ABSTRACT

Grand Unified Theories in Higher Dimensions:
From the Heterotic String to Randall-Sundrum

G. Benjamin Dundee, B.S.

Advisor: Gerald B. Cleaver, Ph.D.

We explore the phenomenology of effective field theories in two different cases. First, we look at a mirror universe model, using an orbifold compactification of the heterotic string, in the free fermionic approach. An expected non-chiral Pati-Salam mirror universe model is transformed into a chiral model with enhanced hidden sector gauge symmetry and reduced observable sector gauge symmetry: $[SU(4)_C \times SU(2)_L \times SU(2)_R]^O \times [SU(4)_C \times SU(2)_L \times SU(2)_R]^H$, is necessarily transformed into a chiral $[SU(4)_C \times SU(2)_L]^O \times [SO(10) \times SU(2)_R]^H$.

Second, we look at a non-supersymmetric $\widetilde{SU}(5)$ theory built within the Randall-Sundrum framework. We derive the low energy running of the couplings, and show how the effects of the numbers and types of representations used for model building can be constrained by looking at universal contributions from KK modes to the beta functions. We show that the Yukawa couplings are, in general, exponentially small, leading to an exponentially short proton lifetime.

Grand Unified Theories in Higher Dimensions:
From the Heterotic String to Randall-Sundrum

by

G. Benjamin Dundee, B.S.

A Thesis

Approved by the Department of Physics

Gregory A. Benesh, Ph.D., Chairperson

Submitted to the Graduate Faculty of
Baylor University in Partial Fulfillment of the
Requirements for the Degree
of

Master of Science

Approved by the Thesis Committee

Gerald B. Cleaver, Ph. D., Chairperson

Jay R. Dittmann, Ph. D.

Edwin P. Oxford, Ph.D.

Anzhong Wang, Ph.D.

Walter M. Wilcox, Ph.D.

Accepted by the Graduate School
December, 2006

J. Larry Lyon, Ph.D., Dean

TABLE OF CONTENTS

APPENDICES

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

I would like to thank all of the sources of support that I have had throughout the preparation of this work. My adviser Dr. Gerald Cleaver, for his patience in explaining complex concepts more than once, especially in his field theory courses, and Dr. Tibra Ali, for his role in my understanding of the more formal aspects of this thesis, and of string theory in general. Had it not been for the patience of these two men, my understanding of string theory would be limited to a few books by Brian Greene. My family has been a constant source of love and support, and a few lines in an obscure book does them no justice. My girlfriend, Jennifer, has made life bearable at times when it was otherwise unbearable. I would like to thank Dean Darnell and Dr. John Perkins, whose questions motivated me to understand these subjects more deeply. The other graduate students in the department contributed greatly to both my understanding of these topics, and of physics in general, especially Sammy Joseph, Richard Obousy, Matt Robinson and Andreas Tziolas. Special thanks to Victor Guerrero for help in generating some of the figures. I would like to thank the Graduate School for making sure that my margins are exactly an inch on the right side, 1.5 inches on the left, and that page numbers are no more than 0.75 inches from the top of each page. I would like to acknowledge the financial contributions of the Texas Space Grant Consortium, under whose support I have worked for most of my time at Baylor. I also owe much to Josh Friess, Amanda Weltman and Joe P., all of whom helped me to open my eyes to the larger community of theoretical physicists. Finally, I would like to thank whoever it was who decided that an undergraduate with a mediocre GPA and a B.S. in Chemistry could possibly be of any value to the Baylor Physics Department.

CHAPTER ONE

Introduction

1.1 The Standard Model of Particle Physics

The Standard Model of particle physics (SM) [2] is the culmination of four decades of theoretical development and experimental verification. Barring some well-hidden new physics, the final pieces of the SM should fall into place when the higgs scalar is discovered in Geneva, by the end of this decade. By specifying a relatively small number of parameters, one is able to understand the dynamics of particle interactions at the electroweak scale, and the conditions in the universe up to its first minutes of existence.

The gauge group of the SM is $SU(3)_C \times SU(2)_L \times U(1)_Y$—three (chiral) generations of fermions transform in the fundamental representations of the gauge group, while the gauge bosons transform in the adjoint representations. Under $SU(3)_C \times SU(2)_L \times U(1)_Y$, the particles transform as follows:

$$\begin{array}{rrcl rcl} \text{Quarks}: & Q_L & = & (\mathbf{3},\mathbf{2},1/6) & Q_R & = & (\bar{\mathbf{3}},\mathbf{1},1/3) \\ \text{Leptons}: & L_L & = & (\mathbf{1},\mathbf{2},1/2) & L_R & = & (\mathbf{1},\mathbf{1},-1/2) \\ \text{Gauge Bosons}: & g & = & (\mathbf{8},\mathbf{1},0) & W^{\pm},Z & = & (\mathbf{1},\mathbf{3},1/2)\,, \end{array} \tag{1.1.1}$$

where the quantum numbers are given as ($SU(3)$ **rep**, $SU(2)$ **rep**, $U\left(1\right)$ charge). Breaking the SM to $SU(3)_C \times U\left(1\right)_{em}$ requires a complex higgs particle,

$$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \tag{1.1.2}$$

which transforms as $(\mathbf{1},\mathbf{2},1/2)$. The SM requires eighteen experimental inputs, like masses of quarks, quark mixings and strengths of forces—once these are set, one is able to make phenomenological predictions of astounding accuracy.

It was realized, in 1973 [3], that the couplings of non-Abelian gauge theories were energy dependent. In general, the renormalization group flow equation is given by

$$\frac{d}{d \log\left[q/\Lambda\right]} g = \beta\left(g\right), \tag{1.1.3}$$

where the parameter Λ is an arbitrary energy scale, and the coupling is given by g. We will generally choose Λ as a UV threshold, above which the runnings cannot be extrapolated. This has been called "parameterizing our ignorance", and corresponds to an energy regime at which the physics is unknown to us. We will see examples of this UV cutoff in several places in the following document.[1]

The function which characterizes the change of the coupling constant with incident energy is called the beta function, $\beta(g)$. The sign of the beta function will tell us how the coupling constant evolves as a function of the energy, q. In a theory like quantum electrodynamics, the beta function is positive. This means that as the energy of the scattering experiment increases, so does the coupling constant. Because the expansion in Feynman diagrams is a perturbation expansion, and only good for values of the coupling constant (much) less than 1, these types of theories may only be treated perturbatively in the IR limit—that is low energies and long distances.

Another class of quantum field theories are the finite field theories, whose beta functions are zero. In two dimensions, one can formulate conformal field theories (CFTs), which have the same beta function properties. These are of vital importance to string theory, in which the world-sheet degrees of freedom are fields governed by a 2-d CFT. There are, however, no physically important examples of 4-d finite field theories.

[1] We can also choose Λ to be an IR (low energy) cutoff, for example M_Z. An IR cutoff is customarily chosen when deriving the runnings of the SM beta functions, Equations (1.1.6). Choosing Λ as M_Z for the SM beta functions means that we are not able to know anything about energies $q \lesssim M_Z$.

Finally, if the beta function is negative, the gauge theory is called asymptotically free. This means that the low energy behavior may not be treated with perturbative methods, but that we can apply those methods in the high energy limit. The most relevant example of this class of field theories is quantum chromodynamics (QCD). At low energies in QCD, perturbation theory breaks down, and one must use complicated numerical techniques to get reasonable results [4].

The general form of the beta function for a non-Abelian gauge theory, with symmetry $SU(N)$, is given by

$$\beta\left(g\right) = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}N - \frac{4}{3}N_f C\left(r\right)\right] + \mathcal{O}\left(g^5\right), \tag{1.1.4}$$

where $C\left(r\right)$ is the Dynkin index of the fermion representation r [5], and N_f is the number of fermions in the theory. The derivation of this equation is tedious but straightforward, and will be done in Appendix A.[2] To find the beta function for QCD, we insert $N = 3$, $N_f = 6$ quarks and $C\left(r\right) = 1/2$ for the fundamental representation,

$$\beta_{QCD} = \frac{-7g^3}{16\pi^2} + \mathcal{O}\left(g^5\right). \tag{1.1.5}$$

Now, we can integrate Equation (1.1.3) using the form of the beta function given in Equation (1.1.4). We choose our Λ as the electroweak symmetry breaking scale, $M_Z = 91.1876 \pm 0.0021$ GeV [6], because $SU(3)_C \times SU(2)_L \times U(1)_Y$ is only a good gauge group above this energy. We will also make the substitution $\alpha = \frac{g^2}{4\pi}$ in natural units, where $\hbar = c = 1$. Then...

$$\alpha^{-1}_{U(1)}\left(E\right) \equiv \alpha^{-1}_1 = c_1 + \frac{b_1}{2\pi}\log\left[\frac{E}{\Lambda}\right] \tag{1.1.6a}$$

$$\alpha^{-1}_{SU(2)}\left(E\right) \equiv \alpha^{-1}_2 = c_2 + \frac{b_2}{2\pi}\log\left[\frac{E}{\Lambda}\right] \tag{1.1.6b}$$

$$\alpha^{-1}_{SU(3)}\left(E\right) \equiv \alpha^{-1}_3 = c_3 + \frac{b_3}{2\pi}\log\left[\frac{E}{\Lambda}\right]. \tag{1.1.6c}$$

[2] The case for Abelian symmetries is a bit easier, as the number of graphs we must compute is smaller. The general form of the coupling constant's dependence on energy is the same.

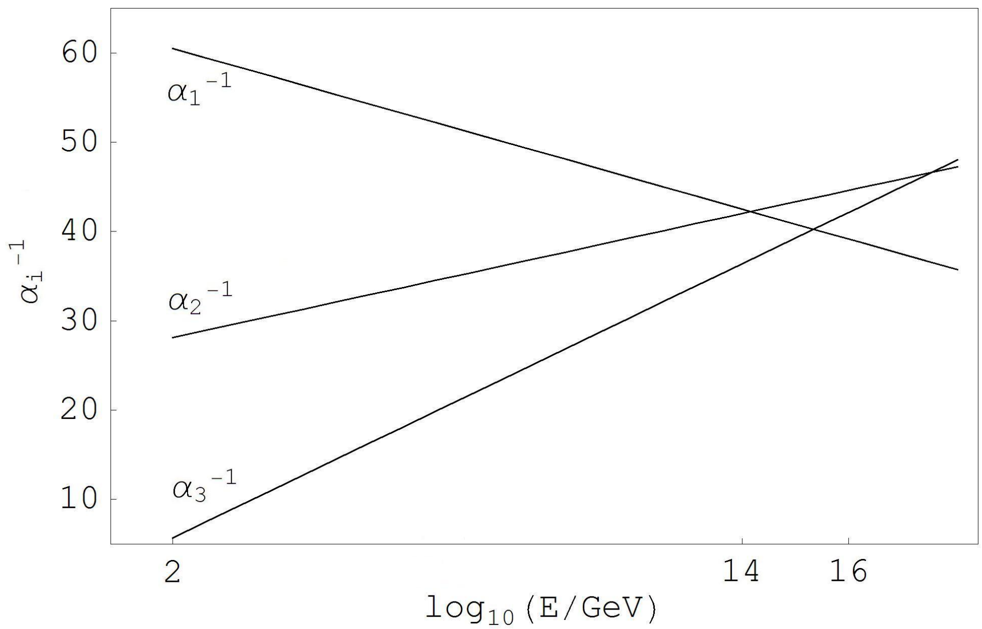


Figure 1.1. The running of the couplings in the Standard Model. Note the approximate unification around 10^{14-16} GeV.

The constants, c_i, are boundary conditions, which we derive from experiments. We know that $\alpha_3\,(M_Z) \cong 0.1187 \pm 0.0020$, $\alpha_2\,(M_Z) \cong 0.033961 \pm 0.000006$, and $\alpha_1 \cong 0.017022 \pm 0.000002$ [6], and from Equation (1.1.4) we know $b_1 = 41/10$, $b_2 = -19/6$, and $b_3 = -7$. Using this information, we can plot how the coupling constants α_i^{-1} change with energy scale E, which we have done in Figure 1.1. It is important to remember that we are looking at the plot of the inverse coupling constants. The strong force, $SU(3)$, starts out at around 0.1 and becomes weaker with increasing energy scale. Conversely, the weakest force, the hypercharge, increases in strength as energy scales increase. These forms of the couplings are essentially an experimental prediction of the SM, assuming that there exists no intermediate scale matter—an "intermediate-scale desert". We will see that different theories tend to predict different runnings of the couplings.

If we want to include exotic matter with mass greater than M_Z (but smaller than $M_{Pl} \sim 10^{18}$GeV in our models, we would have to go back and compute contributions to the graphs in Appendix A. The contributions from this exotic matter will, in general, change the form of the renormalization group equations, and effect the runnings near the new mass scale.

It is important to note that precision measurements of Equations (1.1.6) are pretty limited in scope—that is, they have not been measured past the electroweak symmetry breaking scale. The graphs in Figure 1.1 are extrapolations. This is one exciting idea that the LHC will be looking at [7, 8]—the beta functions put strong constraints on the type of matter that one can have in the theory. The existence of any new matter or forces will change the running of the coupling above the related mass scales for the $\delta\, b_i$, so any variation in the slopes of any of the lines in Figure 1.1 corresponds to new physics. We will try to parameterize the contributions of new matter at some intermediate scale Λ_I as

$$\alpha_i^{-1} = c_i + \frac{b_i + \delta\ b_i}{2\pi} \log\left[\frac{E}{\Lambda_I}\right]. \tag{1.1.7}$$

1.2 The Shortcomings of the Standard Model

After twenty-five years of experimental tests of the SM, we have yet to find any violations. There was some hope, earlier this year, that b-mixing experiments [9, 10], which are extremely sensitive to new physics, would hint at something beyond the SM, but the results only served to verify the SM prediction [11]. The best candidate for a new physics event has been the observation of the muon anomalous magnetic moment [12], which differs from the SM prediction by anywhere between 0.7 σ and 3.2 σ, depending on which experimental and theoretical values one compares [13]. Generally we consider any experimental result that differs from theory by more than 3 σ to be evidence of new physics, but because we don't have a consensus on the calculational end, we cannot be certain that we are seeing something new.

In spite of the tremendous successes of the SM, few accept that it is the final theory. We will focus on five open questions that the SM has nothing to say about [14]: the algebra question, the fermion question, the charge question, the gravity question, and the hierarchy problem. In general, these issues are all "naturalness" arguments, borne from the assumption that the SM is an effective field theory of some more fundamental framework. It is widely held that any satisfactory UV completion of the SM should address all of these issues.

1.2.1 The Algebra Question

In particle physics, we have found that (spin $\frac{1}{2}$) fermions transform as representations of Lie groups [15], with (spin 1) bosons acting as generators and transforming in the adjoint. For example, in QCD, the quarks transform in the fundamental ($\mathbf{3}$) of $SU(3)$, while the gluons transform in the adjoint ($\mathbf{8}$). The same is true for the electroweak theory—fermions come in $SU(2) \times U(1)$ doublets (the fundamental representation), and the W bosons transform in the adjoint. By definition, the gauge bosons must transform as the adjoint representation (they are the generators of the algebra), but it would be a grand coincidence if all of the fermions in the low energy effective field theory just *happened* to transform as the fundamental representations.

What dictates the higgs transformation properties under the various groups? The higgs is a color singlet and a doublet under $SU(2)$. We know that, in order to preserve $SU(3)_C \times U(1)_{em}$ in the low energy limit, the higgs must not break QCD, and must transform as a singlet under $SU(3)_C$. Electroweak symmetry breaking seems to be unique in that the symmetry breaking is accomplished with a higgs in the fundamental rep of $SU(2)$. This is not a generic feature of symmetry breaking—when we study grand unified theories in Chapter 2, we will see that the higgs scalars lie in "very large" representations of the gauge group, generally adjoint or higher. Is there a guiding principle which chooses the which representations the higgses live in? Finally, why do we see the symmetries that we do—why is the effective lagrangian of the SM invariant under the symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$?

1.2.2 The Fermion Question

If we look at the masses of the fermions in the SM, we see the generational structure taking shape. For example, the quark masses seem to follow $m_t, m_b >> m_c, m_s >> m_u, m_d$. If we look at the accepted experimental data [6], the masses obey the (approximate) ratio $1 : 10^{-3} : 10^{-5}$. Further, if we associate the τ lepton with the top/bottom, the μ lepton with the charm/strange and the electron with the up/down, we see the same generational pattern: $m_\tau >> m_\mu >> m_e$.[3] Why does such a pattern exist? And why are there exactly three generations? The SM does not tell us how many generations we should observe—instead there are observational constraints on the number of generations we have.

The conservation of B and L can be proved in the SM [17, 18], with its standard particle content.[4] We generally understand conserved charges as being protected by some unbroken symmetry—for example, we know that electric charge is conserved in low energy scattering experiments because it is protected by the $U(1)_{em}$ which exists below the electroweak symmetry breaking scale. The $B-L$ symmetry is "accidental" in the sense that there is no symmetry in the SM which explicitly protects $B-L$, but it is conserved anyway.

The fermionic sector of the SM also has other issues. We know that neutrinos have mass [19], and aside from explicitly adding mass terms (that are necessarily fine-tuned because of the estimated size of the neutrino mass) to the SM, the most promising way to generate neutrino masses is the seesaw mechanism [20, 21], which requires a (right-handed) neutrino singlet to work, and avoids fine tuning problems. A light, left-handed, Majorana neutrino is naturally obtained when a right-handed

---

[3] Although the neutrino masses are not known exactly, there is the possibility that they, too, obey this approximate ratio. The current *upper bounds* on neutrino masses are $m_{\nu_\tau} < 18.2\ \mathrm{MeV}, \mathrm{m}_{\nu_\mu} < 0.19\ \mathrm{MeV}, \mathrm{m}_{\nu_e} < 3 \times 10^{-3}\ \mathrm{MeV}$ [6]. There are also stricter cosmological bounds on the sum of the three neutrino masses, $\sum m_\nu = 0.56^{+0.30}_{-0.26}\mathrm{eV}$ [16].

[4] Baryons are hadrons that contain three quarks—the proton and neutron are both assigned baryon number of $B = 1$, thus each of the quarks have $B = 1/3$, and the anti-quarks have $B = -1/3$. The electron has lepton number $L = +1$, and the positron has lepton number $L = -1$.

(Majorana) neutrino has a large mass term. The mass terms for the neutrinos are postulated to come from

$$\mathcal{L} = -\frac{1}{2}\left(\bar{\nu}_L\ \bar{\nu}_R\right)\mathcal{M}\begin{pmatrix}\nu_L \\ \nu_R\end{pmatrix} + \text{h.c.} \tag{1.2.1}$$

The mass matrix is given by

$$\mathcal{M} = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix}, \tag{1.2.2}$$

where

$$m_M^R >> m_D >> m_M^L. \tag{1.2.3}$$

The eigenvalues of this matrix give the mass terms of the left-handed and right-handed neutrinos:

$$m_{\nu_L} \sim \left| m_M^L - \frac{m_D^2}{m_M^R} \right|, m_{\nu_R} \sim m_M^R. \tag{1.2.4}$$

Either way, a massive left-handed neutrino means that the SM must be modified with at least the addition of another lepton, with the same generational mixings as the other leptons and the quarks.

Finally, the strong CP problem of QCD either represents a tremendous fine-tuning of coupling constants, or a symmetry which we have not been clever enough to discover. In general, all renormalizable, mass dimension 4, gauge invariant terms that can be added to the lagrangian should be added. Of all of these possible terms in the SM lagrangian, only one is not observed—that is the CP violating term of QCD. Charge C, Parity P and Time Reversal T are all discrete symmetries of a quantum field theory. The symmetry CP is maximally violated in the electroweak model, but no violations have been found in the strong force. From the QCD field strength tensor, we can make the following (renormalizable) combinations

$$\mathcal{L}_{QCD} \supset F \wedge *F + F \wedge F = \frac{-1}{4}\text{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{N_f g^2 \theta}{32\pi^2}\text{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{1.2.5}$$

where θ is the CP-violating angle. $*F$ is the Hodge dual of F, and is given by

$$*F = \tilde{F}_{\rho\sigma} = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}. \tag{1.2.6}$$

One expects that

$$\begin{aligned} -\frac{N_f g^2\theta}{32\pi^2} &\sim -\frac{1}{4} \\ \theta \sim \frac{8}{N_f g^2} &\sim \mathcal{O}\left(\frac{1}{g^2}\right). \end{aligned} \tag{1.2.7}$$

The best limits come from measurements of the electron's dipole moment, and give $\theta < 1.5 \times 10^{-10}$ [22]. Why is QCD any different from the electroweak model, in which CP is maximally violated? Why should CP violations be so small? There are beautiful correlations between CP violations and the matter/anti-matter asymmetry of the universe. The fact that we see only matter (versus a 50/50 mix of matter/anti-matter) in this universe is a consequence of this CP violation [23].

1.2.3 The Charge Question

Another cosmic coincidence in the SM is the difference in charge between the electron and the proton—the difference is less that 1 part in 10^{-20} [24]. There is no mechanism predicting the charges of particles in the SM, just as there are not predictions of masses or couplings in the SM. Charge quantization may be explained by the condition that the SM be free from anomalies, and that a right handed neutrino exists [25]. The right handed neutrino, however, is generally not considered part of the SM.

1.2.4 The Gravity Question

Gravity is the most important force on scales larger than about 1 m, and on scales on the order of the Planck length ($\sim 10^{-35}$ m) but it is completely without mention in the SM. Any attempts to quantize gravity in the way that we quantize the (non-)Abelian symmetries in the SM leads to disastrous consequences. There have

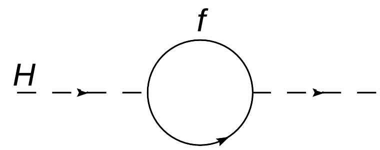


Figure 1.2. In general, the higgs will couple to all of the fermions in the theory. The one loop graph is quadratically divergent.

been attempts to resum the graviton propagator [26], but the regimes in which gravity becomes strong are still poorly understood, at best. This conflict can be understood by noting that the gravitational scattering of two particles scales as $(E/M_{Pl})^2$ at tree level, which diverges for arbitrarily large energies. This divergence is a power-law divergence, arising because the theory of gravity at large energies has no UV cutoff.[5]

Finally, the propagators of all of the other forces are spin one bosons, but the (hypothetical) graviton must have spin two. This suggests that perhaps there is some fundamental difference in the way we must treat the forces.

1.2.5 The Hierarchy Problem

In order for the electroweak symmetry breaking to be consistent, we must postulate the existence of a scalar higgs field. $SU(2)_L \times U(1)_Y$ is broken to $U(1)_{em}$ when the higgs scalar takes on a vacuum expectation value (vev), which is dictated by the minimum of some potential.[6] The higgs vev, v, is related to the higgs mass m_H and the higgs self-coupling, λ, by $v = \sqrt{\frac{m_H^2}{\lambda}}$. The SM data give the higgs mass as 113^{+56}_{-40} GeV [6], giving a vev of around 174 GeV. We know that the higgs mass must be larger than 114.4 GeV, via direct searches at Fermilab. The problem remains that the SM offers no way to determine the shape of the higgs potential, and thus no way to firmly predict the vev that the higgs field will take.

[5] Gravity could, of course, be a low energy effective field theory of some larger theory, but without some fundamental length, this is just a *reductio ad infinitum*—one could continue such extensions forever.

[6] One can imagine a ball rolling down the side of a hill—the place where the ball eventually comes to rest corresponds to the stable minimum where the higgs field will “settle”, or its vev.

We can estimate the higgs mass by looking at the terms from mass renormalization. The higgs will couple to all of the fermions in the theory, as in Figure 1.2, and the higgs mass is quadratically divergent:

$$\Delta m^2 = -\frac{g^2}{8\pi^2}\Lambda_{UV}^2. \tag{1.2.8}$$

Λ is a UV cutoff, and is the scale at which the SM is violated—naturally the Planck scale where gravity becomes strong. We know that the SM cannot hold at arbitrarily high energies where gravity becomes important—this means that the UV cutoff *should* be the Planck scale, 10^{18-19} GeV, if there is no new intermediate scale physics. This calculation tells us that the higgs mass will scale as the UV cutoff, which cannot be the case, because the higgs gains a mass at the electroweak scale, which we *know* from experiments. There should, therefore, be some mechanism which protects the higgs from gaining a vev before the electroweak scale.

1.2.6 Answers

Throughout this thesis, we will examine different ways to resolve these issues in the context of model building. We will take the position that *every* quantum field theory is some low energy effective approximation [27] of a higher theory, until we understand the UV completion of the final theory. We will see that different model building approaches are particularly well-suited for solving particular problems, at the expense of introducing new problems.

There is one final, albeit bleak, possibility. It could be that the constants are the way they are by chance, and there are no first principles from which they may be derived—this is the natural paradigm if we accept that there is no UV completion to the SM. We could accept the higgs mass as fine tuned, that renormalization is the only way to solve our UV divergences, and realize that we are living in a very narrow strip of the SM parameter space that allows us to exist [28]. We may be forced to acknowledge that quantum gravity is too hard of a problem for us to solve, and that the energies involved are just too inaccessible. If precision measurements of SM parameters continue to agree with theory as well as they have in the past—for example, if we fail to find any new physics and only a single higgs at the LHC—and we can find no evidence of anything new in our next generation cosmological experiments [29], then this paradigm may be inevitable.

1.3 Supersymmetry and the Minimal Supersymmetric Standard Model

Supersymmetry (or SUSY) [30–33] has been shown to be the last possible symmetry that one can add to the S matrix. SUSY is the famous exception to the Coleman-Mandula theorem [34], which assumes that:

(1) the S-matrix is based on a local, relativistic quantum field theory in four-dimensional space-time,

(2) there are only a finite number of different particles associated with one-particle states of a given mass, and

(3) there is an energy gap between the vacuum and the one particle states.

The theorem asserts that symmetries of the S-matrix are limited to the Poincaré group, $\mathcal{P}$, and some internal symmetries, which are semisimple Lie groups $\mathcal{G}$, (with additional $U(1)$ factors) whose generators obey

$$[T_a, T_b] = i f_{abc} T_c. \tag{1.3.1}$$

The Poincaré group contains the energy-momentum operator, $P_i \equiv -i\partial_i$, and the Lorentz rotation generator, J_{ij}, which obey the following algebra:

$$[P_i, P_j] = 0, \tag{1.3.2a}$$

$$[P_i, J_{jk}] = g_{ij}P_k - g_{ik}P_j \tag{1.3.2b}$$

$$[J_{ij}, J_{kl}] = -\left(g_{ik}J_{jl} + g_{jl}J_{ik} - g_{ij}J_{kl} - g_{il}J_{jk}\right). \tag{1.3.2c}$$

The symmetries of the S matrix must be a direct product of the two groups, $\mathcal{P}\otimes\mathcal{G}$—in other words,

$$[T_a, P_k] = [T_b, J_{ij}] = 0. \tag{1.3.3}$$

Typically, Lie algebras are defined in terms of commutators, but SUSY avoids the constraints of the Coleman-Mandula theorem by generalizing the idea of a Lie algebra—super-Lie algebras are defined in terms of an anti-commutator. In the following section, we first review supersymmetry, and then the supersymmetric extension to the Standard Model.[7]

1.3.1 The Supersymmetry Algebra and Superspace

All of the symmetries in the SM are internal symmetries, in the sense of the Coleman-Mandula theorem—that is, there exist operators which transform bosons into bosons and fermions into fermions. Symmetries which relate bosons to fermions are supersymmetries. The fundamental requirement of supersymmetry is that we have corresponding fermionic (ψ_α) and bosonic (ϕ) states. The supersymmetry is generated by an operator Q_α, called the supercharge, which takes bosonic states into fermionic states and fermioininc states into bosonic states.

First, note that P^m transforms in the $(\frac{1}{2},\frac{1}{2})$ representation of the Lorentz group, $SO(4) \cong SU(2)_L \times SU(2)_R$. Now, the only symmetry generators consistent with the Coleman-Mandula theorem are the P^m and the J^{mn}, which transform as $(1,0)\oplus(0,1)$, and are the proper Lorentz transformations. The set of operators P and J generate the Poincaré group. Dirac fields (and the supercharge, Q) always transform in the $(\frac{1}{2},0)$ and the $(0,\frac{1}{2})$ representations [35]. The anti-commutator, $\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\}$, must transform as

$$\left(\frac{1}{2},0\right)\otimes\left(0,\frac{1}{2}\right)\sim\left(\frac{1}{2},\frac{1}{2}\right). \tag{1.3.4}$$

[7] In what follows, we will need to develop some formalism, which is done in Appendix B.

The only operator which lies in this representation is P^m, which has a space-time index m. In order to take care of the spinor indices (and preserve Lorentz invariance), we must have

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^{\ m}_{\alpha\dot{\beta}} P_m. \tag{1.3.5}$$

The commutator of $(\frac{1}{2}, \frac{1}{2})$ and $(0, \frac{1}{2})$ must be in a linear combination of $(\frac{1}{2}, 1)$ and $(\frac{1}{2}, 0)$, but because the $(\frac{1}{2}, 1)$ representation of the Lorentz is reducible, we will not consider it. Then, the indices tell us the correct form of the commutator:

$$\left[\sigma^{\ m}_{\alpha\beta} P_m, Q^\alpha\right] = cQ_\beta. \tag{1.3.6}$$

We will state without proof that the constant $c = 0$, giving

$$\left[\sigma^{\ m}_{\alpha\beta} P_m, Q^\alpha\right] = 0. \tag{1.3.7}$$

This is equivalent to the statement that the supercharge is independent of the space-time coordinates.

Finally, consider the anti-commutation of a supercharge with itself—two $(0, \frac{1}{2})$ tensors must form a linear combination of $(0, 0)$ and $(0, 1)$. The $(0, 1)$ generators are linear combinations of the J^{mn}, but because the Q's commute with the P's, so must their anti-commutators. The Poincaré algebra, however, tells us that no linear combination of J^{mn} commutes with P^m. The $(0, 0)$ tensors correspond to internal symmetries [35], and for $\mathcal{N} > 1$ SUSY, we have

$$\{Q_{\alpha r}, Q_{\beta s}\} = e_{\alpha\beta} Z_{rs}, \tag{1.3.8}$$

where r, s run over the SUSYs $(r, s = 1, 2, ..., \mathcal{N})$. The central charges of the group, Z, obey $Z_{rs} = -Z_{sr}$ which is trivially zero for $\mathcal{N} = 1$ SUSY. The e's are Clebsch-Gordan coefficients. Thus we have

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0. \tag{1.3.9}$$

The Equations (1.3.5), (1.3.7), and (1.3.9) comprise the $\mathcal{N} = 1$ SUSY algebra. If one wanted to study >1 SUSY, one would have to include new indices on the supercharges Q.

It was realized, by Salam and Strathdee [36], that the space acted on by the supersymmetry generator is a generalized version of the space on which P^m acts. Just as P^m generates translations in $\mathbb{R}^4$, the Q^α generate translations in superspace. Just as the Poincaré algebra is described by commutation relations and acts on commuting coordinates, the supersymmetry algebra is described by anti-commutation relations and acts on anti-commuting coordinates—points in superspace are identified by $z = \left(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\right)$. The θ's are the (anti-commuting) Grassman numbers, and are defined as

$$\left\{\theta^\alpha, \theta^\beta\right\} = \left\{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\right\} = \left\{\theta^\alpha, \bar{\theta}_{\dot{\beta}}\right\} = 0. \tag{1.3.10}$$

The superspace coordinates also commute with the $\mathbb{R}^4$ coordinates:

$$\left[x^\mu, \theta^\alpha\right] = \left[x^\mu, \bar{\theta}_{\dot{\alpha}}\right] = 0. \tag{1.3.11}$$

Consider a function of the superfield coordinates, $\Phi\left(x, \theta, \bar{\theta}\right)$.[8] Because of the anticommutation relationships in Equation (1.3.10), we have $\theta^1\theta^1\theta^2 = -\theta^1\theta^1\theta^2 = 0$ — any combination of more than two Grassman numbers will always be zero. Note that we are allowed combinations like $\theta\theta\bar{\theta}\bar{\theta}$, because the spinor indices of each θ $\left(\bar{\theta}\right)$ can be different. The most general form of a superfield is given by

$$\begin{aligned} \Phi\left(x, \theta, \bar{\theta}\right) &= f\left(x\right) + \theta\phi\left(x\right) - i\bar{\theta}\bar{\chi}\left(x\right) \\ &+ \theta\theta m\left(x\right) - \bar{\theta}\bar{\theta}n\left(x\right) \\ &- \theta\sigma^m\bar{\theta}v_m\left(x\right) + \theta\theta\bar{\theta}\bar{\lambda}\left(x\right) \\ &+ \bar{\theta}\bar{\theta}\theta\psi\left(x\right) + \theta\theta\bar{\theta}\bar{\theta}d\left(x\right). \end{aligned} \tag{1.3.12}$$

[8] From this point forward, we will (mostly) suppress the spinor indices.

