges: The excess charge beyond zero, $\Delta$ for the $x_i, x_{i+1}$ pairs must be shifted to the complex and real fermions. Every unit of charge to be shifted will be applied to both members of either the $y_i/w_i$ or $y_{i+1}/w_{i+1}$ pairs. Each of the chosen $y/w$

Table 2.3: Our example model [49] possess these fermion pairings. Fields 3-20 correspond to LM and higher numbers are RM. The leftmost block contains the pairs corresponding to the 3 $U(1)$ charges ($S_{i\,i+1}$). The second block shows the remaining $y/w$ complex fermions and the rightmost set lists the real fermions.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 9 | 15 | 4 | 5 | 10 | 7 | 8 | 13 | 14 | 14 | 17 |
| 6 | 12 | 18 | 20 | 11 | 19 | 38 | 44 | 40 | 46 | 41 | 47 |

Table 2.4: The worldsheet fermion pair charges of the fields from our example model [49] .

| Field | $x_{1,2}$ | $x_{3,4}$ | $x_{5,6}$ | $y_1, w_6$ | $w_{1,3}$ | $y_{3,6}$ | $y_2$ | $w_2$ | $y_4$ | $w_4$ | $y_5$ | $w_5$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\psi_3$ | $\frac{1}{2}$ | 0 | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| $\psi_7$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | 0 | $-\frac{1}{2}$ |
| $\psi_{12}$ | $\frac{1}{2}$ | 0 | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ |
| $\psi_{27}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 |

terms, independent of their pairing partners, receive a $\pm 1$ contribution with the sign chosen as needed to satisfy the fermion pair requirements (2.a and 2.b). This process is repeated by redistributing all the excess $x_i, x_{i+1}$ charges.

The picture-changing rules can best be illustrated through example. Let us take a model investigated in another work [49] whose fermion pairings can be seen in Table 2.3.

Consider an order four superpotential term containing the following fields: $\Phi_3, \Phi_7, \Phi_{12}, \Phi_{27}$. The corresponding correlator would appear as: $\psi_3, \psi_7, \phi_{12}, \phi_{27}$ where the last two fields have been bosonized without loss of generality. The corresponding charges for the fields under the fermions can be seen in Table 2.4.

Following our algorithm, the first step is to bosonize the last two fields by applying the gravitino operator which shifts $\psi_{12}$ according to a Ramond category 1 and $\psi_{27}$ as category 2. This adapts our model to fit the form of our desired correlator and allows us to analyze the term according to the picture-changing rules. It changes the charges as seen in Table 2.4 to those found in Table 2.5.

Table 2.5: After bosonization of the last two fields, the charges are summed for the various complex fermions. The real fermion charges are counted as in 2.b.

| Field | $x_{1,2}$ | $x_{3,4}$ | $x_{5,6}$ | $y_1, w_6$ | $w_{1,3}$ | $y_{3,6}$ | $y_2$ | $w_2$ | $y_4$ | $w_4$ | $y_5$ | $w_5$ | # mod 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\psi_3$ | $\frac{1}{2}$ | 0 | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | – |
| $\psi_7$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | 0 | $\frac{-1}{2}$ | – |
| $\phi_{12}$ | 0 | $\frac{-1}{2}$ | $\frac{-1}{2}$ | 0 | 0 | $\frac{-1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{-1}{2}$ | – |
| $\phi_{27}$ | $\frac{-1}{2}$ | 0 | $\frac{-1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{-1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | – |
| $\sum_{cols}$ | 0 | 0 | $-1$ | 1 | 0 | $-1$ | – | – | – | – | – | – | – |
| – | – | – | – | – | – | – | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{-1}{2}$ |
| – | – | – | – | – | – | – | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ |
| – | – | – | – | – | – | – | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

The sum and counting rules for steps 1, 2.a and 2.b yield the bottom half of Table 2.5. From the nonzero sum for the $x_{5,6}$ fermion pair, we can see that there is one unit of charge to distribute to the pair(s) containing $y_5/w_5$ or $y_6/w_6$. The $y_6/w_6$ worldsheet fields are both paired with RM and would normally shift one of the values to create any additional 1s, 1/2's or $-1/2$'s that are needed for the various modular counting. Obviously, this step is not needed and so we can proceed to investigate the possibilities provided by shifting the charge of the $y_5/w_5$ pair(s). Since the combinations $y_5/w_6$ and $y_3/y_6$ have total charge of $1(-1)$, we can shift them by $-1(1)$ using the $x_5/x_6$ charge. This can be done by subtracting one from $\psi_7$'s $y_5/w_6$ charge of 1/2 while adding unity to $\psi_3$'s $y_3/y_6$ charge of $-1/2$. This is a satisfactory result and completes the picture-changing process. It is not vital to record the exact fields to which the picture-changing is applied, unless one desires to calculate the full coupling constant of the fields. This is often unreasonably difficult to perform. The usual custom is to approximate [9] the scale of the various superpotential couplings and this practice will be continued here as well.

### *2.1.3 Moduli*

For generic (heterotic) string models, the radii of the tori on which the extra dimensions are compactified form a continuously connected family of physically consistent realizations. Each member of this family is an equivalent degenerate string vacua and there is no *a priori* method of choosing a desirable configuration. This limits the predictive power of the heterotic theory and it is anticipated that non-perturbative methods from M-theory investigations will allow researchers to find desirable string vacua without any tuning. In terms of an EFT, there must be a way to lift this degeneracy. This is done through the choice of the flat directions of potentials that consist of scalar fields known as moduli. The choice of which flat direction solution must come from non-perturbative effects which may also help break SUSY dynamically or spontaneously [8]. The VEVs of these moduli represent the different radii (and related angles between the various radii[5]) of the toroidal compactification at low energy. Indeed the other string theories also possess moduli spaces though their mathematical interpretation may be different. The free fermionic language of the heterotic string is performed at a fixed point, the self-dual radius, of the tori[6]. Therefore, the moduli that are present in those models come from compactification shape degrees of freedom.

Much of the work in string phenomenology is to characterize and study the different moduli stabilizations of the string models. Perturbative string analysis is unable to predict or explain how to properly lift one (or a subset) of these degenerate vacua. This is because the moduli do not possess a potential at any order in perturbative string theory. Therefore their values must be predicted by non-perturbative M-theory which is not well understood. This is also the case for flat direction VEVs where a degeneracy of solutions exist and it is not clear

[5]In the free fermionic language of string theory, the angular moduli become the primary source souce of parameter freedom since the radius is fixed at the self-dual point: R = 1 in string units.

[6]Since compactification occurs at this fixed point, the compactifications scale, $M_{\text{comp}}$, is naturally close to the string scale.

why nature precludes the choice of many realistic VEV sets. It is important to remember that even though these perturbative VEV directions may not be actual representations of reality, they can be analyzed for phenomenological viability. This can in turn illustrate correlations in the non-perturbative theory. This is, of course, still just a hope but it is a working assumption for practically all phenomenologists.

### *2.1.4 The Dilaton*

A well known stringy moduli field is the dilaton. It is this field that, when VEVed, gives the numerical values of the various gauge coupling constants: $g^2 M_{\text{string}} = 2\langle f(\phi_{\text{dilaton}})\rangle$ [8,22]. Unfortunately, in terms of perturbative theories, the dilaton cannot be stabilized with a VEV since the dilatonic potential is all-order flat according to the non-renormalization theorems.

The dilaton field is stabilized usually through a potential initiated by hidden sector gaugino condensation [8]. Therefore, SUSY breaking is a hallmark of dilaton stabilization. Despite this relationship, it is surprising that the VEV of the dilaton is largely unaffected by hidden sector physics if the dilaton potential is formed by strongly coupled Yang-Mills ($SU(N)$) gauge groups. A side-effect of dilatonic soft SUSY breaking is the reduction in flavor changing neutral currents. This is not the case if other moduli[7] fields lead to SUSY breaking. As discussed elsewhere [8], this form of SUSY breaking is also a result of weakly-coupled heterotic modular invariance and would allow for testable proof for the efficacy of the heterotic string.

The potential induced by the gaugino needs to possess some non-trivial, global minimum in order to stabilize the dilaton. This will result in the dilaton VEV which also defines the effective string coupling which is used in the string perturbative expansion. This string coupling also helps to define the naturalness of lower energy parameters since all fundamental constants can be expressed in terms of this coupling. The dilaton strength also relates to the

[7]Such as non-trivial radial moduli. Therefore, this does not apply to WCCFHS.

powerful 'S-duality' of string theory. This allows weakly-interacting regimes to be connected to strongly-coupled dynamics and has been used to relate the various five string theories.

### *2.2 The Nitty-Gritty: The Usual Approach to Flat Directions*

The model construction consists of two parts: model generation and flat direction analysis. Early work done in these two areas was quite successful using a combination of computational (FORTRAN) and analytic techniques [44, 52]. However, a fully automated search was not possible until recent advances in supercomputing technology. Most recently, the development of the C++ based Free Fermionic (FF) and Gauge Frameworks at Baylor University have allowed for the generation of large sets of WCFFHS models. The second aspect of model building, flat direction analysis, has yet to be fully automated. This is an area of active research at Baylor University and the remainder of this article will focus on flat direction calculations.

The importance of flat directions should not be overlooked as all of the interaction and mass information is revealed only though these calculations. Therefore, a 'true' string model can exist only after the flat directions have been investigated. Often the term 'model' is used loosely among string phenomenologists, so context must guide the reader as to the intended definition. Here, we will use the term to mean a set of models specified by a given collection of gauge groups which have the same charges. In other words, a model is a charge matrix. This matrix can yield many actual models due to the freedom allowed by post-compactification moduli stabilization.

The following sections will outline the usual approach to flat direction analysis. First we will discuss the central piece of data needed for calculations: the charge matrix. Then we will discuss, briefly, the field VEVs calculated through D-flatness. Lastly, F-flatness will be illustrated as the usual end point for the flatness analysis.

### 2.3 Charge Matrix

To study flat directions, analysis of a charge matrix, $Q$, is required. This encodes the actual gauge symmetry information of the model where the columns are the various quantum fields which are charged under the gauge groups of the theory (the rows). The value for the charge is the dimension of the specific representation of the gauge group under which it is charged. Therefore, $Q_{ij} \in \mathbb{Z}$.

### 2.4 General Structure

An $M \times N$ charge matrix will consist of $N$ fields charged under $M$ gauge groups. Since $M < N$, $Q$ will usually be singular. Typically, a model will consist of 50 to 100 fields charged under 5 to 22 gauge groups. This yields: $M \in [50, 100]$, $N \in [5, 22]$, and $M - N \in [28, 95]$.

Some fields have charges that can be paired. These paired fields are called "vector-like" and they represent particle-antiparticle pairing. For Abelian groups or complex NA representations, fields are pairable if they have opposite (non-trivial, non-singlet) charges. For real representations of NA groups, the charges must be the same for both fields. Let the number of paired fields in a given model be: $N_p$, with $N_s$ and $N_{\text{NA}}$ representing the unpaired NA singlet and non-singlet fields, respectively. It is conventional to rearrange and relabel the fields as:

$$\overbrace{\Phi_1, \ldots, \Phi_{N_p}}^{\text{vector-like}}, \overbrace{\Phi_{N_p+1}, \ldots, \Phi_{N_p+N_s}}^{\text{NA singlet}}, \overbrace{\Phi_{N_p+N_s+1}, \ldots, \Phi_N}^{\text{NA}}$$

#### 2.4.1 D-flat Solutions

The equations for solving for D-flatness are: Eq 1.24 - 1.27. It is relatively straightforward to solve this linear system for the $|\Phi_k|^2$ in the case of Type I, or singlet, scenario and that is the usual approach. The only issue is that the system is often under-determined. Therefore, the solution space will likely be unbounded. In the past, searches have been done in a piecewise manner requiring additional constraints which include specifying allowable field

VEV ranges before D-flat directions are even calculated. These constraints could be nicely applied to visible sectors since there would some physically observable data with which to compare. For the hidden sector, however, this may need to be relaxed. In some cases, by controlling these ranges it is possible to tune the order at which SUSY is broken.

