ABSTRACT

Detecting Sigmoidal Trajectories in Structured Latent Curve Models: A Fit Measure Performance and Parameter Recovery Simulation Study

Kevin E. Wells, Ph.D.

Chairperson: Grant B. Morgan, Ph.D.

This simulation study examined parameter estimate recovery and model selection of structured latent curve models under varying conditions to provide recommendations on how to properly format longitudinal research when there is an *a priori* hypothesis of sigmoidal growth. To examine model selection, sigmoidal models were generated using the first order Taylor series approximation method detailed in Browne and du Toit (1991) and analyzed as sigmoidal, linear, quadratic, and cubic. Eleven fit measures were assessed to determine their performance in selecting true sigmoidal models over competing incorrect models. The information criteria examined were the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Bayesian Information Criterion with the Sclove sample size penalty adjustment (aBIC), Consistent AIC (CAIC), Draper Information Criterion (DIC), the Hannan and Quinn adjustment of AIC (HQ), and Sugiura's adjustment of the AIC (AICc). Other fit measures investigated were the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), the Root Mean Square Error of Approximation (RMSEA), and the Standardized Root Mean Residual (SRMR). Parameter estimates were recovered from converged correctly specified sigmoidal models to assess the amount of bias present. Manipulated design factors for this study included sample size (50, 100, 200, 500, 1,000, and 1,500), the number of repeated measures (six, eight, and 10), the location of the inflection point within the measurement window ($\delta^* = .25, .375, .5$), and the rate of change at the inflection point ($\rho^* = .125, .1875, .25$). In addition to model selection and parameter estimate bias, convergence proportions, coverage, and standard error bias were also investigated. Results and recommendations are provided within.

Detecting Sigmoidal Trajectories in Structured Latent Curve Models: A Fit Measure Performance and Parameter Recovery Simulation Study

by

Kevin E. Wells, B.A.

A Dissertation

Approved by the Department of Educational Psychology

Terrill F. Saxon, Ph.D., Chairperson

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Approved by the Dissertation Committee

Grant B. Morgan, Ph.D., Chairperson

Susan Johnsen, Ph.D.

Terrill Saxon, Ph.D.

Tracey Sulak, Ph.D.

Robin Wakefield, Ph.D.

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J. Larry Lyon, Ph.D., Dean

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DEDICATION

To Sunny, Nolan, and Sabrina - "Team Wells"

CHAPTER ONE

Introduction

Latent growth curve models are one of several common methods used in educational, psychological, and social science research to analyze longitudinal data. Since the inclusion of growth curve models within a structural equation modeling framework (Meredith & Tisak, 1984, 1990), longitudinal data have typically been modeled as one of the family of polynomial functions. That is, they have been modeled as linear, quadratic, or cubic functions (Bollen & Curran, 2006; Meredith & Tisak, 1990; Preacher, Wichman, MacCallum, & Briggs, 2008). Subsequently, structured latent curve models (SLCM) (Browne & du Toit, 1991) have been introduced that allow for modeling of a variety of nonlinear functions including sigmoidal (S-shaped) growth patterns. These sigmoidal growth models have several advantages over traditional linear/quadratic/cubic modeling of data when used appropriately. First, unlike some higher-order polynomial models, sigmoidal growth patterns are easily interpreted and can easily map onto developmental theory. Among the parameters sigmoidal models can estimate are the total change, the maximum rate of change, the timing of the maximum change, and the amount of change that occurred prior to the timing of the maximum change. Second, unlike the linear, quadratic, and cubic functions that are unbounded, the sigmoidal models are bounded. Sigmoidal models have upper and lower asymptotes whereas unbounded functions trend towards positive or negative infinity given ample time. This is an important distinction because certain types of longitudinal trends found in educational and social science research are bounded on the upper and lower ends and do not continue *ad infinitum*. It is, therefore, more consistent with the nature of these constructs, when applicable, to model bounded growth with an easily interpreted bounded model.

Background of the Problem

Analysis of longitudinal trajectories are frequently used in educational and social science research and have been for many years (Bollen & Curran, 2006). The development of latent growth curve models and their incorporation into the structural equation modeling framework have greatly increased the utility of longitudinal modeling. One of the prime advantages to using latent growth curve models to analyze patterns of change over time is the ability to assess global model fit. Unfortunately, with latent growth curve models, the types of growth trajectories modeled in the vast majority of research have been limited to polynomial type growth patterns, i.e. linear, quadratic, and cubic. These models, while generally useful in describing local trajectories, do not accurately represent growth within a bounded system. They also offer only limited predictive ability because polynomial trajectories do not necessarily map onto theoretical expectations of how real world growth occurs (Preacher, 2010). Nonlinear growth trajectories have seen limited use in educational and social science research, but have been widely used in other fields. Many of the nonlinear trajectories used have been sigmoidal (S-shaped) functions. Of these sigmoidal functions, a family of models including that includes the Gompertz curve (Gompertz, 1825), Logistic curve (Verhulst, 1845), and the generalized logistic curve, also known as the Richards curve, (Richards, 1959) has been commonly used for modeling bounded growth patterns. Sigmoidal models, unlike some of the polynomial models, align well with developmental theory and can be meaningfully interpreted. Among the parameter estimates that these types of models can recover are the lower and upper bounds of growth, total change, the maximum rate of change, the timing of the maximum change, and the amount of change that occurred prior to the timing of the maximum change.