Now, suppose that we perform a transformation in superspace, such that

$$\Phi\left(x^m,\theta,\bar\theta\right)\to\Phi\left(x^m+\xi^m,\theta+\epsilon,\bar\theta+\bar\epsilon\right), \tag{1.3.13}$$

where $\xi^m = -i\epsilon\sigma^m\bar\theta + i\bar\epsilon\bar\sigma^m\theta$ is the general form of a translation in superspace for x^m. The superspace translations are generated by the Q's, and we may write the transformation as:

$$\Phi\to\left(\epsilon Q+\bar\epsilon\bar Q\right)\Phi \tag{1.3.14}$$

Inserting the definition of Φ into Equation (1.3.14), and using (1.3.13), we find

$$Q_\alpha\Phi \;=\; \partial_{\theta^\alpha}\Phi - i\sigma_{\alpha\dot\alpha}^{\ \ m}\bar\theta^{\dot\alpha}\partial_m\Phi \tag{1.3.15a}$$

$$\bar Q_{\dot\alpha}\Phi \;=\; -\partial_{\bar\theta^{\dot\alpha}}\Phi + i\theta^\alpha\sigma_{\alpha\dot\alpha}^{\ \ m}\partial_m\Phi. \tag{1.3.15b}$$

By imposing different conditions on the general field Φ, we can construct the different representations of the SUSY algebra, which will be useful when we construct a supersymmetric field theory.

When working with internal symmetries in $\mathbb{R}^4$, we must define the notion of a covariant derivative in order to ensure that the lagrangian transforms in such a way as to leave the action invariant. In superspace, the super-covariant derivatives are given by:

$$\mathcal{D}_\alpha \;=\; \partial_{\theta^\alpha} + i\sigma_{\alpha\dot\alpha}^{\ \ m}\bar\theta^{\dot\alpha}\partial_m, \tag{1.3.16a}$$

$$\bar{\mathcal{D}}_{\dot\alpha} \;=\; -\partial_{\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma_{\alpha\dot\alpha}^{\ \ m}\partial_m. \tag{1.3.16b}$$

Because they are so similar to the Q's, we might expect that the super-covariant derivatives obey the same algebra. This is indeed the case: the differential operators obey the same anti-commutator algebra that the supercharges do, and even anti-commute with the supercharges:

$$\left\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\right\} \;=\; -2\sigma_{\alpha\dot\beta}^{\ m}P_m \tag{1.3.17a}$$

$$\left\{\mathcal{D}_\alpha,\mathcal{D}_\beta\right\} \;=\; \left\{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\right\}=0 \tag{1.3.17b}$$

$$\left\{\mathcal{D}_\alpha,Q_\beta\right\} \;=\; \left\{\bar{\mathcal{D}}_{\dot\alpha},\bar Q_{\dot\beta}\right\}=\left\{\mathcal{D}_\alpha,\bar Q_{\dot\beta}\right\}=\left\{\bar{\mathcal{D}}_{\dot\alpha},Q_\beta\right\}=0 \tag{1.3.17c}$$

Referring to Equation (1.3.12), we see that the general superfield contains left-handed fermionic fields ϕ and right-handed fermionic fields χ. If we separate the left- and right-handed parts from Φ, we find:

$$\mathcal{D}_\alpha \Phi_L = 0 \quad (1.3.18a)$$

$$\bar{\mathcal{D}}_{\dot{\alpha}} \Phi_R = 0. \quad (1.3.18b)$$

We can write Φ_L and Φ_R in a more compact form, choosing $y^m = x^m + i\theta\sigma^m\theta$. Then

$$\Phi_L = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \quad (1.3.19a)$$

$$\Phi_R = A^*\left(y^\dagger\right) + \sqrt{2}\bar{\theta}\bar{\psi}\left(y^\dagger\right) + \bar{\theta}\bar{\theta}F^*\left(y^\dagger\right). \quad (1.3.19b)$$

The F field, called an auxiliary field, will not appear in the effective field theory, and will be eliminated using its equation of motion:

$$\frac{\partial \mathcal{L}}{\partial F_k} = F_k + \lambda_k + m_{ik}A_i + g_{ijk}A_iA_j = 0, \quad (1.3.20a)$$

$$\frac{\partial \mathcal{L}}{\partial F_k^*} = F_k + \lambda_k^* + m_{ik}^*A_i^* + g_{ijk}^*A_i^*A_j^* = 0. \quad (1.3.20b)$$

It is now quite clear that the chiral superfield relates a Weyl spinor (spin $\frac{1}{2}$ fermion), ψ with a complex (spin 0) scalar, A. Finally, the super-covariant derivatives are:[9]

$$\mathcal{D}_\alpha \Phi_L = \partial_{\theta^\alpha}\Phi_L + 2i\sigma_{\alpha\dot{\alpha}}^{\ \ m}\bar{\theta}^{\dot{\alpha}}\partial_m\Phi_L, \quad (1.3.21a)$$

$$\bar{\mathcal{D}}_{\dot{\alpha}} \Phi_R = -\partial_{\bar{\theta}^{\dot{\alpha}}}\Phi_R, \quad (1.3.21b)$$

Another representation of the SUSY algebra that we will use is the vector superfield, V, defined to be real:

$$V = V^\dagger. \quad (1.3.22)$$

Expanding in the Grassman parameters, the most general form of a vector superfield

[9] The spatial derivatives are taken with respect to y: $\partial_m = \frac{\partial}{\partial y^m}$.

is given by

$$
\begin{aligned}
V(x,\theta,\bar{\theta}) \;=\;& C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) \\
&+ \frac{i}{2}\theta\theta[M(x) + iN(x)] - \frac{i}{2}\bar{\theta}\bar{\theta}[M(x) - iN(x)] \\
&- \theta\sigma^m\theta v_m(x) + i\theta\theta\bar{\theta}[\bar{\lambda}(x) + \frac{1}{2}\bar{\sigma}^m\partial_m\chi(x)] \\
&- i\bar{\theta}\bar{\theta}\theta[\lambda(x) + \frac{i}{2}\sigma^m\partial_m\bar{\chi}(x)] \\
&+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \frac{1}{2}\Box C(x)]. \qquad (1.3.23)
\end{aligned}
$$

The component fields, C, D, M, N and the vector field v_m are all real. In the Wess–Zumino gauge, we set $C, \chi, M,$ and N to zero, thus removing much of the complexity of Equation (1.3.23). The resulting vector superfield is given by

$$
V = -\theta\sigma^m\bar{\theta}v_m(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \qquad (1.3.24)
$$

As is typical, we have fixed the gauge and lost a manifest symmetry (supersymmetry). This representation of the supersymmetry algebra relates a (spin 1) vector field, v_m, and a left-handed (spin $\frac{1}{2}$) fermion, λ. D is an auxiliary field like F, and will be eliminated in the effective theory using its equation of motion:

$$
D^a - A_i^\dagger T^a_{ik} A_k \;=\; 0, \qquad (1.3.25a)
$$

$$
(D^a)^\dagger - A_i T^a_{ik} A_k^\dagger \;=\; 0, \qquad (1.3.25b)
$$

where $T^a = T^{a\,\dagger}$. This vector superfield is invariant under a local gauge transformation [30]:

$$
V \rightarrow V + \Phi + \Phi^\dagger, \qquad (1.3.26)
$$

and ensures a supersymmetric version of gauge invariance.

1.3.2 Supersymmetric Quantum Field Theories

Our supersymmetric quantum field theory will be built from the superfields that we constructed in the last section. The superfields contain both bosonic and fermionic degrees of freedom—it is in this way that we will relate bosons to fermions in the effective field theory. These fields live in superspace, and are acted on by the super-covariant derivatives, $\mathcal{D}$ and $\bar{\mathcal{D}}$, defined in (1.3.16). The superspace is parameterized by non–commuting Grassman coordinates, θ and $\bar{\theta}$.[10]

The lagrangian will be constructed from two parts—a part containing chiral superfields and a part containing vector superfields. As always, we want the action to be invariant under transformations of the lagrangian. By Gauss' theorem, this means that any transformation can change the lagrangian by, at most, a total derivative. If we look at the way a chiral superfield transforms under the supersymmetry transformation, Equation (1.3.14), we see that only the $\theta\theta$ ($\bar{\theta}\bar{\theta}$) components transform as a total derivative. By direct multiplication, we see that the product of any number of chiral superfields is also a superfield. Then we define a function $\mathcal{P}$, called the superpotential, which transforms as a total derivative under (1.3.14), as

$$\mathcal{P} = \left\{ \lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \right\} \Big|_{\theta\theta \text{ component}} \tag{1.3.27}$$

Note that we could, in general, have more terms in our superpotential. A quick check of the mass dimensions of the higher order terms, however, shows that they are non-renormalizable. As in non-supersymmetric quantum field theory, one could add such non-renormalizable terms to the lagrangian—this is an interesting possibility, and these higher order terms are of much interest in the heterotic, free-fermionic string phenomenology that we will discuss later, in Chapter 3.

Now, consider the product $\Phi^\dagger \Phi$. By direct multiplication, one can show that this product is not a superfield, however, the coefficients of the $\theta\theta\bar{\theta}\bar{\theta}$ term have the correct transformation properties to be included in the lagrangian, namely the term changes by an overall derivative.[11]

[10] In what follows, we will assume a supersymmetric, Abelian field theory, with coupling g.

[11] One can also count mass dimensions—in order to construct a renormalizable lagrangian, all terms in that lagrangian must have mass dimension 4. The product $\Phi^\dagger \Phi$ has mass dimension 2, and $\theta\theta\bar{\theta}\bar{\theta}$ has mass dimension -2, thus the coefficient of this term must have mass dimension 4.

The full chiral superfield lagrangian can then be expressed in terms of the superpotential:

$$\mathcal{L} = \Phi_i^\dagger \Phi_i \bigg|_{\theta\theta\bar{\theta}\bar{\theta}} + \frac{1}{2}\sum_{i,k}\left\{\frac{\partial^2 \mathcal{P}}{\partial\Phi_i \partial\Phi_k}\psi_i\psi_k + \text{ h.c.}\right\} - \sum_i \left|\frac{\partial \mathcal{P}}{\partial \Phi_i}\right|^2 . \tag{1.3.28}$$

In this lagrangian, we have eliminated the auxiliary fields F and D using Equations (1.3.20) and (1.3.25).

In order to construct the vector lagrangian, we make the observation that it must be gauge invariant. The only constituent fields of V which are gauge invariant are λ_α and D. Let us use $\mathcal{D}$ and $\bar{\mathcal{D}}$ to define projection operators, that keep only the λ_α from V:

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}_\alpha V, \\ \bar{W}_{\dot{\alpha}} &= -\frac{1}{4}\mathcal{D}\mathcal{D}\bar{\mathcal{D}}_{\dot{\alpha}} V. \end{aligned} \tag{1.3.29a}$$

By the commutation relationships in Equations (1.3.17), we immediately see that

$$\bar{\mathcal{D}}_{\dot{\alpha}} W_\beta = \mathcal{D}_\alpha \bar{W}_{\dot{\beta}} = 0. \tag{1.3.30}$$

This means that the W fields are chiral (*cf* Equations (1.3.18)). They are also gauge invariant—under the gauge transformation (1.3.26):

$$\begin{aligned} W_\alpha &\rightarrow -\frac{1}{4}\mathcal{D}\mathcal{D}\bar{\mathcal{D}}_{\dot{\alpha}}\left(V + \Phi + \Phi^\dagger\right) \\ &= W_\alpha - \frac{1}{4}\bar{\mathcal{D}}\left\{\bar{\mathcal{D}}, \mathcal{D}\right\}\Phi = W_\alpha. \end{aligned} \tag{1.3.31}$$

If we were to write out the form of W_α, we would see that it contain the gauge invariant fields D and λ_α, as well as the (gauge invariant) Abelian field strength:

$$v_{mn} \equiv \partial_m v_n - \partial_n v_m. \tag{1.3.32}$$

Again, by either looking at the transformation properties, or by counting mass dimensions, one finds that the kinetic terms for a vector and spinor field are contained in

$$W^\alpha W_\alpha \bigg|_{\theta\theta} = -2i\lambda\sigma^m\partial_m\bar{\lambda} - \frac{1}{2}v^{mn}v_{mn} + D^2 + \frac{i}{4}v^{mn}v^{lk}\epsilon_{mnlk}. \tag{1.3.33}$$

The D fields are eliminated using their equations of motion, (1.3.25). The kinetic part of the vector lagrangian is, then:

$$\mathcal{L}_{kin} = \frac{1}{4}\left\{ W^\alpha W_\alpha \Big|_{\theta\theta} + \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\Big|_{\bar\theta\bar\theta} \right\}. \tag{1.3.34}$$

This is the only renormalizable term allowed in the vector lagrangian.

We will not derive the form of the non-Abelian supersymmetric lagrangian here, but note that this can be done by looking at the chiral superfield transformations:

$$\Phi_i \to e^{-igT_i\Lambda}\Phi_i, \tag{1.3.35}$$

and the vector superfield transformations:

$$W_\alpha \to e^{ig\Lambda} W_\alpha e^{-ig\Lambda}. \tag{1.3.36}$$

The final, supersymmetric, gauge invariant, lagrangian has the form

$$\begin{aligned} \mathcal{L} \;=\; & \Phi_i^\dagger e^{-2gTV}\Phi_i \Big|_{\theta\theta\bar\theta\bar\theta} + \frac{1}{2}\left\{ W^\alpha W_\alpha\Big|_{\theta\theta} + \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\Big|_{\bar\theta\bar\theta}\right\} + \frac{1}{2}\sum_{i,k}\left\{ \frac{\partial^2\mathcal{P}}{\partial\Phi_i\partial\Phi_k}\psi_i\psi_k + \text{ h.c.}\right\} \\ & - \sum_i \left|\frac{\partial\mathcal{P}}{\partial\Phi_i}\right|^2 - \frac{1}{2}\sum_a \left| g_a \sum_i A_i^\dagger T^a A_i\right|^2. \end{aligned} \tag{1.3.37}$$

The last line is the so-called "scalar potential", consisting of the F terms and D terms, which we have left explicit:

$$\begin{aligned} \left|F^i\right|^2 &= \left|\frac{\partial\mathcal{P}}{\partial\Phi_i}\right|^2, \\ |D^a|^2 &= \frac{1}{2}\left| g_a\sum_i A_i^\dagger T^a A_i\right|^2. \end{aligned} \tag{1.3.38}$$

1.3.3 The Minimal Supersymmetric Standard Model

In formulating a realistic (i.e., phenomenologically viable) supersymmetric extension to the SM, we must limit ourselves to $\mathcal{N} = 1$ SUSY. There are tight experimental limits on the masses of the supersymmetric particles, and if we had $\mathcal{N} > 1$ SUSY, we would have observed it already. Further, we know the SM to be chiral.

Higher SUSYs lead to transformation laws between left-handed fermions and their right-handed partners, destroying chirality in the effective field theory—left-handed and right-handed Majorana-Weyl spinors combine to form Dirac spinors. In $\mathcal{N} = 1$ SUSY, there is one bosonic state for each SM fermion state, and one fermion for each SM boson. This means that we must double the spectrum of the SM to incorporate SUSY[12] —which is now called the Minimal Supersymmetric Standard Model, or MSSM [37].

Because we have not observed such a symmetry in our experiments yet [6], we must assume that the symmetry breaks at some scale above the electroweak scale. The main motivation for SUSY, as we will see in Section 1.3.4, is that it solves the hierarchy problem—if the SUSY breaking scale is very much above the electroweak scale, then it can no longer protect the hierarchy. This means adding more parameters to our theory, in the form of a SUSY breaking potential. In gauge mediated SUSY breaking, the potential is communicated from a hidden sector by a "messenger" gauge group, usually an additional (possibly anomalous) $U(1)$ [38]. Alternately, gravity could act as the messenger—this is known as gravity mediated SUSY breaking [39], and requires a local formulation of SUSY, known as supergravity. Whatever the final answer, the ultimate supersymmetry-breaking parameter cannot belong to any of the MSSM supermultiplets—a D-term vev for $U(1)_Y$ does not lead to an acceptable spectrum, and there is no candidate gauge singlet in the MSSM whose F-term could develop a vev [40]. So, if we accept that nature is supersymmetric (and if we believe string theory, this must surely be the case), then we must accept that the MSSM as *a priori* incomplete.

The fermions in the MSSM are assigned to chiral superfields, and the spin one bosons come from vector superfields. The superpartners (spartners) of the fermions

[12] One might wonder whether the bosons and fermions in the SM satisfy supersymmetric transformations. The answer is no—the particle spectrum of the SM does not have the correct quantum numbers.

Table 1.1. Particle Content of the MSSM. Reproduced from [1].

Superfield	Bosons		Fermions		$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Gauge							
$\mathbf{G}^a$	gluon	g^a	gluino	$\tilde{g}^a$	$\mathbf{8}$	$\mathbf{1}$	0
$\mathbf{V}^k$	weak	W^k $(W^\pm, W^0)$	wino, zino	$\tilde{w}^k$ $(\tilde{w}^\pm, \tilde{w}^0)$	$\mathbf{1}$	$\mathbf{3}$	0
$\mathbf{V}'$	hypercharge	B (γ)	bino	$\tilde{b}\,(\tilde{\gamma})$	$\mathbf{1}$	$\mathbf{1}$	0
Matter							
$\mathbf{L}_i$	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	$\mathbf{1}$	$\mathbf{2}$	-1
$\mathbf{E}_i$		$\tilde{E}_i = \tilde{e}^c_L$		$E_i = e^c_L$	$\mathbf{1}$	$\mathbf{1}$	2
$\mathbf{Q_i}$	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	$\mathbf{3}$	$\mathbf{2}$	$1/3$
$\mathbf{U}_i$		$\tilde{U}_i = \tilde{u}^c_L$		$U_i = u^c_L$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-4/3$
$\mathbf{D}_i$		$\tilde{D}_i = \tilde{d}^c_L$		$D_i = d^c_L$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$2/3$
Higgs							
$\mathbf{H_1}$	Higgses	$H_{\mathbf{1}}$	higgsinos	$\tilde{H}_{\mathbf{1}}$	$\mathbf{1}$	$\mathbf{2}$	-1
$\mathbf{H_2}$		$H_{\mathbf{2}}$		$\tilde{H}_{\mathbf{2}}$	$\mathbf{1}$	$\mathbf{2}$	1

are scalars (sfermions), and the spartners of the gauge bosons are fermions (gauginos). The higgs sector is now doubled—two higges live in chiral superfields and have fermionic partners called the higgsinos. The particle content, quantum numbers and naming conventions for the spartners are given in Table 1.1.

The superpotential for the MSSM is given by

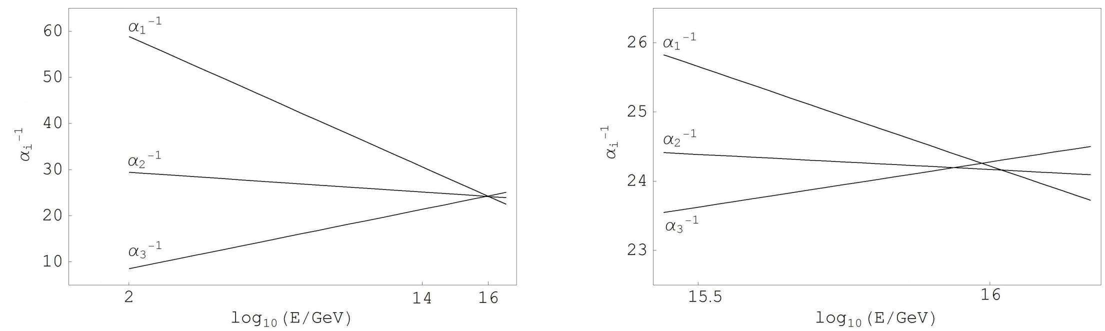
$$\mathcal{P}_{MSSM} = \mathbf{y}_u \mathbf{U}\mathbf{Q}\mathbf{H}_u - \mathbf{y}_d \mathbf{D}\mathbf{Q}\mathbf{H}_d - \mathbf{y}_e \mathbf{E}\mathbf{L}\mathbf{H}_e + \mu \mathbf{H}_u \mathbf{H}_d. \tag{1.3.39}$$

The fields that make up this superpotential are chiral superfields, and the dimensionless Yukawa couplings, $\mathbf{y}$, are matrices in family space.

Note that this is *not* the most general form of a superpotential that one can write. One could also add (gauge invariant, renormalizable) terms that violate conservation of lepton number or baryon number, and introduce flavor changing neutral currents at unacceptable rates:

$$\mathcal{P}_{\Delta L=1} = \frac{1}{2}\lambda^{ijk}\mathbf{L}_i\mathbf{L}_j\mathbf{E}_k + \lambda'^{ijk}\mathbf{L}_i\mathbf{Q}_j\mathbf{D}_k + \mu'^i\mathbf{L}_i\mathbf{H}_u, \tag{1.3.40a}$$

$$\mathcal{P}_{\Delta B=1} = \frac{1}{2}\lambda''^{ijk}\mathbf{U}_i\mathbf{D}_j\mathbf{D}_k, \tag{1.3.40b}$$



(a) (b)

Figure 1.3. The running of the couplings in the Minimal Supersymmetric Standard Model. The graph in Figure 1.3(a) has been magnified around the apparent unification scale, which is an artifact of the thickness of the lines. In order to have perfect unification, we must rely on GUT scale threshold effects, such as those calculated in Section 4.3.1.

where ijk are family indices. The couplings, λ, must either be exponentially suppressed, or set to zero by some symmetry. Generally, phenomenologists invoke R parity [40], a discrete symmetry of the MSSM, given by

$$P_R = (-1)^{3(B-L)+2s}, \tag{1.3.41}$$

where s is the spin of the field in the supermultiplet. As a symmetry of the MSSM, all of the interactions in the MSSM must respect R parity. This means that P_R projects the terms (1.3.40) out of the lagrangian. These terms are certainly an undesirable feature of the MSSM—baryon and lepton number in the SM are conserved because there exist no dimension 4 (or higher) operators that produce baryon or lepton decay. Here we have crudely imposed some unnatural symmetry on the model to make it match the low energy measurements.

The b_i values (see Equation (1.1.7)) for the MSSM beta functions are [41]

$$\begin{aligned} b_1 &= \tfrac{33}{5} \\ b_2 &= 1 \\ b_3 &= -3. \end{aligned} \tag{1.3.42}$$

The beta functions are plotted in Figure 1.3(a).

The apparent unification of the forces at the energy scale $\sim 2.5 \times 10^{16}$ GeV is actually a trick, as seen in Figure 1.3(b). One generally relies on threshold effects to save the day. The general form of a grand unified theory is

$$\alpha^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \log\left[\frac{E}{M_{GUT}}\right] + \Delta_i = \alpha_{GUT}^{-1} + \frac{b_i + \delta\, b_i}{2\pi} \log\left[\frac{E}{\Lambda}\right]. \quad (1.3.43)$$

The Δ_i's are found by requiring that the beta functions agree with experiment at M_Z.

1.3.4 Answers from the MSSM

We set out several issues that a successful extension to the SM should address in Section 1.2. What does the inclusion of SUSY do for us? First of all, between spartner masses, new mixings and new Yukawa couplings, the MSSM has 124 free parameters. These free parameters correspond to experimental observables, much like the aforementioned 18 free parameters of the SM. SUSY says nothing about the algebra question—we can make supersymmetric quantum field theories that have all kinds of gauge groups. Because there are fermionic superpartners of the gauge bosons, we now have fermions transforming in the adjoint representations of $SU(3)_C \times SU(2)_L \times U(1)_Y$. But now we also have a bunch of scalars transforming in fundamental representations—we have introduced new particles and have (perhaps) made our situation a bit more complicated. Further, we can change things in the SM (like the number of generations, or the representations in which the fermions transform) with no restrictions from SUSY. Finally, supersymmetry has no answers for the fermion question, the charge question, or the gravity question.

The main motivation for SUSY is that it contains a mechanism to keep the higgs mass at an acceptable value.[13] The suppression of the higgs mass works because the higgs couples to the new scalar sparticles that SUSY postulates, whose loop

[13] SUSY also contains a suitable dark matter candidate, but this is outside the scope of this thesis [42, 43].

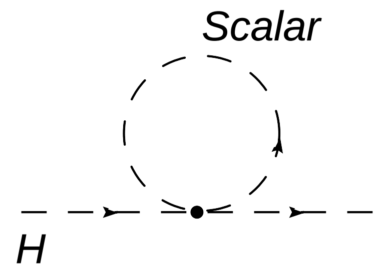


Figure 1.4. If our theory contains any extra scalars, the higgs receives radiative corrections from the four-point interaction vertex.

contributions tend to cancel those in Equation (1.2.8). For example, when complex scalars couple to the higgs (the sparticles of the MSSM) as in Figure 1.4, the higgs mass gets corrected to a reasonable value [40]:

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \log\left(\frac{\Lambda_{UV}}{m_S}\right) - ...\right]. \tag{1.3.44}$$

But the hierarchy problem has been hidden in a new problem—namely the scale of SUSY breaking. In order to protect the higgs mass, the SUSY breaking scale must be somewhere around the electroweak breaking scale. The expected SUSY breaking scale, by the same types of naive calculations that we used in estimating the higgs mass, is around the Planck scale. The breaking of SUSY could be relegated to some non-perturbative, Planckian regime, and thus listed under the gravity question, in which case we don't expect an answer from any perturbative formulation of quantum field theory, supersymmetric or not. In this vein, the best hope is a local formulation of SUSY, called supergravity, or SUGRA.

The near-unification of forces after the inclusion of SUSY, illustrated in Figure 1.3(a), is the strongest evidence that we should be studying some supersymmetric unified theory—two random lines generally always intersect, but three random lines almost never intersect at the same place. Threshold corrections $\sim 5\%$ are sufficient to achieve unification in SUSY, whereas one needs corrections $\sim 20\%$ to get unification in the SM. If one incorporates SUSY into some larger theoretical framework, such as a GUT or string theory, one finds solutions to most of the problems mentioned in

Section 1.2. It would be a grand coincidence if this near-unification were an accident, and nature really respected $SU(3)_C \times SU(2)_L \times U(1)_Y$ at the highest energies.

CHAPTER TWO

Grand Unified Theories

The idea of unification is not a new one—Howard Georgi wrote down the first version of an $SU(5)$ Grand Unified Theory (GUT) in 1974 [44], and he even co-discovered the embedding of the SM in $SO(10)$, along with Fritzsch and Minkowski in the same year [45, 46]. Let us first review the motivations for unification, the possible unification scenarios, and then some specific examples.

2.1 General Arguments for Unification

Matter in our universe interacts according to the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$. It is only logical to imagine how the three gauge groups of the SM may be embedded into some larger gauge group. We are also quite encouraged when we compute the coupling constant's dependence on energy, and find that they tend to a common value at an energy scale of around 10^{16} GeV, as in Figures 1.1 and 1.3(a). The question is, can we address any of the other issues of the Standard Model by postulating unification?

2.1.1 The Algebra Question

There is no answer to the algebra question in most GUTs. The breaking of the GUT is chosen so that the low energy effective field theory contains the SM. (The exception to this is the $\widetilde{SU}(5)$ partial GUTs—this situation is discussed in some detail in Section 2.5.) We really just restate the gauge problem—we *choose* a GUT symmetry so that we get desirable phenomenology. We will see this explicitly in the following sections. This is known as model building from a bottom-up approach—we look at (low energy) experimental data, and make an educated guess as to what nature is like at higher energy scales, working through the problem backwards. Some could argue that traditional 4-d GUTs make this problem worse, as they need to introduce *another* higgs (or a combination of new higgses) to break the grand unified gauge group. Higgs vevs always correspond to free parameters of the theory, so we run the risk of introducing too many degrees of freedom in our theory. The fermions still transform in the fundamental reps of the gauge group, but sometimes the bifundamental is needed. In this sense, the problem is not addressed at all.

2.1.2 The Fermion Question

GUTs typically do not address the generational structure of the SM. The violation of $B-L$, however, is a generic prediction of all GUTs, which tend to introduce higher dimensional operators which can lead to proton decay. At the GUT scale, there are tree level processes like Figure 2.1(a). The relevant term in the lagrangian (after gauge fixing) is

$$\mathcal{L} \sim g_5^2 \left\{ \bar{u}\gamma^\mu u \left(\frac{g_{\mu\nu}}{k^2 - M_X^2} \right) \bar{e}\gamma^\nu d \right\}. \tag{2.1.1}$$

For $M_X^2 >> k^2$, we can write

$$\mathcal{L} \sim \frac{g_5^2}{M_X^2} \left\{ \bar{u}\gamma^\mu u g_{\mu\nu} \bar{e}\gamma^\nu d \right\}. \tag{2.1.2}$$

The process is suppressed by two powers of the GUT scale, which is typically around 10^{16} GeV. Still, the experimental searches for proton decay [47] can provide strong constraints on new physics—for example, the simplest versions of non-supersymmetric and supersymmetric $SU(5)$ are already ruled out [48]. Baryon "conservation" is really baryon "longevity", which is understood as a consequence of Equation (2.1.2). Another generic prediction of GUTs is neutron oscillation, $N \leftrightarrow \bar{N}$—current bounds on this process are $> 1.3 \times 10^8$ s [6]. Indeed, if we observe such things as proton decay or neutron oscillation, we will have strong evidence of some grand unified theory.

The neutrino mass problem is elegantly addressed by many GUTs via the "see-saw" mechanism [20, 21], which is generally incorporated quite naturally. Questions

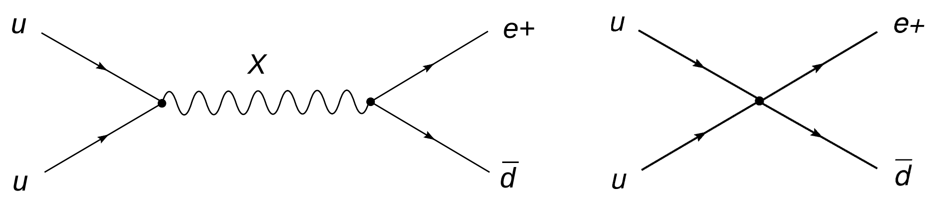


Figure 2.1. The operators which cause proton decay. In (a) we have the Feynman graph which comes from the lagrangian. In (b), we have the effective graph, after integrating over the X boson.

about number of generations and generational mass ratios are not generally addressed within the framework of GUTs.

2.1.3 The Charge Question

Grand Unified Theories naturally explain the quantization of charge via anomaly cancellation. The Pati-Salam GUTs even contain a natural definition of electric charge [49]. This, along with the near-unification of the coupling constants at some large energy scale M_{GUT} (as in Figure 1.1), are perhaps the main motivation for studying GUTs.

2.1.4 The Gravity Question

In general, the unification scale, $M_{GUT} \sim 10^{16}$ GeV is much less than the Planck scale, $M_{Pl} \sim 10^{18-19}$, where we believe gravity becomes important, so our understanding of GUTs gives us no intuition about how to treat gravity at the relevant energy scales. However, if we embed our GUTs in some larger framework, like string theory, then we can quite naturally understand all of the issues associated with the gravity question. On the other hand, in the heterotic string GUTs (as in Chapter Three), all of the breaking occurs at the string scale. In this case, the gravity question is answered by string theory.

2.1.5 The Hierarchy Problem

The hierarchy problem is generally not addressed by GUTs. The higgs vevs (both the electroweak higgs and the GUT higgs) are still free parameters in the theory, and there exist no convincing arguments from which either of the higgs potentials could be derived. On the contrary, it introduces *another*, smaller hierarchy between the GUT scale ($\sim 10^{16}$ GeV) and the Planck scale ($\sim 10^{18-19}$ GeV)—why is there a factor of > 100 difference between the Planck scale and the GUT scale? In our string constructions, all of the symmetry breaking occurs at $M_{str} \sim 5 \times 10^{17}$ GeV [50]. We have investigated [51] some string based solutions to this question. Specifically, Giedt's "Optical Unification" proposal [52] gives conditions whereby $M_{str} = M_{GUT}$ by putting constraints on intermediate scale matter. This intermediate scale matter changes the δb_is in Equation (1.1.7) in such a way that all three lines intersect at one point, at the Planck scale. This work is not included here.

Generally, model builders are content to invoke supersymmetry as a solution to the hierarchy problem, which consequently solves some of the non-supersymmetric GUT problems like proton decay. The idea is to embed the MSSM, as in Section 1.3.3 into a larger symmetry in the same manner as we embed the SM into a larger symmetry. We do not discuss SUSY GUTs in this thesis, however, some may find the review by Raby in [6] useful.

2.2 Candidates for Unification

What types of Lie Groups are suitable for embedding the SM? First, let's look at the four dimensional[1] simple and exceptional groups which contain $SU(3)$. This rules out groups like $SO(5)^2$ and $SU(2)^4$, both of which do not contain $SU(3)$ [5]. Further, we need complex representations for the fermions—this rules out $SO(4)$, $SO(9)$, $Sp(8)$, G_2^2, and F_4. We are left with $SU(5)$!

[1]Because the gauge group of the SM is four dimensional ($SU(3) \times SU(2) \times U(1)$), this is the smallest dimension for a unified gauge group.

But what other groups are suitable for unification? We will briefly discuss $SO(10)$ and E_6 GUTs, but there are many more possibilities [53]. The most studied GUTs are, by far, those based on the groups $SO(10)$ and $SU(5)$. The $SU(5)$ GUTs are discussed in some detail in the next section and the appendices—here we will briefly review the breaking patterns of $SO(10)$.