### *2.4.2 Introductory Description: Stringent F-flat*

It is helpful to introduce the mathematical formulation of stringent F-flatness using a straightforward, brute-force methodology. This is not the way our calculations are performed but previous work [2,30,52] has pursued this course and so it is a logical introduction. Since stringent F-flatness is solely concerned with the presence of the VEVed and non-VEVed fields the solutions space can be represented in terms of powers of the individual F-terms:

$$\Phi_1^{r_1} \ldots \Phi_M^{r_M} \longrightarrow \vec{\mathrm{R}} \equiv \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{pmatrix}. \tag{2.13}$$

The mapping we need to investigate is:

$$Q \cdot \vec{\mathrm{R}} = \vec{0}. \tag{2.14}$$

In other words we need to consider the nullspace of $Q$. The values of the components of $\vec{\mathrm{R}}$ are of no consequence except for those $r_i$ that correspond to solutions of the D-flat equation (Eq 3.8) where $x_i = 0$. Let the columns of $Ns(Q)$ be the basis vectors that span the nullspace($Q$). The investigation of potentially dangerous F-terms can then be recast into two possibilities. The first can be seen as satisfying Eq 1.28 and is represented as:

$$Ns(Q)_{red} \cdot P = \mathbb{I} \tag{2.15}$$

where $Ns(Q)_{red}$ is the reduced nullspace basis for the charge matrix constructed by keeping the rows of $Ns(Q)$ which correspond to $x_i = 0$ and removing all others. The columns of $P$, $col(P)$, will be the coefficients to be used with the $Ns(Q)$ basis vectors to see if that combination stays in the nullspace:

$$Q \cdot \sum_i P_{ij}\ col(Ns(Q))_i \stackrel{?}{=} \vec{0} \tag{2.16}$$

where $j$ is equal to the number of $x_k$ that are equal to zero and $i \in [0, n-l]$ as dictated by Eq 3.10.

The second F-term constraint that must be tested is the null case, which is related to Eq 1.29 (the supergravity constraint):

$$Ns(Q)_{red} \cdot \vec{p} = \vec{0}. \tag{2.17}$$

This case is more difficult since it may be severely under-determined. If there were a finite number of solutions, they could be assembled into the matrix $P$ as before and tested as in Eq 2.16. In general, there will be zero, one or infinite solutions to Eq 2.17. However, after integerizing the solutions, only a finite number should exist within the physically realizable order range of the superpotential: 3 - 17. In the case of all order infinite solutions existing beyond order seventeen, we find ourselves severely handicapped at how to approach SUSY breaking without relying on some other mechanism such as, gravity-mediation, kinetic mixing or shadow charges [53, 54].

For either case (Eqs 2.15,2.17), if a particular set of nullspace basis vector coefficients, $col(P)$, is shown to be mapped outside of the nullspace, then we have found a potentially dangerous term which will then be tested against worldsheet selection rules [15]. If it survives, the vector with the smallest value of $\sum_i |r_i|$ is the energy scale of SUSY breaking closest to 1 TeV for that particular set of D-flat field VEVs.

#### *2.4.3 Basic Flat Directions*

The last two sections outline a straightforward method for finding flat directions. They are summarized here in outline form:

1. D-flat (Hidden Sector):
   (a) Investigate singlet D-flatness by canceling the FI-term
   (b) (If needed) Generate NA D-flat solutions
      i. Partial NA case: Consider only $U(1)$ charges but retain the multiplicity inherent in the NA multiplets.
      ii. Full NA case: Calculate gauge group generators and solve coupled linear system.
   (c) Scale of FI term determines the VEV scale for hidden gaugino condensates.
2. F-flat (Using D-flat VEVs)
   (a) Generate a given order of superpotential and apply VEVs (or generate potentially dangerous terms and check for survival in stringy models)
   (b) Continue to next order until fails
   (c) Calculate the scale of SUSY breaking
   (d) If SUSY is kept past $17^{\text{th}}$ order, SUSY must be broken via other mechanisms[8]
3. Phenomenology
   (a) From the surviving superpotential interactions and after inserting VEVs, determine:
      i. hidden sector mass textures
      ii. effective Yukawa terms
   (b) Cosmological constant calculated from the lightest stable hidden sector matter

[8]This can be relaxed just slightly: SUSY can be kept up to $16^{\text{th}}$ or $17^{\text{th}}$ order but broken at the next highest. This is an acceptable scenario and would explain SUSY breaking via flat directions at reasonable energy scale without requiring another SUSY breaking mechanism.

Step two will have applied the hidden sector VEVs to the superpotential. This results in an EFT that is on the energy scale of the hidden sector condensates. This is still far removed from our SM physics. In order to approach these lower energies the SM(GUT)-like matter must be allowed to take on VEVs but at a lower energy scale. In a similar manner to that outlined above, some of the observable (including Higgs) fields will take on VEVs. This will break the GUT gauge groups (ideally) into the SM. The corresponding energy scales can be analyzed and will relate, like step three, the masses for generational matter.

CHAPTER THREE

Methods

### *3.1 The New Approach*

At the end of the last section, it was shown how a typical flat direction analysis is performed. The technique implemented here is different. Steps 1 and 2 remain intact within themselves but they will be performed in the opposite order. This does not seem possible given that the F-flatness seems to rely directly on the VEVs of the various fields. However, F-flatness itself only considers whether a fields has a non-zero VEV or not. Therefore, by considering the on/off combinations of the field VEVs, it is possible to know whether a given superpotential term will have a zero VEV.

We will outline the course below and then, in the later sections, describe how this method is implemented into a software package.

1. F-flat
   (a) Generate a given order of superpotential, keeping only terms that survive stringy constraints
   (b) Toggle (hidden sector) VEVs on/off and find the minimal set of zero VEVs needed to make term VEV zero
   (c) Continue to next order
      i. Find superpotential terms
      ii. Create VEV constraints
      iii. Combine previous order constraints with current
   (d) Check combined constraints, if too many field VEVs are needed to be zero then maximum SUSY breaking order has been found
   (e) If SUSY is kept past $17^{\text{th}}$ order, SUSY must be broken via other mechanisms[1]

[1] This can be relaxed as in the usual prescription of flat direction analysis outlined earlier.

2. D-flat (Using hidden sector field VEVs from F-flatness, SM/GUT-like left unspecified)
   (a) Hybrid I/II case: Consider only $U(1)$ charges but retain the multiplicity inherent in the NA multiplets (see 3.6.1), or
   (b) Full NA case: Calculate gauge group generators and solve coupled linear system.
   (c) Scale of FI term determines the VEV scale for hidden gaugino condensates.
3. Phenomenology (Apply VEVs to superpotential terms)
   (a) From the surviving superpotential interactions
      i. hidden sector mass textures
      ii. effective Yukawa terms
   (b) Cosmological constant calculated from the lightest stable hidden sector matter

## *3.2 The Julia Difference*

The language of choice for our software package is Julia Technical Computing Language [55]. Appearing in 2012, Julia is a relatively new computer coding language and on the surface similar to MATLAB. Therefore, it is very fast computationally and has simple syntax. Julia was designed to balance the in-fashion use of dynamic languages with the scientific communities need for numerically robustness and efficiency. This makes Julia attractive to physical and mathematical scientists who do not wish to also become computer scientists. This can be seen in Julia's extensive use of multiple dispatch for function calls. This means that functions can/are designed to handle different variations of arguments. Adding two vectors will work in the intuitive and expected way while the same adding function call will also add scalars correctly. This is an almost trivial thing in an analytic approach but it quickly becomes difficult and frustrating to program in many languages.

One of the most touted aspects of Julia is that of its speed. In simple benchmarking, Julia approaches that of Fortran and C, while moves much beyond MATLAB itself [55]. The language is also geared for simple methods of parallelization and for performing distributed calculations. As physical system modeling becomes more complex, supercomputing has become a requirement rather than a request. Julia is built to answer that need without requiring very much knowledge about node and processor system architecture, as well as the various types of information passing.

Optional and inferential type casting make it a very forgiving and extensible language. The language is also homoiconic, meaning that its source code it written in the language itself. This aids in learning the language, as well as makes it possible to create your own speed improvements should the need arise. For those who need more control over types, Julia allows for user-defined types that are reportedly as efficient as those built in to to the language. In general, types can also be promoted and converted as needed. This allows the data to be tailored to different applications as needed. The characteristics allow for Julia to be a one-stop language rather than just a nice front-end interface. The use of well-known, stable and fast libraries, like Linear Algebra PACKage (LAPACK), give the Julia user the ability perform many needed and familiar operations without having to add extra libraries. However, users can add their own libraries and build macros to allow for maximum extensibility.

Julia can interface with Fortran and C codes giving it effective and simple front-end programming usability and since Julia is an open-source language, new packages are being added and improved upon constantly. For example, statistical libraries (such as DataFrames) allow for the built in functionality that handles missing data similar to the statistics-oriented language, R. Other packages allow for interfacing with gnuplot or Python's plotting library, for performing calculus and Linear Programming. Because of the design features many people are contributing to (and testing) the overall Julia project.

One major difficulty from which the initial versions of the code suffered was the limit of eight processors per node. In communicating, for example with Baylor's Kodiak super-computing cluster, the built-in Torque batch system requires that the number of nodes and processors per node be specified at startup. Unfortunately, many Julia functions will not recognize more than one node at a time, therefore only eight processors can be utilized at once. A macro and module built for Julia by Douglas Moore is able to circumvent this problem by manually adding processors to the job up to the total number specified (#of nodes $\times$ # processors/node). By recording the addresses of which processors are part of the batch call, this Julia macro is able to pass data between processors that live on different nodes. This increases the size of allowable jobs and is of great use, especially in F-flatness calculations. Since the design is a macro call, all one has to do is to wrap the existing code within it. All the work is done for the user thus maintaining the simple nature of the Julia architecture.

### *3.3 Algorithmic Layout*

In this section, we seek to expand upon the outline given at this beginning of the section. In a step-by-step process we hope to guide the reader through the model analysis process and in so doing justify our approach which differs from traditional flat direction analysis. While a difficult and computational intensive procedure, our methodologies have made the flat direction analysis tenably performed in finite time. This has been done in a way to offer studies that are fully automated and simple to initiate. Despite Julia's simple syntax and code, our goal is to limit the necessary interface of researchers, thus giving them time to analyze results rather than learn how to use the software. This structure is still in its infancy but it is a main focus that drives the design of the program. It is import to balance ease of use with options for analysis and that remains a constant struggle.