Browne and du Toit (1991) introduced SLCMs that allow nonlinear functions to be modeled within a latent growth curve framework. By using a first-order Taylor series approximation, the partial derivatives with respect to the latent random coefficients only depend on fixed-effect parameters. This has the effect of making the model linear with respect to the latent variables and thus able to be estimated using common structural equation modeling software. Although these models can be estimated using common software, these functions are not native, not easily implemented, and have not been well studied in spite of their potential usefulness.

Of the studies that have used SLCMs, most have done so as demonstrations of the method. Applied examples have sample sizes ranging from just over 100 to as large as 20k+. Yet, to date, there have been no published methodological studies examining how easily or accurately SLCMs can be selected over competing incorrect models, nor the extent to which recovered parameter estimates may be biased. This Monte Carlo simulation provides recommendations for model selection over competing linear and nonlinear models and provides guidance on properly structuring studies to minimize parameter estimate bias. This simulation examined four fully crossed factors: number of repeated measures, sample size, maximum rate of change, and the timing of the inflection point. Each dataset was generated with a true sigmoidal growth pattern and then analyzed as linear, quadratic, cubic, and sigmoidal. A number of fit measures were used to select the best-fitting model and parameter estimates of the sigmoidal models were recovered.

Prior to modeling applied data using SLCM models, it is important to understand how they work. Finding the conditions under which parameter estimates may become excessively biased and the conditions under which a sigmoidal model can be correctly selected over incorrect competing models provides guidance for proper use of this type of model. This study provides recommendations for how to properly structure longitudinal research to ensure adequate sample size and number of repeated measures when there is an *a priori* hypothesis of sigmoidal growth as well as recommendations for model selection when completing *post hoc* analysis of existing data.

Purpose of Study

The purpose of this study was twofold. First, this study provides guidance and recommendations regarding model selection through the use of fit measures to select the true model over competing incorrect models. Two different families of models were estimated in this study: polynomial and sigmoidal. The polynomial models were linear, quadratic, and cubic. The sigmoidal models in this study were first-order Taylor series approximations of the Richards curve (Browne, 1993; Browne & du Toit, 1991). A total of seven different information criterion, as well as four absolute or incremental fit measures, were used to select the best-fitting model between linear, quadratic, cubic, and sigmoidal models when the true simulated model was sigmoidal. The second purpose of this study was to examine the quality of parameter estimates recovered from correctly specified sigmoidal models under a variety of conditions and provide recommendations for researchers in designing longitudinal studies when theory suggests nonlinear bounded growth.

Overview of Procedures

A Monte Carlo simulation design was used to examine four fully crossed factors with 1,000 converged replications per cell. The factors this study examines were: the number of repeated measures (6, 8, 10), sample size (50, 100, 200, 500, 1,000, 1,500), maximum rate of change (.125, .1875, .25), and the timing of the inflection point expressed as a proportion of the number of repeated measures (.25, .375, .50). Each dataset was generated with a true sigmoidal growth pattern and then analyzed as linear, quadratic, cubic, and sigmoidal for a total of 162 cells, 162,000 datasets, and 648,000 analyzed models. Rates of convergence were recorded and discussed.

For each replication, the best-fitting model was selected for each fit measure. For each condition, the proportion of times the true model was selected for each fit measure was recorded. Higher proportions indicate better fit measure performance. A logistic regression was run on the summary results to determine how the individual manipulated factors contributed to fit measure performance. A variety of fit measures will be used in model selection. These measures fall into two broad categories: comparative fit measures and absolute or incremental fit measures. The comparative fit measures, as the name implies, are used for comparing competing models. The model with the lowest value on a particular information criterion is considered better fitting than those competing models with higher values. Comparative fit measures that were used in this study were Akaike Information Criterion (AIC), (Akaike, 1973), Bayesian Information Criterion (BIC), (Schwarz, 1978), and Bayesian Information Criterion with the (Sclove, 1987) sample size penalty adjustment (aBIC). While these are the fit indices most typically used and reported, several other information criteria will be reported as well: Consistent AIC (CAIC), (Bozdogan, 1987), Hannan and Quinn (1979) adjustment of AIC (HQ), Draper Information Criterion (DIC), (Draper, 1995), and Sugiura's adjustment of AIC (AICc), (Sugiura, 1978). Four absolute or incremental fit measures were examined for this study, the comparative fit index (CFI) (Bentler, 1990), the Tucker-Lewis Index (TLI) (Tucker & Lewis, 1973), The Root Mean Square Error of Approximation (RMSEA) (Steiger, 1990), and finally, the Standardized Root Mean Residual (SRMR) (Jöreskog & Sörbom, 1981).