A generation of SM fermions fit into the **16** of $SO(10)$. Higgsing to the SM depends on the breaking pattern, but electroweak symmetry breaking is generally accomplished by a higgs in the fundamental representation of the gauge group, as is required by the standard Glashow-Weinberg-Salam theory. Generally, one breaks $SO(10)$ in one of four ways:

$$\begin{array}{rlcl} SO(10) & \rightarrow & SO(6) \times SO(4) & [49], \\ & \rightarrow & SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1) & [54], \\ & \rightarrow & SU(5) \times U(1) & [54], \\ & \rightarrow & \widetilde{SU}(5) \times U(1)_{\tilde{Y}} & [55]. \end{array} \tag{2.2.1}$$

Note that the last two breaking patterns are distinct, as will be demonstrated in Section 2.5. The first option, the so-called Pati-Salam models, have some interesting characteristics. For example, not only do they give a simple form of the electric charge, but they naturally accommodate the see-saw mechanism, and they contain a beautiful explanation of the chirality of the SM. We will discuss these types of models, which will come up in the context of our string constructions, in Section 2.3. The second breaking pattern, often referred to as the Left-Right Symmetric (or LRS) model, shares many of the same features as the first—the difference is how the symmetries form the SM. In the first case, one observes that $SO(6)$ and $SO(4)$ are isomorphic to $SU(4)$ and $SU(2)_L \times SU(2)_R$, respectively. The $SU(4)$ then breaks to $SU(3)_C \times U(1)$. The higgsing from $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ to the SM in both cases is then achieved in the same manner. The third breaking pattern will not be explored here, but, as already mentioned, the simplest versions of this model have been eliminated. We do note, however, that the breaking of $SU(5)$ is discussed in Section 2.4, and in more detail in Appendix C. The final breaking pattern of $SO(10)$ are the flipped $SU(5)$ models of Barr and others [55, 56]. These will be explained in Section 2.5, although their embeddings in $SO(10)$ will not be discussed.

There has also been some exploration of the E_6 models [57], which generally break in one of two ways:

$$\begin{aligned} E_6 &\rightarrow SO(10) \times U(1)_X \quad [58], \\ &\rightarrow SU(3)_c \times SU(3)_L \times SU(3)_R \quad [59]. \end{aligned} \tag{2.2.2}$$

The $U(1)_X$ has the interpretation of a family symmetry: family replication is achieved naturally, which is one of the main motivations for embedding $SO(10)$ in E_6. In the previous examples, one must add generational copies of the representations to the spectrum—so, for example, in typical $SO(10)$ constructions, one adds a copy of each **16** for each generation of fermions. The E_6 models contain a natural resolution to some of the issues associated with the fermion problem (see Section 1.2.2), but these models will not be discussed any further.

2.3 Pati-Salam and Partial Unification

In Section 1.1, it was noted that unification of all forces in the SM required threshold corrections on the order of about 20%. What if we only want to unify *two* of the forces? So, for example, $SU(2)$ and $U(1)$ unify at about 10^{14} GeV, and we could conceivably embed these forces into an extra $SU(3)$, making the gauge group $SU(3) \times SU(3)_C$ at $E \gtrsim 10^{14}$ GeV. This partial unification creates a small hierarchy between the masses of the partial GUT higgs and the GUT higgs, just as we saw a hierarchy between the GUT higgs and the electroweak symmetry breaking higgs. We might be willing to accept this new, smaller hierarchy if the partial unification scheme does other things for us.

The idea that only two of the forces unify at some intermediate scale is known as partial unification. If some smaller gauge group, G is embedded into a single larger gauge group, $\mathcal{G}$, at some mass scale M_I, then the coupling constants must obey

$$g_G\left(M_I\right) = g_{\mathcal{G}}\left(M_I\right). \tag{2.3.1}$$

A less trivial case is when the generators of the smaller group G are a linear combination of the generators of several larger groups $\prod_i \mathcal{G}_i$. For t_i^a a generator of $\mathcal{G}_i$, the generators of G are given by

$$\lambda_G^b = \sum_{i,a} c_{i,a}^b t_i^a, \tag{2.3.2}$$

where a, b run over the dimensions of the respective groups. The matching condition in this case is then

$$\frac{1}{g_G^2\left(M_I\right)} = \sum_{i,a,b} \frac{c_{i,a}^b c_{i,a}^b}{g_{\mathcal{G}_i}^2\left(M_I\right)} \tag{2.3.3}$$

Let us motivate the Pati-Salam constructions by noticing an asymmetry in the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$, and the SM spectrum, in Equation (1.1.1). Left-handed particles seem to be treated preferentially—that is, the SM is a chiral theory. Some would view this as a non-issue, just a statement about how nature works, but we will see how addressing this issue of chirality can solve some of the other issues that we had with the SM, as per Section 1.1.

Pati and Salam proposed, in 1974, that $SO(10)$ contains $SO(6) \times SO(4)$ [49, 60, 61], now called the Pati-Salam Group, or PS. We have seen before that $SO(4)$ is isomorphic to $SU(2)_L \times SU(2)_R$ in the derivation of the Supersymmetry Algebra, in Appendix B. Further, one can show that $SO(6) \cong SU(4)$. After higgsing from the $SO(10)$ GUT we have the intermediate gauge group

$$SU(4)_C \times SU(2)_L \times SU(2)_R. \tag{2.3.4}$$

The $SU(4)_C$ then breaks to QCD $\times\, U(1)_{B-L}$:

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \tag{2.3.5}$$

Besides the appealing feature of left-right symmetry, these models contain a "natural" definition of the hypercharge quantum number, $U(1)_Y$, of the SM, and a natural explanation for conservation of the $B-L$ quantum number (see Section 1.2.2). There is no derivation of the hypercharge quantum number in the SM, just an *ad hoc* assignment of values. In the Pati-Salam models, the hypercharge is defined in terms of the $SU(2)_L \times SU(2)_R$ isospins, and the $B-L$ quantum number—the electric charge is given by [62, 63]:

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}, \tag{2.3.6}$$

which can be shown from Equation (2.3.3). Finally, the PS models contain a right-handed neutrino, which gives small masses to the left-handed neutrinos via the seesaw mechanism [20, 21].

As in the SM, the quarks and leptons come in doublets. In writing out the quantum numbers under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the symmetry between left-handed and right -handed states is clear:

$$\begin{aligned} Q_L &= (\mathbf{2}, \mathbf{1}, 1/3) \quad & Q_R &= (\mathbf{1}, \mathbf{2}, 1/3) \\ L_L &= (\mathbf{2}, \mathbf{1}, -1) \quad & L_R &= (\mathbf{1}, \mathbf{2}, -1) \\ W_L &= (\mathbf{3}, \mathbf{1}, 0) \quad & W_R &= (\mathbf{1}, \mathbf{3}, 0)\,. \end{aligned} \tag{2.3.7}$$

The covariant derivative is just a generalization of that of the Glashow-Weinberg-Salam theory:

$$\begin{aligned} D_{L\mu} + D_{R\mu} \equiv\ & \frac{1}{2}\left\{\partial_\mu - ig_L \sigma \cdot \mathbf{W}_{L\mu} - ig_{B-L} Y_{B-L} B_\mu\right\}_L \\ & + \frac{1}{2}\left\{\partial_\mu - ig_R \sigma \cdot \mathbf{W}_{R\mu} - ig_{B-L} Y_{B-L} B_\mu\right\}_R . \end{aligned} \tag{2.3.8}$$

Notice that the covariant derivative is symmetric under $L \leftrightarrow R$ when $g_L = g_R$—this shows explicitly the equal treatment of left- and right-handed particles. The σ are the generators of $SU(2)$, and Y_{B-L} is the $B-L$ quantum number.

Higgsing to the SM is accomplished using a minimal higgs sector—two higgses in the adjoint of $SU(2)_L \times SU(2)_R$, Δ_L and Δ_{R}, and one scalar "bi-doublet", ϕ:

$$\begin{aligned} \Delta_L &= (\mathbf{3}, \mathbf{1}, 2)\,, && \text{(2.3.9a)} \\ \Delta_R &= (\mathbf{1}, \mathbf{3}, 2)\,, && \text{(2.3.9b)} \\ \psi &= (\mathbf{2}, \mathbf{2}, 0)\,. && \text{(2.3.9c)} \end{aligned}$$

The higgs gain vevs of the form:

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \quad , \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & v_R \\ 0 & 0 \end{pmatrix}, \tag{2.3.10a}$$

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} e^{i\alpha}. \tag{2.3.10b}$$

The scalars give the extra gauge bosons mass in the standard way: one can derive the gauge boson masses by inserting the form of Equations (2.3.10) into the definition of the covariant derivative, Equation (2.3.8). The gauge boson masses at the energy scale where (2.3.5) is a good symmetry can then be found using the matching condition that $g_{2L}\,(M_{LRS}) = g_{2R}\,(M_{LRS})$.

The upper limit on the mass of the Z' boson (from $SU(2)_R$) is $M_{Z'} > 860$ GeV at 95% confidence limit [6]. One can also put bounds on the mass of the extra W_R bosons from astrophysical observation. If the neutrinos have Majorana mass terms (of the form $m_\nu \nu_L \bar{\nu}_L$ + h.c.), we can use data from SN1987a [64, 65] to constrain $M_{W_R} \gtrsim 1000$ GeV. These bounds should be significantly improved if no evidence of an extra Z' boson is found when the LHC data is analyzed, by the end of this decade.

2.4 $SU(5)$ Unification

It is well known that the SM matter fits into the **1**, the $\overline{\mathbf{5}}$ ($\overline{\chi}$) and **10** (ψ) reps of SU(5) [44]:

$$\bar{\chi} = \begin{pmatrix} \bar{d}^1 \\ \bar{d}^2 \\ \bar{d}^3 \\ e \\ \nu_e \end{pmatrix}_L \qquad \psi = \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & -u_1 & -d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -\bar{e} \\ d_1 & d_2 & d_3 & \bar{e} & 0 \end{pmatrix}_L \qquad \bar{\nu}_e = \mathbf{1} \tag{2.4.1}$$

Traditionally, the breaking of $SU(5)$ is achieved when some scalar field takes on a vev. In the simplest example, a scalar Σ, transforming as the **24** (adjoint) of $SU(5)$, takes on a vev of the form

$$\langle\Sigma\rangle = v \begin{pmatrix} -\frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.4.2}$$

This form of the vev for the scalar Σ is justified *ex post facto*—it leaves the gluons, the W's and the B all massless and preserves the SM at energies much less than M_{GUT}, while giving the other 12 generators of $SU(5)$ (the X's and Y's) masses of $\frac{25}{9}v^2 \sim M_{GUT}$.[2] Finally, a higgs in the fundamental (**5**) representation achieves electroweak symmetry breaking, giving the $W^\pm$ and the Z^0 boson's mass $\sim$ 100 GeV, fourteen orders of magnitude smaller than the masses of the X and Y bosons.

2.5 $\widetilde{SU}(5)$ Unification

Flipped SU(5) [55, 56] (or $\widetilde{SU}(5)$) unification has been studied extensively in the context of string model building [66]. The first realistic examples of a string derived

[2] See Appendix C for details.

MSSM came from $SO(10)$ embeddings of $\widetilde{SU}(5)$, obtained from the free fermionic heterotic string [67].

The fermions still fit into the $\mathbf{1}$, $\overline{\mathbf{5}}$ and $\mathbf{10}$ reps of $SU(5)$. The only difference is that the neutrino and the electron, the up- and down-type quarks, and their respective anti-particles, exchange places in the corresponding reps. So, we have

$$\overline{\chi} = \begin{pmatrix} \bar{u}^1 \\ \bar{u}^2 \\ \bar{u}^3 \\ \nu_e \\ e \end{pmatrix}_L , \quad \psi = \begin{pmatrix} 0 & \overline{d}_3 & -\overline{d}_2 & -d_1 & -u_1 \\ -\overline{d}_3 & 0 & \overline{d}_1 & -d_2 & -u_2 \\ \overline{d}_2 & -\overline{d}_1 & 0 & -d_3 & -u_3 \\ d_1 & d_2 & d_3 & 0 & -\nu_e \\ u_1 & u_2 & u_3 & \nu_e & 0 \end{pmatrix}_L , \quad \bar{e} = \mathbf{1}. \tag{2.5.1}$$

Two things are important here: First, it is immediately obvious that the hypercharges of the fields in the reps (*cf* the $\overline{\mathbf{5}}$ and the $\mathbf{1}$) do not trace to zero. This means that we must add another generator (proportional to the identity) to ensure freedom from anomalies. The actual gauge group of this model is

$$SU(5) \times U(1)_{\tilde{Y}}. \tag{2.5.2}$$

Under the $\widetilde{SU}(5)$ gauge group, the fermion reps transform as follows:

$$\bar{\chi} \;=\; (\bar{\mathbf{5}}, -3/2)\,, \quad \psi \;=\; (\mathbf{10}, 1/2)\,, \quad \bar{e} \;=\; (\mathbf{1}, 5/2)\,. \tag{2.5.3}$$

Second, because there is now a color singlet in the $\mathbf{10}$, we may use this non-adjoint rep for higgsing the GUT. Historically, this represents the first time a representation with a dimension less than that of the adjoint was used for symmetry breaking in *any* GUT. We still need a higgs in the fundamental rep to accomplish the electroweak symmetry breaking. The higgs reps transform as

$$H \;=\; (\mathbf{10}, 1/2)\,, \tag{2.5.4a}$$

$$h \;=\; (\mathbf{5}, -1)\,. \tag{2.5.4b}$$

Much of the phenomenology, as well as many of the predictions of traditional $SU(5)$ GUTs, are altered in $\widetilde{SU}(5)$. It is well known that the standard prediction for down-type quarks and their associated leptons in the standard $SU(5)$ GUT is $\frac{m_d}{m_e} = \frac{m_s}{m_\mu}$. One way to address this (wrong) prediction is to add more GUT scale higgses to the model, generating new mass terms. For example in the case of $SU(5)$, we can add a higgs in the **45** [68, 69]. This representation of the gauge group has the property that

$$H^p_{sq} = -H^p_{qs}, \tag{2.5.5a}$$

$$\sum_{p=1}^{5} H^p_{pq} = 0. \tag{2.5.5b}$$

The $SU(5)$ lagrangian now contains new Yukawa couplings of the form

$$\mathcal{L}_Y = \lambda \bar{\psi}_p H^{p\,\dagger}_{sq} \chi_{sq} + \cdots \tag{2.5.6}$$

If we let this higgs take on a vev of the form

$$\left\langle H^i_{i5} \right\rangle = \frac{-1}{3} \left\langle H^4_{45} \right\rangle = \Lambda, \tag{2.5.7}$$

one can then show that this new theory, including the **45** higgs gives

$$\frac{m_e}{m_\mu} = \frac{1}{9} \frac{m_d}{m_s}, \tag{2.5.8}$$

which is quite well-satisfied by experiment.[3]

The other way that we might think to address this problem is to make the observation that the mass ratio predictions are GUT-scale predictions, and we can only observe the fermion masses around 1 GeV. The fermion mass at energy scale μ is related to the fermion mass at energy scale M by

$$\log \frac{m_{f_1}(\mu)}{m_{f_2}(\mu)} = \log \frac{m_{f_1}(M)}{m_{f_2}(M)} + \int \sum_i \left[Y^i_{f_1} - Y^i_{f_2} \right] \frac{d\mu'}{\mu'} \tag{2.5.9}$$

[3] We could also add higgses in the **10** or **50** to fix this [14], and we have no *a priori* reason to exclude these scalars from the theory, however, the attitude is often one of minimalism—we *could* add as many free parameters to our theory as we like, but with enough free parameters, we could model *anything*!

The Y^i's are quantum numbers under some group G_i. In the case of the down and strange quarks, and the electron and muon, the quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y$ will all be the same, so the ratio holds at all scales $M < M_{GUT}$.

In $\widetilde{SU}(5)$ the problem is worse—because the neutrino has exchanged places with the electron, the same product of representations ($\mathbf{10} \times \mathbf{5} \times \mathbf{5} \supset \mathbf{1}$) generate mass terms for the neutrino $\frac{m_u}{m_{\nu_e}} = \frac{m_s}{m_{\nu_\mu}}$. We will get realistic neutrino masses only when we have a supersymmetric formulation of the theory. The current limits on neutrino masses do not rule out this ratio, however, if one does insert Majorana mass terms for the neutrino by hand into the SM, one runs into fine tuning problems.

We know that $SU(5) \supset SU(3) \times SU(2) \times U(1)$. (We will denote this $U(1)$ as $U(1)_{Y'}$.) The form of the $\widetilde{SU}(5)$ partial GUT suggests that $U(1)_{Y'} \times U(1)_{\tilde{Y}}$ breaks to $U(1)_Y$. The matching condition for the coupling constants at mass scale M_* is

$$\frac{1}{g^2_{SU(3)}(M_*)} = \frac{1}{g^2_5(M_*)}, \tag{2.5.10a}$$

$$\frac{1}{g^2_{SU(2)}(M_*)} = \frac{1}{g^2_5(M_*)}, \tag{2.5.10b}$$

$$\frac{1}{g^2_Y(M_*)} = \frac{1/25}{g^2_5(M_*)} + \frac{24/25}{g^2_{\tilde{Y}}(M_*)}. \tag{2.5.10c}$$

The form of $\frac{1}{g^2_Y(M_*)}$ contains terms weighted by the generators of $SU(5)$ (24), and the generators of $U(1)$, as per Equation (2.3.3).

The beta functions for the standard $\widetilde{SU}(5)$ are given by [56]:

$$\alpha^{-1}_{SU(3)}(M_Z) = \alpha_5^{-1}(M_*) + \frac{1}{2\pi}\left[11 - \frac{4}{3}N_f\right] \log\frac{M_Z}{M_*}, \tag{2.5.11a}$$

$$\alpha^{-1}_{SU(2)}(M_Z) = \alpha_5^{-1}(M_*) + \frac{1}{2\pi}\left[\frac{22}{3} - \frac{4}{3}N_f - \frac{1}{6}N_H\right] \log\frac{M_Z}{M_*}, \tag{2.5.11b}$$

$$\alpha^{-1}_{U(1)}(M_Z) = \alpha_1^{-1}(M_*) - \frac{1}{2\pi}\left[\frac{4}{3}N_f + \frac{1}{10}N_H\right] \log\frac{M_Z}{M_*}. \tag{2.5.11c}$$

We can solve the Equations (2.5.10) and (2.5.11), using $M_* \sim 3\times 10^{15}$ GeV, $\alpha_5^{-1}(M_*) \cong \frac{1}{37}$, and $\alpha_1^{-1}(M_*) \cong \frac{1}{41}$. The beta functions are plotted in Figure 2.2, showing explicitly the partial unification at the intermediate scale, which we have chosen to be

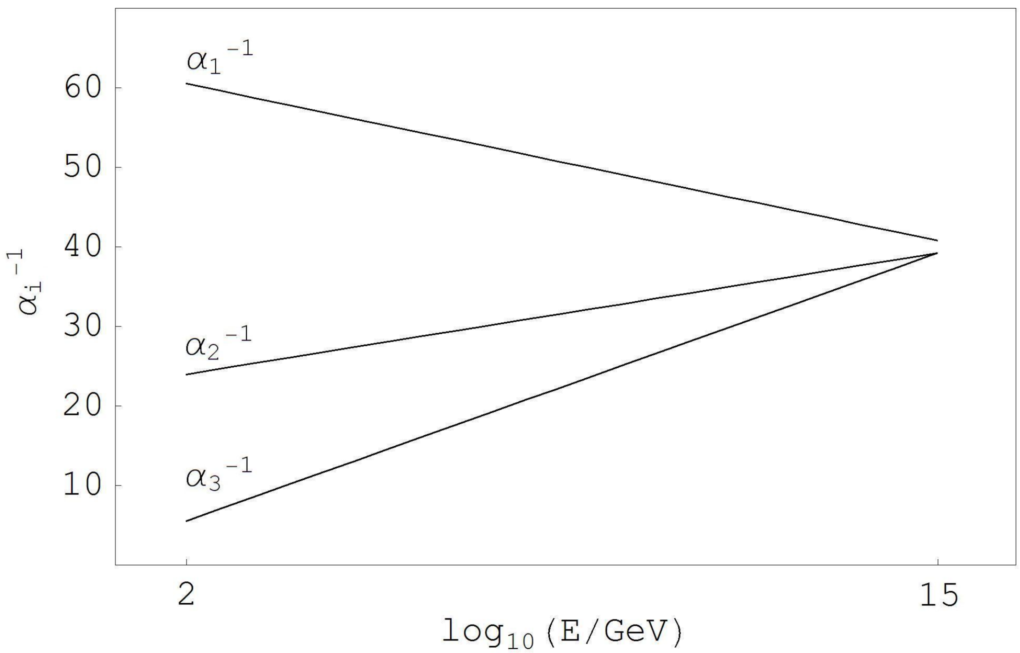


Figure 2.2. The running of the couplings in the minimal $\widetilde{SU}(5)$ model. We have chosen the partial unification of $SU(3) \times SU(2)$ to occur at $\sim 3 \times 10^{15}$ GeV.

3×10^{15} GeV. In general, this scale can be anywhere from 10^{14} GeV up to about 10^{17} GeV absent any Planck-scale threshold effects [70].

CHAPTER THREE

Heterotic Strings and Observable/Hidden Sector Symmetry

In this chapter, we present results from our investigation into the mirror universe proposal [71]. These models were constructed in the free-fermionic formulation of the heterotic string. In Section 3.1 we will first review the bosonic string, and then introduce the heterotic string and the GSO projections. The free-fermionic formulations and applications to model building will be discussed next. An introduction to the idea of mirror universes, along with an overview of our models is presented in Section 3.2. We look more closely at the string scale symmetry breaking in Section 3.3. Finally, we comment on acceptable hypercharge definitions in Section 3.4.

3.1 A Brief Overview of String Theory

General relativity is a classical theory, as mentioned before in Section 1.2.4. Attempts to quantize this theory have led to disastrous problems—for example, a naive counting of the degree of divergence of the graviton propagator gives mass dimension +4. This means that the one-loop corrections to the tree-level graviton-graviton scattering amplitudes give quadratic divergences.

While there have been some other attempts to quantize gravity [72], string theory [73, 74] is by far the most well-studied.[1] Early on it was realized that there were five consistent theories, with varying degrees of freedom—it was not until 1995 [75] that the dualities were recognized, and the long sought UV completion of string theory, M-theory, began to take shape. The five theories (plus 11-d supergravity) were realized as limits of this larger (11-dimensional) theory, whose details still are not well-known.

[1] Steven Weinberg has said “I see all of the things that are wrong with the present situation, but I still think string theory is the only game in town.”

Because a full review of string theory is beyond the scope of this thesis, we will concentrate our discussion on the two so-called “heterotic” string theories, whose (space-time) gauge groups are $SO(32)$ and $E_8 \times E_8$, and which are most relevant to our model building process. We will also attempt to make the distinction between world-sheet degrees of freedom and space-time degrees of freedom explicit.

3.1.1 The Bosonic String

Let us briefly review the bosonic string theory, in order that we might introduce some language that will be vital to further development. The bosonic string theory is a toy theory—it is known to be inconsistent because of the presence of a state of negative mass-squared, called a tachyon, and because it contains no fermions. It will give us hints, however, about how to proceed in more complex cases. We will start with a string (which can be either open or closed), propagating in flat space, parameterized by X^μ. The index μ runs over the dimensions of space-time, upon which we will put no constraints, for now. The string sweeps out a world-sheet, defined by σ, a “space” coordinate and τ, a “time” coordinate. [2]

If we were dealing with a particle, we could construct its action by looking at its world-line, and then use the Euler-Lagrange equations to find the extrema of the action, giving us the equations of motion. We will proceed in that manner here—the simplest action, the Nambu-Goto action, depends only on the area of the world-sheet. The induced metric of the world-sheet is given by

$$h_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = \partial_a X^\mu \partial_b X_\mu. \tag{3.1.1}$$

The indices a, b run over the world-sheet coordinates σ, τ. The Nambu-Goto action

[2] It is often convenient to map the world-sheet coordinates to the upper-half complex plane, also called the Teichmüller space: $z = \exp[\sigma + i\tau]$ and $\bar{z} = \exp[\sigma - i\tau]$. The standard convention for derivatives with respect to world-sheet coordinates is $\partial \equiv \frac{\partial}{\partial z}$ and $\bar{\partial} \equiv \frac{\partial}{\partial \bar{z}}$. When we deal with closed strings, we call z a right-moving coordinate and $\bar{z}$ a left-moving coordinate. In this chapter, the over-bar $(\bar{\ })$ will denote a left-moving degree of freedom.

is given by:

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int_W d\tau d\sigma \left(-\det h_{ab}\right)^{\frac{1}{2}}, \tag{3.1.2}$$

where W is the world-sheet. α' is the string coupling, related to the tension in the string by $T = \frac{1}{2\pi\alpha'}$.

One can now ask about the symmetries of this action—that is, in what ways can one change the lagrangian such that the action remains invariant. It turns out that there are two such transformations:

(1) the isometry group of flat space-time, the Poincaré group in D-dimensions, corresponding to translations and Lorentz transformations, and

(2) two-dimensional (world-sheet) diffeomorphism invariance, which tells us that the action does not depend on the manner in which we choose our coordinates.

Both (1) and (2) will ensure that we can always give our world-sheet a Lorentz metric, called γ:

$$\gamma_{ab} = \mathrm{diag}\left(-+\right). \tag{3.1.3}$$

One can rewrite Equation (3.1.2) in terms of this Lorentzian metric:

$$S_P = -\frac{1}{4\pi\alpha'} \int_W d\tau d\sigma \left(-\det \gamma_{ab}\right)^{\frac{1}{2}} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu. \tag{3.1.4}$$

This is the Polyakov action, in which a new symmetry has become manifest—Weyl invariance. This is essentially a scale-invariance of the world-sheet metric, which has no analog in the Nambu-Goto action. It is also interesting to note that Equation (3.1.4) is the action of D bosonic fields living in two dimensions—we will sometimes call these “world-sheet bosons”, and they will be of vital importance to us shortly. This fact, along with the appearance of scale-invariance, is closely tied to the fact that a two-dimensional conformal field theory lives on the world-sheet, as was eluded to in Section 1.1.

One can now find the equations of motion by varying the Polyakov action:

$$\partial_a \left\{ (-\det \gamma_{ab})^{\frac{1}{2}} \gamma^{ab} \partial_b X^\mu \right\} \equiv (-\det \gamma_{ab})^{\frac{1}{2}} \nabla^2 X^\mu = 0. \tag{3.1.5}$$

Here is where we will have to distinguish between open strings and closed strings. In order to solve this (second order) differential equation for $X^\mu(\tau, \sigma)$, we will need two boundary conditions. Open strings are taken (by convention) with $\sigma \in [0, \pi]$, and we will consider von Neumann boundary conditions:[3]

$$\partial_\sigma X^\mu(\tau, 0) = 0, \ \partial_\sigma X^\mu(\tau, \pi) = 0. \tag{3.1.6}$$

If we are studying closed strings, $\sigma \in (0, 2\pi]$, and we must have

$$\partial_\sigma X^\mu(\tau, 0) = \partial_\sigma X^\mu(\tau, \pi), \tag{3.1.7a}$$

$$X^\mu(\tau, 0) = X^\mu(\tau, \pi) \tag{3.1.7b}$$

Because we will not be dealing with any open string theories in what follows, we will concentrate our discussion from here on the closed string solutions.

The closed string's right- and left-moving modes decouple—$X^\mu(z, \bar{z}) = X_R^\mu(z) + X_L^\mu(\bar{z})$. If we express the equations of motion in z and $\bar{z}$, the right- and left-moving modes, we can solve the equations of motion using a Fourier transform.

$$X_R^\mu(z) = \frac{1}{2}x^\mu - i\left(\frac{\alpha'}{2}\right)^{1/2} \alpha_0^\mu \log z + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu z^{-n} \tag{3.1.8a}$$

$$X_L^\mu(\bar{z}) = \frac{1}{2}x^\mu - i\left(\frac{\alpha'}{2}\right)^{1/2} \bar{\alpha}_0^\mu \log \bar{z} + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \bar{\alpha}_n^\mu \bar{z}^{-n} \tag{3.1.8b}$$

The α_n's and $\bar{\alpha}_n$'s are the Fourier expansion coefficients, and will have the interpretation of operators upon quantization. For now, they represent right- and left-moving waves in the string. The zero-mode of the closed string is given by

$$\alpha_0^\mu = \bar{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu, \tag{3.1.9}$$

[3] The open strings can also have Dirichlet boundary conditions—this led to an important development in sting theory, namely the discovery of D(irichlet)-branes [76].

The space-time vectors x^μ and p^μ correspond to the position and momentum of the string—one can interpret these as center of mass quantities.

The classical Hamiltonian of the closed string can be found via the Legendre transformation of the lagrangian, and is given by

$$H = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left[\alpha^\mu_{-n}\cdot\alpha_{n\,\mu} + \bar{\alpha}^\mu_{-n}\cdot\bar{\alpha}_{n\,\mu}\right]. \tag{3.1.10}$$

From this point forward we will suppress the space-time index on the oscillator mode when it is summed over. We can define the Virasoro operators (which can be derived by imposing certain physical constraints on the energy-momentum tensors) in terms of the oscillator modes as well:

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty}\alpha_{m-n}\cdot\alpha_n, \tag{3.1.11a}$$

$$\bar{L}_m = \frac{1}{2}\sum_{n=-\infty}^{\infty}\bar{\alpha}_{m-n}\cdot\bar{\alpha}_n. \tag{3.1.11b}$$

Note that the Hamiltonian is then

$$H = L_0 + \bar{L}_0. \tag{3.1.12}$$

The final point we will make, before quantization, is that using X^μ and its conjugate momentum, Π^μ, we can derive Poisson brackets for the oscillator modes α. Quantizing the closed string is then reduced to promoting Poisson brackets to commutators and coordinates to operators, à la Dirac. A much more sophisticated process is to use the Feynman approach with path integrals.

Upon quantization of the string, we find

$$[\alpha^\mu_m, \alpha^\nu_n] = [\bar{\alpha}^\mu_m, \bar{\alpha}^\nu_n] = m\delta_{m+n,0}\eta^{\mu\nu} \tag{3.1.13a}$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu};\ [\alpha^\mu_m, \bar{\alpha}^\nu_n] = 0, \tag{3.1.13b}$$

where we have suppressed factors of c and $\hbar$. Because the α's do not commute, the zero-mode Virasoro operator now has an extra term from an ordering ambiguity that looks quite dangerous:

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \frac{1}{2}(D-2)\sum_{n=1}^{\infty} n. \tag{3.1.14}$$

The last term can be expressed as the Riemann zeta function $\zeta(-1)$, which is not analytic for negative integers. The problem is very similar as one that appears when calculating scattering at one-loop in quantum field theory, where one obtains expressions like $\Gamma(0)$.[4] By using similar regulation methods, one can sum the last term of Equation (3.1.14), and find $\sum_{n=1}^{\infty} n = -\frac{1}{12}$. We have several alternate derivations of this result, so we should not dwell on this point too long. We will re-emphasize, however, that we have done nothing different mathematically than to analytically continue a function outside of its radius of convergence, a common procedure in the process of dimensional regulation.

After quantization, the Virasoro algebra is given by:

$$\begin{aligned} [L_m, L_n] &= (m-n) L_{m+n} + \frac{D}{12}\left(m^3 - m\right)\delta_{0,m-n}, && (3.1.15a)\\ \left[\bar{L}_m, \bar{L}_n\right] &= (m-n) \bar{L}_{m+n} + \frac{D}{12}\left(m^3 - m\right)\delta_{0,m-n}, && (3.1.15b)\\ \left[L_m, \bar{L}_n\right] &= 0. && (3.1.15c) \end{aligned}$$

where D is the dimension of space-time. Physical states are required to obey

$$\begin{aligned} (L_0 - a)\,|\phi\rangle &= 0, && (3.1.16a)\\ \left(\bar{L}_0 - \bar{a}\right)|\phi\rangle &= 0. && (3.1.16b) \end{aligned}$$

The value of the "normal ordering" constant a depends on the content of our theory. For the bosonic string, the form of a is given by:

$$-a = \frac{1}{2}(D-2)\sum_{n=1}^{\infty} n = \frac{D-2}{24}. \tag{3.1.17}$$

By applying the requirement that the string action be Lorentz invariant, and then looking at the commutation relations obeyed by the Lorentz generators, one can prove that $D = 26$, and then that $a = 1$.

---

[4]See Appendix A for an example of such a calculation.

The appearance of the tachyon is easy to see from here. The momentum was defined in terms of the zero oscillator mode, in Equation (3.1.9). Using Equation (3.1.14), we find, for the ground state $|\psi\rangle$:

$$(L_0 - a)\,|\psi\rangle = \left(\frac{\alpha'}{2}p^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n\right)|\psi\rangle = 0. \tag{3.1.18}$$

The ground state contains no oscillator modes, and using the mass-shell condition $M^2 = -p^2$, we find

$$M^2 = -\frac{2a}{\alpha'} = -\frac{2}{\alpha'}. \tag{3.1.19}$$

This was one of the original motivations for the abandonment of string theory as a theory for strong gauge dynamics, in the early 1970's. Along with a tachyon, the theory contained an unexplained spin two boson—it took Green and Schwartz, and Susskind, to reinterpret the failed competitor of QCD as a quantum theory of gravity.

3.1.2 The Heterotic String

In order to get fermions into the spectrum of bosonic string theory, we supersymmetrize the (world-sheet) action, adding fermionic partners for each of the bosons. When we do this, we find that the critical dimension is no longer 26—anomaly cancellation and unitarity require that the superstring live in $D = 10$. The transformation between the fermions and their bosonic superpartners is given by:

$$\delta X^\mu = i\epsilon\psi^\mu, \tag{3.1.20a}$$

$$\psi^\mu = \epsilon\bar{\delta}X^\mu. \tag{3.1.20b}$$

This is a *world-sheet* supersymmetry, and should not be taken to correspond to a *space-time* supersymmetry. If, however, we make a GSO projection on the left- and right-moving modes, we end up with a space-time supersymmetry.