### *3.4 Input Necessities and other Initializing Steps*

As mentioned in the Introduction, a model is not a true phenomenological model until a set of flat direction VEVs are found. In process to that, we must first know the field theoretic and stringy characteristics of a given pre-model model. This information includes:

- A list of fields
- Charges under the various NA gauge groups and the orthogonal $U(1)$'s
  - NA charges are the representations in which the field lives
  - The $U(1)$ charges will have been rotated such that only one $U(1)$ is anomalous
- Worldsheet current information
  - RNS category which is the first pass performed prior to the full picture-changing calculation
  - The various real and complex pairings will need to be supplied for the picture-changing process.
  - Sector (or sets of linear combinations of BVs) information to distinguish between SM/GUT-like versus hidden sector (this is done in conjunction with the NA representations)

### *3.4.1 Accessible Information*

A slew of information is readily available before any major program functionality is utilized. A list is included below:

- Before Pairing
  - Gauge group names and rank (*e.g.* $\mathrm{A}_n$, $\mathrm{D}_n$, $\mathrm{E}_n$)
  - Number of NA and $U(1)$ groups
  - Dimensionality of the real representations for the NA groups[2]
  - Names of the fields
  - Sector data
  - Unpaired charge matrix (both $U(1)$ and NA)

[2] A Julia Module written by Douglas Moore allows for the calculation of the Lie algebraic characteristics of highest weight representations of a given group. All possible highest weights can be calculated given the Kač-Moody level of an algebra. These, using Moore's code, can be used to determine whether a representation is real or complex.

  - RNS categories

- After Pairing (see next section)

  - Pairs list
  - Number of vector-like fields
  - Paired Sector data
  - Paired charge matrix (both $U(1)$ and NA)
  - Paired RNS categories
  - Separation of SM/GUT-like fields from hidden matter

### *3.4.2 Pairing*

The first functionality required in the program is the pairing of fields. When calling on some of the data listed in the above section, pairing is automatically performed. In fact, that operation is the default for the program. Vector-like pairs have reference to particle/antiparticle pairs and will be important for generating mass matrices after field VEVs are found. Additionally, if a given partner of a pair takes on a VEV, its anti-partner pair will not necessarily do so as well.

Pairing is performed by comparing these three pieces of data:

1. RNS categories

   - fields must carry the same RNS worldsheet charges in order to be paired

2. $U(1)$ charges

   - during the pairing process, the $U(1)$ charges are compared. If the charges of the two fields differ by a global minus sign then they can be paired.

3. NA charges

   - real(complex) representations require the same(negative) value for candidate fields to be paired.

If a two fields meet all three of the above criteria then they are paired. The algorithm that performs the pairings will pair the first two fields satisfying the constraints. This will possibly leave unpaired later fields. For example, if field $\phi_1$ can be paired with $\{\phi_3, \phi_4, \phi_5\}$ then $\phi_1$ and $\phi_3$ will be paired while $\{\phi_4, \phi_5\}$ will be untouched and left to be paired with other fields.

After pairing, the fields are reordered and relabeled. If there are $M$ fields resultant from the pairing process, where $i(k)$ of them are vector-like singlet(NA) fields, then the original number of fields was: $M + i + k$. When the pairing array is called it will be listed in this order:

$$\overbrace{\Phi_1, \ldots, \Phi_i,}^{\text{vector-like singlet}} \overbrace{\Phi_{i+1}, \ldots, \Phi_{i+j},}^{\text{unpaired singlet}} \overbrace{\Phi_{i+j+1}, \ldots, \Phi_{i+j+k}}^{\text{vector-like NA}} \overbrace{\Phi_{i+j+k+1}, \ldots, \Phi_M}^{\text{unpaired NA}} . \tag{3.1}$$

The pairing array will have $M$-rows and 3 columns. Its purpose is to translate between the original field numbering (unpaired) and the new numbering (paired). The $l^{\text{th}}$ row is the $l^{\text{th}}$ field and the $1^{\text{st}}$ column contains the original number of that field while the $2^{\text{nd}}$ column would contain the original number of the paired partner. A zero entry in this second column indicates an unpaired field. The last column is '1' if the field is non-trivially charged under the NA gauge groups and is '0' if it is a NA singlet field An example of a pair matrix is found in Table 3.1. Relating this table to list of fields in Eq 3.1, we find that: $M = 40$, $i = 17$, $j = 3$ and $k = 11$.

Using the pairing array allows the redefinition of the charge matrix, sectors and field names using the new field labeling. It is this labeling that is used throughout the flat direction analysis. If a vector-like field takes on a VEV or appears in a superpotential term then the presence of a negative sign indicates the partner field as the field in question. *e.g.* a $< \Phi_{1(-1)} >$ indicates that the original(antiparticle) field is being VEVed.

Table 3.1: An example of the pair fields list which relates the new label to the original field labels.

| New # (row) | Original # | Partner Original # | NA bool |
|---|---|---|---|
| 1 | 1 | 2 | 0 |
| 2 | 3 | 5 | 0 |
| 3 | 4 | 6 | 0 |
| 4 | 7 | 10 | 0 |
| 5 | 8 | 11 | 0 |
| 6 | 9 | 12 | 0 |
| 7 | 13 | 14 | 0 |
| 8 | 15 | 16 | 0 |
| 9 | 17 | 18 | 0 |
| 10 | 19 | 20 | 0 |
| 11 | 21 | 22 | 0 |
| 12 | 23 | 24 | 0 |
| 13 | 25 | 26 | 0 |
| 14 | 27 | 28 | 0 |
| 15 | 29 | 34 | 0 |
| 16 | 30 | 33 | 0 |
| 17 | 31 | 32 | 0 |
| 18 | 35 | 0 | 0 |
| 19 | 36 | 0 | 0 |
| 20 | 37 | 0 | 0 |
| 21 | 38 | 62 | 1 |
| 22 | 39 | 64 | 1 |
| 23 | 40 | 63 | 1 |
| 24 | 44 | 61 | 1 |
| 25 | 45 | 59 | 1 |
| 26 | 46 | 58 | 1 |
| 27 | 47 | 60 | 1 |
| 28 | 48 | 49 | 1 |
| 29 | 50 | 56 | 1 |
| 30 | 51 | 55 | 1 |
| 31 | 52 | 57 | 1 |
| 32 | 53 | 54 | 1 |
| 33 | 41 | 0 | 1 |
| 34 | 42 | 0 | 1 |
| 35 | 43 | 0 | 1 |
| 36 | 65 | 0 | 1 |
| 37 | 66 | 0 | 1 |
| 38 | 67 | 0 | 1 |
| 39 | 68 | 0 | 1 |
| 40 | 69 | 0 | 1 |

### 3.4.3 Particle Identification

Particle Identification (PID) in the context of string phenomenology is that of separating hidden matter fields from any SM, GUT or Higgs fields. To determine this, a simple set of boolean checks can be performed. Schematically this will loop over the set of desired observable gauge groups: $G = \{SO(10), SU(5),$ Pati-Salam$(SU(4) \times SU(2)_L \times SU(2)_R)$, SM$(SU(3) \times SU(2) \times U(1)\}$ checking for the appropriate generations of observable matter and any Higgs content. If not enough representations (minimum of 3 for matter), or if they are in the wrong configuration, the next gauge group configuration in $G$ is checked.

In order to identify the hidden sector, it is vital to first find and separate the observable NA symmetries. Once those are found, it is straightforward to associate any remaining gauge groups with the hidden sector and find fields that are non-trivially charged under those groups. The various representations sought are listed in Table 3.2.

There is a difficulty in choosing the appropriate $U(1)$ to be paired with $SU(5)$ (to make flipped $SU(5)$) or to be the SM $U(1)$. This is due to the ambiguity in defining hypercharge. Determining whether any $U(1)$'s are broken through flat direction VEVs will allow for this hypercharge calculation. At this stage all $U(1)$'s are treated as hidden. Those left unbroken, after VEVs are chosen, will be considered as candidates for the observable sector. This is done at a later stage, yet is vitally important for the case of specifying $SU(5)$ as flipped or unflipped.

## 3.5 F-flatness and Constraints

Once the fields are properly paired and the hidden sector fields are identified, the F-flatness analysis can begin. Generating the superpotential is really just a case in gross combinatorics. For example, an order $n$ term from model with $m$ fields of which $k$ are vector-like will have $\left(\binom{m+k}{n}\right)$ superpotential terms. This is readily recognizable as combination with replacement and is equivalent to:

Table 3.2: This table lists the gauge group representations for the matter and Higgs content needed to determine if a NA gauge group is observable. If fewer than three copies of matter representations appear, then the gauge group under investigation belongs in the hidden sector.

| Field Type | $D_5$ $SO(10)$ | $A_4$ $SU(5)$ | $A_3 \times A_1^2$ $SU(4) \times SU(2)^2$ | $A_2 \times A_1$ $SU(3) \times SU(2)$ |
|---|---|---|---|---|
| Matter | **16** | **-5**,**10**,**1** | **(4,2,1)**(**-4,2,1**)(**6,1,1**) (**1,1,3**)(**1,3,1**) | (**-3,1**)(**1,2**) (**3,2**)(**1,1**) |
| Higgs | **10** | **24**,**5**,**-5** | (**4,1,2**)(**-4,1,2**) | (**1,2**) |

$$\left(\binom{m+k}{n}\right) = \binom{m+k+n-1}{n}. \tag{3.2}$$

The sheer multitude of superpotential terms, as illustrated by Eq 3.2, warrants two computational approaches: parallel processing and finding a cut-off. The first point is relatively straightforward (albeit possibly difficult to implement depending on computer architecture and coding language) but the second is a question of physics. Let us discuss each of these in turn.

### *3.5.1 F-term Generation*

As discussed above, the number of terms is quite staggering. The approach taken here is to use a recursive algorithm to generate all possible ways of putting $m$ fields with $k$ vector-like partners in $n$ (the order) slots. Once a term is generated it must be tested for gauge invariance. It must also pass the stringy constraints: RNS category rules and picture-changing. Gauge invariance is satisfied according to Eq 2.14 and the string rules are given in Sections 2.1.2.1 and 2.1.2.2. Once a good term is found to satisfy those rules, it is written to file where it will later contribute to the zero VEV constraints which will be needed to satisfy F-flatness.

In order to make the allowed superpotential terms in finite time, it will be necessary to perform the calculations in a distributed way across the nodes of a supercomputing cluster. As cited in the section on the Julia computing language, it is relatively straightforward to perform basic parallelization of a given set of code. Using the native *pmap* (parallel mapping) function, all one has to do is split up the given data into chunks and have a function which can operate on that set of input data. *pmap* is called and uses as its arguments both the input chunks and a function which will operate on the data. A process is created which applies that function to a given chunk of data and is done simultaneously on as many processors that the cluster or user will allow. Note that the parallelization macro, also mentioned in the Julia section, is utilized here so that construction of superpotential terms is broken up between a large number of processors.

### *3.5.2 F-Class Creation*

The second issue mentioned at the beginning of the section was a question of physics: is there an allowable cut-off whereby one can ignore higher orders of the superpotential thus limiting the overall number of terms which need to be generate? This directly relates to the calculable scale of SUSY breaking, but it also leads into a discussion of fields VEV constraints.