Without a methodological basis to guide model selection, there is no evidence as to the probability of correct model selection under any set of conditions. By simulating a wide range of conditions and examining model selection using a variety of fit measures, this research provides guidelines and recommendations on how to appropriately structure longitudinal research. The results of this study help to ensure adequate sample size and number of repeated measures when there the hypothesis of sigmoidal growth exists. It also provides guidance for correct model selection once the data have been collected.

Parameter Recovery

After 1,000 converged replications were generated, parameter estimates were recovered from the correctly specified sigmoidal model results. The parameters that were recovered were the upper and lower asymptotes which describe the overall growth of the latent curve, the maximum rate of growth, the timing of the maximum rate of growth (the inflection point) as it occurs within the measurement window, and the relative asymmetry of the latent curve. Using measures of raw and relative bias, the quality of the recovered parameters was examined to determine the conditions under which negligible bias occurs. ANOVAs were run to understand how the manipulated factors contributed to overall parameter recovery quality.

Delimitations

This study provides a first step towards understanding the conditions under which parameter estimates are relatively unbiased as well as the conditions that sigmoidal SLCMs can be properly selected as the best-fitting model over incorrectly specified competing models. However, like any Monte Carlo simulation, the findings are generalizable only to the conditions that were simulated in the study. Typically, a simulation of this nature would draw upon prior applied studies to guide the direction of the research and inform the decisions made regarding the conditions examined within the study. Unfortunately, the body of applied work done with these types of models is extremely sparse, so it is difficult to know what conditions would be realistic. As a result, the conditions examined within this study may be overly broad, with future research designed to focus on conditions under which correct model selection becomes problematic.

CHAPTER TWO

Review of the Literature

Development of Growth Modeling

Latent growth curve models are a set of statistical methods that allow for the estimation of inter-individual variability in intra-individual patterns of change over time within a structural equation modeling framework (Bollen & Curran, 2006; Curran, Obeidat, & Losardo, 2010; Newsom, 2015; Preacher et al., 2008). The development of structured latent curve modeling has its roots in exploratory factor analysis before being reconceptualized within a confirmatory factor analysis framework, and later, extended to incorporate Taylor series approximations of nonlinear functions. This is a brief summary of the significant developments in the past 60 years leading to structured latent curve models.

In an attempt to model longitudinal data, Baker et al. (1954) used factor analysis to extract factors from a correlation matrix and then analyzed them by treating the different factors as differing stages of development. Although there were many practical issues with his methodology, he was one of the first to tie factor analysis to longitudinal growth. In 1958, two independent papers (Rao, 1958; Tucker, 1958) both provided formal presentations of growth curve models within a factor analytic framework. While still based on exploratory factor analysis and principal components analysis, these were the first models to begin to look like modern latent growth curve analysis. The factors of these models were conceptualized as growth components as opposed to psychometric components. These early attempts to tie longitudinal growth to exploratory factor analysis faced the problem of rotational indeterminacy where there are an infinite number of solutions with identical fit.

The formulation of confirmatory factor analysis (CFA) (Jöreskog, 1969) allowed factors to be defined with specific properties and eliminated the indeterminacy issue. Meredith and Tisak (1984, 1990) extended the previous work of Rao (1958) and Tucker (1958) but instead used CFA as the foundation of their work. This not only allowed the latent growth curves to have a specified shape but allowed for model fit testing as with other structural equation models. Furthermore, Meredith and Tisak (1990) demonstrated the implementation of latent growth curve models using common software of the era. This led to a great deal of interest in latent growth curve models and led to many extensions of the original.

One such extension was the introduction of structured latent curve models by Browne and du Toit (1991). These models allowed nonlinear functions to be modeled within a latent growth curve framework. To incorporate nonlinear growth into latent growth curve analysis, first-order Taylor series approximations were used. Partial derivatives with respect to the individual parameters of a function were used to represent individual latent growth factors. Rather than, for example, having two latent growth factors for a linear model (intercept and slope), there could be more that represented the different aspects of growth of a nonlinear function. The factor loadings of the latent growth factors themselves were complex nonlinear functions (Grimm, Ram, & Estabrook, 2010) but only varied with time. This allowed estimation using Mplus and other structural equation modeling software packages.