The heterotic string theory [77] is a theory of a closed string. We will take the left moving modes as the bosonic string, which live in 26 dimensions, and the right moving modes as the superstring, which live in 10 dimensions.[5] We will take the left moving sector of the string to be purely bosonic, and the right moving sector to be a superstring. We want our string world-sheet to propagate in ten large space-time dimensions (for now), so we will compactify 16 of the bosonic directions on a torus, T^{16}.[6] When we want to see what kind of low energy phenomena that we can get out of string theory, we will compactify another six dimensions (or seven, if we're dealing with M-theory), leaving us with four large space-time directions.

Let us count the degrees of freedom—equivalently we could phrase this "let's count the bosons and complex fermions". In the left-moving (bosonic) sector (denoted by $\bar{\partial}$), we have ten bosons ($\mu = 0, ..., 9$) plus 32 real fermions ($A = 1, ..., 32$) = 16 bosons, and in the right-moving (supersymmetric) sector we have ten bosons and their ten (real fermionic) superpartners. Note that the bosonic theory would have had 26 bosons right out—we have "fermionized" some of the bosons! This does *not* mean that we have added degrees of freedom to the theory—just that we have reparameterized some of the bosons as fermions.[7]

The action of the heterotic string, as we expect from our counting exercise, is given by

$$\mathcal{S} = \int d^2 z \left[\frac{2}{\alpha'} \partial X^\mu \bar{\partial} X_\mu + \tilde{\psi}^\mu \bar{\partial} \tilde{\psi}_\mu + \sum_{A=1}^{32} \lambda^A \bar{\partial} \lambda_A \right], \tag{3.1.21}$$

where the overbar ($^-$) denotes a left-moving coordinate, and the space-time coordinate $\mu \in \{0, ..., 9\}$. The λ_A's are 32 real = 16 complex fermions from the left-moving sector, and the ψ's are the (10 real fermionic) superpartners of the right-moving bosons. As per our convention, we see that X^μ is neither left-moving nor righ- moving, but left-

[5] If you're keeping track, this means that the string sees space-times of two different dimensionalities, either 10 or 26, depending on which way the mode is propagating.

[6] In general, the term "compactify" (and its variants) will mean that we choose the topology of a space dimension to be finite and periodic—in the present case, we have made 16 large space dimensions finite with the ansatz that those directions are circular.

[7] See Section 3.1.4 for the continuation of this idea.

moving *and* right-moving. The objects $\bar{\partial} X^\mu$ and ∂X^μ are left- and right-moving degrees of freedom, respectively.

The world sheet of the heterotic string has an $SO(9,1) \times SO(32)$ symmetry. The $SO(9,1)$ symmetry is just the Lorentz symmetry in 9+1 dimensions, and the $SO(32)$ is an internal (or, "worldsheet") symmetry acting on the real left-moving (internal) fermions, the λ's. The $SO(32)$ cannot be a space-time symmetry because there are still unphysical tachyonic states in the bosonic sector—the tachyon has not been destroyed in the process of fermionization, and still remains in the theory.

The $SO(32)$ is an *internal gauge symmetry*, which acts on the 32 left-moving λ's when they all have the same boundary conditions. (A more subtle possibility is that the λ's do not obey the same boundary conditions: this is the case in the $E_8 \times E_8$ heterotic string theory.) We can break the $SO(32)$ symmetry by assigning different boundary conditions to some of these fermions—for example, Lorentz invariance tells us that fermions can either be periodic or anti-periodic, so we could assign some λ's even boundary conditions and some of them odd boundary conditions. Finally, it is notable that the appearance of a non-Abelian internal symmetry is a purely stringy effect. If we had been dealing with point particles, the internal symmetry would have been Abelian—$U(1)^{16}$. The appearance of non-Abelian space-time symmetries is a post-diction of string theory—while we do not understand why we have a low energy effective field theory which obeys $SU(3)_C \times SU(2)_L \times U(1)_Y$, we are quite confident that there exists a stringy vacuum with that symmetry.

3.1.3 The GSO Projection Operator

It is well-known that string theory is only consistent when we add world-sheet fermions to the spectrum—bosonic string theory suffers from a state of negative mass squared, called the tachyon (*cf* Equation (3.1.19)), which is most certainly a bad thing for particle theory. It is only when we add fermions and a GSO projection

[78] do we recover a tachyon-free theory, with space-time supersymmetry and gauge symmetries. The GSO projection can also be motivated as a statement of modular invariance, or as a statement of unitarity.

Complex world-sheet degrees of freedom can, in principle, have any boundary conditions, whereas real world-sheet states are either periodic or anti-periodic. In the Ramond (R) sector, the fermions have periodic boundary conditions and are called Ramond fermions. The anti-periodic sector is called the Neveu-Schwartz (NS) sector, and world-sheet fermions with anti-periodic boundary conditions are called Neveu-Schwartz fermions. We will look at the cases of both Ramond and Neveu-Schwartz boundary conditions for the left-moving states. In the supersymmetric (right-moving) sector, we will first treat the Ramond case, and then the Neveu-Schwartz case.

For simplicity, let us assume that all of the internal (left-moving) $\bar{\lambda}^A$'s have the same boundary conditions. The fermions $\bar{\lambda}$ (with $\tau =$ constant) from each sector can be written as follows:

$$\bar{\lambda}^A_R(\sigma) = \sum_{n\in\mathbb{Z}} \bar{\lambda}^A_n e^{-2in\sigma}, \tag{3.1.22a}$$

$$\bar{\lambda}^A_{NS}(\sigma) = \sum_{r\in\mathbb{Z}+1/2} \bar{\lambda}^A_r e^{-2ir\sigma}. \tag{3.1.22b}$$

The $\bar{\lambda}^A_n$'s are the fermionic equivalent of the bosonic ladder operators, which appeared in the context of the bosonic string, Equation (3.1.8).[8] Much like the Grassman variables that were invoked when we introduced supersymmetry in Section 1.3, these ladder operators obey anti-commutation relations:

$$\left\{\bar{\lambda}^A_m, \bar{\lambda}^B_n\right\} = \delta^{AB}\delta_{m+n,0}\,, \quad \left\{\bar{\lambda}^A_r, \bar{\lambda}^B_s\right\} = \delta^{AB}\delta_{r+s,0}. \tag{3.1.23}$$

We can separate L_0 for the heterotic string into two parts—the zero-modes and the non-zero modes. We absorb the higher oscillator modes into a number operator, N. For the supersymmetric (right-moving) Ramond sector of the heterotic string, the

[8] We have used the over-bar ($\bar{\ }$) to remind us that these are the the left-moving states.

number operator N_R, in the light-cone gauge ($\mu = 1, ..., 8$) is defined as:

$$N_R = \sum_{n=1}^{\infty} \left(\alpha^{\mu}_{-n} \cdot \alpha_{n\,\mu} + n\psi^{\mu}_{-n} \cdot \psi_{n\,\mu}\right). \tag{3.1.24}$$

$\bar{N}$ is defined differently for the (fermionic) R and NS sectors:

$$\bar{N}_R = \sum_{n=1}^{\infty} \left(\bar{\alpha}^{\mu}_{-n} \cdot \bar{\alpha}_{n\,\mu} + n\bar{\lambda}^A_{-n}\bar{\lambda}^A_n\right), \tag{3.1.25a}$$

$$\bar{N}_{NS} = \sum_{n=1}^{\infty} \bar{\alpha}^{\mu}_{-n} \cdot \bar{\alpha}_{n\,\mu} + \sum_{r\in\mathbb{Z}+1/2}^{\infty} r\bar{\lambda}^A_{-r}\bar{\lambda}^A_r. \tag{3.1.25b}$$

This number operator serves the same purpose as the familiar example from quantum mechanics—it counts the level of the oscillator of the closed string. Now, L_0 can be put in the following form:

$$L_0 = \frac{p^2\alpha'}{4} + N_R, \tag{3.1.26a}$$

$$\bar{L}_0 = \frac{p^2\alpha'}{4} + \bar{N}_{R,NS}. \tag{3.1.26b}$$

In the heterotic string, we have a similar statement about physical states as Equations (3.1.16). The normal ordering constants depend on the content of our theory: this was mentioned in Section 3.1.1. In general, we are dealing with bosonic coordinates (X) and fermionic coordinates ($\bar{\lambda}, \psi$). A bosonic coordinate contributes a factor of $1/24$ to a, an integrally moded fermionic coordinate (or Ramond fermion) contributes a factor of -1/24, and a half-integrally moded fermionic coordinate (or Neveu-Schwartz fermion) contributes a factor of 1/48. These contributions can be derived from conformal field theory considerations—we are basically counting contributions to the central charges of the conformal field theory which lives on the world-sheet.[9] In the supersymmetric (right-moving) sector, the contributions to a from the bosonic coordinates and fermionic coordinates cancel—after choosing the

[9] The general contribution, for any boundary condition is $a = \frac{1-12Q^2}{24}$ for complex world-sheet fermions, and $a = \frac{1-12Q^2}{48}$ for real world-sheet fermions, where Q is a $U(1)$ charge, defined in Equation (3.1.45).

light-cone gauge there are eight complex fermions and eight bosonic degrees of freedom (see Equation (3.1.21)), giving $a = 0$. In R and NS sectors, we have:

$$\bar{a}_R = \frac{8}{24} + \frac{32}{48} = -1, \tag{3.1.27a}$$
$$\bar{a}_{NS} = \frac{8}{24} - \frac{32}{48} = 1. \tag{3.1.27b}$$

In the supersymmetric (right-moving) sector, the physical state requirement, Equation (3.1.16), gives

$$p^2 = \frac{-4N}{\alpha'} = -M^2. \tag{3.1.28}$$

$p^2 = p^\mu p_\mu$ is the (center of mass) space-time momentum of the string. Because N is positive semi-definite, this tells us that p is a space-like Lorentz vector, and that no tachyons are present. The lowest state is the massless one.

Next, combining the two Equations (3.1.16) gives the following relationship for the massless states:

$$0 = N_R + \bar{N}_R + 1, \tag{3.1.29a}$$
$$0 = N_R + \bar{N}_{NS} - 1. \tag{3.1.29b}$$

Massless states must have $N_R = 0$—that is, we expect them to be the lowest lying states in the (right-moving) spectrum, and contain no oscillator modes. This implies that $\bar{N}_R = -1$. Equations (3.1.25) tell us that $\bar{N}$ is positive definite, thus there are no massless states in the R sector.[10]

The condition that $-N = \bar{N}_{NS} - 1$ has some interesting consequences. N_R defined in Equation (3.1.24) has integer eigenvalues, whereas $\bar{N}_{NS}$ has integer *or* half-integer values (see, for example, Equation (3.1.25b)). The half-integer values cannot, therefore, contribute to the physical spectrum, because there is no suitable choice of right-moving state that would enable one to obtain $N_R = \bar{N}_{NS} - 1$. This

[10] While the R sector contains no massless modes, we cannot just discard it outright. Unitarity in one-loop scattering processes requires that we keep both sectors.

condition forces us to discard states with odd numbers of λ_{NS}^{A} oscillators. This is the $R-NS$ sector of the heterotic string, and we will argue below that the physical states we keep correspond to space-time fermions.

Next, let us treat the case where we assign the right-moving fermions NS boundary conditions. In this case, we have 8 bosonic degrees of freedom and eight NS fermions, giving us $a = \frac{1}{2}$. The relationships between the left-moving and right-moving number operators are given by

$$0 = N_{NS} + \bar{N}_R + \frac{1}{2}, \quad (3.1.30a)$$

$$0 = N_{NS} + \bar{N}_{NS} - \frac{3}{2}. \quad (3.1.30b)$$

Again, we find that there is no massless state in the R sector. In the NS sector, we have a similar requirement as before: the values of $\bar{N}_{NS}$ are limited to integer values by the requirement that N_{NS} is integrally valued. We find ourselves discarding half of the candidate states because they are unphysical. This is the $NS-NS$ sector of the heterotic string, and we will argue below that the physical states we keep correspond to space-time bosons.

Now we will make a connection between physical (space-time) states and all of this oscillator counting. We will assign all states a quantum number F, called "fermion number", where

$$F = 2N \bmod 2 = 0, 1. \quad (3.1.31)$$

There is one final observation to make: in the supersymmetric (right-moving) R sector, there are consistency requirements that the vacuum have a spinor structure, and thus has $F_V = 1$. The fermion number of a particular state is then $F_R + F_L + F_V = F$.

In the $R-NS$ sector, which gives space-time fermions, we have the condition that

$$N_R = 0 = \bar{N}_{NS} - 1. \quad (3.1.32)$$

Table 3.1. Summary of physical states in the heterotic string.

State	Sector	F_R mod 2	F_L mod 2	F_V mod 2	F mod 2
Space-time bosons	$NS-NS$	1	0	0	1
Space-time fermions	$R-NS$	0	0	1	1

F for the right-moving (Ramond) states is $F_R = 0$, while for the left-moving (Neveu-Schwartz) states, $F_L = 2 = 0$ mod 2, because we are limited to even numbers of oscillator modes. In the $NS-NS$ sector, which gives space-time bosons, we have

$$N_{NS} - \frac{1}{2} = \bar{N}_{NS} - 1 = 0. \tag{3.1.33}$$

The lowest level has $F_R = 1$ and $F_L = 2 = 0$ mod 2. These results are sumarized in Table 3.1.3.

This is the GSO projection. We assign states a quantum number F which essentially tells us how many λ^A's and α's make up each state, as well as the index structure of the vacuum. We know that the physical states have $F = 1$ under this symmetry, while unphysical states have $F = 0$. Because they are unphysical, they cannot contribute to the low energy phenomenology, and are of no interest to us.

3.1.4 The Free-Fermionic Formulation

In Equation (3.1.21), we "fermionized" some coordinates in the left-moving (bosonic) sector of the theory. In the free-fermionic formulation of the heterotic string, we fermionize the rest of the internal bosons in the same vein—that is we turn each boson into two real (or equivalently one complex) fermions. We can rewrite the "free-fermionic" version of the heterotic string action, Equation (3.1.21). First, we will choose the light-cone gauge. This eliminates two non-physical degrees of freedom from the action and simplifies our calculations by eliminating the related ghosts, but destroys manifest Lorentz invariance. We write:

$$\begin{aligned}\mathcal{S} &= \frac{2}{\alpha'}\int d^2z\Bigg\{\left[\sum_{\mu=+,-}\partial X^\mu\bar{\partial}X_\mu+\psi^\mu\partial\psi_\mu\right]\\ &\quad+\left[\sum_{j=4}^{9}\partial X^j\bar{\partial}X_j+\psi^j\partial\psi_j\right]\\ &\quad+\sum_{A=1}^{32}\lambda^A\bar{\partial}\lambda_A\Bigg\}.\end{aligned}\tag{3.1.34}$$

Now we will count left-moving (LM) and right-moving (RM) degrees of freedom, in terms of real (Majorana-Weyl) fermions. We have:

$$\begin{array}{rll}\text{Large Space Time Dim}\Big\{ & \sum_{\mu=+,-}\partial X^\mu\bar{\partial}X_\mu & 4\text{ LM} + 4\text{ RM}\\ & \sum_{\mu=+,-}\psi^\mu\partial\psi_\mu & 2\text{ RM}\\ \text{Compactified Dim}\Big\{ & \sum_{j=4}^{9}\partial X^j\bar{\partial}X_j & 12\text{ LM} + 12\text{ RM}\\ & \sum_{j=4}^{9}\psi^j\partial\psi_j & 6\text{ RM}\\ \text{Bosonic Sector}\Big\{ & \sum_{A=1}^{32}\lambda^A\bar{\partial}\lambda_A & 32\text{ LM.}\end{array}\tag{3.1.35}$$

We have left out the contributions of the right-moving (world-sheet) bosons X^μ, in the first line above, as they do not contribute to the massless states. They do not contribute to the massless spectrum, and thus are of no interest in the effective field theory. Our ultimate goal is to rewrite all of the internal complex bosons in terms of complex fermions—these are the states which contribute to the massless sector, and thus the low energy phenomenology. The left-moving (world-sheet) bosons in the first line of Equation (3.1.35) only contribute to the graviton (and gravitino) modes. They do not effect the low energy phenomenology, and will be ignored in what follows. This gives us a total of 20 LM + 44 RM = 64 states to reparameterize as fermions.

The fermions are split into two parts: those that parameterize the specific compactification that we are working with, and those that give us space-time gauge symmetries. First, the (6+6) compactified bosons get split into (12+12) real fermions:

$$\begin{aligned} \sum_{j=4}^{9} \partial X^j \bar{\partial} X_j \quad &\equiv \quad \sum_{B,C=1}^{6} \left\{ \bar{y}^B \partial \bar{y}_B + \bar{\omega}^C \partial \bar{\omega}_C \right\}_{LM} \\ &\quad + \sum_{B,C,D=1}^{6} \left\{ y^B \bar{\partial} y_B + \omega^C \bar{\partial} \omega_C \right\}_{RM} . \end{aligned} \tag{3.1.36}$$

These fermions parameterize the compactified space-time dimensions. The topological properties of the internal (six-dimensional) manifold govern much of the low energy phenomenology, in terms of the number of supersymmetries that we expect. In the specific class of compactifications that we will be interested in, our internal space is an orbifolded torus, $T^6/\mathcal{Z}_2 \times \mathcal{Z}_2$, which ensure that we have $\mathcal{N} = 1$ SUSY. Changing the boundary conditions on these fermions will allow us considerable range in adjusting the properties of the low energy effective field theories that we are working with.

The compactified fermions (spartners of the compactified bosons) from the right-moving sector are renamed:

$$\sum_{j=4}^{9} \psi^j \partial \psi_j \equiv \sum_{D=1}^{6} \chi^D \bar{\partial} \chi_D . \tag{3.1.37}$$

The 32 left-moving λ's are split into

$$\sum_{A=1}^{32} \lambda^A \bar{\partial} \lambda_A \equiv \sum_{D=1}^{6} \eta^D \bar{\partial} \eta_D + \sum_{I=1}^{5} \psi^I \bar{\partial} \psi_I + \sum_{M=1}^{8} \phi^M \bar{\partial} \phi_M . \tag{3.1.38}$$

The χ's and the η's will govern the generational structure of our effective field theory—they will become the generators of an Abelian ($U(1)^3$) when we talk about our model building processes. The five ψ's will form the generators of the observable sector $SO(10)$. Finally, the eight ϕ's will form the root lattice of the hidden sector E_8, which can then be broken down to some smaller symmetries.[11]

Now we can write down the full heterotic string action, in terms of free fermions, where the y's, ω's, η's and χ's are (world-sheet) Majorana-Weyl spinors, while the

[11] Note that we have not assigned boundary conditions just yet, so the space-time gauge symmetry (after the GSO projection) is still technically $SO(32)$. We will assign different boundary conditions to the fermions in Section 3.1.5.

ϕ's and ψ's are complex.

$$\begin{aligned}\mathcal{S}_{ff} &= \int d^2z \left\{ \sum_{B,C,D=1}^{6} \left[\bar{y}^B \partial \bar{y}_B + \bar{\omega}^C \partial \bar{\omega}_C + \bar{\eta}^D \partial \bar{\eta}_D \right] + \sum_{M=1}^{8} \bar{\phi}^M \partial \bar{\phi}_M + \sum_{I=1}^{5} \bar{\psi}^I \partial \bar{\psi}_I \right\}_{LM} \\ &\quad + \int d^2z \left\{ \sum_{B,C,D=1}^{6} \left[y^B \bar{\partial} y_B + \omega^C \bar{\partial} \omega_C + \chi^D \bar{\partial} \chi_D \right] + \sum_{\mu=+,-} \psi^\mu \bar{\partial} \psi_\mu \right\}_{RM} . \end{aligned} \tag{3.1.39}$$

Remember we have done nothing more than reparameterize our original action, Equation (3.1.21), in terms of a single specie, namely fermions. Because we have neither gained nor lost any degrees of freedom (except in the choice of gauge), we are allowed to do this—a check of the number of degrees of freedom in the action (3.1.39) and in Equation (3.1.35) will prove this. Recall, also, that we are dealing with *world-sheet* fermions, and not *space-time* fermions. In obtaining the latter, we must ensure that no unphysical states survive in the IR limit, and this means choosing the correct GSO projection.

3.1.5 The NAHE Set and Model Building

In order to construct a realistic, four-dimensional model from the free-fermionic string, one must specify two things: first, a set of basis vectors $\{\mathbf{V}_i\}$, which generate a finite, additive Abelian group Ξ, and a GSO projection matrix, $\mathbf{k}$ [79–82].

Modular invariance will be the principle which we use to guide our model building. In order to understand what modular invariance is, we will consider the simplest example—the torus. The torus is completely specified with a flat metric and a complex structure, τ. A torus in the complex z plane is pictured in Figure 3.1. Each point outside of the region $\mathcal{T}$ is equivalent to a point inside $\mathcal{T}$ by the equivalence relations

$$\begin{aligned} z &\sim z + 2\pi n, \\ z &\sim z + 2\pi m\tau, \end{aligned} \tag{3.1.40}$$

where $n, m \in \mathbb{Z}$.

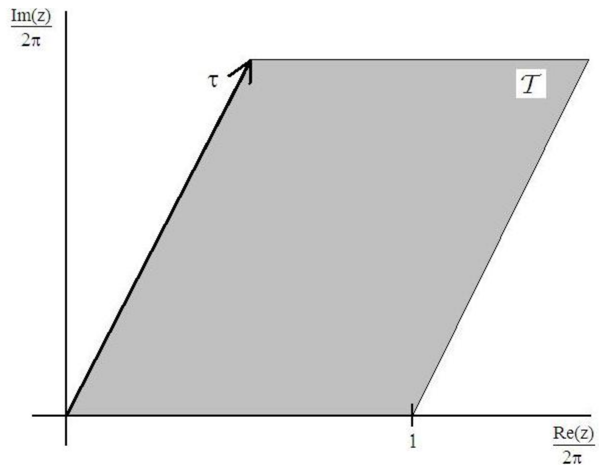


Figure 3.1. A torus in the complex plane. Each point outside of $\mathcal{T}$ is equivalent to a point inside $\mathcal{T}$ by the equivalence relations in Equation (3.1.40).

The full family of equivalent tori, parameterized by their complex structure τ, are described by the modular transformations:

$$\begin{aligned} T &: \tau \to \tau + 1, \\ S &: \tau \to -\frac{1}{\tau}. \end{aligned} \tag{3.1.41}$$

These transformations generate the group $SL(2,\mathbb{Z})$. This group is defined as the group of 2×2 matrices with integer elements with determinant of unity. The complex structure modulus, τ, is invariant under

$$SL(2,\mathbb{Z}) : \tau \to \frac{a\tau + b}{c\tau + d} \tag{3.1.42}$$

The moduli space of inequivalent tori $\mathcal{M}$ is the upper-half complex plane M_g (also called the Teichüller space) modded out by the group $SL(2,\mathbb{Z})$—$\mathcal{M} \cong M_g/SL\,(2,\mathbb{Z})$, which is shown in Figure 3.2. Modular invariance is the requirement that all tori with the same complex structure modulus are equivalent, and therefore that the physical properties of any model should be invariant under the $SL(2,\mathbb{Z})$ equivalences.

It can be proved [79, 81] that modular invariance in the one loop vacuum to vacuum polarization graph ($\mathcal{O}\left(g_s^2\right)$), along with an additional constraint from the two loop graph, implies modular invariance to all orders in the perturbation expansion.

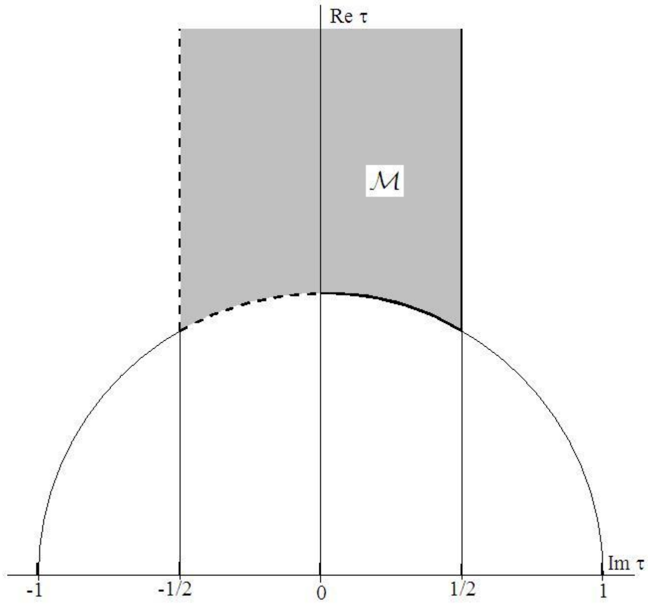


Figure 3.2. The moduli space of the torus is $\mathcal{M} \cong M_g/SL\,(2,\mathbb{Z})$, where M_g is the Teichüller space and is isomorphic to the upper half of the complex plane. All tori (specified by their complex structure, τ) correspond to points in this space.

The one loop graph is always isomorphic to a torus (T^2), which has homotopy group $\pi_1 \cong S^1 \oplus S^1$, and thus has two non-contractible one cycles, denoted a and b.[12] Upon transport around either a or b, the fermion fields pick up a phase that is in general complex:

$$a: \quad \psi \mapsto \quad -e^{\pi i \alpha(\psi)}\psi, \tag{3.1.43a}$$

$$b: \quad \psi \mapsto \quad -e^{\pi i \beta(\psi)}\psi. \tag{3.1.43b}$$

Real fermions must remain real under this transformation, and thus $\alpha\,(\psi)$ $(\beta\,(\psi))$ must be an integer so we don't introduce a complex phase. Note that this is stricter than the general constraint that the internal fermions be invariant under an $SO(32)$ rotation, and amounts to the requirement that we limit ourselves to the $R - NS$ or the $NS - NS$ sectors of the heterotic string.

[12] In English, this means that there are two ways to wrap a piece of string around the surface of a donut such that the loops cannot be shrunk to a point.

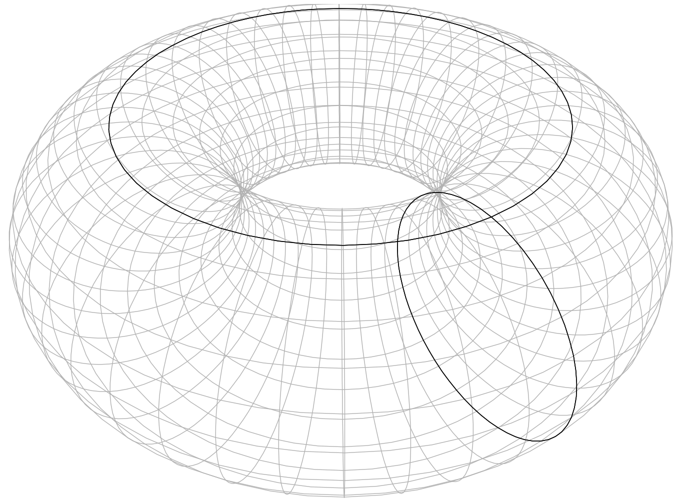


Figure 3.3. The heavy black lines are the two non-contractible one-cycles on the torus. The fact that one can find two one-cycles on the surface of the torus means that its homotopy group is $\pi_1 \cong S^1 \oplus S^1$.

We can specify the boundary conditions on each of the 64 fermions from Equation (3.1.39) by choosing two 64 component vectors, called $\vec{\alpha}$ and $\vec{\beta}$, which will be denoted as

$$\begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix}. \tag{3.1.44}$$

Now, complex fermions, $\psi_c^p = \psi^{r_1} + i\psi^{r_2}$, form a charge lattice, $\mathbf{Q}_\alpha$. For each 64 component boundary condition sector $\vec{\alpha}$, we have

$$(Q_\alpha)_p = \frac{\alpha_p}{2} + F_p, \tag{3.1.45}$$

where F is the fermion number operator, as in Section 3.1.3. F_p has eigenvalues of $\{0, -1\}$ for real fermions and $\{0, \pm 1\}$ for complex fermions.

The one loop vacuum to vacuum partition function may be written as

$$Z = \int_{\mathcal{M}} \frac{d^2\tau}{[\mathrm{Im}(\tau)]^2} Z_B(\tau\,\overline{\tau}) \sum_{\vec{\alpha},\vec{\beta}} C\begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix} Z\begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix}, \tag{3.1.46}$$

where $\mathcal{M} \cong M_g/SL\,(2,\mathbb{Z})$ is the moduli space of T^2, and is shown in Figure 3.2. The $C\binom{\vec{\alpha}}{\vec{\beta}}$ are called the spin structure coefficients, or phase weights, and are generated by a finite, additive Abelian group, Ξ. We may define a basis of Ξ, $\{\mathbf{V}_i\}$, such that

$$\begin{aligned} \vec{\alpha} &= \sum_{j=0}^{n} a_j \mathbf{V}_j \in \Xi, \quad a_j \in \mathbb{Z}, \\ \vec{\beta} &= \sum_{i=0}^{n} b_i \mathbf{V}_i \in \Xi, \quad b_i \in \mathbb{Z}. \end{aligned} \tag{3.1.47}$$

Then we can show that the phase weights obey [80]

$$C\binom{\vec{\alpha}}{\vec{\beta}} = (-1)^{s_{\vec{\alpha}}+s_{\vec{\beta}}} \exp\left\{\pi i \sum_{i,j} b_i (k_{ij} - \tfrac{1}{2}\mathbf{V}_i \cdot \mathbf{V}_j) a_j\right\}. \tag{3.1.48}$$

The $s_{\vec{\alpha}}$ and $s_{\vec{\beta}}$ are the four dimensional space-time components of $\vec{\alpha}$ and $\vec{\beta}$ (the boundary conditions of $\psi_\pm$ and $\bar{\psi}_\pm$ from Equation (3.1.39), and the $k_{i,j}$ are the components of the GSO projection matrices, which were discussed in Section 3.1.3, and ensure that no unphysical states survive in the low energy effective field theory.

Modular invariance imposes constraints on the basis vectors $\mathbf{V}_i$ and on the GSO projection matrix $\mathbf{k}$, with elements $k_{i,j}$:

$$k_{i,j} + k_{j,i} = \tfrac{1}{2}\mathbf{V}_i \cdot \mathbf{V}_j \ \mathrm{mod}(2), \tag{3.1.49a}$$

$$N_j k_{i,j} = 0, \ \mathrm{mod}(2), \tag{3.1.49b}$$

$$k_{i,i} + k_{i,0} = -s_i + \tfrac{1}{4}\mathbf{V}_i \cdot \mathbf{V}_i \ \mathrm{mod}(2). \tag{3.1.49c}$$

One can rewrite Equations (3.1.49) in terms of $\{\mathbf{V}_i\}$ and show:

$$N_{i,j}\mathbf{V}_i \cdot \mathbf{V}_j = 0, \ \mathrm{mod}(4) \tag{3.1.50a}$$

$$N_i \mathbf{V}_i \cdot \mathbf{V}_i = 0, \ \mathrm{mod}(8) \tag{3.1.50b}$$

The number of real fermions simultaneously periodic for any three basis vectors is even. (3.1.50c)

$N_{i,j}$ is the lowest common multiple of N_i and N_j, since Equation (3.1.50c) still applies when two or more of the basis vectors are identical. Thus, each basis vector must have an even number of real periodic fermions.

Table 3.2. The NAHE Basis Set.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\psi}'^{1,...,5}$	$\bar{\eta}'^1$	$\bar{\eta}'^2$	$\bar{\eta}'^3$
1	1	1	1	1	1,...,1	1	1	1	1,...,1	1	1	1
S	1	1	1	1	0,...,0	0	0	0	0,...,0	0	0	0
$\mathbf{b}_1$	1	1	0	0	1,...,1	1	0	0	0,...,0	0	0	0
$\mathbf{b}_2$	1	0	1	0	1,...,1	0	1	0	0,...,0	0	0	0
$\mathbf{b}_3$	1	0	0	1	1,...,1	0	0	1	0,...,0	0	0	0

	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$
1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
S	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}_1$	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}_2$	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
$\mathbf{b}_3$	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

The NAHE set [66] is composed of 5 basis vectors which give excellent low energy phenomenology [83]. The low energy theories, quite generally, are compactifications on an orbifolded six-torus ($T^6/\mathcal{Z}_2 \times \mathcal{Z}_2$). They have the gauge group $SO(10) \times SO(6) \times E_8'$, $\mathcal{N}$=1 (space–time) SUSY and 48 spinorial **16**'s of $SO(10)$, sixteen from each sector $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$. The NAHE sector reduces the gauge symmetry in the observable sector E_8 to $SO(10) \times U(1)^3$, and does not affect initial hidden sector E_8' of the uncompactified 10-dimensional model. The $U(1)^3$ are then embedded in an enhanced $SO(6)^3$. The five sectors are listed in Table 3.2.

3.1.6 Answers from String Theory

We have mentioned, in several places, that string theory is the most well-studied candidate for a UV completion of particle physics. The appearance of a spin two boson (a graviton), as well as large, non-Abelian groups and chiral fermions tell us that string theory almost certainly contains a description of our universe. Further, string theory requires space-time SUSY, and while there exist no real mechanisms by which to predict the SUSY breaking scale, string theory is robust enough to offer several different possibilities. It is hoped that a full, non-perturbative formulation of string theory would post-dict the values of the experimental inputs of the MSSM.