If a given order of the superpotential has to have all of its fields take on zero VEVs then any higher order calculation will be unnecessary. This is apparent when considering the calculation of fermion mass textures, or the difference in mass scales between matter generations. In order to be able to calculate the fermion mass matrices, it is necessary to find superpotential terms of the form:

$$\begin{aligned} \text{SM/GUT-like fields:} \quad & \langle A \rangle f \bar{f} \langle h \rangle \\ \text{Hidden fields:} \quad & \langle B \rangle f \bar{f}. \end{aligned} \tag{3.3}$$

In Eq 3.3, $A$ and $B$ are collection of fields which receive non-zero VEVs. Since it is the Higgs mechanism that imparts mass to observable matter, the Higgs field must also take on a non-zero VEV for the SM/GUT case. A generalized Higgs effect is allowed for the Hidden sector matter. The numerical coefficient left by the VEVed fields is inputted into a mass matrix and is used to calculate the mass eigenstates of the various fermionic particles. For the first pass of the flat direction analysis, it is important to remember that only hidden sector fields will have the ability to take on a VEV. This removes the SM/GUT-like and Higgs fields from the remainder of the discussion. The reason that this is possible is because hidden sector mediated SUSY breaking occurs at relatively high energies compared to SM/GUT flat directions.

Of particular relevance to our current discussion is that at least two hidden sector fields, the particle and antiparticle ($F$ and $\bar{f}$, respectively), cannot both receive VEVs. This puts a minimum limit of two fields which do not take on zero VEVs in the constraint-finding process. In order to maintain the greatest freedom in ranging over the VEV moduli space, it is necessary to choose the minimal set of zero-valued fields VEVs which satisfy F-flatness. Therefore, if all the fields are required to be set to zero, we have an over-constrained model and SUSY breaking must occur at the given, or a previous, order of the superpotential. This gives us a natural and physical cut-off for superpotential term generation. Due to this approach may not be necessary to investigate all 17 orders. Additionally, this approach offers the greatest flexibility in investigating VEV parameter spaces, since the constraints have only specified which fields must be zero. All others are left free to range over values allowed by D-flatness. This will make it possible to help tune the matter masses. One possibility is to encode a pre-mass matrix containing all the fields that will contribute VEVs to a given element. After VEVs are specified by D-flatness those values can be quickly applied to the mass matrix in order to see if they are a desirable set.

The set of zero VEV constraints will basically be a boolean array where a *true*(*false*) at a given array index indicates that the corresponding field will have a zero(an unspecified) VEV. In this language of hidden sector flatness, all SM/GUT-like and Higgs fields will have an unspecified VEV. Therefore, their corresponding array elements will be 0s. As mentioned above, only the arrays with the least number of 1s will be kept so as to not over constrain our VEV space when moving on to D-flatness. These arrays are written to file and are collectively called *F-Classes*, as they partition the VEV-space in equivalence classes where each member of a given class generates the same F-flatness solution.

#### *3.5.3 Generating F-Class Constraints*

A straight forward approach to generating F-Classes would be a 'brute-force' *for*-loop over the integers from 1 to $2^N$, where $N$ is the number of fields in a given model. In this case, the 1s in the binary representation of an integer represent a field VEV going to zero. For small $N$, this technique is satisfactory as it will lead to a complete set of minimal zero VEVs. However, as $N$ increases performance suffers drastically due to the multiples of 2. Figure 3.2, is constructed from a quick test using Julia *for*-loops. For a loop over 30 fields, the average is approximately 13 seconds. This may seem reasonable but since most models have anywhere between 40 to 70 fields, this quickly becomes too time-consuming. For example, at 13 seconds to scan through $2^{30}$ possibilities, $2^{60}$ would take: 1.3958E+10 seconds or 442.63 years.

Since a straight brute-force technique is unrealistic, an alternative approach is necessary. This new method has roots in set and graph theory. When parsed down, the problem we are trying to solve is that of the minimal covering set of fields where each F-term contains at least one of these fields. Another view point is to consider all the F-terms are vertices of a graph. The edges represent the fields and are formed by connecting vertices which contain a given field. The goal is to find the minimum spanning tree (or the minimal set of edges

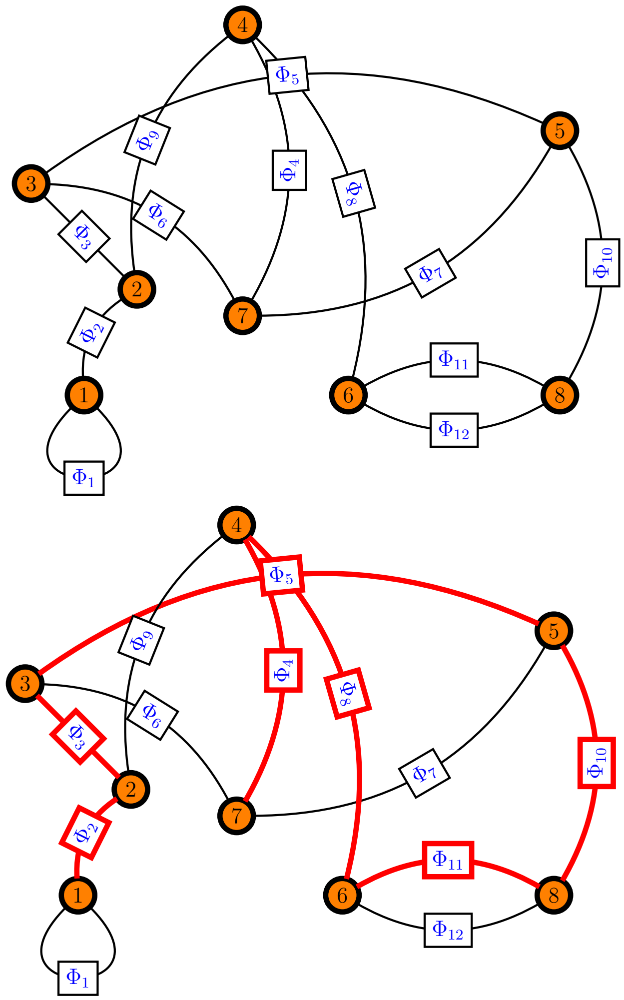


Figure 3.1: The top figure illustrates, as an example, the field ($\Phi_i$) interconnectivity between various F-terms (numbered nodes). A minimal spanning tree is the shortest possible path that touches each node once. The emboldened red path in the bottom figure represents this minimal set of fields. These fields along this path receive zero VEVs and are chosen to force all the F-terms VEVs to zero. Note that this path chose is not unique. It is our goal to find the full set of minimal spanning trees for a given order of superpotential terms. Each tree constitutes an F-Class.

where every vertex is touched). While it can be shown that these solutions exist, it is much harder to find such a solution (see Figure 3.1). Instead, we try to get as close to a minimal set as possible by considering the multiplicities of each field with respect to the number of F-terms in which they sit.

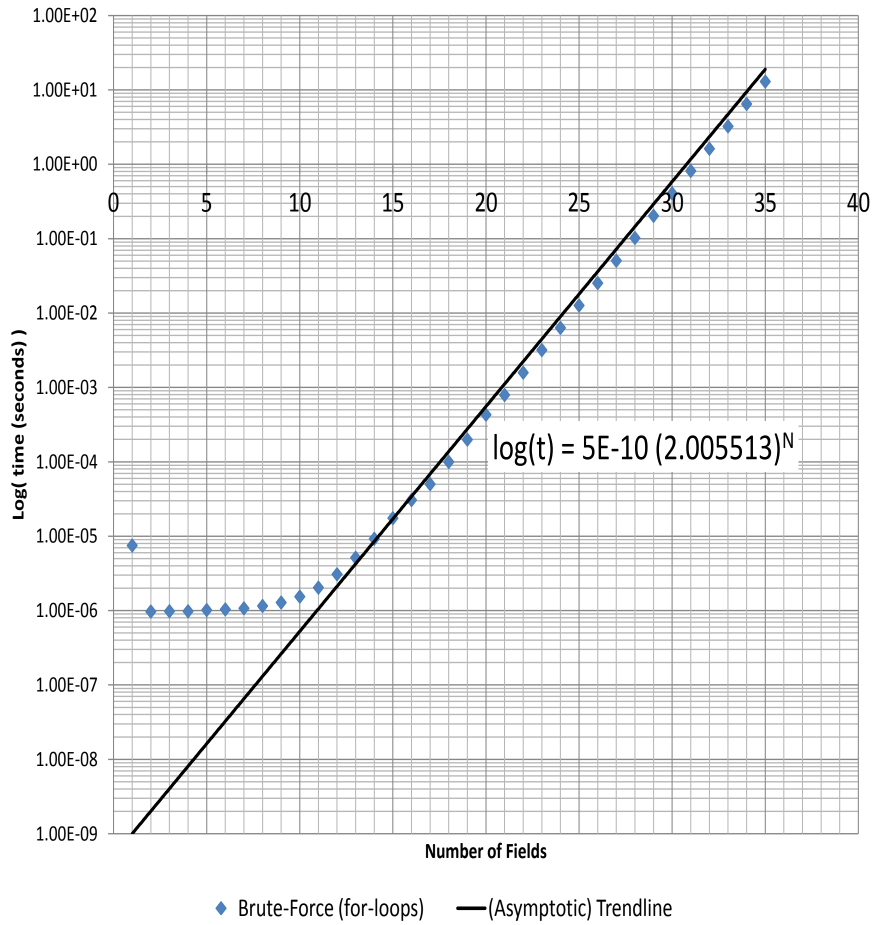


Figure 3.2: A logarithmic graph showing the increase in time for *for*-loops ranging from 1 to $N$, the number of fields. An asymptotic trendline shows the 2-folds relationship. For $N \in \{1, 27\}$, the calculations were averaged over a 1000 trials. The range of $\{28, 35\}$ is comprised of 10 trials a piece.

By counting and ordering the multiplicities, it is easy to see which fields will have the greatest impact if they are zeroed. Of course, some of these higher-multiplicity fields may also overlap so we must be careful to find the minimal amount acceptable. The steps of counting multiplicities, minimizing overlap and removing fields must be an iterative procedure which will approach the minimal covering. Once some fields have been zeroed out the remaining fields can be brute-force checked. The parameters that must be balanced are the allowed overlap as well as the maximum number of unspecified fields. This must be done while remembering that when the number of fields goes above 30, run time increases dramatically for brute-force techniques.

*3.5.3.1 The* ZeroOutFields *Algorithm.* The workhorse function in performing F-Classes calculations is called *ZeroOutFields*. Schematically, it works as follows:

1. Input:
   (a) maximum allowed overlap
   (b) field list
   (c) F-term list
2. Calculates multiplicities
3. Increments from highest to lowest multiplicity until a set of fields is found with overlap less than the specified maximum overlap
4. Output:
   (a) Removed Fields
   (b) Remaining Fields
   (c) F-terms with removed fields zeroed ('Reduced F-terms')

This function is used multiple times to find the maximum desirable overlap as well as to subtract out any zeroed fields.

*3.5.3.2 F-Class Algorithm.* The whole F-Class algorithm is summarized here:

1. Generate F-terms from given order
2. Find any F-terms made up of one distinct field $\longrightarrow$ zero out fields and reduce F-terms/Fields
3. Count how many fields exist in F-terms: Required Fields
4. if # Required Fields $< 20 \longrightarrow$ Brute-Force check
5. else
   (a) increment max overlap in *ZeroOutFields* until # of reduced fields $\leq 25$
   (b) return max overlap
6. Run *ZeroOutFields* with max overlap $\longrightarrow$ Removed Fields, Remaining Fields, Reduced F-Terms
7. Toggle Removed Fields
   (a) turn field VEVs back on one-by-one
   (b) if all (originally killed) F-Terms still go to zero, add those fields back into Remaining Fields
8. Brute-Force constraint generation over Remaining Fields F-Classes for the Reduced F-Terms

### 3.5.4 Subsequent Orders and Cumulative Constraints

Once the superpotential term and F-Classes are found for a given order, the same calculations can be performed for the next highest order. Order $N$s superpotential constraints will have the F-Classes from order $(N-1)$ applied before the F-Class algorithm is run (see 3.5.3.1 and 3.5.3.2). After F-Classes are found it is a relatively simple process to check for SUSY breaking (by causing too many fields to have a zero VEV). If this is not the case, then the next order is investigated by performing the methods outline in the previous sections. This process is repeated until SUSY is forced to be broken by an offending F-Class VEV constraint.