One of the biggest problems with M-theory, as it currently stands, is the absence of a mechanism which selects a vacuum—M-theory almost certainly contains a description of our universe, but it seems that our universe is no more likely than any of the other $10^{100-500}$ possible universes. Moreover, there may be many descriptions of our universe that are nearly degenerate—that is, they may predict the same values of fundamental constants to a precision beyond our present or future measurement capabilities. This has been dubbed the landscape [84], and there is some debate as to whether this is a problem or whether it as a new paradigm for physics.[13]

There are other problems with string theory which manifest themselves in cosmological observations. For example, if our universe continues in the current trend of accelerating expansion, then the equations of motion tend toward an approximately de-Sitter vacuum, however, such late-time solutions are quite difficult to construct and do not appear to be a natural feature of M-theory. There has been some progress along these lines [85], however, these constructions suffer from the seemingly universal landscape problem of M-theory, and the late-time evolution of the universe is still considered an open question.

The emergence of a naturally small cosmological constant from M-theory seems unlikely, and is perhaps the biggest problem facing the string community [86]. On the other hand, anti-de Sitter vacua come about quite naturally as solutions to M-theory [87], and one could consider this as a prediction of M-theory—that the current inflationary trend will reverse, and that at some point in the history of the universe the expansion will change course. In Chapter 4, we will look at specific constructions of field theories on anti-de Sitter spaces.

[13] This was also a widely noticed problem with string theory, before Witten's 1995 discovery of dualities [75]. It was hoped that these dualities between the various string theories would solve the problem of vacuum selection, but this has turned out not to be the case.

3.2 Introduction to Free Fermionic Mirror Universes

In general string constructions, there exists quite naturally a hidden sector, whose matter will only interact with the SM matter via gravity. Immediately, one recognizes this hidden sector matter as a possible candidate for the so-called "dark matter", which composes somewhere around 24% of the universe [88]. In the string model building process, the hidden sector arises much in the same way as the visible sector, with (possibly) non-chiral matter living in representations of non-Abelian gauge groups, which were broken from some larger non-Abelian gauge group. Motivated by the science fiction texts, we ask the question: If the symmetry breaking occurs in the same way between the hidden sector and the visible sector, what are the consequences?

The mirror universe models have been proposed in the context of neutrino physics (specifically by attempts to understand the nature of a sterile neutrino) and in the context of superstring/M-theory [89–93]. As a means to reconcile the constraints of big bang nucleosynthesis, the reheating temperature of the mirror universe after inflation was postulated to be lower than that in the observable universe [89]. From this it was shown that the asymmetric reheating can be related to a difference of the electroweak symmetry breaking scales in the two sectors, as required for a mirror solution to the neutrino puzzle. In such models it was shown that the baryon asymmetry is greater in the mirror universe than in the observable universe and that the mirror baryons could provide the dominant dark matter in the bulk universe [92].

These mirror universes have been with us all along—M-theory compactified on a line segment gives exactly $E_8 \times E_8$ heterotic string theory [94], in which the E_8's do not interact.[14] At the time of the preparation of this thesis, however, string-derived quasi-realistic three generation mirror universe models in four-dimensions have, to our knowledge, not been constructed. Rather, in typical quasi-realistic string

[14] Indeed, our model can be pictured as such—the hidden and observable symmetry groups can be thought of as confined to separate branes, on the boundary of some internal space.

models, the ten-dimensional mirror symmetry[15] is broken by compactification to four dimensions. In free fermionic [79, 81] models, mirror symmetry breaking generally results from corresponding asymmetric boundary conditions of observable and hidden sector world-sheet fermions. In this chapter an obstacle to mirror model construction in weakly coupled heterotic strings is discussed, which produces an interesting physical effect in its own right. Mirror symmetry breaking can necessarily still occur with symmetric boundary conditions. This will be demonstrated, and the implications for such models will be considered.

During the string/M-derived mirror universe investigation, we found that even when symmetric world-sheet fermion boundary conditions are imposed, breaking of mirror symmetry is sometimes mandated by an asymmetry in the GSO projections. Two example models demonstrate how GSO projections can necessitate mirror symmetry breaking of observable (O) and hidden sector (H) Pati-Salam [49] gauge groups

$$[SU(4)_C \times SU(2)_L \times SU(2)_R]^O \times [SU(4)_C \times SU(2)_L \times SU(2)_R]^H \ . \tag{3.2.1}$$

In the first example, the GSO projections reduce the observable sector gauge group to

$$[SU(4)_C \times SU(2)_L]^O \, , \tag{3.2.2}$$

by transferring $SU(2)^O_R$ to the hidden sector. In the process the

$$[SU(4)_C \times SU(2)_L]^H \tag{3.2.3}$$

subgroup of the hidden sector Pati-Salam gauge symmetry is enhanced to

$$[SO(10)]^H \ . \tag{3.2.4}$$

[15]Note that we are using this term somewhat outside of its standard usage in string theory—"Mirror Symmetry", in the context of Type IIa or Type IIb string theory, is a correspondence between two Calabi-Yau manifolds of different complex structure. See [95] for a review of this fascinating topic.

Table 3.3. Broken Mirror Model 1 GSO Matrix $\mathbf{k}$

	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	$\mathbf{b}'_1$	$\mathbf{b}'_2$	$\mathbf{b}'_3$	$\mathbf{a}$	$\mathbf{a}'$
$\mathbf{1}$	0	0	1	1	1	1	1	1	0	0
$\mathbf{S}$	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	1	1	1	1	1	0	0	0	0	1
$\mathbf{b}_2$	1	1	1	1	1	0	0	0	0	1
$\mathbf{b}_3$	1	1	1	1	1	0	0	0	0	1
$\mathbf{b}'_1$	1	1	0	0	0	1	1	1	1	0
$\mathbf{b}'_2$	1	1	0	0	0	1	1	1	1	0
$\mathbf{b}'_3$	1	1	0	0	0	1	1	1	1	0
$\mathbf{a}$	0	0	0	0	0	0	0	0	1	*1
$\mathbf{a}'$	0	0	0	0	0	0	0	0	*0	1

Additionally, the initial shadow (S) sector (corresponding to charges carried by both observable and hidden states) gauge group, $[SU(2)^3 \times SU(3) \times U(1)^7]^S$, absorbs $[SU(2)_R]^H$ to become $[SU(2)^3 \times SU(5) \times U(1)^6]^S$ (see Table D.1)

The second example differs from the first in some of its $k_{i,\alpha}$ matrix elements (and corresponding $k_{i',\alpha'}$). For model 2, the GSO projections reduce the observable sector Pati-Salam gauge group to $[SU(4)_C \times SU(2)_R]^O$ by alternately rendering $SU(2)^O_L$ to the hidden sector. The hidden sector $[SU(4)_C \times SU(2)_R]^H$ subgroup of the Pati-Salam group is similarly enhanced to $[SO(10)]^H$. In this version, the shadow sector gauge group remains of rank 12 and does not absorb $SU(2)^H_L$ (see Table D.4).

These models make use of the free fermionic construction, and are an example of NAHE based models reviewed previously in Section 3.1.4 (see Table 3.2). A presentation of the gauge groups and GSO matrices for our two models, along with tables listing the full matter states of both models are found in Appendix D.

3.3 Symmetry Breaking of Mirror Models

Mirror models with matching observable and hidden sector symmetries and states may be created from NAHE-based models by adding mirror basis vectors, $\mathbf{b}'_1$, $\mathbf{b}'_2$ and $\mathbf{b}'_3$, as defined in Table 3.5. Due to the symmetry between $\mathbf{b}_i$ and $\mathbf{b}'_i$, these

Table 3.4. Broken Mirror Model 2 GSO Matrix

	$\mathbf{1}$	$\mathbf{S}$	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	$\mathbf{b}'_1$	$\mathbf{b}'_2$	$\mathbf{b}'_3$	$\mathbf{a}$	$\mathbf{a}'$
$\mathbf{1}$	0	0	1	1	1	1	1	1	0	0
$\mathbf{S}$	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	1	1	1	1	1	0	0	0	0	0
$\mathbf{b}_2$	1	1	1	1	1	0	0	0	0	0
$\mathbf{b}_3$	1	1	1	1	1	0	0	0	0	0
$\mathbf{b}'_1$	1	1	0	0	0	1	1	1	0	0
$\mathbf{b}'_2$	1	1	0	0	0	1	1	1	0	0
$\mathbf{b}'_3$	1	1	0	0	0	1	1	1	0	0
$\mathbf{a}$	0	0	1	1	1	0	0	0	1	*1
$\mathbf{a}'$	0	0	0	0	0	1	1	1	*0	1

mirror vectors break the hidden sector E_8 in the same manner as the NAHE set breaks the observable sector E_8 into $SO(10) \times U(1)^3$. Each $\mathbf{b}'_i$ produces 16 copies of **16**'s of the hidden sector $SO(10)$.

The right-moving components of the $\mathbf{b}_i$ and the $\mathbf{b}'_i$ basis vectors that are simultaneously non-zero form a subset of $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\cdots,6}$. That is, $\mathbf{b}_i$ and $\mathbf{b}'_i$ states will both carry some $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\cdots,6}$ "shadow sector" charges. Within the NAHE set the only two bosonic sectors that can produce gauge states are $\mathbf{0}$ and $\mathbf{1}+\mathbf{b}_1+\mathbf{b}_2+\mathbf{b}_3$. All observable and "shadow sector" $SO(n)$ and $U(1)$ generators, along with the hidden sector **120** rep of $SO(16) \in E_8$, originate in $\mathbf{0}$, while the hidden sector **128** rep of $SO(16) \in E_8$ originates in $\mathbf{1} + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$. The GSO projections from the mirror sectors $\mathbf{b}'_1$, $\mathbf{b}'_2$, and $\mathbf{b}'_3$ remove the **128** rep of $SO(16)$, reducing the hidden sector E_8 symmetry to $SO(16)$. Further, the mirror sectors reduce the hidden sector $SO(16)$ to an $SO(10)\times U(1)^3$ symmetry, matching the observable sector. (Similarly the GSO projections of $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$ remove any adjoint contribution to the observable gauge group from the $\mathbf{1} + \mathbf{b}'_1 + \mathbf{b}'_2 + \mathbf{b}'_3$ sector.)

The hidden sector $U(1)^3$ generation charges and the observable sector $U(1)^3$ generation charges combine with the shadow sector charges in a like manner. Simultaneously, the $\mathbf{b}'_i$ sector GSO's reduce the $SO(4)^3$ shadow sector symmetry to $U(1)^6$.

Table 3.5. The Mirror Set

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\psi}'^{1,...,5}$	$\bar{\eta}'^1$	$\bar{\eta}'^2$	$\bar{\eta}'^3$
$\mathbf{b}'_1$	1	1	0	0	0,...,0	0	0	0	1,...,1	1	0	0
$\mathbf{b}'_2$	1	0	1	0	0,...,0	0	0	0	1,...,1	0	1	0
$\mathbf{b}'_3$	1	0	0	1	0,...,0	0	0	0	1,...,1	0	0	1

	$y^{1,...,4}$	$\bar{y}^{1,...,4}$	$y^{5,6}, \omega^{1,2}$	$\bar{y}^{5,6}, \bar{\omega}^{1,2}$	$\omega^{3,...,6}$	$\bar{\omega}^{3,...,6}$
$\mathbf{b}'_1$	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}'_2$	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
$\mathbf{b}'_3$	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

The net result is a $SU(3) \times SU(2)^3 \times U(1)^7$ shadow sector symmetry, whose charges are carried by representations of the observable $SO(10)$ and of the hidden $SO(10)$. The additional $SU(3) \times SU(2)^3$ generators originate in several additional massless gauge sectors formed from linear combinations of $\mathbf{1}$, $\mathbf{b}_i$, and $\mathbf{b}'_i$. All of these sectors have massless vacua and only carry $\bar{\eta}_i$, $\bar{\eta}'_i$, and $\bar{y}$, $\bar{\omega}$ shadow charges. While most massless gauge states from these sectors are projected out, a few are not. Those that survive mix the $SO(6)^3$ containing the $\bar{\eta}$-charges carried by the observable states and the shadow sector $SO(6)^3$ containing the $\bar{\eta}'$-charge carried by the hidden sector states. Thus, at this stage mirror symmetry still exists, as should be expected. Note in particular that $\mathbf{k}$ is invariant under exchange of $\mathbf{b}_i$ with corresponding $\mathbf{b}'_i$.

To break each of the $SO(10)$ to their corresponding Pati-Salam $SO(6) \times SO(4)$, two additional sectors $\mathbf{a}$ and $\mathbf{a}'$ are added to the model, where $\mathbf{a}$ and $\mathbf{a}'$ are mirror sectors, as shown in Table 3.5. The set of ten basis vectors $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}'_1, \mathbf{b}'_2, \mathbf{b}'_3, \mathbf{a}, \mathbf{a}'\}$, produce a model with mirror symmetry boundary conditions for the observable and hidden sectors. However, an asymmetry *necessarily* develops among the GSO projections with the addition of $\mathbf{a}$ and $\mathbf{a}'$. Although all new degrees of freedom of $k_{\alpha,\beta}$ where $\alpha \in \{\mathbf{a}, \mathbf{a}'\}$ and $\beta \in \{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}'_1, \mathbf{b}'_2, \mathbf{b}'_3\}$, are chosen to be invariant under simultaneous exchange of primed and unprimed sectors for both α and β, symmetry

Table 3.6. $SO(10) \times SO(10)$ Breaking and Generation Reduction

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$y, \bar{y}, \omega\bar{\omega}$	$\bar{\psi}'^{1,\ldots,5}$	$\bar{\eta}'^1$	$\bar{\eta}'^2$	$\bar{\eta}'^3$
$\mathbf{a}$	1	1	1	1	0,0,0,1,1	1	1	1	$\vec{0}$	0,0,0,0,0	0	0	0
$\mathbf{a}'$	1	1	1	1	0,0,0,0,0	0	0	0	$\vec{0}$	0,0,0,1,1	0	0	0

breaking between $k_{\mathbf{a},\mathbf{a}'}$ and $k_{\mathbf{a}',\mathbf{a}}$ occurs automatically. Only one of $k_{\mathbf{a},\mathbf{a}'}$ and $k_{\mathbf{a}',\mathbf{a}}$ is a degree of freedom; the other is specified by (3.1.49a). As Table 3.6 shows, $\mathbf{a} \cdot \mathbf{a}' = 10$, which from (3.1.49a) yields

$$k_{\mathbf{a},\mathbf{a}'} + k_{\mathbf{a}',\mathbf{a}} \;\; = 1 \mod(2). \tag{3.3.1}$$

Thus, $k_{\mathbf{a},\mathbf{a}'}$ and $k_{\mathbf{a}',\mathbf{a}}$ cannot be equal (mod 2). Instead, either $k_{\mathbf{a},\mathbf{a}'} = 1$ and $k_{\mathbf{a}',\mathbf{a}} = 0$ or vice versa (since all components of $\mathbf{a}$ and $\mathbf{a}'$ are either anti-periodic or periodic).

The addition of $\mathbf{a}$ and $\mathbf{a}'$ to the model generates several new massless gauge sectors of the form $\mathbf{a} + \mathbf{a}' + ...$ However, in both model variations presented herein, the GSO projections remove all possible gauge states from such sectors, except those coming from $\mathbf{a} + \mathbf{a}'$. The $\mathbf{a} + \mathbf{a}'$ sector has all anti-periodic components except for four periodic associated with the two complex fermions generating the observable $SO(4) = SU(2)_L \times SU(2)_R$ and the two complex fermions generating the hidden sector $SO(4) = SU(2)_L \times SU(2)_R$. Thus, in the $\mathbf{a} + \mathbf{a}'$ sector, massless gauge states require one anti-periodic fermionic (with $Q = \pm 1$) excitation.

The GSO projection from $\mathbf{a}$ acts on observable $SO(4)$ spinors while $\mathbf{a}'$ acts on hidden $SO(4)$ spinors. Since $k_{\mathbf{a},\mathbf{a}'}$ and $k_{\mathbf{a},\mathbf{a}'}$ differ by 1 (mod 2), so do $k_{\mathbf{a},\mathbf{a}+\mathbf{a}'}$ and $k_{\mathbf{a}',\mathbf{a}+\mathbf{a}'}$. Thus, a state in the $\mathbf{a} + \mathbf{a}'$ sector survives both the $\mathbf{a}$ sector GSO projections and $\mathbf{a}'$ sector GSO projections if and only if its observable and hidden $SO(4)$ spinors have opposite chirality. That is, the net number of down spins among the four spinors must be odd, implying that an $\mathbf{a} + \mathbf{a}'$ gauge state will either carry observable $SU(2)_L \in SO(4)$ charge and hidden $SU(2)_R \in SO(4)$ charge or vice-versa.

For model 1, the additional $k_{i,\mathbf{a}}$ and $k_{i,\mathbf{a}'}$ require $\mathbf{a}+\mathbf{a}'$ states to always have an even number of observable $SO(4)$ $-\frac{1}{2}$ spins and an odd number of hidden $SO(4)$ $-\frac{1}{2}$ spins, linking observable $SU(2)_R \in SO(4)$ reps with hidden $SU(2)_L \in SO(4)$ reps. The remaining GSO projections on $\mathbf{a}+\mathbf{a}'$ gauge states require the $Q = \pm 1$ anti-periodic fermion excitation charge to be from the hidden sector $SO(6) \sim SU(4)$. Thus, the surviving $\mathbf{a}+\mathbf{a}'$ simple root connects observable $SU(2)_R$ roots with the hidden sector $SO(6)$ and $SU(2)_L$ roots, thereby regenerating a hidden sector $SO(10)$:

$$[SU(2)_R]^O \times [SO(6) \times SU(2)_L]^H \rightarrow [SO(10)]^H \tag{3.3.2}$$

as in Table D.1.

In addition, a sector $1+\mathbf{b}_1+\mathbf{b}_1'+\mathbf{a}$ gauge state with eight complex spinors links the shadow gauge states with the hidden sector $SU(2)_R$, increasing the shadow sector gauge symmetry,

$$\left[SU(3) \times SU(2)^3 \times U(1)^7\right]^S \times (SU(2)_R)^H \rightarrow \left[SU(5) \times SU(2)^3 \times U(1)^6\right]^S . \tag{3.3.3}$$

The final model 1 gauge group is, therefore,

$$[SU(4) \times SU(2)_L]^O \times \left[SU(5) \times SU(2)^3 \times U(1)^6\right]^S \times [SO(10)]^H . \tag{3.3.4}$$

While maintaining symmetry under exchange of primed and unprimed components of $\mathbf{k}$, model two differs from model 1 in some choices of $k_{\mathbf{a},\beta}$ and $k_{\mathbf{a}',\beta}$ for $\beta \in \{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_1', \mathbf{b}_2', \mathbf{b}_3'\}$. Model 2 GSO projections require an odd number of observable $SO(4)$ $-\frac{1}{2}$ spins and an even number of hidden $SO(4)$ $-\frac{1}{2}$ spins in the $\mathbf{a}+\mathbf{a}'$ sector. This links observable $SU(2)_L \in SO(4)$ reps with hidden $SU(2)_R \in SO(4)$ reps. The remaining GSO projections again require the $Q = \pm 1$ anti-periodic fermion excitation charge of an $\mathbf{a}+\mathbf{a}'$ simple root to be from the hidden sector $SO(6)$. Thus, for model 2 the gauge boson from $\mathbf{a}+\mathbf{a}'$ again regenerates a hidden sector $SO(10)$:

$$[SU(2)_L]^O \times [SO(6) \times SU(2)_R]^H \rightarrow [SO(10)]^H . \tag{3.3.5}$$

However, in model 2, no additional gauge state is produced to mix the shadow sector with the hidden sector $SU(2)_L$. The final model 2 gauge group is, therefore,

$$[SU(4) \times SU(2)_R]^O \times \left[SU(3) \times SU(2)^3 \times U(1)^7\right]^S \times [SO(10) \times SU(2)_L]^H \ . \quad (3.3.6)$$

For aesthetic reasons, we exchange the definitions of left and right-handedness in the observable sector and of related SM states transforms this into

$$[SU(4) \times SU(2)_L]^O \times \left[SU(3) \times SU(2)^3 \times U(1)^7\right]^S \times [SO(10) \times SU(2)_L]^H \ . \quad (3.3.7)$$

3.4 Hypercharge Definitions

An important issue for these models is whether an acceptable definition of hypercharge can be found, since the conventional hypercharge is missing either its equivalent T_3^R contribution in model 1 or its equivalent T_3^L contribution in model 2. In standard NAHE-based models the hypercharge is formed as

$$Y = \tfrac{1}{3}\tilde{Q}_C + \tfrac{1}{2}\tilde{Q}_L, \quad (3.4.1)$$

(for $Y\left(Q_L\right) = \frac{1}{3}$ normalization), where $\tilde{Q}_C = \sum_{m=1}^{3} Q_{\bar{\psi}^m}$ is the associated charge trace of $U(1)_C \equiv \left[\bar{\psi}^{1*}\bar{\psi}^1 + \bar{\psi}^{2*}\bar{\psi}^2 + \bar{\psi}^{3*}\bar{\psi}^3\right]$ and $\tilde{Q}_L = \sum_{m=4}^{5} Q_{\bar{\psi}^m}$ is the associated charge trace of $U(1)_L \equiv \left[\bar{\psi}^{4*}\bar{\psi}^4 + \bar{\psi}^{5*}\bar{\psi}^5\right]$. Since $SU(4)_C \rightarrow SU(3)_C \times U(1)_{B-L}$, $\frac{1}{3}\tilde{Q}_C = \tilde{Q}_{B-L}$, as Table 3.7 indicates. Similarly, since $SO(4) = SU(2)_L \times SU(2)_R \rightarrow SU(2)_L \times U(1)_L$, $\frac{1}{2}\tilde{Q}_L = 2T_3^R$. Thus,

$$Y = \tilde{Q}_{B-L} + 2T_3^R, \quad (3.4.2)$$

which yields electromagnetic charge

$$\tilde{Q}_{EM} = T_3^L + \tfrac{1}{2}Y = T_3^L + T_3^R + \tfrac{1}{2}\tilde{Q}_{B-L}. \quad (3.4.3)$$

Hence, model 1 requires a replacement for T_3^R, while model 2 needs a replacement for T_3^L.

Under $SU(4)_C \times SU(2)_L$ the SM $SU(3)_C \times SU(2)_L$ left handed reps combine into

Table 3.7. Hypercharge Components for Broken Mirror Models

	$\tilde{Q}_{B-L}$ $\frac{1}{3}\tilde{Q}_C$	$2T_3^R$ $\frac{1}{2}\tilde{Q}_L$	$2T_3^L$	$\tilde{Q}_{EM}$	$= T_3^L + T_3^R + \frac{1}{2}\tilde{Q}_{B-L}$ $= T_3^L + \frac{1}{2}Y$
Q_L	$\frac{1}{3}$	0	± 1		$\frac{2}{3}$, $-\frac{1}{3}$
d_L^c	$-\frac{1}{3}$	1	0		$\frac{1}{3}$
u_L^c	$-\frac{1}{3}$	-1	0		$-\frac{2}{3}$
L_L	-1	0	± 1		0, -1
e_L^c	1	1	0		1
ν_L^c	1	-1	0		0

- $Q_L = (\mathbf{3},\mathbf{2})_{2T_3^R=0} \oplus L_L = (\mathbf{1},\mathbf{2})_{2T_3^R=0} \rightarrow (QL)_L = (\mathbf{4},\mathbf{2})_{2T_3^R=0}$,
- $e_L^c = (\mathbf{1},\mathbf{1})_{2T_3^R=1} \oplus d_L^c = (\mathbf{3},\mathbf{1})_{2T_3^R=1} \rightarrow (d^c e^c)_L = (\mathbf{4},\mathbf{1})_{2T_3^R=1}$,
- $\nu_L^c = (\mathbf{1},\mathbf{1})_{2T_3^R=-1} \oplus u_L^c = (\mathbf{3},\mathbf{1})_{2T_3^R=-1} \rightarrow (u^c \nu^c)_L = (\mathbf{4},\mathbf{1})_{2T_3^R=-1}$.

Model 1 contains three generations of pairs of $(q^c l^c)_i^n = (\mathbf{4},\mathbf{1})$ states (with generation index specified by $i = 1$ to 3, pair element index specified by $n = 1, 2$, and left-handed index L implicit), where $(q^c l^c)$ denotes either $(d^c e^c)$ or $(u^c \nu^c)$ (see Table D.2). These states are also respective doublets under the three generation $SU(2)_i$ of the shadow sector. As discussed, for a three generation model, the $SU(2)_i$ must be broken to the generational $U(1)_i$ of standard NAHE models by additional GSO projections from further sectors. When each $SU(2)_i$ is broken to $U(1)_i$ in this manner, one component of each $SU(2)_i$ doublet is also projected out. If these additional GSO projections can be chosen such that the up-spin component of $SU(2)_i$ for $(q^c l^c)_i^{n=1}$ survives along with the down-spin component of $SU(2)_i$ for $(q^c l^c)_i^{n=2}$, then for $Y = \tilde{Q}_{B-L} + 2\,(T_3)_i$ the $(q^c l^c)_i^{n=1}$ become the $(d^c e^c)_i$ states and the $(q^c l^c)_i^{n=2}$ become the $(u^c \nu^c)_i$ states. Under $L \leftrightarrow R$ exchange, the same process can be applied to create a consistent three generation hypercharge for model 2.

For both models, this is the only possible choice for $(q^c l^c)$ hypercharge, since in each model the extra Abelian charges carried by the $(q^c l^c)_i^n$ are independent of the index n for each generation i. That is, no hypercharge definition involving only the extra $U(1)_k$ could yield valid hypercharge for both $(d^c e^c)$ and $(u^c \nu^c)$ reps. For model 1, this posses a difficulty for the MSSM Higgs. In model 1, the only additional $SU(2)_L$ doublets are singlets under all $SU(2)_i$. These are the pairs of states h_i^n $(n = 1, 2)$ and the more exotic H_i^n, which are also $\mathbf{5}$ reps of $SU(5)^S$. For a given generation i, the extra $U(1)_k$ charges are independent of the index n. Thus, no hypercharge definition could yield both a $(Y = -1)$-charged up-Higgs and a $(Y = +1)$-charged down-Higgs from an $h_i^{n=1}$ and $h_i^{n=2}$ pair. Instead, a hypercharge definition is required such that the $U(1)_k$ hypercharge contribution to QL and ql states is zero, while it is +1 for at least one h_i^n pair and -1 for at least one other $h_{i'}^n$ pair. Applying the six QL and $(q^c l^c)$ constraints prevents any $U(1)_k$ from appearing in a general hypercharge definition. Thus, model 1 cannot provide a suitable definition for hypercharge, unless the Cartan subalgebra of $SU(5)^S$ can contribute to the hypercharge of H_i^n components after $SU(5)^S$ is broken.

In contrast, the MSSM Higgs of model 2 come in the standard h_i and $\bar{h}_i$ pairs for each generation i. However, each h_i and $\bar{h}_i$ is also an $SU(2)_i$ doublet (see Table D.5). Thus, if under $SU(2)_i$ breaking by GSO projections from additional sectors, the $SU(2)_i$ up-spin component of h_i survives and the $SU(2)_i$ down-spin component of $\bar{h}_i$ (or vice-versa) then h_i becomes down-Higgs and $\bar{h}_i$ becomes up-Higgs (or vice versa). Thus, a viable hypercharge definition for model 2 is

$$Y = \tfrac{1}{2}\tilde{Q}_{B-L} + \sum_{i=1}^{3} T_3^i. \tag{3.4.4}$$

This would produce generational Higgs pairs, which is a common occurrence in NAHE-based models. This often provides for mass hierarchy between generations since the physical Higgs usually becomes a weighted (by several orders of magnitude) linear combination of generational Higgs. MSSM matter states then couple differently by generation to the physical Higgs, producing a large mass hierarchy, even when all mass couplings in the superpotential are third order.

Models 1 and 2 both contain an anomalous $U(1)_A$, and it is unlikely that additional basis vectors would change this for either model. In fact, additional sectors generally increase the anomaly. For model 2 the charge traces of the seven $U(1)_k$ is

$$\mathrm{Tr}\,\vec{Q} = (0, -144, 96, 0, 0, -192, 0)\ . \tag{3.4.5}$$

(as can be computed from Table D.5). Thus, the anomaly may be rotated into

$$U_A = [-3Q_2 + 2Q_3 - 4Q_6] \tag{3.4.6}$$

for which the trace is 1392. The orthogonal

$$U'_2 = 2Q_2 + 3Q_3 \tag{3.4.7}$$

$$U'_3 = -3Q_2 + 2Q_3 + (13/4)Q_6\,. \tag{3.4.8}$$

become non-anomalous (traceless).

Model 2 is an example [96, 97] in which non-Abelian fields must necessarily take on vevs to cancel the Fayet-Iliopoulos (FI) term,

$$\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2}\mathrm{Tr}\,Q^{(A)} = \frac{g_s^2 M_P^2}{192\pi^2}1392, \tag{3.4.9}$$

generated in the U_A D-term by the Green-Schwarz-Dine-Seiberg-Witten anomalous U(1) breaking mechanism [98–100]. To see this, first note from Table D.5 that singlet states S_1^n through S_6^n carry the non-anomalous charge $Q_1 = 3$, while the remaining singlets have $Q_1 = 0$. Thus, D-flatness for $U(1)_1$ cannot be maintained if only singlets receive vevs and one or more of the fields S_1^i through S_6^i are among those that do. Next, the singlets S_7 and $\bar{S}_7$ do not carry anomalous charge (only Q_4 and Q_5 charge), and so cannot help cancel the F-I term. The remaining singlets are simply S_8 and S_9, and their vector partners of opposite charges. S_8 carries anomalous charge $Q_A = 4$ and non-zero non-anomalous charges $Q_4 = 1$, $Q_5 = 4$, $Q'_2 = 6$, $Q'_3 = -42$, while S_9 carries anomalous charge $Q_A = 4$ and non-zero non-anomalous charges $Q_4 = -1$, $Q_5 = -4$, $Q'_2 = 6$, $Q'_3 = -42$. $\bar{S}_8$ and $\bar{S}_9$ carry respective opposite charges. Thus, we see that no combination of $S_{7,8,9}$ and $\bar{S}_{7,8,9}$ vevs can simultaneously cancel the anomalous D_A-term contribution from the trace of U_A and keep the D-terms for Q'_2 and Q'_3 flat. Therefore, some non-Abelian fields must take on vevs in the process of canceling the $\mathrm{Tr}\, Q_A$ contribution to D_A to maintain D-flatness. Of particular interest is whether $SU(2)_i$-charged fields take on vevs in the parameter space of flat directions. Analysis of flat directions is, however, beyond the scope of this chapter.

3.5 Summary

This chapter has demonstrated, under certain conditions, mirror symmetry is necessarily broken between observable and hidden sector gauge groups of heterotic string models with mirror boundary conditions for observable and hidden sector world-sheet fermions. The observable/hidden sector gauge group mirror breaking occurs because of an unavoidable asymmetry in GSO projections. This effect can be induced in free fermionic models though an observable/hidden sector mirror-like pair of basis vectors, $\mathbf{a}$ and $\mathbf{a}'$, with the properties that:

- Their vector sum yields new, independent gauge sectors (possibly after further basis vectors are added) $\mathbf{a} + \mathbf{a}' + ...$
- They do not overlap with non-zero components in the observable and hidden sectors.
- Their inner product does not equal 0 mod(4).

Under these conditions the observable sector and hidden sector gauge states from some $\mathbf{a} + \mathbf{a}' + ...$ sector (or sectors) will not be mirror images since the observable sector gauge states surviving the $k_{\mathbf{a},\mathbf{a}+\mathbf{a}'+...}$ GSO projections will be different from the hidden sector gauge states surviving the corresponding $k_{\mathbf{a}',\mathbf{a}+\mathbf{a}'+...}$ GSO projections. In the examples shown, starting with mirror Pati-Salam gauge groups $[SU(4)_C \times SU(2)_L \times SU(2)_R]^O \times [SU(4)_C \times SU(2)_L \times SU(2)_R]^H$, the observable sector $SU(2)_{R(L)}$ was transformed to the hidden sector by this necessary asymmetry of the GSO projections, enhancing the hidden sector gauge group to $[SO(10) \times SU(2)_R]^H$. This transference of gauge rank from the observable sector to the hidden sector acts favorably for coupling strength renormalizations, allowing non-Abelian hidden sector coupling strengths to run upward faster than observable sector coupling strengths as energy scales run downward from the string scale, leading to the formation of generally advantageous intermediate scale hidden sector condensates.

CHAPTER FOUR

Randall-Sundrum Unification

In this chapter, we study the novel aspects of unification within the Randall-Sundrum (RS) proposal. We first briefly review the RS proposal in Section 4.1. In Section 4.2 we look at models that have been constructed using RS, almost exclusively based of $SU(5)$. We present our model [101], based on $\widetilde{SU}(5)$ in Section 4.3. In Section 4.3.1 we compute the beta function of our model, using the results of [102–105]. Finally, we show in Section 4.3.3 that our model leads to an extremely small lifetime for the proton if the Yukawa couplings are not fine-tuned.