## 3.6 D-flatness

The F-Classes built from our hidden sector F-flat search allow us to now consider what values the field VEVs will actually take. Rewritten here are the equations for D-flatness:

1. For the case of Type-I ($U(1)$-charged) fields:

$$D^A = \sum_k Q_k^{(A)} |\langle \Phi_k \rangle|^2 + \frac{g^2 \mathrm{Tr}(Q_A)}{192\pi^2} M_{\mathrm{Pl}}^2; \tag{3.4}$$

$$D^\alpha = \sum_k Q_k^{(\alpha)} |\langle \Phi_k \rangle|^2 \ , \ \alpha \neq A \ ; \tag{3.5}$$

2. For Type-II fields, the corresponding equation is:

$$D_a^\alpha = \sum_k \Phi_k^\dagger T_a^\alpha \Phi_k \tag{3.6}$$

In all cases, D-flatness requires: $\langle D \rangle = 0$. As mentioned before the full Type-II scenario is difficult to perform since all the gauge generators must be calculated based on the dimensionality of the given field, $\Phi_k$.

### *3.6.1 Hybridized Type I/II Case*

It was originally believed that the space of gauge invariant monomials is in one-to-one correspondence to the space of D-flat stabilizing moduli [26]. However, this was shown to be too strong of a condition [32] because this mapping is, in fact, surjective: there exist D-flat solutions which do not correspond to gauge invariant monomials. In our current build approach, we take the middle ground by requiring D-flatness, through the hybridized $U(1)$ flatness, and will ensure that superpotential terms are gauge invariant. This means that our searches will not be exhaustive for the moment. Extension of the framework is certainly possible and there are efforts in the works to require full NA D-flatness.

The hybrid scenario which bridges the two cases is possible because a field, $\Phi_j$, is non-trivially charged under the NA gauge groups. Therefore, each of its components must satisfy Type-I D-flatness. If there are $n_j$ components to field $\Phi_j$ then that will add to our system of equation $n_j$ copies of the same equation. The net effect is that the entire $j^{\text{th}}$ column of the original charge matrix is multiplied by $n_j$.

With the $\{Q_{ij}\}$ constituting the charge matrix, $Q$, we can re-express Eq 3.4 - 3.6 as a system of linear equations. For convenience, assume that the first $N_p$ fields are vector-like pairs (both singlet and NA) and that there are $M$ fields. We must now 'hybridize' the charge matrix according to the NA field component multiplicity. For concreteness, let us assume that the $9^{\text{th}}$ field of a given model is charged as $\bar{\mathbf{3}}$ of $SU(3)$, a $\mathbf{4}$ of $SU(4)$, and a $\mathbf{1}$ of $SO(7)$. Then all of the $U(1)$ charges of field 9 would be multiplied by $3 \times 4 \times 1 = 12$. Schematically, redefining our charge matrix applying a multiplicity vector to the original charge matrix:

$$Q \longrightarrow Q_{hybrid} = Q \cdot (\overbrace{n_1, n_2, \ldots n_i}^{\text{vector-like singlet}}, \overbrace{n_{i+1}, \ldots, n_{i+k}}^{\text{vector-like NA}}, \overbrace{1, \ldots, 1}^{\text{unpaired}})^T, \tag{3.7}$$

where the $n_j$ are the appropriate multiplicity factors as explained above.

If we define $x_i \equiv |\langle\phi_i\rangle|^2$ then our linear system of D-flatness equations becomes:

$$\begin{aligned} & Q \cdot \vec{x} = \begin{pmatrix} -\xi \\ \vec{0} \end{pmatrix} \\ & x_i \in \mathrm{R},\ 1 \leq i \leq N_p \\ & x_j \in \{\mathrm{R} \geq 0\}\,,\ N_p < j \leq M. \end{aligned} \tag{3.8}$$

The last two lines may cause a question. The positivity constraint for the singlet fields is a result of our convention: a negative norm-squared of the VEV represents the pair-partner (antiparticle) field gaining a VEV rather than the original field.

### *3.6.2 SVD*

Due to the singular nature of $Q$, it is necessary to construct a pseudo-inverse in order to solve the system of equations. Additionally, this singularness will allow for multiple (usually, infinite) solutions. A useful method in dealing with matrix equations of this form is Singular Value Decomposition (SVD). This process decomposes a given matrix into the product of three matrices:

$$\begin{aligned} Q &= U\Sigma V^T \\ \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{m1} & \cdots & q_{mn} \end{pmatrix} &= \begin{pmatrix} \vec{u}_1, \cdots, \vec{u}_m \end{pmatrix} diag\,(\sigma_1, \cdots, \sigma_l, 0, \cdots, 0) \begin{pmatrix} \vec{v}_1^T \\ \vdots \\ \vec{v}_n^T \end{pmatrix}. \end{aligned} \tag{3.9}$$

Given that the diagonal matrix, $\Sigma$, has $l$ nonzero elements, the first $l$ column vectors of $U$: $\{\vec{u}_1, \cdots, \vec{u}_l\}$ then form a basis over the range of $Q$. Similarly, the last $n - l$ columns of $V$: $\{\vec{v}_{l+1}, \cdots, \vec{v}_n\}$ are a basis for the nullspace of $Q$. Using SVD to solve a system of equations will yield one solution and additional solutions can be generated by adding to it linear combinations of the nullspace of $Q$.

Letting $\vec{x}_i^{null} \equiv \vec{v}_{l+i}, \forall i \in [1, n-l]$, then any solution to Eq 3.8 can be represented by:

$$\vec{x} = \vec{x}_{\mathrm{SVD}} + \sum_{i=1}^{n-l} \alpha_i \vec{x}_i^{null}. \tag{3.10}$$

The coefficients,$\{\alpha_i\}$, are not without constraint. According to Eq.3.8, any given vector-like field can take on both negative and positive while non-vector-like fields must be strictly positive. This means that the allowed range of the $\{\alpha_i\}$ will be determined by

$$\begin{aligned} &\sum_{j=1}^{l} x_{ij}^{null} \alpha_j \geq -x_i^{\mathrm{SVD}}, \\ &N_p + 1 \leq i \leq M, \\ &\alpha_j \in \mathrm{R}. \end{aligned} \tag{3.11}$$

It is readily apparent that the coefficient space will likely be unbounded. Therefore, fully automated searches can not yield exhaustive solutions. Physically meaningful results may come from specifying allowable VEV ranges for the fields before D-flat directions are even calculated. These constraints would be nicely applied to visible sectors but could be relaxed, in varying degrees, to the hidden sector. Investigations into methods to *a priori* constraining the coefficient space (for example, using Integer Programming (IP) or Linear Programming (LP)) are on-going.

### *3.6.3 D-Flat Solutions*

With respect to the software, D-flatness requires very little input and it may seem that the 'heavy-lifting' has already been performed by the F-flatness and F-Class generation.

That is not entirely so. As alluded to in the last section, the accessible VEV space may be rather large unless exact solutions can be found. Since SVD will often yield least-squares solutions, the nullspace will need to be explored to try to extend into physically allowable regions of the VEV space. This problem is mathematically represented in Eq 3.12 and forces us to split the vector-like pairs from the unpaired fields.

Additionally, since observable SM NA fields will not yet take on VEVs, they must be removed from the calculations as well. This is done in a straightforward manner since the F-Classes provide boolean arrays which indicate whether a fields has taken on a zero(nonzero) VEV. This is represented by a 1(0). In dealing with this observable matter the F-Classes left those fields unspecified, which resulted in their VEVs being kept 'off'. When it comes to D-flatness though, the meaning of these boolean arrays change meaning slightly. The charge matrix only needs to know which fields are going to take on non-zero VEVs. The SM/GUT and Higgs fields are not yet allowed to take on a VEV at this energy scale. That makes these fields and the fields which have already been sent to a zero VEV equivalent in the D-flatness analysis. The boolean F-Class arrays will have all observable field elements now turned 'on'. It is a straightforward process to create reduced charge matrices, which are created from the columns corresponding to hidden sector fields whose VEVs are unspecified:

$$Q_{\text{reduced}} = Q \cap \{\Phi_i | \Phi_i \in \{\text{hidden sector} \cap \{\langle\Phi_j\rangle \neq 0\}\}\}\,. \tag{3.12}$$

Using SVD, a pseudoinverse is constructed, $Q^{-1}_{\text{reduced}}$ in order to solve:

$$Q_{\text{reduced}} \cdot \vec{x}_{\text{reduced}} = \begin{pmatrix} -\xi \\ \vec{0} \end{pmatrix}. \tag{3.13}$$

This will result in a solution that only represents hidden sector fields with potentially nonzero VEVs: $\vec{x}^{\,\text{SVD}}_{reduced}$. If the reduced SVD solution is found to have non-vector-like components which are negative, the nullspace perturbation must occur. Now any given extension (including the trivial or zeroth extension) of the original SVD solution must survive the following constraints:

1. Any element which corresponds to a non-vector-like field must be positive and
2. Eq 3.13 is satisfied to some acceptable numerical accuracy.

Thus we can extend the SVD solution with linear combinations of the nullspace in a manner similar to Eq 3.10, whereupon we find:

$$\vec{x}_{\text{reduced}} = \vec{x}^{\text{SVD}}_{\text{reduced}} + \sum_{i=1}^{n-l} \alpha_i \vec{n}_i^{\text{reduced}}. \tag{3.14}$$

Here, the $\vec{n}_i^{\text{reduced}}$ constitute the nullspace basis of $Q_{\text{reduced}}$.

When perturbing the $\vec{x}^{\text{SVD}}_{reduced}$ using Eq 3.14, it is necessary to bifurcate the problem once again. This can be seen as we split the above solution into its vector-like and unpaired constituents:

$$\begin{pmatrix} \vec{x}^{\text{vl}}_{\text{red}} \\ \vec{x}^{\text{nvl}}_{\text{red}} \end{pmatrix} = \begin{pmatrix} \vec{x}^{\text{vl,svd}}_{\text{red}} \\ \vec{x}^{\text{nvl,svd}}_{\text{red}} \end{pmatrix} + \sum_{i=1}^{n-l} \alpha_i \begin{pmatrix} \vec{n}_i^{\text{vl,red}} \\ \vec{n}_i^{\text{nvl,red}} \end{pmatrix}. \tag{3.15}$$

By considering only the non-vector-like fields, it will be possible to find the $\{\alpha_i\}$ that will force $\vec{x}_{\text{reduced}}$ to satisfy both demands (D-flatness and positivity). Consider the matrix formed by the non-vector-like portion of the reduced nullspace, $[n^{\text{nvl,red}}]_{ij}$ and the vector $\vec{\alpha} = (\alpha_1, \ldots, \alpha_i, \ldots, \alpha_{M-N_p})$ from Eq 3.15. This vector and matrix combination will need to solve the following positivity equation (where the positivity requirement must be satisfied row-by-row):

$$n^{\text{nvl,red}} \vec{\alpha} \geq_{\text{row}} -\vec{x}^{\text{nvl,svd}}_{\text{red}}. \tag{3.16}$$

At this point, it seems possible to solve for $(n^{\text{nvl,red}})^{-1}$ using SVD but doing so does not yield any additional information. This can be seen when comparing Eq. 3.16 to the lower half of Eq. 3.15:

$$\begin{aligned}
\vec{x}^{\,\mathrm{nvl}}_{\mathrm{red}} &= \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} + \sum_{i=1}^{n-l} \alpha_i \vec{n}^{\,\mathrm{nvl,red}}_i \\
&= \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} + n^{\mathrm{nvl,red}} \vec{\alpha} \\
&\approx \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} + n^{\mathrm{nvl,red}} \vec{\alpha}^{\mathrm{svd}} \\
&= \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} \\
&= \vec{0},
\end{aligned}$$

where the last line are a result from the use of $\vec{\alpha}^{\mathrm{svd}}$. This is a minimal case, if acheivable, but it does not help us to know how to perturb the coefficients in the right 'direction' since a matrix form of *greater than* is not well-defined. Finding sets of non-trivial solutions requires a paradigm shift. One that forces us to reconsider our starting point.

To find a good set of coefficient vectors, we must choose to use a different form of the positivity requirement by looking at the actual non-vector-like solution $\vec{x}^{\,\mathrm{nvl}}_{\mathrm{red}}$:

$$\begin{aligned}
\sum_{i=1}^{n-l} \alpha_i \vec{n}^{\,\mathrm{nvl,red}}_i &= \vec{x}^{\,\mathrm{nvl}}_{\mathrm{red}} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} \\
n^{\mathrm{nvl,red}} \vec{\alpha} &= \vec{x}^{\,\mathrm{nvl}}_{\mathrm{red}} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} \\
&\approx \vec{\zeta} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}},
\end{aligned}$$

where $\vec{\zeta} \in \{\zeta_i | [\vec{\zeta} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}}]_i \geq 0 \;\; \forall\, i\}$. $\vec{\zeta}$ is then the variable(s) to be perturbed. The coefficient vector is found by applying the SVD inverse, $(n^{\mathrm{nvl,red}})^{-1_{\mathrm{svd}}}$, to the linear combination: $\vec{\zeta} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}}$. If $\vec{\xi}$ is allowed to range over variety of values then the resulting set of coefficient vectors, $\vec{\alpha}^{\mathrm{svd}}$, will still be a solution to Eq. 3.14:

$$\begin{aligned}
\vec{x}_{\mathrm{reduced}} &= \vec{x}^{\,\mathrm{SVD}}_{\mathrm{reduced}} + n^{\mathrm{reduced}} (n^{\mathrm{nvl,red}})^{-1_{\mathrm{svd}}} \left( \vec{\zeta} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} \right) \\
&= (Q_{\mathrm{reduced}})^{-1_{\mathrm{svd}}} \begin{pmatrix} -\xi \\ \vec{0} \end{pmatrix} \\
&\quad + n^{\mathrm{reduced}} (n^{\mathrm{nvl,red}})^{-1_{\mathrm{svd}}} \left( \vec{\zeta} - \vec{x}^{\,\mathrm{nvl,svd}}_{\mathrm{red}} \right).
\end{aligned} \tag{3.17}$$

The full solution to the hidden sector $U(1)$ D-flat equation for a given F-class is, therefore, given by the Eq. 3.17.

All D-flat solutions to a given F-Class are written to file. These can then be applied to the superpotential terms for access to phenomenological data. Since we are not yet prepared for an exhaustive solution search, only representative solutions will be found. These solutions are found by choosing a zeroth perturbation point such that $[\vec{\zeta} - \vec{x}_{\text{red}}^{\,\text{nvl,svd}}]_i \geq 0 \quad \forall\ i$. A convenient choice for this perturbation is: $\vec{\zeta} = |\vec{x}_{\text{red}}^{\,\text{nvl,svd}}|_{\text{component-wise}} + \vec{s}$, with stepsize vector components, $s_i \in \left\{0, min(|\vec{x}_{\text{red}}^{\,\text{nvl,svd}}|)\right\}$. Note that the 'component-wise' absolute value of a vector is an absolute value operation performed on all indices individually and does not represent a measure of length. To allow for some degrees of freedom, the stepsize vector is not uniform (*i.e.* it is not proportional to $\vec{1}$). Instead all possible combinations of zero and nonzero entries for its components are considered in our calculations.

The resultant VEVs are not a complete set but represent classes of solutions. Future work may entail a way of specifying various relationships between fields VEVs and then testing to see if they are accessible. It will also be interesting to consider the input perturbation space to output VEV correlations, if any exist. This will likely be a candidate for future analysis.

### *3.7 Phenomenology*

After isolating a set of likely F-Classes and their corresponding collection of field VEVs, it is appropriate to try to connect the models to realistic physics. The studies mentioned at the end of the previos section are important since the hidden sector field VEVs can be used to study a variety of phenomenology. The most realistic VEV sets can then be further analyzed and, potentially, provide indirect experimental predictions once the observable matter field VEVs are considered. The salient calculations to perform, in the future, are:

1. Hidden Sector Fermion Masses,
2. Gaugino Condensation Scales, and

3. Cosmological Constants.

We will discuss these items briefly.

*3.7.0.1 Mass Matrices.* The mass matrix is one of the most important pieces of data to produce since it contains the first potentially observable information on a given model. VEVs are the most fundamental of phenomenological data and particle masses are only of the most tangible. Unfortunately, there is no direct correlation between them outside of the sifting through the superpotential to find contributing terms. Eq. 1.23 highlights which terms in the superpotential are sought. After gathering the relevant terms, mass eigenvalues are calculated and, from which, gaugino condensate scales can be determined.

In order to generate a model's mass matrix, $M^2$, it is essential to have knowledge of a model's field VEVs as well a large range of superpotential terms. As can be seen from Eq. 1.23, the requirement for the a mass term contribution is that there must exist at least $n-2$ nonzero-valued field VEVs in an order $n$ collection of fields. Since the mass matrix is an element of the scalar potential (see Eq. 1.19), this collection of fields is created from the model's F-terms. By investigating the notation of Eq. 1.19, it can be seen that the F-terms, $F_{\phi_i}$, are vectors. The mass contributions will then arise from a dot product where the components are sum of field monomials.

These field monomial are generated from many orders of the superpotential. In fact, there will be infinite contributions due to infinite order. However, we are interested only in general features of these models, and will consider only the largest contributions. Here that equates to the smallest superpotential order that gives rise to a non-trivial entry in the mass matrix. Therefore, various components of the matrix will then derive from different energy scales of the theory. Ideally, this scale is manifest in the superpotential term coefficients as well as in the values of the field VEVs. These coefficients are notoriously difficult to calculate but their scale is related to the superpotential order from which the term arises. Since higher orders of the superpotential contribute less than lower orders, these care called suppressed contributions.

The determination of coefficient suppression factor is the inverse to the process for finding the scale of SUSY breaking (as was described in 1.11). At $3^{rd}$ through $4^{th}$ order, the suppressions scale is unity. For each superpotential order increase, the suppression is multiplied by a factor of the Planck mass [9]. Once the entire matrix is found, the mass eigenvalues are found through matrix diagonalization.

Applied to our current approach, mass contributions will arise from F-terms which possess fewer than two F-Class fields at all superpotential orders. One F-Class field per term is guaranteed until the SUSY-breaking scale. At this order, the W-terms are not necessarily forced to zero and therefore, may possess one or zero F-Class fields. An additional constraint on mass-contributing terms will be the presence of at least one field with unit multiplicity. It is this field, when combined with a field from another F-term, which is to 'receive' the mass contribution. This process will be a straightforward addition to the Flat Direction Framework and is the next calculational tool which will be implemented.

*3.7.0.2 Gaugino Condensates.* The condensation scales can be found using Eq. 1.17. Including the one-loop $\beta$-function for an $SU(N_c)$ group, this equation becomes

$$\langle \lambda\lambda \rangle = \mu^3 \exp\left(\frac{-24\pi^2}{(-3N_c + N_f)\, g^2(\mu)}\right), \tag{3.18}$$

where $N_f$ is the number of *flavors*, the number of vector-like pairs which carry the $SU(N_c)$ charge. These fields become strongly coupled ($g(\mu)$ becomes large) at the condensation scale, $\Lambda$, when $N_c > N_f$. Since we are dealing with an EFT, we consider contributions to $N_f$ from only those fields whose masses, $m$, are much less than $\Lambda$. Therefore, it is imperative to first calculate the various field masses.

The condensate now picks up a VEV: $\langle\lambda\lambda\rangle = \frac{\Lambda^3}{m}\left(\frac{m}{\Lambda}\right)^{\frac{N_f}{N_c}}$. This endows the superpotential with a non-trivial VEV which in turn also finalizes the gravitino mass calculation (see [56]):

$$\begin{aligned} m_{3/2} &= \langle e^K W \rangle \\ &= \langle e^K \rangle N_c \Lambda^3 \left(\frac{m}{\Lambda}\right)^{\frac{N_f}{N_c}} , \end{aligned}$$

where $K$ is the Kähler potential [56].

*3.7.0.3 Cosmological Constants.* The scale of the cosmological constant, in a given model, will be equal to hidden sector SUSY breaking scale, if one exists.

# CHAPTER FOUR

## Results

In this chapter, we highlight the results of the Flat Direction Framework. The code was constructed utilizing three models as tests. The models are summarized here:

- Model 1
  - $SO(10)_{obs} \times SU(8)_{hid} \times SU(2)_{hid} \times U(1)^6$
  - 69 fields (41 Singlets)
- Model 2
  - $SU(5)_{obs} \times SU(8)_{hid} \times U(1)^8$
  - 59 fields (37 Singlets)
- Model 3
  - $SU(5)_{obs} \times SU(4)_{hid} \times SU(2)^4_{hid} \times U(1)^8$
  - 83 fields (26 Singlets)

These models were chosen for test subjects since they possess a variety of potentially interesting phenomenology. They were also covered in more detail in our reduced-higgs spectrum paper: [49]. Model 1 here corresponds to the $SO(10)$ model in the reduced-higgs paper while Models 2 and 3 are the flipped $SU(5)$ models 2 and 3 from that publication. Interested readers can find all the necessary gauge charge information as well as data on their Type-I D-flat directions there.

### *4.1 Superpotential and F-term Constraints*

The task of calculating viable superpotential terms is incredibly time-consuming (for either man or machine) and so requires the necessity to expedite the process by any reasonable method. Therefore, it is important to characterize the various constraints that are in place to generate the W-terms. The constraints come in two varieties: Gauge Invariance and RNS Picture Changing. The former is true whether using or not using string theory. The latter, is a string theoretic effect.

Table 4.1: The time taken generate superpotential terms for our three example models. Calculations were performed by the given number of processors and was done using the original order: gauge invariance, RNS and picture changing.