4.1 An Introduction to the Randall-Sundrum Scenario

The Randall-Sundrum (RS) proposal [106, 107] represents a beautiful geometrical solution to the hierarchy problem, outlined in Section 1.2.5. Specifically, by embedding a four dimensional Minkowski space in a higher dimensional anti-de Sitter (AdS) bulk, one is able to suppress the weak scale by a factor of $e^{-k\pi r_c}$, with k and r_c the warp factor and compactification radius, respectively. The normal argument goes like this: suppose we have an AdS space, whose measure is given by

$$ds^2 = e^{-2kz}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2, \tag{4.1.1}$$

where the fifth dimension (z) is an orbifolded circle ($S^1/\mathcal{Z}_2$). Suppose also that we have a bulk scalar in our theory, H, whose action is given by:

$$\mathcal{S} \supset \int d^4x \int_0^{\pi r_c} dz\sqrt{-G}\left(G^{MN}\partial_M H^\dagger \partial_N H + m^2 H^\dagger H\right), \tag{4.1.2}$$

where G_{MN} is the full 5-dimensional metric and is given by

$$G_{MN} = e^{-2kz}\mathrm{diag}(-1, +1, +1, +1, \mathrm{e}^{2kz}) \;, \tag{4.1.3a}$$

$$G^{MN} = e^{2kz}\mathrm{diag}(-1, +1, +1, +1, \mathrm{e}^{-2kz}). \tag{4.1.3b}$$

If our universe exists in the IR limit of (4.1.2), then we evaluate the action on the IR brane, namely where $z = \pi r_c$, and find

$$\mathcal{S}_{eff} \sim \int d^4x \left(e^{-2k\pi r_c} \partial^\mu H^\dagger \partial_\mu H + e^{-4k\pi r_c} m^2 H^\dagger H \right) . \tag{4.1.4}$$

Redefining the scalar field $H \to e^{-k\pi r_c} H$ we see

$$\mathcal{S}_{eff} \sim \int d^4x \left(\partial^\mu H^\dagger \partial_\mu H + e^{-2k\pi r_c} m^2 H^\dagger H \right) . \tag{4.1.5}$$

The mass parameter m (which is on the order of the Plank mass, as we expect from dimensional analysis) is now weighted by the warp factor on the IR brane where we would observe it, $m \to m e^{-k\pi r_c}$, and can be tuned to give phenomenologically acceptable values. The stabilization of the brane separation has already been addressed in [108], so that the choice of

$$k\pi r_c = \log \left[\frac{M_{UV}}{M_{IR}} \right] \sim 11\pi \tag{4.1.6}$$

is well-motivated.[1]

If we compare the MSSM with the RS proposal, we begin to see the advantages of the latter. SUSY protects the higgs mass with 124 free parameters, whereas RS only needs one, the warp factor $k\pi r_c$, and the existence of a finite extra dimension, z. Although, in some sense, the RS proposal is an alternative to SUSY, it can incorporate SUSY—then the warp factor serves to protect the SUSY breaking scale, which in turn protects the higgs mass.[2] The SUSY breaking terms are generated on the UV brane, and communicated (either gravitationally or via a "messenger" $U(1)$) to the IR brane. These models will not be discussed in this thesis.

The presence of a compact extra dimension necessarily means that there will be Kaluza-Klein modes [109, 110]. Calculating the shape of the wave-function in the fifth

---

[1] We see that the hierarchy problem has been replaced with a fine-tuning problem. This can be seen from Equation (4.1.3)—the exponential factor is $\sim 10^{-15}$, while all the other entries in the matrix are ~ 1.

[2] Indeed, if we want to embed RS into string theory, then it *must* be consistent with SUSY.

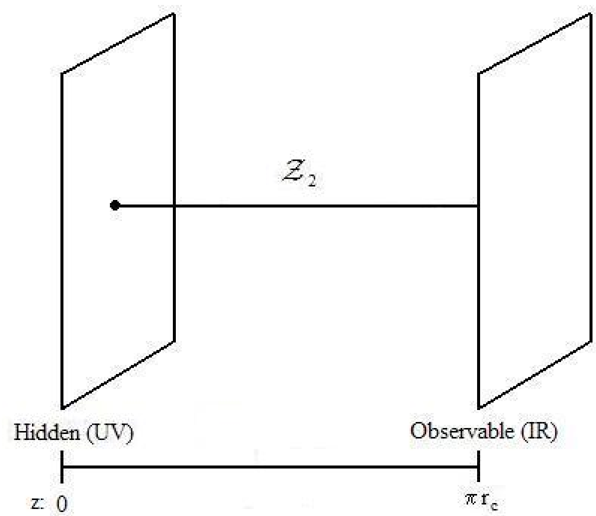


Figure 4.1. In the Randall-Sundrum scenario, one starts with a five dimensional anti-de Sitter space. The fifth dimension, z, is an orbifolded circle, $S^1/\mathcal{Z}_2$, with three-branes at the orbifold fixed points, 0 and πr_c. If we start with a circle of radius R, parameterized by the angle θ, the $\mathcal{Z}_2$ discrete symmetry identifies θ with $2\pi - \theta$.

dimension is relatively straightforward, using the method of separation of variables. If one varies the action, Equation (4.1.2), one finds that the equation of motion of the scalar field H is given by:

$$\partial^2 H + e^{2kz}\partial_5\left(e^{-4kz}\partial_5 H\right) - m_H^2 e^{-2kz} H = 0. \tag{4.1.7}$$

To solve this differential equation, we express the higgs field as composed of two parts, a four dimensional part $(\Phi(x))$ and a one dimensional part $(\phi(z))$:

$$H = \frac{1}{\sqrt{\pi r_c}} \sum_n \Phi^{(n)}(x)\, \phi^{(n)}(z). \tag{4.1.8}$$

The general solution for the zero mode, whose mass is given by $m^2 = ak^2$, for k the warp factor, is given by

$$\phi^{(0)}(z) = c_1 e^{(2-\alpha)kz} + c_2 e^{(2+\alpha)kz}, \tag{4.1.9}$$

where $\alpha = \sqrt{4+a}$. c_1 and c_2 are arbitrary constants. If one were to try to solve for the arbitrary constants with simple von Neumann or Dirichlet boundary conditions, one would find that there is no zero mode solution [111]. We must modify the boundary actions as with a mass term:

$$\mathcal{S}_{boundary} = \int d^4x dz \sqrt{-g} 2bk \left[\delta(\, z\,) - \delta(\, z - \pi r_c\,)\right] |H|^2 \,, \tag{4.1.10}$$

where b is a dimensionless number which parameterizes the boundary mass of the field. The modified von Neumann boundary condition[3] is given by

$$\left[\partial_5 \phi^{(0)} - bk\phi^{(0)}\right]\Big|_{0,\pi r_c} = 0. \tag{4.1.11}$$

The fields are assigned von Neumann (+) or Dirichlet (–) boundary conditions about the branes at the ends of the AdS space. Equivalently, these are the parities under the $\mathcal{Z}_2$ discrete symmetry which characterizes the orbifold. These boundary conditions are specified in doublets, with the first entry corresponding to the UV brane, and the second corresponding to the IR brane. Application of these new boundary conditions across both of the orbifold fixed points to Equation (4.1.7) tells us that the scalar zero modes with von Neumann boundary conditions must look like

$$\phi^{(0)} \propto e^{-bkz}. \tag{4.1.12}$$

Note that the b values are not calculable from the low energy effective field theory, just as quark masses are not calculable in the SM. These represent free parameters of the theory, that will be constrained by experiments if the RS proposal is validated in the future.

One can now add spin $\frac{1}{2}$ fermions [112] and spin 1 bosons [113, 114] to the spectrum in much the same way. In typical RS constructions, fermion fields are allowed to live in the bulk, and the individual fields in each representation are allowed to take on different boundary conditions. We find that the spin one bosons have flat wavefunctions in the fifth dimension, for either von Neumann or Dirichlet boundary conditions.

[3] A similar process can be followed to find modified Dirichlet boundary conditions.

A spin $\frac{1}{2}$ fermion whose wavefunction is symmetric about the $\mathcal{Z}_2 \times \mathcal{Z}_2$ orbifold symmetry has the general form:

$$\Psi_5 \sim e^{\left(\frac{1}{2}-c\right)kz}\psi_4. \tag{4.1.13}$$

Here, Ψ_5 (ψ_4) is the five- (four-) dimensional wavefunction. The $c-$values ($c \in [0, 1]$) dictate about which brane the wavefunction is localized, and thus the observed mass in the low energy effective field theory. By picking numbers of $O(\frac{1}{2})$, one is able to generate mass hierarchies which are put into the standard model (SM) by hand.

4.2 Unification in Five Dimensions: The GUTs of Randall-Sundrum

The question of unification in RS models has been addressed in several places, and in several different incarnations. The first examples of unification in RS were based on $SU(5)$ GUTs [115]. Supersymmetric $SU(5)$ unification has been investigated [116] and more recently, some detailed investigations of $SO(10)$ GUTs were preformed [117].

Generally, one first replaces the $S^1/\mathcal{Z}_2$ orbifold with a new topology: $S^1/\mathcal{Z}_2 \times \mathcal{Z}_2$. One must now assign boundary conditions across each of the orbifold fixed points (at 0 and πr_c), which correspond to parities about the branes. The breaking of the GUT symmetry down to the SM is achieved by assigning different boundary conditions to the fields appearing in the representation. In the five-dimensional bulk, the fields respect the GUT symmetry, say $SU(5)$. On the boundaries, the representations (and thus the GUT symmetry) are broken by the orbifold parities—in general, no GUT scale scalars are needed to break the symmetries.

Breaking the GUT symmetry with boundary conditions requires that one add copies of representations to the spectrum since only fields with $(+\,+)$ boundary conditions are physical, and the quarks and leptons (from the same generation) in the SM come from different representations in the underlying symmetry group. So, taking one of the models in [115], one has

$$\overline{\mathbf{5}}_1 = L_1^{++} + d_1^{+-} \tag{4.2.1a}$$

$$\overline{\mathbf{5}}_2 = L_1^{+-} + d_1^{++} \tag{4.2.1b}$$

The SM states have $(+\ +)$ boundary conditions while the other fields have $(+\ -)$ boundary conditions. While all of these models elegantly incorporate the features of the RS proposal, they all suffer from this seemingly universal problem of representation proliferation. For example, the $SO(10)$ model in ref. [117] required 6 copies of each **16** for each generation! This is an issue associated with any model in which the GUT symmetry is broken with boundary conditions.

This proliferation of representations does solve some problems. The experimentalists have given us strict bounds on the proton's lifetime, $\tau_p > 6.7 \times 10^{33}$ y [47], and one must be wary of higher dimensional operators in the effective field theory which violate these bounds. For example, in 5-d one could write the following operator down (from $\overline{\mathbf{5}} \times \mathbf{5} \times \overline{\mathbf{10}} \times \mathbf{10} \supset \mathbf{1}$) [111]:

$$\int d^4x dz \sqrt{-G} \frac{\overline{\Psi}_{\overline{\mathbf{5}}} \Psi_{\mathbf{5}} \overline{\Psi}_{\overline{\mathbf{10}}} \Psi_{\mathbf{10}}}{M_5^3}. \tag{4.2.2}$$

We can evaluate this integral on the IR brane, using (4.1.13)...

$$\frac{2k}{M_5^3} \frac{1}{N_i N_j N_k N_l} \frac{e^{\pi k r_c (4-c_1-c_2-c_3-c_4)} - 1}{4 - c_1 - c_2 - c_3 - c_4} \int d^4x \overline{\psi}_{\overline{\mathbf{5}}} \psi_{\mathbf{5}} \overline{\psi}_{\overline{\mathbf{10}}} \psi_{\mathbf{10}}, \tag{4.2.3}$$

where

$$N_i = \frac{1/2 - c_i}{e^{(1-2c_i)k\pi r_c} - 1}. \tag{4.2.4}$$

The $c-$values are less than 1, making the coupling constant in the effective field theory exponentially large. These operators are responsible for phenomena such as proton decay, and such large couplings will cause proton lifetime to be exponentially short. By requiring quarks and leptons to come from different generations, there exist no physical (i.e. on the IR brane) quark-lepton mixing, unlike in traditional GUTs.

It seems that there are no conventional baryon decay modes in this type of model, so the prediction is that experiments like Super-Kamiokande [47] will *never* see $p \rightarrow$ leptons + mesons. If such decays were observed, then (to avoid fine tuning, as per the RS paradigm) one would either have to find some other way to suppress these processes, or accept the exponentially tuned Yukawa couplings required by terms like (4.2.3). Finally, it should be noted that there have been investigations into breaking the GUT by turning on the vev of a bulk scalar field [104, 115].

4.3 A $\widetilde{SU}(5)$ Model in the RS Scenario

In the following model, we consider a non-supersymmetric $\widetilde{SU}(5)$ partial GUT living in the background of Equation (4.1.1). The breaking of $\widetilde{SU}(5)$ is accomplished with one bulk scalar field, called Φ, that takes on a vev at some intermediate scale M_*. Note that, in general, M_* is less than the GUT scale, M_{GUT}, but it is not completely unreasonable that one push M_* up to M_{GUT}—this would eliminate the embedding of $\widetilde{SU}(5)$ into some larger symmetry, like $SO(10)$. We take a minimal matter content, as in Equation (2.5.1).

SUSY $\widetilde{SU}(5)$ unification in extra dimensions has been studied in [118, 119], but this analysis was in the presence of a flat extra dimension, on an orbifolded circle ($S^1/\mathcal{Z}_2 \times \mathcal{Z}_2'$). The main difference between these two approaches is the form of the Kaluza-Klein modes and their contributions to the β functions (see [102, 103] for more details)—in the RS proposal the masses of the KK modes are solutions to combinations of Bessel functions, whereas in the flat case

$$m_n^2 = m^2 + \frac{n^2}{R^2}, \tag{4.3.1}$$

for $n = 0, 1, 2, ...$ Generally, one would like to unify the $\widetilde{SU}(5)$ theory in some higher dimensional GUT, like $SO(10)$. Here we will only work at the partial GUT level, and leave the problem of $SO(10)$ embedding to a future study [120].

4.3.1 Gauge Coupling Renormalization

Calculating the one-loop corrections to the vacuum-to-vacuum polarizations is relatively straightforward, and has been done for the (extremely popular) case of scalar QED in several places [102–105]. The difference between the standard QFT calculation and the RS1 calculation is the appearance of a tower of KK modes. For fields that are even about the orbifold $\mathcal{Z}_2 \times \mathcal{Z}_2$ symmetry, the KK masses are solutions to [111]

$$b_\alpha \left(m_n\right) = b_\alpha \left(m_n e^{\pi k r_c}\right), \tag{4.3.2}$$

where

$$b_\alpha \left(m_n\right) = -\frac{\left(-r + \frac{s}{2}\right) J_\alpha \left(\frac{m_n}{k}\right) + \frac{m_n}{k} J'_\alpha \left(\frac{m_n}{k}\right)}{\left(-r + \frac{s}{2}\right) Y_\alpha \left(\frac{m_n}{k}\right) + \frac{m_n}{k} Y'_\alpha \left(\frac{m_n}{k}\right)}, \tag{4.3.3}$$

and the constants are given by

$$\alpha = \begin{cases} \sqrt{4+a} & \text{spin } 0 \\ \left|c \pm \frac{1}{2}\right| & \text{spin } \frac{1}{2} \\ \sqrt{1+d} & \text{spin } 1 \end{cases}, \quad r = \begin{cases} b & \text{spin } 0 \\ \mp c & \text{spin } \frac{1}{2} \\ 0 & \text{spin } 1 \end{cases}, \quad s = \begin{cases} 4 & \text{spin } 0 \\ 1 & \text{spin } \frac{1}{2} \\ 2 & \text{spin } 1 \end{cases}. \tag{4.3.4}$$

In this study, we are concerned with fields which have $(+\,+)$ boundary conditions on the $\mathcal{Z}_2 \times \mathcal{Z}'_2$ orbifold symmetry—there are similar expressions for the odd case. The constants a, b, c and d come from the wavefunction's "boundary mass", and parameterize the field's profile in the fifth dimension—for example, we have already seen how c is defined in Equation (4.1.13). The constant a for a scalar field is given by $a \equiv \frac{m_\Phi^2}{k^2}$, where m_Φ is the five dimensional scalar mass. The constant $b = 2 + \alpha$, and $d \equiv \frac{M^2}{k^2}$, where M is the gauge boson's mass—possibly zero. Finally, $n = 1, 2, \ldots$.

Before we begin, it can be shown [122] that the leading contribution to the running of the couplings is logarithmic, as we find in the standard 4-d case—the corrections should contain terms $\sim \log\left[\frac{p}{\Lambda}\right]$, as long as we consider the regime where $p << \Lambda \lesssim k$, where p is some intermediate energy scale. The general form of the

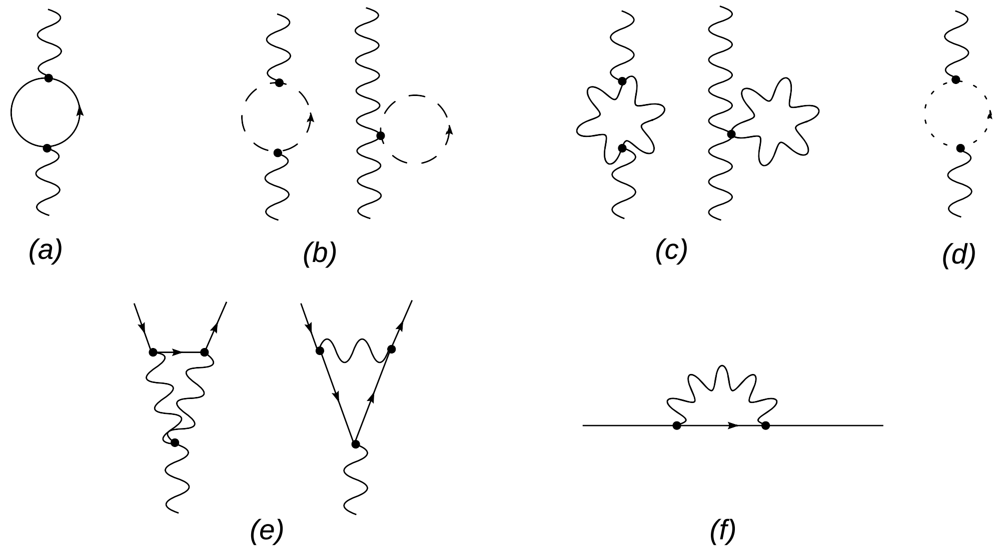


Figure 4.2. Possible graphs for calculating the massive contribution to the runnings of the couplings, α_i^{-1}. We must include contributions to the vector boson self-energy from fermions (a), scalars (b), $\widetilde{SU}(5)$ gauge bosons (c), and ghosts (d). We also get contributions from the fermion-fermion-boson counterterm (e) and the fermion self-energy (f). Note that we must include the KK mode sum in each of the loops. Note that fields which lie in complete representations of the GUT do not contribute to the differential runnings of the couplings. These graphs were generated using [121].

running coupling constants in an RS background is given by [102–104]:

$$\frac{1}{g_i^2\left(p^2\right)} = \frac{1}{kg_5^2} + \Delta_{UV} + \Delta_{IR} + \frac{1}{8\pi^2}\left\{\Delta_{1-\text{loop}} + \frac{\gamma_i}{24\pi^3}\Lambda\pi r_c + b_i \log\left[\frac{\Lambda}{p}\right]\right\}. \quad (4.3.5)$$

The g_5 is the bulk coupling constant, the Δ_{UV} and Δ_{IR} come from the couplings of the Maxwell tensors localized on the branes, and the $\Delta_{1-\text{loop}}$ are the one-loop contributions from the graphs in Figure 4.2–they arise because of the presence of Kaluza-Klein modes in the spectrum. The linear divergences are regularization scheme dependant, and cannot be calculated within our effective field theory. They are of $\mathcal{O}(\frac{M_*}{k})$ and will be ignored in what follows.[4] The last term is the familiar non-Abelian beta function contributions.

In general, Δ_{UV}, Δ_{IR} and g_5 are incalculable—they depend on some completion of the theory (possibly string theory). We will take the incalculable parts of Equation

[4] Equivalently, one could include these contributions in the redefinition (4.3.6), as in [103].

(4.3.5) to be [103]

$$\frac{1}{kg_5^2} + \Delta_{UV} + \Delta_{IR} \cong \frac{1}{g^2_{\widetilde{SU}(5)}} + \mathcal{O}\left(\frac{1}{8\pi^2}\right). \tag{4.3.6}$$

Now, let's compute the contribution of a massive scalar and its KK modes. The relevant graphs are shown in Figure 4.2, (b). The Feynman rules for a scalar transforming in an arbitrary representation of a non-Abelian symmetry are just a straightforward modification of the rules for scalar QED. We take our bulk scalar to have $(+\,+)$ boundary conditions. If we compute the amplitude of the graphs in Figure 4.2, (b), we find that the one loop correction $\Delta^{\text{scalar}}_{1-\text{loop}}\left(q^2\right)$, for a massive scalar and its KK excitations is given by:[5]

$$\Delta^{\text{scalar}}_{1-\text{loop}}\left(q^2\right) = \frac{g^2 C\left(r\right)}{\left(4\pi\right)^{\frac{D}{2}}}\Gamma\left(2-\frac{D}{2}\right)\int_0^1 dx \sum_{\text{KK modes}}\left(\frac{\mu^2}{K_n}\right)^{2-\frac{D}{2}}\left(1-2x\right)^2, \tag{4.3.7}$$

where

$$K_n = m_n^2 + x\left(1-x\right)\left(-q^2\right) \equiv m_n^2 + \chi^2. \tag{4.3.8}$$

$C(r)$ is the Dynkin index of the representation of the scalar field. Sums of this form have been evaluated in [104, 105], and Equation (4.3.7) can be shown to be equal to

$$\begin{aligned}\sum_n K_n^{\frac{D}{2}-2} = \frac{1}{2} &+ \left(\frac{D}{2}-2\right)\left[\log\left[f\left(i\chi\right)\right] + \log\left[\chi\pi\frac{e^{\frac{k\pi r_c}{2}}}{k}\right] + \log\chi\right] \\ &+ \mathcal{O}\left(D-4\right)^2,\end{aligned} \tag{4.3.9}$$

where

$$\chi = \sqrt{x\left(1-x\right)^2\left(-q^2\right)}. \tag{4.3.10}$$

For our case, a 5-d bulk scalar with $(+\,+)$ boundary conditions whose zero mode has mass m_Φ, we have

$$f\left(m_n\right) = \frac{1}{\pi\alpha}\left(\frac{e^{k\pi r_c}}{k^2}\right)^{\alpha-1}\left(\frac{m_\Phi^2}{m_n^2} + \frac{2+\alpha}{2\alpha-2}\right). \tag{4.3.11}$$

[5] As was mentioned above, if the only field in the theory is the zero mode $(n=0)$ scalar, then the sum consists of just one term—reducing the integral then gives the standard result.

Because we are interested in the effect on the low energy effective field theory ($\sim$ TeV) due to the (massive) KK modes, we have used the asymptotic form of the Bessel functions ($m_n \to 0$), and the fact that $e^{k\pi r_c} >> 1$ [104]. If we consider $k >> m_\Phi$, we can use this form of (4.3.11) in (4.3.9) to evaluate the integral in Eq. (4.3.7), the correction to the coupling is given by:

$$\Delta^{\text{scalar}}_{1-\text{loop}}\left(q^2\right) = -\frac{C\left(r\right)}{6}\left\{\left(\alpha - 1\right)k\pi r_c + \log\frac{\mu}{k}\right\}. \tag{4.3.12}$$

From this analysis, we can construct the full form of the (energy dependent) SM couplings. Luckily, this "rather tedious" analysis has already been done, and we will adapt these results to fit our purposes. The arbitrary mass scale which was introduced in the regularization μ (*cf* Eq. 4.3.7) becomes M_*—the scale of partial unification, and our cutoff. Using Equation (4.3.6), we find [103]:

$$\begin{aligned}
\alpha_i^{-1}\left(p\right) &= \frac{1}{\alpha_{\widetilde{SU}(5)}} + \mathcal{O}\left(\frac{1}{2\pi}\right) \\
&+ \frac{\frac{3}{2}}{12\pi}\left\{-k\pi r_c + \log\frac{k}{M_*}\right\} \\
&+ \frac{5}{24\pi}\left\{22k\pi r_c + 21\log M_*\pi r_c\right\} \\
&+ \frac{b_i}{2\pi}\log\frac{M_*}{p} + \mathcal{O}\left(\frac{1}{2\pi}\frac{M_*^2}{k^2}\right)k\pi r_c \\
&+ 5 - \text{d threshold effects.}
\end{aligned} \tag{4.3.13}$$

The second line gives the contributions to the couplings from the massive scalar in the bulk—if we wish to modify the spectrum of the theory by adding additional higgses, we can add terms similar to these. The next line gives the contributions from the X and Y gauge bosons of $\widetilde{SU}(5)$, and the next line is of the familiar form. Note that the second two lines give a universal (to each of the beta functions, independent of p) correction to $\alpha^{-1}_{\widetilde{SU}(5)}$, suggesting that we take

$$\alpha^{-1}_{eff} = \alpha^{-1}_{\widetilde{SU}(5)} + \Delta\alpha^{-1}. \tag{4.3.14}$$

The correction $(\Delta\alpha^{-1})$ is ~ 45 for the model presented here, and we will take $\alpha_{eff}^{-1} \sim 61$.[6]

We also notice the familiar SM runnings in the fourth line. Because the fermions form complete representations in the GUT, they have no contributions to the runnings. The SM gauge bosons, however, do contribute. They are given as...

$$\begin{array}{llcl} U(1) & b_1 & = & 0 \\ SU(2) & b_2 & = & -\frac{22}{3} \\ SU(3) & b_3 & = & -11. \end{array} \tag{4.3.15}$$

The last $\mathcal{O}\left(\frac{1}{2\pi}\frac{M_*^2}{k^2}\right) k\pi r_c$ are 5-d mass splittings that are calculable, but of sub-sub-leading order and we do not calculate them here.

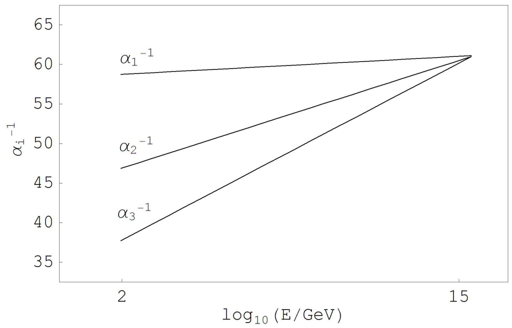
Finally, there are the 5-d threshold effects [115] that are assumed to give the corrections needed for unification. In order to break $\widetilde{SU}(5)$, we need the bulk scalar fields to take on vevs. This is done by choosing a suitable potential for the fields, with minima at the desired mass scale. It is this 5-d potential that gives the threshold corrections needed for unification.

We see that the leading logarithm in each term of Equation (4.3.13) is exactly as expected, from [122]. Also notice that the terms proportional to $k\pi r_c$ are effects due to KK modes. If we were to eliminate these states from the spectrum, we would recover the standard form of a coupling constant plus threshold corrections.[7]

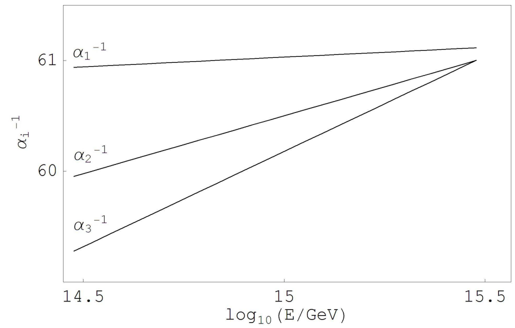
Let us compare the runnings of the couplings in our model to those of the SM. In the $\widetilde{SU}(5)$ models, $SU(5) \supset SU(3)_C \times SU(2)_L$. We take the effective $SU(5)$ coupling,

[6] This value of α_{eff}^{-1} has been chosen because we want a value for the partial GUT coupling that is *less than* the value of $\alpha_{U(1)}^{-1}(M_*)$, so we get unification of $SU(2) \times SU(3)$ before they unify with $U(1)$. See Figure 4.3 below.

[7] See, for example, [123].



(a)



(b)

Figure 4.3. SM beta functions in our model, with corrections from KK modes, GUT scalars, and GUT-mass bosons. Note that the unification at $M_* \sim 3 \times 10^{15}$ GeV is not exact—only $SU(3)$ and $SU(2)$ unify here, as expected.

$\alpha_{eff}^{-1} = 61$. The runnings of $SU(3)$ and $SU(2)$ are given by

$$\alpha_3^{-1}(E) = \alpha_{eff}^{-1} + \frac{b_3}{2\pi} \log \frac{M_*}{E} + \frac{\delta b_3}{2\pi} \log \frac{M_*}{E}, \tag{4.3.16a}$$

$$\alpha_2^{-1}(E) = \alpha_{eff}^{-1} + \frac{b_2}{2\pi} \log \frac{M_*}{E} + \frac{\delta b_2}{2\pi} \log \frac{M_*}{E}. \tag{4.3.16b}$$

One can calculate the δb_i's needed for unification using $\alpha_3(M_Z) \cong 0.1187 \pm 0.0020$, $\alpha_2(M_Z) \cong 0.033961 \pm 0.000006$, and $\alpha_1 \cong 0.017022 \pm 0.000002$ [6], and find

$$SU(3) : \delta b_3 \cong 0.761, \tag{4.3.17a}$$

$$SU(2) : \delta b_2 \cong 0.185. \tag{4.3.17b}$$

These expressions are plotted in Figure 4.3, using $k \sim 10^{18}$ GeV and $M_* \sim 3 \times 10^{15}$ GeV. We have added threshold effects on the order of about 10% to the $U(1)_Y$ in the SM, because we expect that the corrections to the coupling are of the same size as those of the other graphs.

4.3.2 Constraining Randall-Sundrum GUTs

We will now briefly comment on an interesting contstraint revealed in our investigations of this model. It is hoped that the constraints shown here will be of use to anyone wishing to construct Randall-Sundrum GUTs. Further, while it was not

Table 4.1. We look at the scalar content of various non-supersymmetric $SU(5)$ constructions, to get some idea of the representations that are important for model-building. As we include larger and larger bulk scalar reps, the universal contributions to the beta functions approach zero. Calculated here are the contributions from an adjoint (**24**) of gauge bosons, and scalars in various reps.

Model	Bulk Scalars	$\sum_r C(r)$	M_{GUT} (GeV)	k (GeV)	$\Delta/2\pi$
RSFSU5	**10**	1.5	3×10^{15}	10^{18}	44.796
Georgi [44]	**24**	5	3×10^{15}	10^{18}	38.956
Dorsner [124]	**15**,**24**	8.5	3×10^{15}	10^{18}	33.143
HHM I [125]	**24**, **45**	17	3×10^{15}	10^{18}	19.026
HHM II [125]	**45**, **75**	37	3×10^{15}	10^{18}	-14.191

explicitly invesitgated, we believe that these constraints apply also to more general 5-d orbifold GUTs, as well as any construction that gives universal corrections to the beta functions of the model. In general, the contributions to the beta functions coming from the KK modes of the bulk scalars and the GUT gauge bosons are:

$$\frac{\Delta_{1-\mathrm{loop}}}{2\pi} \equiv \frac{C(r)}{12\pi}\left\{-k\pi r_c + \log\frac{k}{M_*}\right\} + \frac{C(N)}{24\pi}\left\{22k\pi r_c + 21\log M_*\pi r_c\right\}. \quad (4.3.18)$$

$C(r)$ is the Dynkin index of the scalar field, and $C(N)$ is the quadratic Casimir operator of the algebra. In what follows, we will look exclusively at $SU(5)$, where $C(N) = 5$.

In some sense, the threshold effects due to the presence of a bulk scalar "compete" with the corrections due to the $SU(5)$ gauge bosons—they give contributions of opposite sign. The scalar loops are weighted by their Dynkin index, a group theory factor that depends on the representation in which the scalars transform. One can compute these, or just look them up [5], and find that larger representations (generally) have larger Dynkin indices.

The bulk field content of the model will govern the types of values that one can obtain for M_* and $\alpha^{-1}_{\widetilde{SU}(5)}$, via universal contributions to the beta functions from the bulk matter, as in Equation (4.3.14). Likewise, any constraints on $\alpha^{-1}_{\widetilde{SU}(5)}$ will tell us the maximum contributions from KK threshold effects, as per [103]. Any effects from some higher unification scale, at M_{GUT}, would still enter the beta function as a correction to the effective α_{eff}^{-1}, and be of the same form as Equation (4.3.13). This will also put constraints on the size and number of bulk fields introduced in the GUT model. In Table 4.1 we have looked at the scalar content of some 4-d $SU(5)$ theories. We note that these models were not built within the RS framework, but we have looked at these examples to get an idea for the important scalar reps used in model building.

In the model presented in this paper, the corrections due to scalars and vectors are ~ 45. Requiring that our threshold effects be on the order of 10% means that $55 \lesssim \alpha_{\widetilde{SU}(5)}^{-1} \lesssim 62$, which in turn forces $10 \lesssim \alpha_{\widetilde{SU}(5)}^{-1} \lesssim 17$. A more interesting case is when the corrections due to the KK modes are negative, as in the case in the HHM II model [125], and is the case when we include larger (or more) scalar reps in our models. In this case, the value of $\alpha_{SU(5)}^{-1}$ must be at least as large as the corrections coming from the bulk scalar and vector representations in order to ensure the positivity of α_{eff}^{-1}, as per Equation (4.3.14). If we were to build an $SU(5)$ RS GUT, placing the matter content of HHM II [125] in the bulk, we could plot Equation (4.3.14), showing where $\alpha_{SU(5)}^{-1}$ becomes negative—see Figure 4.4. This gives the possibility of excluding this model, based on estimates of the size of α_5^{-1}, or equivalently g_5 from Equation (4.3.6).