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| order | 1 proc | 1 proc | 1 proc |
| 3 | 5.160323 | 6.343136 | 2.69076 |
| 4 | 14.68306 | 9.10767 | 10.639666 |
| 5 | 211.228039 | 117.822991 | 162.027935 |
| 6 | 2815.900753 | 1187.53881 | 2530.68283 |
| 7 | 34187.42118 | 13003.23664 | 35083.00057 |
| 8 | 390820.2807 | 136413.9423 | 463257.3039 |
| 9 | – | 637440.328 | – |
| | 41 procs | 56 procs | 74 procs |
| 3 | 2.660907 | 2.500445 | 2.796697 |
| 4 | 3.365721 | 2.749135 | 5.393435 |
| 5 | 56.905457 | 31.395642 | 125.322772 |
| 6 | 871.869913 | 397.412848 | 2147.145389 |
| 7 | 10353.52543 | 4362.929003 | 34443.62293 |
| 8 | 111709.3807 | 41577.47633 | 449727.4724 |
| 9 | 1.24E+06 | 357768.0003 | – |

As mentioned in Sections 2.1.2.1 and 2.1.2.2, the RNS Picture Changing constraints can be broken down into a simple first-pass test using Boolean constraints (generally called, RNS constraints) and a more-complicated, second test. This second test is actually the full RNS Picture Changing process and, so, gives the same constraints as the first-pass test in addition to others. The separation, therefore, is artificial in the physical sense but extremely important computationally. This will be illustrated shortly.

The above two superpotential term generating constraints can be expanded into three:

1. Gauge Invariance
2. RNS (Boolean)

3. Picture Changing

The ordering of these constraints would, naively, follow the numbering above as well. This is understandable since the gauge invariance would be assumed to remove more terms than a stringy effect. This coincides with our three example models (Tables 4.3, 4.4 and 4.4). Hence, it was our first choice to perform the operations in the order described earlier. The run times for serial and parallel runs can be found in Table 4.1.

Table 4.2: The number of superpotential terms generated with and without constraints for our three example models.

| | Model1 1 | | Model 2 | | Model 3 | |
|---|---|---|---|---|---|---|
| Order | No Test | Full Test | No Test | Full Test | No Test | Full Test |
| 3 | 57155 | 128 | 35990 | 5 | 98770 | 2 |
| 4 | 1028790 | 0 | 557845 | 30 | 2123555 | 5 |
| 5 | 15020334 | 474 | 7028847 | 40 | 36949857 | 0 |
| 6 | 185250786 | 0 | 74974368 | 106 | 541931236 | 108 |
| 7 | 1984829850 | 17871 | 696190560 | 250 | 6890268572 | 0 |
| 8 | 18855883575 | 0 | 5743572120 | 1416 | 77515521435 | 2717 |
| 9 | 1.61323E+11 | 527970 | 42757703560 | 3826 | 7.83768E+11 | 0 |

The story changes when one utilizes a better metric: the number removed per unit time. In our case, we have chosen to illustrate the point with percent removed per second. These results show that it is the RNS first pass test that should be run first as it removes the most terms per time (see figures 4.1, 4.2 and 4.3). Next, comes the gauge invariance check. Lastly, any models surviving are picture changed. It is interesting to note that the most powerful superpotential term constraint comes from string theory in this case.

The total number of terms for each of the models can bee seen in Table 4.2. The observed growth from order to order warrants the investigation into the placement of the various constraints relative to one another. This become apparent when comparing Table 4.1 (the original ordering) to the W-terms time column in Tables 4.7, 4.9 and 4.9 where the ordering has been changed.

Table 4.3: Model 1. The number of terms removed per order by each of the constraints.

| Order | Gauge Inv. | RNS | Pict. Change |
|---|---|---|---|
| 3 | 56934 | 48646 | 0 |
| 4 | 1027176 | 941067 | 1028702 |
| 5 | 15010192 | 13475121 | 15005727 |
| 6 | 185188829 | 161726031 | 185243696 |
| 7 | 1984477806 | 1688888871 | – |
| 8 | 18853989837 | 15627103527 | – |
| 9 | – | 1.33008E+11 | – |

Table 4.4: Model 2. The number of terms removed per order by each of the constraints.

| Order | Gauge Inv. | RNS | Pict. Change |
|---|---|---|---|
| 3 | 35956 | 30982 | 0 |
| 4 | 557712 | 481830 | 555428 |
| 5 | 7028475 | 5920932 | 7020308 |
| 6 | 74972974 | 60401658 | 74921524 |
| 7 | 696186612 | 551448130 | 695966136 |
| 8 | 5743557571 | 4460927740 | – |
| 9 | 42757661111 | 33354221895 | – |

Table 4.5: Model 3. The number of terms removed per order by each of the constraints.

| Order | Gauge Inv. | RNS | Pict. Change |
|---|---|---|---|
| 3 | 98714 | 85535 | 0 |
| 4 | 2123340 | 1991255 | 2121703 |
| 5 | 36949033 | 34087557 | 36924258 |
| 6 | 541928030 | 488546436 | 541613411 |
| 7 | 6890256587 | 6062093672 | 6890268572 |
| 8 | 77515474364 | 66418825160 | 77515521435 |

## *4.2 F-Class and D-Flatness*

After the superpotential terms are generated it becomes essential to limit the allowed set of VEVs since, in most cases, they are already unbounded. Additionally, in future extensions of the framework the order at which superpotential terms break SUSY will be determined by the F-Class constraints. This is anticipated to curtail unnecessary calculation of higher order

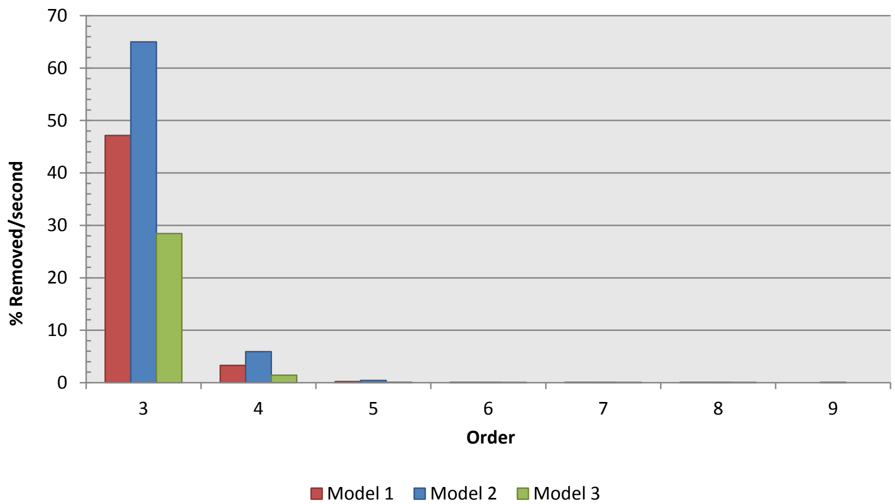


Figure 4.1: The percent of W-terms removed by gauge invariance constraints as a function of superpotential order.

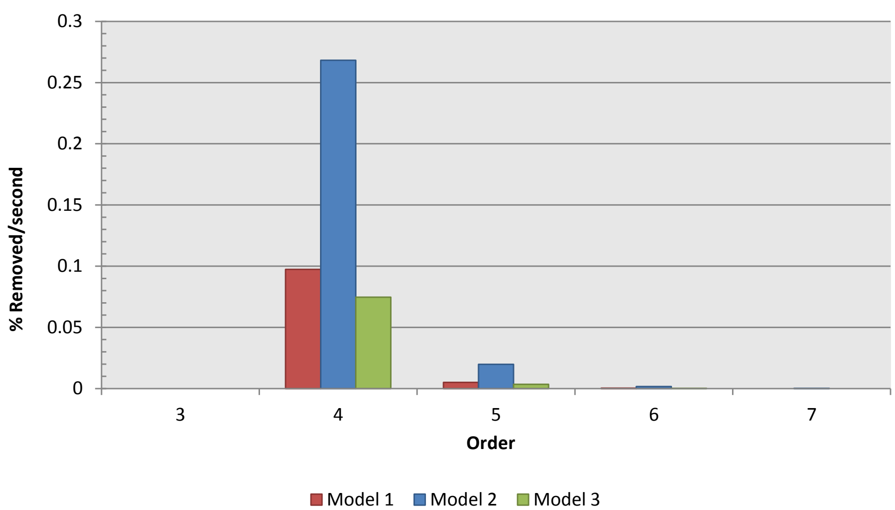


Figure 4.2: The percent of W-terms removed by RNS (first-pass) constraints as a function of superpotential order.

W-terms which, as can be seen, is very costly in terms of computational time. The F-Classes are generated in such a way as to represent which fields are necessarily zero VEVed so that SUSY preserved at the given order. From the runs of our example models, the majority of

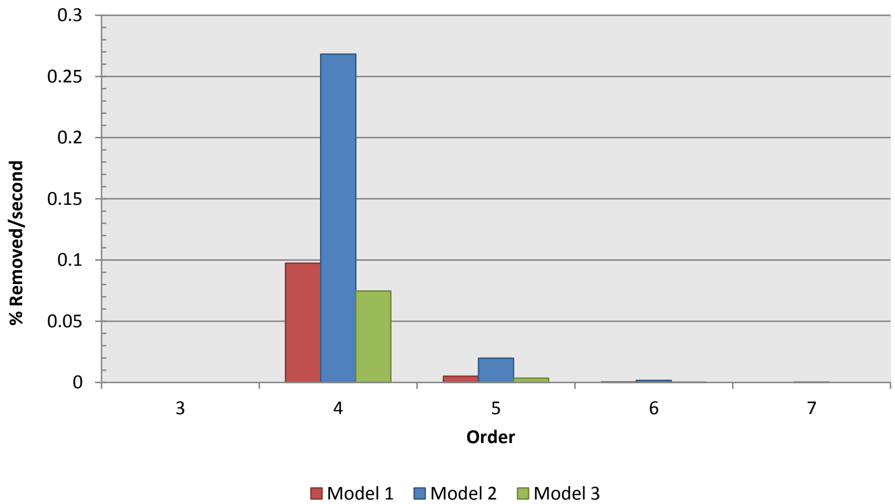


Figure 4.3: The percent of W-terms removed by Picture Changing constraints as a function of superpotential order.

fields are determined at order 3 or 4. Higher orders seem to be mostly subsumed by or add only a few field constraints at a time.

Table 4.6: The number of zero-VEVed fields at each order of F-Class for the three example models.

| Order | Model 1 69 fields | Model 2 59 fields | Model 3 83 fields |
|---|---|---|---|
| 3 | 45 | 8 | 2 |
| 4 | 45 | 20 | 10 |
| 5 | 45 | 22 | 10 |
| 6 | 45 | 23 | 10 |
| 7 | 46 | 24 | 10 |

### *4.2.1 Model 1*

The superpotential, F-Class and D-flat solution generating times can be found in Table 4.7. In Model 1, the 'heavy-lifting' for constraint calculation is performed at order 3 and yielding 45 of 69 fields with zero VEVs. No new constraints are added until order 7, where one field is added to the quadruple set of F-Classes. This correlates well with the observed run time, which remains fixed until the addition of the extra field at order 7.

Table 4.7: Model 1. 11 Processors, 10 perturbation steps. Columns 2,3 and 5 are all in units of seconds.

| Order | W-term time | F-Class time | # F-Classes | Dflat time | # Dflat |
|---|---|---|---|---|---|
| 3 | 2.549295 | 79.625326 | 4 | 182.195236 | 122881 |
| 4 | 2.393977 | 0.092686 | 4 | 180.638033 | 122881 |
| 5 | 18.306367 | 0.043227 | 4 | 180.193417 | 122881 |
| 6 | 254.372428 | 0.007365 | 4 | 178.062005 | 122881 |
| 7 | 3005.142267 | 0.843381 | 4 | 78.777953 | 0 |

For the D-flat directions in this model, we see from the last column of Table 4.7, SUSY will be broken at order 7 since no flat directions were found. It is possible that in some parameter space, far removed from our perturbation starting point, flat directions may be found but they may yield unrealistic inter-field VEV ratios.