4.3.3 Proton Lifetime

As was mentioned in Section 4.2, without some exponential tuning of Yukawa couplings, proton decay will be a problem as per Equation (4.2.3). We have solved the hierarchy in the higgs sector of the model by introducing the warp factor, and we must ask ourselves if we are willing to introduce another fine-tuning in the form of an exponentially small Yukawa coupling. Ideally we would like to explain all things in terms of $\mathcal{O}(1)$ parameters, and if we insist on using bulk scalar fields to break $\widetilde{SU}(5)$,

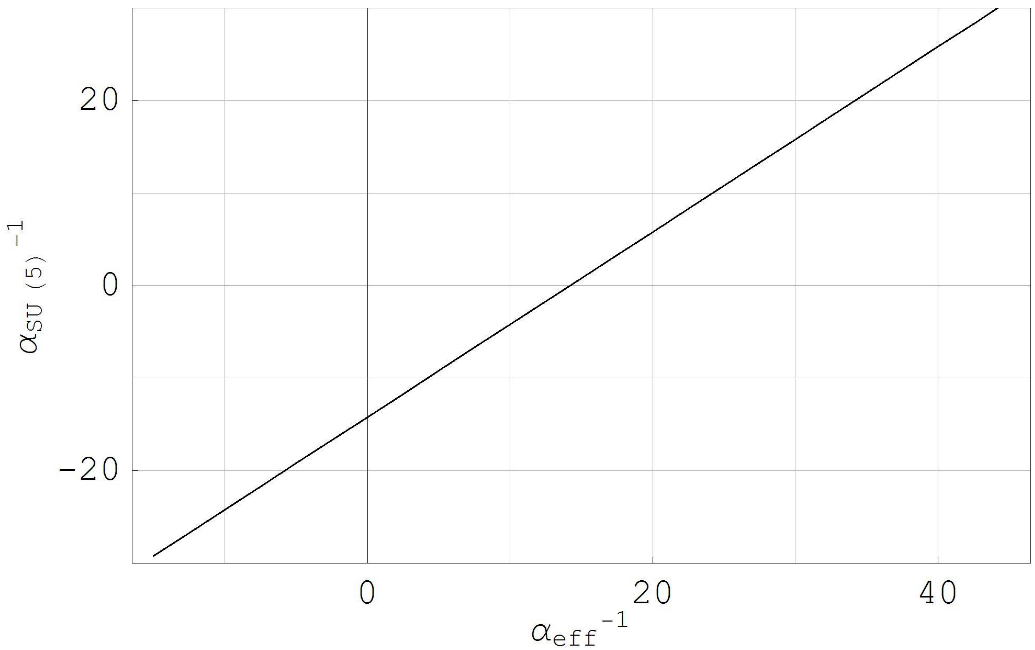


Figure 4.4. A plot of Equation (4.3.14) for the model HHM II [125]. If we choose α_{eff}^{-1} less than 15, we get a negative value for $\alpha_{SU(5)}^{-1}$, which is unphysical.

we may have to take a more creative approach, invoking some (possibly discrete) symmetry to protect baryons in the low energy effective field theory by forbidding terms like Equation (4.2.3). If proton decays are observed in a next generation experiment, we will almost certainly have to accept some fine tuning.[8]

Further, in the RS scenario, one must be careful to check *all* of the possible decay modes of the proton—there will be new decays through KK mode exchange. In general, the KK modes will have masses on the order of a few TeV. The SM fermions may interact with these KK modes to violate bounds on proton decay, and may even produce flavor changing neutral currents at an unacceptable rate [117]. The problem is not limited to RS GUTs, but also to the RS formulation of the SM [111]. The only way to eliminate these problems is to break the GUT symmetry with boundary conditions [115, 116], or invoke some discrete or global symmetry which protects baryon and lepton number.

[8] We already accept such fine-tunings when dealing with the strong CP problem of QCD.

In the standard $\widetilde{SU}(5)$ models, the predominant baryon decay operator is given by [126]:

$$\mathcal{L} \sim \frac{g_5^2}{M_*^2}\left\{-\bar{d}\gamma^\mu d\bar{u}\gamma_\mu \nu + \bar{d}\gamma^\mu u\bar{u}\gamma_\mu \ell^-\right\}, \tag{4.3.19}$$

where ℓ^- is a linear combination of the three (left-handed) leptons. The typical calculation, using $\ell \sim e$, puts the $\widetilde{SU}(5)$ prediction for the proton lifetime

$$\tau_{p\to e^+\pi^0} \sim 10^{33-37}\ \mathrm{y}, \tag{4.3.20}$$

which safely evades the current lower bounds. Let us quickly estimate the proton lifetime using our choice of constants, that is, $\alpha_{eff}^{-1} \sim 61$ and $M_* \sim 3\times 10^{15}$ GeV. Ellis, Lopez and Nanopoulos [127] have estimated the lifetime of the proton in $\widetilde{SU}(5)$ via the decay channel $p \to e^+\pi^0$:

$$\tau_{p\to e^+\pi^0} \cong 1.5\times 10^{33}\left(\frac{M_*}{10^{15}\ \mathrm{GeV}}\right)^4\left(\frac{0.042}{\alpha_5}\right)^2\ \mathrm{y}. \tag{4.3.21}$$

Using our values, $M_* = 3\times 10^{15}$ GeV and $\alpha_{eff}^{-1} = 55$, we find

$$\tau_{p\to e^+\pi^0} \sim 8.0\times 10^{35}\ \mathrm{y}. \tag{4.3.22}$$

Now, Equation (4.3.19) and Equation (4.2.3) give the same term in the effective lagrangian. We may write:

$$\mathcal{L}_{\Delta B\neq 0} \sim \left(\frac{2k}{M_5^3}\frac{\lambda}{N_iN_jN_kN_l}\frac{e^{\delta k\pi r_c}}{\delta}\right)\bar{\psi}_{\bar{\mathbf{5}}}\psi_{\mathbf{5}}\bar{\psi}_{\overline{\mathbf{10}}}\psi_{\mathbf{10}}, \tag{4.3.23}$$

where $\delta > 1$ in general, and λ is some (dimensionless) coupling in the fundamental theory. Comparing with Equation (4.3.19), with the replacement $M_{Pl} \to M_*$, we have[9]

$$g_5^2 \sim \frac{\lambda}{N_iN_jN_kN_l}\frac{e^{\delta k\pi r_c}}{\delta}. \tag{4.3.24}$$

[9] We will leave out factors of $\mathcal{O}(1)$, for comparison's sake.

Because the value of δ must be calculated from the UV completion of the theory, we cannot determine an accurate value for the proton lifetime. We know, however, that δ is $\mathcal{O}(\frac{1}{2})$, and using the relation between 4-d and 5-d mass scales, M_5 is within two or three orders of magnitude of M_*, along with the parameters from Section 4.3.1, we find

$$\lambda \sim 10^{-44}, \tag{4.3.25}$$

in order to match the predictions of the $\widetilde{SU}(5)$ GUTs. One can now see why we were so cavalier with two or three orders of magnitude and a few numbers of order 1! We have created a fine tuning problem thirty orders of magnitude worse than the QCD CP problem from Section 1.2.2. If there exists a (D=6) baryon decay term like Equation (4.2.3) in the GUT, then the Yukawa coupling must be tremendously small to keep the proton sufficiently long lived. Thus, one most likely must invoke discrete symmetries to remove such finely tuned terms.

4.4 Summary

In this Chapter, we have shown how $\widetilde{SU}(5)$ partial unification is obtained in the framework of the Randall-Sundrum proposal. The higgsing from $\widetilde{SU}(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$ was obtained by taking the minimal higgs sector, namely a bulk scalar transforming in the **10** of $SU(5)$. We derived the evolution of the coupling constants on the IR brane, relying on threshold effects on the order of 10%, which are motivated from the GUT symmetry breaking higgs potential. We showed how the threshold effects from bulk fields produced a correction to the unification scale on the IR brane, and were able to derive bounds on the higgs sector by arguing that the five dimensional coupling must be positive. Finally, we showed that, with Yukawa couplings of $\mathcal{O}\left(1\right)$, the proton decays much too fast—if this model is correct, we must accept exponentially tuned $\mathcal{O}\left(e^{k\pi r_c}\right)$ couplings, or invoke some discrete symmetry which prevents the term from occuring in the five-dimensional lagrangian. We have not discussed the anticipated problem of flavor changing neutral currents (FCNC) or neutrino masses, but leave these to future investigations. Because violations of FCNC bounds come from operators like Equation (4.2.3), we expect that we will experience a similar fine-tuning problem as was experienced with proton decays. Finally, the neutrino mass issue is expected to be resolved when a supersymmetric version of this model is formulated [128].

APPENDICES

APPENDIX A

Derivation of the Beta Function for a non-Abelian Gauge Theory

To calculate the SM beta functions, we need to compute the contributions to the gauge boson self energy to $\mathcal{O}(g^2)$, and the corrections to the fermion propagator to $\mathcal{O}(g^3)$. The graphs we need to compute are the same graphs that are listed in Figure A.1.

The counter-terms that we must calculate are given in Figure A.2. The form of these counter-terms are given by

$$\begin{aligned} X &= -i\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\delta_1 \\ Y &= ig\lambda^a\gamma^\mu\delta_2 \\ Z &= ip^\mu\gamma_\mu\delta_3. \end{aligned} \tag{A.0.1}$$

The beta function for a non-Abelian gauge theory is given, to lowest order, by

$$\beta(g) = g\Lambda\frac{\partial}{\partial\Lambda}\left(\frac{1}{2}\delta_1 - \delta_2 + \delta_3\right), \tag{A.0.2}$$

where Λ is the renormalization scale.

A.1 The Corrections to the Boson Self Energy

The first graph is the fermion loop, in Figure A.1, (a). We will consider the fermions (of mass m) to be in representation r of $SU(N)$, generated by λ_a. The

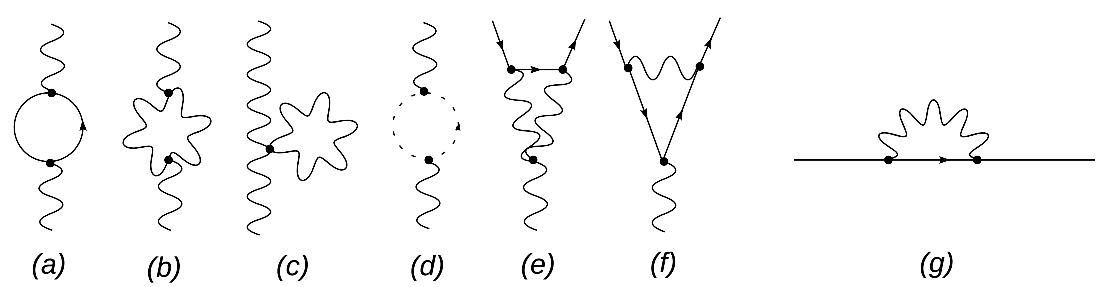


Figure A.1. Graphs needed to compute the beta functions for non-Abelian gauge groups.

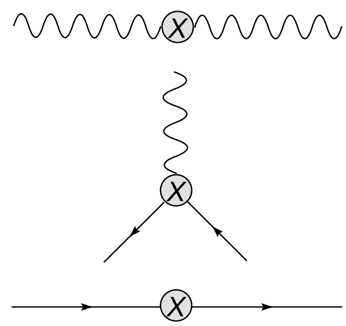


Figure A.2. The relevant counterterms for computing the beta function of a non-Abelian gauge theory.

amplitude is given by:

$$\begin{aligned}(a) \quad &= \quad i\mathrm{Tr}\left[\lambda^a \lambda^b\right]\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\frac{-g^2}{(4\pi)^{D/2}} \\ &\times \int_0^1 dx\ 8x\ \ (1-x)\,\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(m^2 - x\ \ (1-x)\ \ q^2\right)^{2-D/2}}. \end{aligned} \quad \text{(A.1.1)}$$

Now, there will, in general, be a similar contribution for each of the fermions in the theory. Further, we're really only interested in the divergent parts of the above expression—the logarithmic divergences are hidden in the denominator of the integrand of Equation (A.1.1), and are always of the form $\Gamma\left(2-\frac{D}{2}\right)/\Delta^{2-D/2}$, where Δ is defined as

$$\Delta = x\ \ (1-x)\ \ q^2 \quad \text{(A.1.2)}$$

Δ is a function of the momenta invariants, and can always be set to $\Delta = \Lambda^2$ in these calculations. The final contribution to the counterterm X is (taking Equation (A.0.1) into consideration):

$$(a) = \frac{g^2}{16\pi^2}\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}}\left[\frac{4}{3}N_f C\left(r\right)\right]. \quad \text{(A.1.3)}$$

The next task is to compute the boson and ghost loops, as in Figure A.1, (b), (c), and (d). After a bit of algebra and a Wick rotation (b) can be expressed as,

$$\begin{aligned}
(b) \quad &= \quad \tfrac{ig^2}{(4\pi)^{D/2}} C_2(G) \int_0^1 dx \tfrac{1}{\Delta^{2-D/2}} \\
&\times \quad \Big\{ \Gamma\left(1-\tfrac{D}{2}\right) g^{\mu\nu} q^2 \left[\tfrac{3}{2}(D-1)\ x\ (1-x)\right] \\
&+ \quad \Gamma\left(2-\tfrac{D}{2}\right) g^{\mu\nu} q^2 \left[\tfrac{1}{2}(2-x)^2 + \tfrac{1}{2}(1+x)^2\right] \\
&- \quad \Gamma\left(2-\tfrac{D}{2}\right) q^{\mu} q^{\nu} \left[\left(1-\tfrac{D}{2}\right)(1-2x)^2 + (1+x)(2-x)\right] \Big\}.
\end{aligned} \tag{A.1.4}$$

The quantity $C_2(G)$ is called the quadratic Casimir operator of the adjoint representation. For $SU(N)$ algebras, we know $C_2(G) = N$. The next graph, in Figure A.1, (c) is next. We do the same process with this graph—after we introduce a Feynman parameter and Wick rotate, we find

$$\begin{aligned}
(c) \quad &= \quad \tfrac{ig^2}{(4\pi)^{D/2}} C_2(G) \int_0^1 dx \tfrac{1}{\Delta^{2-D/2}} \\
&\times \quad \Big\{ -\Gamma\left(1-\tfrac{D}{2}\right) g^{\mu\nu} q^2 \left[\tfrac{1}{2} D (D-1)\ x\ (1-x)\right] \\
&- \quad \Gamma\left(2-\tfrac{D}{2}\right) g^{\mu\nu} q^2 \left[(D-1)(1-x)^2\right] \Big\}.
\end{aligned} \tag{A.1.5}$$

Finally, the ghost contribution to the process is given by

$$\begin{aligned}
(d) \quad &= \quad \tfrac{ig^2}{(4\pi)^{D/2}} C_2(G) \int_0^1 dx \tfrac{1}{\Delta^{2-D/2}} \\
&\times \quad \Big\{ -\Gamma\left(1-\tfrac{D}{2}\right) g^{\mu\nu} q^2 \left[\tfrac{1}{2} x (1-x)\right] \\
&- \quad \Gamma\left(2-\tfrac{D}{2}\right) q^{\mu} q^{\nu} \left[x\ (1-x)\right] \Big\}.
\end{aligned} \tag{A.1.6}$$

Now, we integrate these three expressions, making the replacement $\Delta \to \Lambda^2$, as we did in Equation (A.1.3). The sum of these four expressions gives the total contribution to the beta function:

$$\delta_1 = \frac{g^2}{16\pi^2} \frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}} \left[\frac{5}{3} N - \frac{4}{3} N_f C(r)\right]. \tag{A.1.7}$$

A.2 The Corrections to the Fermion-Boson Vertex

The corrections to the fermion-boson vertex, as in Figure A.2 Y, are computed in the same manner as the previous two examples. First, we'll compute Figure A.1, (e). The graph has three vertices, and enters at $\mathcal{O}\left(g^3\right)$. After some algebra, we find

$$(e) = \frac{ig^3}{16\pi^2}\left[C_2\left(r\right) - \frac{1}{2}C_2\left(G\right)\right]\lambda^a\gamma^\mu\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}}. \tag{A.2.1}$$

The other graph which contributes is Figure A.1, (f). All said and done, we find

$$(f) = \frac{ig^3}{16\pi^2}\left[\frac{3}{2}C_2\left(G\right)\right]\lambda^a\gamma^\mu\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}}. \tag{A.2.2}$$

Now, these two expressions combine to give the counterterm in Figure A.2 Y. After a bit of Dirac algebra, and arranging of group theory factors, we find the form of Y:

$$\delta_2 = \frac{g^2}{16\pi^2}\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}}\left[N + C_2\left(r\right)\right]. \tag{A.2.3}$$

A.3 The Corrections to the Fermion Propagator

The final graph which we must compute is the correction to the fermion self-energy, Figure A.2, Z. The relevant graph is found in Figure A.1, (g). After going through the usual procedure, we find:

$$(g) = \frac{ig^2}{16\pi^2}k_\mu\gamma^\mu C_2\left(r\right)\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}}, \tag{A.3.1}$$

and comparing to Equation (A.0.1), we find

$$\delta_3 = \frac{g^2}{16\pi^2}C_2\left(r\right)\frac{\Gamma\left(2-\frac{D}{2}\right)}{\left(\Lambda^2\right)^{2-D/2}}. \tag{A.3.2}$$

A.4 The Complete One-Loop Result

We can now sum all three contributions, inserting Equations (A.1.7), (A.2.3), and (A.3.2) into Equation (A.0.2), and find the desired result, namely

$$\beta\left(g\right) = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}N - \frac{4}{3}N_f C\left(r\right)\right] + \mathcal{O}\left(g^5\right). \tag{A.4.1}$$

APPENDIX B

SUSY Formalism

We will specialize our discussion to $\mathcal{N} = 1$ SUSY, wherein we have one bosonic state for each fermionic state in the spectrum. $\mathcal{N} > 1$ SUSY is of interest in some more theoretical constructions, but is generally not of phenomenological importance—$\mathcal{N} = 4$ Yang-Mills theory is very important in formulating the AdS/CFT correspondence [129], for example, but such discussions are outside the scope of this work.

Consider[1] a matrix $M \in SL(2,\mathbb{C})$. The matrix, its complex conjugate M^*, its transpose inverse $\left(M^T\right)^{-1}$ and its hermitian conjugate inverse $\left(M^\dagger\right)^{-1}$ are all representations of $SL(2,\mathbb{C})$—they represent the action of the Lorentz group on two-component Weyl spinors.

The two-component Weyl spinors transform as follows under M:

$$\begin{array}{rclcrcl} \psi'_\alpha &=& M_\alpha{}^\beta \psi_\beta &;& \bar{\psi}'_{\dot{\alpha}} &=& (M^*)_{\dot{\alpha}}{}^{\dot{\beta}}\, \psi_{\dot{\beta}}; \\ \psi'^\alpha &=& (M^{-1})^\alpha{}_\beta\, \psi^\beta &;& \bar{\psi}'^{\dot{\alpha}} &=& \left((M^*)^{-1}\right)^{\dot{\alpha}}{}_{\dot{\beta}}\, \psi^{\dot{\beta}}. \end{array} \tag{B.0.1}$$

The Lorentz group in four dimensions is $SO(4)$, which is isomorphic to $SU(2)_L \times SU(2)_R$. Spinors are denoted with Greek indices[2] —the dotted indices indicate that the spinor transforms under the $\left(0, \frac{1}{2}\right)$ representation of the Lorentz group $(SU(2)_L \times SU(2)_R)$, whereas undotted indices transform under the $\left(\frac{1}{2}, 0\right)$ conjugate representation of the Lorentz group. Note that it is customary to give the Lorentz "spins" of the tensors, as opposed to the representation, as was done for the SM states in Equation (1.1.1).

The Dirac matrices, in the Weyl basis, are given by

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix}, \tag{B.0.2}$$

---

[1] This section follows a similar development as in [130].

[2] Latin indices are space-time indices.

where $\sigma^1, \sigma^2, \sigma^3$ are the Pauli matrices, and

$$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{B.0.3}$$

Now, the Pauli matrices,[3] plus σ^0, form a basis for $SL(2,\mathbb{C})$, and we may always expand any hermitian matrix P as

$$P = P_m \sigma^m = \begin{pmatrix} -P_0 + P_3 & P_1 - iP_2 \\ P_1 + P_2 & -P_0 - P_3 \end{pmatrix} \tag{B.0.4}$$

for P_m real. The matrix P transforms as

$$P' = MPM^\dagger. \tag{B.0.5}$$

We may write this out in terms of Equation (B.0.4), giving

$$\sigma^m P'_m = M\sigma^m P_m M^\dagger. \tag{B.0.6}$$

This shows that the index structure of σ^m is

$$\sigma^m_{\alpha\dot{\alpha}}. \tag{B.0.7}$$

Lorentz scalars can be formed from

$$\psi^\alpha \psi_\alpha, \bar{\psi}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}}, \text{ and } \psi^\alpha \sigma^m_{\alpha\dot{\alpha}} \partial_m \psi^{\dot{\alpha}} \tag{B.0.8}$$

We denote a four-component Dirac spinor as

$$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \tag{B.0.9}$$

and a Majorana spinor as

$$\Psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}. \tag{B.0.10}$$

[3] Or, equivalently, the generators of $SU(2)$ times a half.

We can transform between the two spinor components using the ϵ tensor thusly:

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \tag{B.0.11}$$

where $\epsilon_{21} = \epsilon^{12} = 1, \epsilon_{12} = \epsilon^{21} = -1, \epsilon_{11} = \epsilon_{22} = 0$, and $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_\alpha^\gamma$. In general, the two component spinors ψ and χ commute. Using the spinor summation convention:

$$\psi\chi \equiv \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi, \tag{B.0.12}$$

with a similar expression for the complex conjugates of the $\psi,\ \chi$ fields.

APPENDIX C

Higgsing $SU(5)$ and $\widetilde{SU}(5)$

In this appendix, we will demonstrate how the higgs mechanism works when breaking $SU(5)$ or $\widetilde{SU}(5)$ down to the SM. Because we will need the covariant derivative for calculating the higgsing of $\widetilde{SU}(5)$,[1] we list it here for the **10** [131].

$$D_\mu \Phi = \partial_\mu \Phi - ig\left\{A_{a\mu}\frac{\lambda_a}{2}\Phi + \Phi A_{a\mu}\frac{\lambda_a^T}{2}\right\} - i\tilde{g}\tilde{Y}\tilde{B}_\mu \Phi. \tag{C.0.1}$$

The λ^a's are the 24 generators of SU(5), and the A^a_μ's are the corresponding gauge bosons. The extra $U(1)_{\tilde{Y}}$'s gauge boson is denoted by $\tilde{B}_\mu$.

C.1 Higgsing $SU(5)$

In order to see how $SU(5)$ is broken to the SM, we will look at the gauge kinetic term for our scalar field, Σ:

$$\mathcal{L} = \left(\partial_\mu \sigma_c - \frac{ig}{2}A_{a\mu}\lambda_a\Sigma_c\right)^\dagger \left(\partial^\mu \sigma_c - \frac{ig}{2}A^\mu_a\lambda_a\Sigma_c\right). \tag{C.1.1}$$

The $A_{a\mu}$'s are the (24) gauge bosons of $SU(5)$, while the λ_a's are the generators of the group. One can multiply the matrices out directly, using the form of Σ given in Equation (2.4.2). The matrix $A_{a\mu}\lambda_a \equiv \tilde{A}_\mu$ is given by:

$$\tilde{A}_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} & & & \overline{X}^1_\mu & \overline{Y}^1_\mu \\ & gluons & & \overline{X}^2_\mu & \overline{Y}^2_\mu \\ & & & \overline{X}^3_\mu & \overline{Y}^3_\mu \\ X^1_\mu & X^2_\mu & X^3_\mu & \frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} & W^+_\mu \\ Y^1_\mu & Y^2_\mu & Y^3_\mu & -W^-_\mu & \frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} \end{pmatrix}. \tag{C.1.2}$$

The cross terms cancel, and the gauge boson masses are given by:

$$\mathcal{L}^{mass} = \frac{g^2v^2}{4}\left(\tilde{A}_\mu\Sigma\right)^\dagger\left(\tilde{A}^\mu\Sigma\right) \tag{C.1.3}$$

[1] It is quite difficult to find the proper form of this in the literature, so we have also listed it here for posterity's sake!

The matrix multiplication is straightforward, and one finds

$$\mathcal{L}^{mass} = \frac{25}{9} v^2 g^2 \sum_{i=1}^{3} \left(X^i_\mu \overline{X}^{i\ \mu} + Y^i_\mu \overline{Y}^{i\ \mu} \right). \tag{C.1.4}$$

There is a slightly more elegant way to arrive at this relationship, utilizing some of the properties of the $SU(5)$ algebra. First, we will note that the adjoint representation of $SU(N)$ obeys

$$\begin{aligned} \lambda_a \Sigma_b &= f_{abc} \Sigma_c, \\ [\lambda_a, \lambda_b] &= f_{abc} \lambda_c \end{aligned} \tag{C.1.5}$$

where the f_{abc} are called the structure constants of the algebra. The generators obey

$$\frac{1}{2} \mathrm{Tr}\ \{\lambda_a \lambda_b\} = \frac{1}{2} \mathrm{Tr}\ \left\{\lambda_a^\dagger \lambda_b\right\} = \delta_{ab}. \tag{C.1.6}$$

Now, we note that Equation (C.1.1) can be written as

$$\mathcal{L} = \frac{1}{2} \mathrm{Tr}\ \left\{\lambda_c^\dagger \lambda_k\right\} \left(\partial_\mu \sigma_c - \frac{ig}{2} A_{a\mu} \lambda_a \Sigma_c \right)^\dagger \left(\partial^\mu \sigma_k - \frac{ig}{2} A_i^\mu \lambda_i \Sigma_k \right). \tag{C.1.7}$$

Using Equation (C.1.5), we may write this as

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \mathrm{Tr}\ \left\{\lambda_c^\dagger \lambda_k\right\} \left(\partial_\mu \Sigma_c + \frac{ig}{2} A_{a\mu} \Sigma_b f_{abc} \right)^\dagger \left(\partial^\mu \Sigma_k + \frac{ig}{2} A_i^\mu \Sigma_j f_{ijk} \right), \\ &= \frac{1}{2} \mathrm{Tr} \left\{ \lambda_c^\dagger \left(\partial_\mu \Sigma_c + \frac{ig}{2} A_{a\mu} \Sigma_b f_{abc} \right)^\dagger \left(\partial^\mu \Sigma_k + \frac{ig}{2} A_i^\mu \Sigma_j f_{ijk} \right) \lambda_k \right\}. \end{aligned} \tag{C.1.8}$$

Using the definition of the algebra, Equation (C.1.5), we can write this as

$$\mathcal{L} = \frac{1}{2} \mathrm{Tr} \left\{ \left(\partial_\mu \lambda_c \Sigma_c + \frac{ig}{2} A_{a\mu} \Sigma_b f_{abc} \lambda_c \right)^\dagger \left(\partial^\mu \lambda_k \Sigma_k + \frac{ig}{2} A_i^\mu \Sigma_j f_{ijk} \lambda_k \right) \right\}. \tag{C.1.9}$$

Again, using the $SU(5)$ algebra, we can write

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \mathrm{Tr} \Bigg\{ \left(\partial_\mu \lambda_c \Sigma_c + \frac{ig}{2} \left[A_{a\mu} \lambda_a, \Sigma_b \lambda_b \right] \right)^\dagger \\ &\quad \times \left(\partial^\mu \lambda_k \Sigma_k + \frac{ig}{2} \left[A_j^\mu \lambda_j, \Sigma_k \lambda_k \right] \right) \Bigg\}. \end{aligned} \tag{C.1.10}$$

The mass term is now written as

$$\mathcal{L}^{mass} = \frac{g^2}{4}\mathrm{Tr}\left\{\left[\tilde{A}_\mu, \tilde{\Sigma}\right]^\dagger \left[\tilde{A}_\mu, \tilde{\Sigma}\right]\right\}. \tag{C.1.11}$$

Now, we will insert the form of the vev for Σ, and Equation (C.1.11) can be written as (repeated indices are summed):

$$\mathcal{L}^{mass} = |A^\mu_{ik}|^2 \left(\Sigma_k - \Sigma_i\right)^2, \tag{C.1.12}$$

where $\Sigma = diag\left(\Sigma_1, ..., \Sigma_5\right)$. If we look at the form of the vev for Σ, we find that $\left(\Sigma_k - \Sigma_i\right)^2 = \frac{25}{9}$ or 0, and we find...

$$\mathcal{L}^{mass} = \frac{25}{9}g^2v^2\sum_{i=1}^{3}\left(X^i_\mu \overline{X}^{i\ \mu} + Y^i_\mu \overline{Y}^{i\ \mu}\right). \tag{C.1.13}$$

C.2 Higgsing $\widetilde{SU}(5)$

The higgsing of $\widetilde{SU}(5)$ is achieved in a similar manner as in the previous case. The covariant derivative for the **10** is given by

$$D_\mu\Phi = \partial_\mu\Phi - ig\left\{A_{a\mu}\frac{\lambda_a}{2}\Phi + \Phi A_{a\mu}\frac{\lambda_a^T}{2}\right\} - i\tilde{g}\tilde{Y}\tilde{B}_\mu\Phi. \tag{C.2.1}$$

Note that the actual gauge group of $\widetilde{SU}(5)$ is $SU(5)\times U(1)$. As in the previous case, the terms which contribute to $\mathcal{L}^{mass}$ come from

$$\mathcal{L}^{mass} = \frac{g^2}{4}\left\{A_{a\mu}\lambda_a\Phi + \Phi A_{a\mu}\lambda_a^T\right\}^\dagger\left\{A^\mu_a\lambda_a\Phi + \Phi A^\mu_a\lambda_a^T\right\} + \tilde{g}^2\tilde{Y}^2\tilde{B}_\mu\tilde{B}^\mu\left|\Phi\right|^2. \tag{C.2.2}$$

We have to do a little work to find the correct form of $A^\mu_a\lambda^T_a$, but looking at the form of the generators of $SU(5)$,[2] , the gauge boson matrix, Equation (C.1.2) changes in a simple way:

$$A^\mu_a\lambda^T_a = \frac{1}{\sqrt{2}}\begin{pmatrix} & & & \overline{X}^1_\mu & \overline{Y}^1_\mu \\ & gluons & & \overline{X}^2_\mu & \overline{Y}^2_\mu \\ & & & \overline{X}^3_\mu & \overline{Y}^3_\mu \\ X^1_\mu & X^2_\mu & X^3_\mu & \frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} & W^-_\mu \\ Y^1_\mu & Y^2_\mu & Y^3_\mu & -W^+_\mu & \frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} \end{pmatrix}. \tag{C.2.3}$$

[2] See, for example [14].

One now finds that Equation (C.2.2) gives

$$\mathcal{L}^{mass} \supset \frac{1}{6}v^2 g_5^2 \left\{ \frac{3}{2}B^2 + 6\sum_{i=1}^{3}\left(X_\mu^i \overline{X}^{i\ \mu} + Y_\mu^i \overline{Y}^{i\ \mu} \right) \right\} + \tilde{g}^2 v^2 \tilde{B}^2. \tag{C.2.4}$$

APPENDIX D

Broken Mirror Models: Gauge Groups and States

Table D.1. Broken Mirror Model 1 Gauge Group

Observable	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^3$	$\bar{\psi}^5$
$SU(4)_C$	0	1	-1	0	0
	1	0	-1	0	0
	0	1	1	0	0
$SU(2)_L$	0	0	0	1	-1

Shadow	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{y}_{1,3}$	$\bar{y}_{2,4}$	$\bar{y}_5\bar{w}_1$	$\bar{y}_6\bar{\omega}_2$	$\bar{\omega}_{3,5}$	$\bar{\omega}_{4,6}$	$\bar{\psi'}^4$	$\bar{\psi'}^5$	$\bar{\eta'}^1$	$\bar{\eta'}^2$	$\bar{\eta'}^3$
$SU(2)^3$	$\frac{1}{2}$	-$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	-$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	0	0	-$\frac{1}{2}$	$\frac{1}{2}$	0
	$\frac{1}{2}$	0	-$\frac{1}{2}$	$\frac{1}{2}$	0	-$\frac{1}{2}$	$\frac{1}{2}$	0	-$\frac{1}{2}$	0	0	-$\frac{1}{2}$	0	$\frac{1}{2}$
	0	$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	-$\frac{1}{2}$	0	0	0	0	-$\frac{1}{2}$	$\frac{1}{2}$
$SU(5)$	0	0	0	0	0	0	0	0	0	1	1	0	0	0
	$\frac{1}{2}$	-$\frac{1}{2}$	0	0	-$\frac{1}{2}$	0	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	-$\frac{1}{2}$	0
	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	-$\frac{1}{2}$	0	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
	0	$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	-$\frac{1}{2}$

Hidden	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^3$	$\bar{\psi}^5$	$\bar{\psi'}^1$	$\bar{\psi'}^2$	$\bar{\psi'}^3$	$\bar{\psi'}^3$	$\bar{\psi'}^5$
$SO(10)$	0	0	0	0	0	0	0	0	1	-1
	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-1	$\frac{1}{2}$	-$\frac{1}{2}$
	0	0	0	0	0	0	1	-1	0	0
	0	0	0	0	0	1	-1	0	0	0
	0	0	0	0	0	1	1	0	0	0

Table D.2. Broken Mirror Model 1 States

	$(4_C, 2_L)_O$	$(2^3, 5)_S$	10_H	$4Q_1$	$4Q_2$	$4Q_3$	$4Q_4$	$4Q_5$	$4Q_6$
Singlets									
S_1	(1,1)	(1,1,1,1)	(1)	0	-12	-4	-36	128	80
$\bar{S}_1$	(1,1)	(1,1,1,1)	(1)	0	12	4	36	-128	-80
S_2	(1,1)	(1,1,1,1)	(1)	-4	4	4	-4	176	0
$\bar{S}_2$	(1,1)	(1,1,1,1)	(1)	4	-4	-4	4	-176	0
S_3	(1,1)	(1,1,1,1)	(1)	-4	16	8	32	48	-80
$\bar{S}_3$	(1,1)	(1,1,1,1)	(1)	4	-16	-8	-32	-48	80
Observable									
$QL_1^{n=1,2}$	(4,2)	(1,1,1,1)	(1)	0	-6	-6	6	-40	-80
$QL_2^{n=1,2}$	(4,2)	(1,1,1,1)	(1)	0	0	-4	-56	0	0
$QL_3^{n=1,2}$	(4,2)	(1,1,1,1)	(1)	-6	0	0	0	0	0
$ql_1^{n=1,2}$	(-4,1)	(2,1,1,1)	(1)	-2	-4	12	-12	24	-40
$ql_2^{n=1,2}$	(-4,1)	(1,2,1,1)	(1)	0	-12	8	-28	0	0
$ql_3^{n=1,2}$	(-4,1)	(1,1,2,1)	(1)	0	-6	10	-10	-64	-40
$h_{1,a}^{n=1,2}$	(1,2)	(1,1,1,1)	(1)	0	-6	-10	-50	-40	-80
$h_{2,a}^{n=1,2}$	(1,2)	(1,1,1,1)	(1)	-6	-6	-6	6	-40	-80
$h_{3,a}^{n=1,2}$	(1,2)	(1,1,1,1)	(1)	-6	0	-4	-56	0	0
$H_1^{n=1,2}$	(1,2)	(1,1,1,5)	(1)	2	-8	4	16	80	-16
$H_2^{n=1,2}$	(1,2)	(1,1,1,5)	(1)	2	10	10	-10	-8	-16
$H_3^{n=1,2}$	(1,2)	(1,1,1,5)	(1)	0	-6	6	14	-56	64

Table D.3. Broken Mirror Model 1 States Continued

	$(4_C, 2_L)_O$	$(2^3, 5)_S$	10_H	$4Q_1$	$4Q_2$	$4Q_3$	$4Q_4$	$4Q_5$	$4Q_6$
Shadow									
F_1	(1,1)	(1,1,1,5)	(1)	0	-12	-4	-36	-96	-16
F_2	(1,1)	(1,1,1,5)	(1)	-4	-8	0	-40	80	-16
F_3	(1,1)	(1,1,1,5)	(1)	-4	4	4	-4	-48	-96
$X_1^{n=1,2}$	(1,1)	(2,1,1,1)	(1)	-2	-4	16	44	24	-40
$X_2^{n=1,2}$	(1,1)	(2,1,1,1)	(1)	-2	2	18	-18	64	40
$X_3^{n=1,2}$	(1,1)	(2,1,1,1)	(1)	-2	2	-14	14	112	-40
$X_4^{n=1,2}$	(1,1)	(2,1,1,1)	(1)	-2	20	-8	-12	24	-40
$Y_1^{n=1,2}$	(1,1)	(1,2,1,1)	(1)	0	-6	-18	-2	88	0
$Y_2^{n=1,2}$	(1,1)	(1,2,1,1)	(1)	6	-12	8	-28	0	0
$Y_3^{n=1,2}$	(1,1)	(1,2,1,1)	(1)	0	-6	14	-34	40	80
$Y_4^{n=1,2}$	(1,1)	(1,2,1,1)	(1)	-2	-4	-16	-4	-48	80
$Z_1^{n=1,2}$	(1,1)	(1,1,2,1)	(1)	-2	2	-14	14	-112	40
$Z_2^{n=1,2}$	(1,1)	(1,1,2,1)	(1)	0	-6	14	46	-64	-40
$Z_3^{n=1,2}$	(1,1)	(1,1,2,1)	(1)	0	18	-10	-10	-64	-40
$Z_4^{n=1,2}$	(1,1)	(1,1,2,1)	(1)	6	-6	10	-10	-64	-40
U_1	(1,1)	(2,2,1,1)	(1)	-4	-8	0	40	-24	40
$\bar{U}_1$	(1,1)	(2,2,1,1)	(1)	4	8	0	-40	24	-40
U_2	(1,1)	(2,1,2,1)	(1)	-4	4	4	-4	-48	80
$\bar{U}_2$	(1,1)	(2,1,2,1)	(1)	4	-4	-4	4	48	-80
U_3	(1,1)	(1,2,2,1)	(1)	0	-12	-4	44	24	-40
$\bar{U}_3$	(1,1)	(1,2,2,1)	(1)	0	12	4	-44	-24	40
Hidden									
$T_1^{n=1,2}$	(1,1)	(1,1,1,1)	(16)	-2	-16	0	0	0	0
$T_2^{n=1,2}$	(1,1)	(1,1,1,1)	(16)	-2	2	6	-26	-88	0
$T_3^{n=1,2}$	(1,1)	(1,1,1,1)	(16)	0	0	4	-24	-24	0

Table D.4. Broken Mirror Model 2 Gauge Group

Observable	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^3$	$\bar{\psi}^5$
$SU(4)_C$	0	1	-1	0	0
	1	0	-1	0	0
	0	1	1	0	0
$SU(2)_R$	0	0	0	1	1

Shadow	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{y}_{1,3}$	$\bar{y}_{2,4}$	$\bar{y}_5\bar{w}_1$	$\bar{y}_6\bar{\omega}_2$	$\bar{\omega}_{3,5}$	$\bar{\omega}_{4,6}$	$\bar{\eta'}^1$	$\bar{\eta'}^2$	$\bar{\eta'}^3$
$SU(2)^3$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
$SU(3)$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$

Hidden	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^3$	$\bar{\psi}^5$	$\bar{\psi'}^1$	$\bar{\psi'}^2$	$\bar{\psi'}^3$	$\bar{\psi'}^3$	$\bar{\psi'}^5$
$SO(10)$	0	0	0	0	0	0	0	0	1	1
	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	$\frac{1}{2}$	$\frac{1}{2}$
	0	0	0	0	0	0	1	-1	0	0
	0	0	0	0	0	1	-1	0	0	0
	0	0	0	0	0	1	1	0	0	0
$SU(2)_L$	0	0	0	0	0	0	0	0	1	-1

Table D.5. Broken Mirror Model 2 States

	$(4_C, 2_R)_O$	$(2^3, 3)^S$	$(10, 2_R)^H$	$4Q_1$	$4Q_2$	$4Q_3$	$4Q_4$	$4Q_5$	$4Q_6$	$4Q_7$
Singlets										
$S_1^{n=1,2}$	(1,1)	(1,1,1,1)	(1,1)	12	0	-4	-8	16	0	0
$S_2^{n=1,2}$	(1,1)	(1,1,1,1)	(1,1)	12	0	-4	8	-16	0	0
$S_3^{n=1,2}$	(1,1)	(1,1,1,1)	(1,1)	12	0	0	-6	0	4	24
$S_4^{n=1,2}$	(1,1)	(1,1,1,1)	(1,1)	12	0	0	6	0	4	24
$S_5^{n=1,2}$	(1,1)	(1,1,1,1)	(1,1)	12	0	0	2	-16	4	-24
$S_6^{n=1,2}$	(1,1)	(1,1,1,1)	(1,1)	12	0	0	-2	16	4	-24
S_7	(1,1)	(1,1,1,1)	(1,1)	0	0	0	8	32	0	0
$\bar{S}_7$	(1,1)	(1,1,1,1)	(1,1)	0	0	0	-8	-32	0	0
S_8	(1,1)	(1,1,1,1)	(1,1)	0	0	8	4	16	8	0
$\bar{S}_8$	(1,1)	(1,1,1,1)	(1,1)	0	0	-8	-4	-16	-8	0
S_9	(1,1)	(1,1,1,1)	(1,1)	0	0	8	-4	-16	8	0
$\bar{S}_9$	(1,1)	(1,1,1,1)	(1,1)	0	0	-8	4	16	-8	0
Observable										
$QL_1^{n=1,2}$	(4,2)	(1,1,1,1)	(1,1)	-6	-2	-2	2	-16	0	-8
$QL_2^{n=1,2}$	(4,2)	(1,1,1,1)	(1,1)	-6	-2	-2	-2	16	0	-8
$QL_3^{n=1,2}$	(4,2)	(1,1,1,1)	(1,1)	-6	-2	2	0	0	4	16
$ql_1^{n=1,2}$	(-4,1)	(2,1,1,1)	(1,1)	6	-6	-2	0	0	-4	0
$ql_2^{n=1,2}$	(-4,1)	(1,2,1,1)	(1,1)	6	-6	2	-2	-8	0	0
$ql_3^{n=1,2}$	(-4,1)	(1,1,2,1)	(1,1)	6	-6	2	2	8	0	0
h_1	(1,2)	(2,1,1,1)	(1,1)	0	-8	0	0	0	0	16
$\bar{h}_1$	(1,2)	(2,1,1,1)	(1,1)	0	8	0	0	0	0	-16
h_2	(1,2)	(1,2,1,1)	(1,1)	0	-8	0	-4	8	0	-8
$\bar{h}_2$	(1,2)	(1,2,1,1)	(1,1)	0	8	0	4	-8	0	8
h_3	(1,2)	(1,1,2,1)	(1,1)	0	-8	0	4	-8	0	-8
$\bar{h}_3$	(1,2)	(1,1,2,1)	(1,1)	0	8	0	-4	8	0	8
$H_1^{n=1,2}$	(1,2)	(2,1,1,3)	(1,1)	4	0	4	0	0	0	0
$H_2^{n=1,2}$	(1,2)	(1,2,1,3)	(1,1)	4	0	0	2	8	-4	0
$H_3^{n=1,2}$	(1,2)	(1,1,2,3)	(1,1)	4	0	0	-2	-8	-4	0
$B_1^{n=1,2}$	(6,1)	(1,1,1,1)	(1,1)	0	4	4	6	0	-4	-8
$B_2^{n=1,2}$	(6,1)	(1,1,1,1)	(1,1)	0	4	0	0	0	-8	16
$B_3^{n=1,2}$	(6,1)	(1,1,1,1)	(1,1)	0	4	4	-6	0	-4	-8

Table D.6. Broken Mirror Model 2 States Continued

	$(4_C,2_R)_O$	$(2^3,3)^S$	$(10,2_R)^H$	$4Q_1$	$4Q_2$	$4Q_3$	$4Q_4$	$4Q_5$	$4Q_6$	$4Q_7$
Shadow										
$F_1^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	-4	-8	-4	0	0	0	16
$F_2^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	-4	-8	0	-6	0	4	-8
$F_3^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	-4	-8	0	6	0	4	-8
$F_4^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	-4	8	-4	0	0	0	-16
$F_5^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	-4	8	0	-2	16	4	8
$F_6^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	-4	8	0	2	-16	4	8
$F_7^{n=1,2}$	(1,1)	(1,1,1,3)	(1,1)	-8	0	-8	0	0	0	0
$\bar{F}_7^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	8	0	8	0	0	0	0
$F_8^{n=1,2}$	(1,1)	(1,1,1,3)	(1,1)	-8	0	0	-4	-16	8	0
$\bar{F}_8^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	8	0	0	4	16	-8	0
$F_9^{n=1,2}$	(1,1)	(1,1,1,3)	(1,1)	-8	0	0	4	16	8	0
$\bar{F}_9^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,1)	8	0	0	-4	-16	-8	0
$X_1^{n=1,2}$	(1,1)	(2,2,1,1)	(1,1)	-12	0	0	-2	-8	-4	0
X_2	(1,1)	(2,2,1,1)	(1,1)	0	0	0	-4	8	0	-24
X_3	(1,1)	(2,2,1,1)	(1,1)	0	0	0	4	-8	0	24
$Y_1^{n=1,2}$	(1,1)	(2,1,2,1)	(1,1)	-12	0	0	2	8	-4	0
Y_2	(1,1)	(2,1,2,1)	(1,1)	0	0	0	4	-8	0	-24
Y_3	(1,1)	(2,1,2,1)	(1,1)	0	0	0	-4	8	0	24
$Z_1^{n=1,2}$	(1,1)	(1,2,2,1)	(1,1)	-12	0	4	0	0	0	0
Z_2	(1,1)	(1,2,2,1)	(1,1)	0	0	0	-8	16	0	0
Z_3	(1,1)	(1,2,2,1)	(1,1)	0	0	0	8	-16	0	0
Hidden										
$T_1^{n=1,2}$	(1,1)	(1,1,1,1)	(10,1)	0	-4	4	-2	16	-4	8
$T_2^{n=1,2}$	(1,1)	(1,1,1,1)	(10,1)	0	-4	4	2	-16	-4	8
$T_3^{n=1,2}$	(1,1)	(1,1,1,1)	(10,1)	0	-4	0	0	0	-8	-16
$\bar{F}_1^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,2)	-4	-4	4	0	0	0	-16
$\bar{F}_2^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,2)	-4	-4	0	-6	0	-4	8
$\bar{F}_3^{n=1,2}$	(1,1)	(1,1,1,$\bar{3}$)	(1,2)	-4	-4	0	6	0	-4	8
$D_1^{n=1,2}$	(1,1)	(1,1,1,1)	(1,2)	12	4	4	0	0	0	16
$D_2^{n=1,2}$	(1,1)	(1,1,1,1)	(1,2)	12	4	0	-2	16	-4	-8
$D_3^{n=1,2}$	(1,1)	(1,1,1,1)	(1,2)	12	4	0	2	-16	-4	-8

BIBLIOGRAPHY

[1] A. V. Gladyshev and D. I. Kazakov, Supersymmetry and LHC, (2006).

[2] W. Cottingham and D. Greenwood, *An Introduction to the Standard Model of Particle Physics*, Cambridge University Press, Cambridge, UK, 2005.

[3] D. J. Gross and F. Wilczek, Ultraviolet behavior of non-Abelian gauge theories, Phys. Rev. Lett. **30**, 1343–1346 (1973).

[4] M. Creutz, *Quarks, Gluons and Lattices*, Cambridge University Press, Cambridge, UK, 1985.

[5] R. Slansky, Group theory for unified model building, Phys. Rept. **79**, 1–128 (1981).

[6] S. Eidelman et al., Review of particle physics, Phys. Lett. **B592**, 1 (2004).

[7] A. Heister et al., Measurements of the strong coupling constant and the QCD colour factors using four-jet observables from hadronic Z decays, Eur. Phys. J. **C27**, 1–17 (2003).

[8] D. Kcira, Jet production at HERA and measurements of the strong coupling constant α_s, in *Lake Louise Winter Institute: Fundamental Interactions*, edited by C. E. Carlson, Lake Louise, Alberta, Canada, February 2005.

[9] M. Rescigno, B meson mixing at CDF II, Nucl. Phys. Proc. Suppl. **156**, 43–47 (2006).

[10] T. Bose, b/d mixing and prospects for b/s mixing at D0, Int. J. Mod. Phys. **A20**, 3513–3517 (2005).

[11] M. Bona et al., The 2004 UTfit Collaboration report on the status of the unitarity triangle in the Standard Model, JHEP **07**, 028 (2005).

[12] G. W. Bennett et al., Final report of the muon E821 anomalous magnetic moment measurement at BNL, Phys. Rev. **D73**, 072003 (2006).

[13] M. Passera, The Standard Model prediction of the muon anomalous magnetic moment, J. Phys. **G31**, R75–R94 (2005).

[14] R. N. Mohapatra, *Unification and Supersymmetry: The Frontiers of Quark-Lepton Physics*, Springer, New York, New York, 2003.

[15] H. Georgi, *Lie Algebras in Particle Physics: From Isospin to Unified Theories*, Perseus Books, Reading, Massachusettes, 1999.

[16] S. W. Allen, R. W. Schmidt, and S. L. Bridle, A preference for a non-zero neutrino mass from cosmological data, Mon. Not. Roy. Astron. Soc. **346**, 593 (2003).

[17] S. Weinberg, Non-Abelian gauge theories of the strong interaction, Phys. Rev. Lett. **31**, 494–497 (1973).

[18] D. V. Nanopoulos, Towards a renormalizable unified gauge theory of strong, electromagnetic and weak interactions, Lett. Nuovo Cim. **8**, 873–877 (1973).

[19] Y. Fukuda et al., Evidence for oscillation of atmospheric neutrinos, Phys. Rev. Lett. **81**, 1562–1567 (1998).

[20] M. Gell-Mann, P. Ramond, and R. Slansky, Complex Spinors And Unified Theories, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman, North Holland Publ. Co., Amsterdam, Holland, 1979.

[21] T. Yanagida, Horizontal gauge symmetry and masses of neutrinos, in *The Proceedings of the Workshop on the Baryon Number and of the Universe and Unified Theories*, Tsukuba, Japan, February 1979.

[22] M. V. Romalis, W. C. Griffith, and E. N. Fortson, A new limit on the permanent electric dipole moment of Hg- 199, Phys. Rev. Lett. **86**, 2505–2508 (2001).

[23] G. Senjanovic and F. W. Stecker, Soft CP violation and the global matter - antimatter symmetry of the universe, Phys. Lett. **B96**, 285 (1980).

[24] C. Caprini, S. Biller, and P. G. Ferreira, Constraints on the electrical charge asymmetry of the universe, JCAP **0502**, 006 (2005).

[25] K. S. Babu and R. N. Mohapatra, Quantization of electric charge from anomaly constraints and a Majorana neutrino, Phys. Rev. **D41**, 271 (1990).

[26] B. F. L. Ward, Quantum corrections to Newton's law, Mod. Phys. Lett. **A17**, 2371–2382 (2002).

[27] S. Weinberg, Effective gauge theories, Phys. Lett. **B91**, 51 (1980).

[28] S. Weinberg, Anthropic bound on the cosmological constant, Phys. Rev. Lett. **59**, 2607 (1987).

[29] J.-M. Lamarre et al., The Planck High Frequency Instrument, a 3rd generation CMB experiment, and a full sky submillimeter survey, in *The Procedings of the Workshop on The Cosmic Microwave Background Radiation and its Polarization*, Minneapolis, Minnesota, 19-22 March 2003.

[30] J. Wess and B. Zumino, Supergauge transformations in four dimensions, Nucl. Phys. **B70**, 39–50 (1974).

[31] Y. A. Golfand and E. P. Likhtman, Extension of the algebra of Poincaré group generators and violation of P invariance, JETP Lett. **13**, 323–326 (1971).

[32] D. V. Volkov and V. P. Akulov, Possible universal neutrino interaction, JETP Lett. **16**, 438–440 (1972).

[33] J.-L. Gervais and B. Sakita, Field theory interpretation of supergauges in dual models, Nucl. Phys. **B34**, 632–639 (1971).

[34] S. R. Coleman and J. Mandula, All possible symmetries of the S matrix, Phys. Rev. **159**, 1251–1256 (1967).

[35] R. Haag, J. T. Lopuszanski, and M. Sohnius, All possible of generators of supersymmetries of the S matrix, Nucl. Phys. **B88**, 257 (1975).

[36] A. Salam and J. A. Strathdee, Supersymmetry and Superspace, Fortschr. Phys. **26**, 57 (1978).

[37] H. E. Haber and G. L. Kane, The search for supersymmetry: Probing physics beyond the Standard Model, Phys. Rept. **117**, 75 (1985).

[38] C. F. Kolda, Gauge-mediated supersymmetry breaking: Introduction, review and update, Nucl. Phys. Proc. Suppl. **62**, 266–275 (1998).

[39] R. Barbieri, S. Ferrara, and C. A. Savoy, Gauge models with spontaneously broken local supersymmetry, Phys. Lett. **B119**, 343 (1982).

[40] S. P. Martin, A supersymmetry primer, (1997).

[41] K. R. Dienes, E. Dudas, and T. Gherghetta, TeV-scale GUTs, (1998).

[42] S. Weinberg, Upper bound on gauge fermion masses, Phys. Rev. Lett. **50**, 387 (1983).

[43] H. Goldberg, Constraint on the photino mass from cosmology, Phys. Rev. Lett. **50**, 1419 (1983).

[44] H. Georgi and S. L. Glashow, Unitiy of all elementary forces, Phys. Rev. Lett. **32**, 438–441 (1974).

[45] H. Georgi, Particles and fields, in *The Proceedings of the American Physical Society Division of Particles and Fields*, edited by C. E. Carlson, Albany, New York, August 1974.

[46] H. Fritzsch and P. Minkowski, Unified interactions of hadrons and leptons, Ann. Phys. **93**, 193–266 (1975).

[47] Y. Hayato et al., Search for proton decay through p $\rightarrow \bar{\nu} K^{+}$ in a large water Cherenkov detector, Phys. Rev. Lett. **83**, 1529–1533 (1999).

[48] H. Murayama and A. Pierce, Not even decoupling can save minimal supersymmetric $SU(5)$, Phys. Rev. **D65**, 055009 (2002).

[49] J. C. Pati and A. Salam, Lepton number as the fourth color, Phys. Rev. **D10**, 275–289 (1974).

[50] V. S. Kaplunovsky, Mass scales of the string unification, Phys. Rev. Lett. **55**, 1036 (1985).

[51] J. Perkins et al., Stringent phenomenological investigation into heterotic string optical unification, (2005).

[52] J. Giedt, Optical unification, Mod. Phys. Lett. **A18**, 1625–1633 (2003).

[53] M. Gell-Mann, P. Ramond, and R. Slansky, Color embeddings, charge assignments, and proton stability in unified gauge theories, Rev. Mod. Phys. **50**, 721 (1978).

[54] H. Georgi and D. V. Nanopoulos, Ordinary predictions from grand principles: t quark mass in $O(10)$, Nucl. Phys. **B155**, 52 (1979).

[55] S. M. Barr, A new symmetry breaking pattern for $SO(10)$ and proton decay, Phys. Lett. **B112**, 219 (1982).

[56] J. P. Derendinger, J. E. Kim, and D. V. Nanopoulos, Anti - $SU(5)$, Phys. Lett. **B139**, 170 (1984).

[57] P. K. Mohapatra, R. N. Mohapatra, and P. B. Pal, Implications of E_6 grand unification, Phys. Rev. **D33**, 2010 (1986).

[58] F. Gursey, P. Ramond, and P. Sikivie, A universal gauge theory model based on E_6, Phys. Lett. **B60**, 177 (1976).

[59] A. de Rujula, H. Georgi, and S. Glashow, The Trinification of all forces, in *The Fifth Workshop on Grand Unification*, edited by K. Kang, H. Fried, and P. Frampton, 1984.

[60] R. N. Mohapatra and J. C. Pati, Left-right gauge symmetry and an 'isoconjugate' model of CP violation, Phys. Rev. **D11**, 566–571 (1975).

[61] G. Senjanovic and R. N. Mohapatra, Exact left-right symmetry and spontaneous violation of parity, Phys. Rev. **D12**, 1502 (1975).

[62] A. Davidson, $B - L$ as the fourth color, quark–lepton correspondence, and natural masslessness of neutrinos within a generalized WS model, Phys. Rev. **D20**, 776 (1979).

[63] R. E. Marshak and R. N. Mohapatra, Quark–lepton symmetry and $B - L$ as the $U(1)$ generator of the electroweak symmetry group, Phys. Lett. **B91**, 222 (1980).

[64] R. M. Bionta et al., Observation of a neutrino burst in coincidence with SN1987A in the large Magellinic cloud, Phys. Rev. Lett. **58**, 1494 (1987).

[65] K. Hirata et al., Observation of a neutrino burst from the supernova SN1987A, Phys. Rev. Lett. **58**, 1490–1493 (1987).

[66] I. Antoniadis, J. R. Ellis, J. S. Hagelin, and D. V. Nanopoulos, The flipped $SU(5) \times U(1)$ string model revamped, Phys. Lett. **B231**, 65 (1989).

[67] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, String derived MSSM and M-theory unification, Phys. Lett. **B455**, 135–146 (1999).

[68] P. H. Frampton, S. Nandi, and J. J. G. Scanio, Estimate of flavor number from $SU(5)$ grand unification, Phys. Lett. **B85**, 225 (1979).

[69] H. Georgi and C. Jarlskog, A new lepton–quark mass relation in a unified theory, Phys. Lett. **B86**, 297–300 (1979).

[70] K. R. Dienes and A. E. Faraggi, Gauge coupling unification in realistic free fermionic string models, Nucl. Phys. **B457**, 409–483 (1995).

[71] B. Dundee, J. Perkins, and G. Cleaver, Observable / hidden broken symmetry for symmetric boundary conditions, Int. Jour. of Mod. Phys. A **22**(16), 3367–3387 (2005).

[72] L. Smolin, Loop quantum gravity and Planck scale phenomenology, Lect. Notes Phys. **669**, 363–408 (2005).

[73] J. Polchinski, *String Theory Volume I: An Introduction to the Bosonic String*, Cambridge University Press, Cambridge, UK, 1998.

[74] J. Polchinski, *String Theory Volume II: Superstring Theory and Beyond*, Cambridge University Press, Cambridge, UK, 1998.

[75] E. Witten, String theory dynamics in various dimensions, Nucl. Phys. **B443**, 85–126 (1995).

[76] J. Polchinski, Dirichlet-branes and Ramond-Ramond charges, Phys. Rev. Lett. **75**, 4724–4727 (1995).

[77] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, The heterotic string, Phys. Rev. Lett. **54**, 502–505 (1985).

[78] F. Gliozzi, J. Scherk, and D. I. Olive, Supersymmetry, supergravity theories and the dual spinor model, Nucl. Phys. **B122**, 253–290 (1977).

[79] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Construction of fermionic strin models in four dimensions, Nucl. Phys. **B288**, 1 (1987).

[80] H. Kawai, D. C. Lewellen, J. A. Schwartz, and S. H. H. Tye, The spin structure construction of string models and multiloop modular invariance, Nucl. Phys. **B299**, 431 (1988).

[81] I. Antoniadis, C. P. Bachas, and C. Kounnas, Four dimensional super-strings, Nucl. Phys. **B289**, 87 (1987).

[82] I. Antoniadis and C. Bachas, 4-D fermionic superstrings with arbitrary twists, Nucl. Phys. **B298**, 586 (1988).

[83] A. E. Faraggi, Self-duality and vacuum selection, Int. J. Mod. Phys. **A19**, 5523–5559 (2004).

[84] B. Freivogel and L. Susskind, A framework for the landscape, Phys. Rev. **D70**, 126007 (2004).

[85] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, De Sitter vacua in string theory, Phys. Rev. **D68**, 046005 (2003).

[86] T. Banks, M. Dine, and L. Motl, On anthropic solutions of the cosmological constant problem, JHEP **01**, 031 (2001).

[87] J. P. Gauntlett, D. Martelli, J. Sparks, and D. Waldram, Supersymmetric AdS_5 solutions of M-theory, Class. Quant. Grav. **21**, 4335–4366 (2004).

[88] T. H. Reiprich, The galaxy cluster X-ray luminosity–gravitational mass relation in the light of the WMAP 3rd year data, (2006).

[89] Z. G. Berezhiani, A. D. Dolgov, and R. N. Mohapatra, Asymmetric inflationary reheating and the nature of mirror universe, Phys. Lett. **B375**, 26–36 (1996).

[90] R. N. Mohapatra and V. L. Teplitz, Structures in the mirror universe, Astrophys. J. **478**, 29–38 (1997).

[91] Z. Berezhiani, D. Comelli, and F. L. Villante, The early mirror universe: Inflation, baryogenesis, nucleosynthesis and dark matter, Phys. Lett. **B503**, 362–375 (2001).

[92] Z. K. Silagadze, TeV scale gravity, mirror universe, and ... dinosaurs, Acta Phys. Polon. **B32**, 99–128 (2001).

[93] P. Ciarcelluti, *Cosmology of the mirror universe*, PhD thesis, University of Rome, Rome, Italy, 2003.

[94] P. Horava and E. Witten, Eleven-dimensional supergravity on a manifold with boundary, Nucl. Phys. **B475**, 94–114 (1996).

[95] K. Hori, Trieste lectures on mirror symmetry, in *Superstrings and Related Matters*, Trieste, Italy, 18–26 March 2002.

[96] G. B. Cleaver, A. E. Faraggi, and C. Savage, Left-right symmetric heterotic-string derived models, Phys. Rev. **D63**, 066001 (2001).

[97] G. B. Cleaver, D. J. Clements, and A. E. Faraggi, Flat directions in left-right symmetric string derived models, Phys. Rev. **D65**, 106003 (2002).

[98] M. B. Green and J. H. Schwarz, Infinity cancellations in $SO(32)$ superstring theory, Phys. Lett. **B151**, 21–25 (1985).

[99] M. Dine, N. Seiberg, and E. Witten, Fayet-Iliopoulos terms in string theory, Nucl. Phys. **B289**, 589 (1987).

[100] J. J. Atick, L. J. Dixon, and A. Sen, String calculation of Fayet-Iliopoulos D terms in arbitrary supersymmetric compactifications, Nucl. Phys. **B292**, 109–149 (1987).

[101] B. Dundee and G. Cleaver, Randall-Sundrum and flipped $SU(5)$, BU-HEPP 06-06, CASPER 06-02.

[102] K. Agashe, A. Delgado, and R. Sundrum, Gauge coupling renormalization in RS1, Nucl. Phys. **B643**, 172–186 (2002).

[103] K.-w. Choi and I.-W. Kim, One loop gauge couplings in AdS_5, Phys. Rev. **D67**, 045005 (2003).

[104] R. Contino, P. Creminelli, and E. Trincherini, Holographic evolution of gauge couplings, JHEP **10**, 029 (2002).

[105] W. D. Goldberger and I. Z. Rothstein, Effective field theory and unification in AdS backgrounds, Phys. Rev. **D68**, 125011 (2003).

[106] L. Randall and R. Sundrum, A large mass hierarchy from a small extra dimension, Phys. Rev. Lett. **83**, 3370–3373 (1999).

[107] L. Randall and R. Sundrum, An alternative to compactification, Phys. Rev. Lett. **83**, 4690–4693 (1999).

[108] W. D. Goldberger and M. B. Wise, Modulus stabilization with bulk fields, Phys. Rev. Lett. **83**, 4922–4925 (1999).

[109] S. Kaluza, Zum Unitatsproblem in der Physik (On the problem of unity in physics), Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.) , 966–972 (1921).

[110] O. Klein, Quantum theory and the five-dimensional theory of relativity, Z. Phys **37**, 895–906 (1926).

[111] T. Gherghetta and A. Pomarol, Bulk fields and supersymmetry in a slice of AdS, Nucl. Phys. **B586**, 141–162 (2000).

[112] Y. Grossman and M. Neubert, Neutrino masses and mixings in non-factorizable geometry, Phys. Lett. **B474**, 361–371 (2000).

[113] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Bulk gauge fields in the Randall-Sundrum model, Phys. Lett. **B473**, 43–49 (2000).

[114] A. Pomarol, Gauge bosons in a five-dimensional theory with localized gravity, Phys. Lett. **B486**, 153–157 (2000).

[115] K. Agashe, A. Delgado, and R. Sundrum, Grand unification in RS1, Ann. Phys. **304**, 145–164 (2003).

[116] W. D. Goldberger, Y. Nomura, and D. R. Smith, Warped supersymmetric grand unification, Phys. Rev. **D67**, 075021 (2003).

[117] K. Agashe and G. Servant, Baryon number in warped GUTs: Model building and (dark matter related) phenomenology, JCAP **0502**, 002 (2005).

[118] S. M. Barr and I. Dorsner, Unifying flipped $SU(5)$ in five dimensions, Phys. Rev. **D66**, 065013 (2002).

[119] I. Dorsner, Flipping $SU(5)$ towards five dimensional unification, Phys. Rev. **D69**, 056003 (2004).

[120] B. Dundee and G. B. Cleaver, In progress.

[121] D. Binosi and L. Theußl, JaxoDraw: A graphical user interface for drawing Feynman diagrams, Comp. Phys. Comm. **161**, 76–86 (2004).

[122] A. Pomarol, Grand unified theories without the desert, Phys. Rev. Lett. **85**, 4004–4007 (2000).

[123] G. G. Ross, *Grand Unified Theories*, Westview Press, Oxford, UK, 1984.

[124] I. Dorsner, Minimal realistic SU(5) scenario, (2006).

[125] H. Hayashi, A. Murayama, and M. J. Hayashi, ROLES OF 45 HIGGS IN SU(5) GUT, Phys. Rev. **D26**, 1185 (1982).

[126] J. R. Ellis, J. S. Hagelin, S. Kelley, and D. V. Nanopoulos, Aspects of the flipped unification of strong, weak and electromagnetic interactions, Nucl. Phys. **B311**, 1 (1988).

[127] J. R. Ellis, J. L. Lopez, and D. V. Nanopoulos, Lowering α_s by flipping $SU(5)$, Phys. Lett. **B371**, 65–70 (1996).

[128] S. A. Abel, Neutrino masses in flipped $SU(5)$, Phys. Lett. **B234**, 113 (1990).

[129] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. **2**, 231–252 (1998).

[130] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, Princeton, New Jersey, 1992.

[131] B. M. Oginni, $SU(5)$ Grand Unified Theory, Master's thesis, The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34100 Trieste, Italy, August 2002.