### *4.2.2 Model 2*

Model 2 differs greatly from Model 1, in that the spread of the field VEVs changes drastically among its 59 fields. At every order, the number of fields taking on VEVs changes. The largest change, sensically, coincides with the longest run time. The addition of 12 zeroed fields also expands the number of allowed F-Classes by a factor of five. A long F-Class calculation indicates that there are either large F-term set overlaps or that there are many fields requiring brute-force on/off toggling. At the time of writitng, there was no D-flat information available for this model.

Table 4.8: Model 2. 21 Processors, 10 perturbation steps. Column 2 and 3 are in units of seconds.

| Order | W-term | F-Class time | # F-Classes |
|---|---|---|---|
| 3 | 2.67802 | 2.007713 | 72 |
| 4 | 1.753862 | 4812.16487 | 360 |
| 5 | 10.509974 | 17.590815 | 1 |
| 6 | 136.728789 | 0.065007 | 3 |
| 7 | 1542.159322 | 0.152016 | 1 |

### 4.2.3 Model 3

The most unique aspect of the final example model is the order 4 increase to ten F-Classes with a decrease in run time. The limited constraints on order 3 (two fields, see Table 4.6) indicates a very simple set of W-terms. At order 4, there is a huge number of new constraints generated from a 5-fold increase in the number of zeroed fields. Since the run time is so small, it can be assumed that the order 4 W-terms have a simple form. This is indeed the case. We can see from Table 4.2, that there are only 5 W-terms. A quick inspection of the superpotential terms (not listed) shows that 18 new fields are introduced and with very little overlap among them. Therefore, a quick brute-force calculation results in the 486 possibilities.

Table 4.9: Model 3. 11 Processors, 10 perturbation steps. Columns 2,3 and 5 are all in units of seconds.

| Order | W-term | F-Class time | # F-Classes | Dflat time | # Dflat |
|---|---|---|---|---|---|
| 3 | 2.30279 | 1.695605 | 1 | 1.524483 | 0 |
| 4 | 3.37334 | 0.812725 | 486 | 11.069074 | 0 |
| 5 | 29.463026 | 0.054953 | 486 | 17.413776 | 0 |
| 6 | 513.840001 | – | 486 | 18.371869 | 0 |
| 7 | 8119.36786 | – | 486 | 18.780749 | 0 |

The zero VEV field constraints, unfortunately, do not allow for any solution to the D-flatness equations. Therefore SUSY will be broken at the string scale. Like Model 1, there is a chance that we have chosen the a poor spot in the perturbation parameter space but then it must lie far from the near-minimal starting point we have chosen.

# CHAPTER FIVE

## Discussion

### 5.1 Future Work

In order to access the full benefit of this fully-automated software, it will be necessary to connect it to a model generating system. Baylor's existing FF-framework (see Section 2.2) produces models *en masse* and allows for the user to specify certain desirable features (number of matter generations, gauge groups *etc*...). Having this Flat Direction Framework called directly when a good model is found will be a huge boost to systematic studies of the WCFFH landscape. In this way it will be used to analyze many potentially viable models.

The current state of the Flat Direction Framework is in single model analysis. This functionality will remain in future versions. As with any software, there is room for improvement as well as expansion into other area of phenomenological interest. We summarize some of these ideas below and then offer some concluding remarks.

### 5.2 Extensions

#### 5.2.1 F-flatness

*5.2.1.1 Non-Stringence.* While it is computationally preferable to perform stringent searches, for exhaustive analysis it will become essential to allow all possible types of superpotential term cancellation to occur. For example in [18], it was shown that all-order stringent flat directions produced unrealistic constraints on quark and lepton mass matrices. It was also argued that SUSY breaking at finite, yet sufficiently high, order would be beneficial. In the past it has been too expensive computationally to allow for this full search. The chief obstacle to such a calculation, is not in the sheer number of terms but the couplings for all superpotential terms which would need to be calculated. This is required since each term will have its own coupling strength and cross-cancellation would be limited by these numerical values (see Section 1.11). Hence, it is an extremely difficult problem to solve in its fullness.

Instead of direct calculation it might be possible to extend stringent flat directions into non-stringent. On a model-by-model basis, there may exist correlations between the coefficient space, the D-flat parameters and the F-flat regions. It could then be fruitful to investigate possible relationships between the coefficient spaces of models that lie close to each other in moduli space; this may hint at ways of deforming one model into another.

*5.2.1.2 Numerical polynomial homotopy continuation.* Recent work in adapting Numerical Polynomial Homotopy Continuation (NPHC) to physical systems has shown promising results [57]. The utility of this approach is that it can solve, numerically, a multivariate system of polynomial-like equations. Polynomial-like equations are those with no worse than polynomial non-linearity after a suitable transformation. This is an incredibly powerful tool that can be of use, especially when trying to compare stringent to non-stringent flat directions over a given range of orders in the superpotential. Additionally, this may be of use in solving the positivity constraints found in the D-flatness equations whereby the non-vector-like fields must not receive a negative VEV-squared.

NPHC works by taking a given system of (polynomialized) equations and parametrically adding a series of similar, but easily solved equations. This new set of equations needs to be path-track solved with respect to the new parameter. In solving over various values of the parameter, it is possible to parallelize the code making this method more useful when trying to construct a fast, yet exhaustive search algorithm. Therefore, up to a given, finite order of the superpotential it will be possible to find numerical flat directions.

The difficulty still remains of calculating the W-term coefficients. This can be done rather easily for low orders. Combining the first few orders of F-term constraints with the D-flatness equations could produce a system of questions that may be readily solved. In fact, if multiple solutions are found then it may be possible to extract the F-Classes from which they arise. Those F-Classes may be, creatively, employed at extending the solutions to higher orders of the superpotential. This may be a difficult process to perform but it certainly merits further investigation.

### *5.2.2 D-flatness*

*5.2.2.1 NA Flat Directions.* It is possible to enhance the stringent flat direction search. One method is to incorporate full non-Abelian D-flat directions (Type II) [56]. Abelian D-flat solutions only specify the magnitudes of the VEVs taken on by the fields. It is possible then to use the gauge freedom of the groups to effect $W$ and F-term cancellation [2]. If there exists one or more NA gauge charges carried by some of the flat directions, then it is possible for self-cancellation of F-terms to occur. In the past, this was performed as model-by-model process but the influx of extra flatness data from this program may yield greater insight into the self-cancellation phenomenon.

The case of Type II investigations is not important for observable sectors only, but for the hidden states as well. Hidden sector Higgs fields from triplets of $SU(3)_H$ or doublets from $SU(2)_H$ can contribute to F-term self-cancellation and thus break SUSY in the observable sector well-above the EW scale. The course usually pursued involves investigating the $SU(3)_H$ at intermediate scales since the triplet/antitriplet condensates are more likely to produce EW SUSY breaking than the $SU(2)_H$ states [32].

The full NA D-flat search faces the difficulty of calculating the generators of the non-Abelian groups under consideration (Eq 1.27). Since it is very unlikely to find good phenomenology with using only Type I fields, this extension is not just natural but required. For example, current work has shown that realistic fermion mass hierarchies arise in some Type II flat directions search, while none have been found without inclusion of NA flat directions.

Another challenge posed by Type II searches is the increase in the field VEV parameter space. Each 'field' that is non-trivially charged under a NA gauge group is really a set of fields (*e.g.* a **16** of $SO(10)$ is really 16 fields). In our current calculation, the gauge groups are unbroken so that all the fields in a single representation receive the same VEV. If we allow for these fields to take on different values (thereby, breaking the gauge symmetry), then we increase the overall number of parameters for our F-Class constraint generation. This will be more computationally intensive but we will also be able to understand the breaking of gauge groups in a given model without actually having to calculate the VEVs themselves.

*5.2.2.2 D-flat Parameter Space Crawl.* Independent of the types of D-flat searches performed (Type I, Hybrid or full Type II), it will likely be necessary to still perturb our SVD solutions in order to find representative sets of D-flat induced field VEVs. The current scheme pushes through the nullspace coefficient space in an homogenous way. For reference, consider the discussion under Eq. 3.17.

By extending the case to non-homogenous perturbations, where components of the desired solution vector are changed at different rates, it may allow access to a larger sampling of the solution space. In either case (homogenous or not) it will be important to look at how the nullspace coefficients are affected by the perturbations. This may illustrate how to 'push' our solutions along a preferred direction when when certain sets of VEVs are desired. It will also indicate the stability of different fields and their VEVs over the parameter space. That will give some measure of statistical likelihood and may pose anthropic implications.

*5.2.2.3 Linear Programming.* As seen in Eq. 3.17 and in other discussions of SVD, we learn that the solution space in infinite. However, it may be bounded. The determination of the nullspace coefficients along some dimensions of the parameter space may be curtailed through an extremization algorithm like Linear Programming (LP) or Integer Programming (IP). The brute-force *ad hoc* methods currently in use are used due to simplicity and speed.

It is hoped that one of these methods will help find, at least, parts of a bounding wall for the coefficient space. We use the term 'wall' to mean a surface which does not enclose a volume. Once this bounding wall is established, systematic generation of the coefficients within various subspaces can then be analyzed more readily. Whether or not there exists some bound to the certain areas of the coefficient space, it is still possible to control the systematic searches by investigating the moduli stabilizing population densities. Due to the coprime nature of the VEV's, the greatest number of solutions will exist towards the center of the coefficient space. If one imposed a minimum 'distance' between any two solutions, the search would end after a finite number of solutions were found. Future searches could be implemented to begin where previous investigations left off.

*5.2.2.4 Furthering Phenomenology.* The important first steps of phenomenological analysis were describes in 3.7. Once the mass matrix, gaugino condensate and cosmological constant calculations are in place, it will easy to determine whether a model is worth further investigation. If it is, then there a number of checks that will need to be added.

- Gauge group breaking - the introduction of NA VEVs will have broken gauge symmetries. The new gauge groups will have to be analyzed and cataloged.
- Neutrino masses - many modern theories rely heavily[1] on neutrinos with mass and with higher generations of neutrinos. It will be important to add in program functionality to generate various neutrino masses.
- SM/SM-like EFTs - the hidden sector field VEVs represent a higher-energy scale theory. In order to connect to more observable scales, it will be necessary to find flat directions for the observable sector.

[1]Pun intended.

- Observable sector SUSY breaking - as mentioned above Eq. 1.22, there are various mechanism for communicating hidden sector SUSY breaking to the observable. These with methods will have implemented as optional calculatory tools.

### *5.3 Conclusion*

Fully automated flatness analysis is computationally feasible and within reach. Great strides have been made at Baylor University in creating a Julia framework which will perform stringent searches of the WCFFHS models. As a way of generating flat directions, we have seen that SVD is a fast and reliable method to solving the $U(1)$ D-flatness constraints. Additionally, SVD plays a larger role in the Julia framework in directly producing D-flat solutions than in the previous FORTRAN code. Due to the unboundedness of the D-flat solutions, a quick scan of VEV ranges is initially applied to find a representative set of field VEVs.

The primary goal for a these flat direction searches will be to investigate phenomenologically viable models. Some of the parameters sought are masses (and mass hierarchies), interactions strengths, proton stability and SUSY-breaking scales. This framework has the machinery to perform these calculations on each model generated. The modularity and popularity afforded by Julia will allow for easy extension of the Flat Direction Framework. Systematic searches like these will yield insight into correlations between input and output spaces: moduli and masses, among them. This is essential if we hope to someday find mechanism that allow for certain universes to exist.

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