Latent Growth Curve Model Estimation

There are many excellent texts that describe model specification of latent growth curve models (e.g., Bollen & Curran, 2006; Grimm, Ram, & Estabrook, 2016; Newsom, 2015; Preacher et al., 2008). For this description, I will draw from all of them and attempt where possible to incorporate any standard model notation used.

Growth models, within the structural equation modeling framework, are fit in the same way as restricted common factor models (Meredith & Tisak, 1990) where the latent variables are the components of growth. As an example, in a linear model, the latent variables are the intercept and slope. For a quadratic model they would be intercept, slope, and quadratic. The restricted common factor model can be given as:

$$y_i = \mathbf{\Lambda}\eta_i + \epsilon_i \tag{2.1}$$

where y_i is a $T \times 1$ vector of the manifest scores taken at time T for individual i, $\mathbf{\Lambda}$ is a $T \times p$ matrix of the factor loadings for the latent variables where p is the number of latent growth factors in the model, η_i is a $p \times 1$ vector containing the factor scores for the latent growth factors in the model for individual i, ϵ_1 is a $T \times 1$ vector containing random errors for individual i. The factor scores for the latent variables η can further be expressed as a function of the latent means and individual departures from those means:

$$\eta_i = \alpha + \xi_i. \tag{2.2}$$

As above, η_i is a $p \times 1$ vector of the factor scores for the latent growth factors for individual i, α is a $p \times 1$ vector of the latent means of the growth factors, and ξ_i is a $p \times 1$ vector of the residuals associated with the latent growth factors.

From the common factor model given in Equation 2.1 and the function of the latent means given in Equation 2.2 the mean and covariance structure of the model can be derived. Within structural equation modeling, the mean and covariance structures are used to calculate parameter estimates and measures of model fit. The mean structure is a function that represents the population means for each repeated measure and is given as

$$\boldsymbol{\mu} = \mathbf{\Lambda}\,\boldsymbol{\alpha} \tag{2.3}$$

where $\boldsymbol{\mu}$ is a $T \times 1$ vector representing the population means of the observed variables, $\boldsymbol{\Lambda}$ is a $T \times p$ matrix that represents the factor loadings of the latent growth factors, and $\boldsymbol{\alpha}$ is a $p \times 1$ vector of the latent variable means.

The covariance structure is a function that represents the population variances and covariances of the repeated measures and is given by

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda\Psi\Lambda}' + \boldsymbol{\Theta} \tag{2.4}$$

where $\boldsymbol{\Sigma}$ is a $T \times T$ matrix of the variances and covariances of the observed variables, $\boldsymbol{\Lambda}$ is a $T \times p$ matrix that represents the factor loadings of the latent growth factors, $\boldsymbol{\Psi}$ is a $p \times p$ matrix that represents the variances and covariances of the latent growth factors, and $\boldsymbol{\Theta}$ is a $T \times T$ matrix of the estimated residual variances.

For the covariance and means structure, there are four parameters of interest: $\boldsymbol{\Lambda}, \boldsymbol{\alpha}, \boldsymbol{\Psi},$ and $\boldsymbol{\Theta}$. Sample matrices are shown below for a linear model (two latent growth factors therefore $p = 2$) with four evenly spaced repeated measures ($T = 4$).

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \tag{2.5}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \tag{2.6}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} \Psi_{11} & \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \tag{2.7}$$

$$\boldsymbol{\Theta} = \begin{bmatrix} \theta_\epsilon & & & \\ 0 & \theta_\epsilon & & \\ 0 & 0 & \theta_\epsilon & \\ 0 & 0 & 0 & \theta_\epsilon \end{bmatrix} \tag{2.8}$$

Polynomial Latent Curve Models

Within the context of latent growth curve modeling, the family of polynomial models typically follow the form of:

$$y = x_0 t^0 + x_1 t^1 + x_2 t^2 + \cdots + x_k t^k. \tag{2.9}$$

Linear models are polynomials of the first degree, quadratics are polynomials of the second degree and so on. The higher the degree of the polynomial, the closer the potential fit between the data and the latent curve. In fact, it is possible to perfectly fit a curve with k repeated measures using a polynomial of $k-1$ degrees. The drawback to higher-degree polynomials is that the interpretation is nonsensical. In educational and social science research, the only polynomial functions that are regularly used are the linear, quadratic, and cubic models. Although the interpretation of linear models is straightforward, quadratic and cubic equations can be difficult to interpret, and rarely map onto developmental theory. In fact, Preacher et al. (2008) strongly cautions against selection of a quadratic growth model over a linear model for the sole reason that they fit better. The rationale is that the improved fit may be purely because of idiosyncrasies in the sample data and there is rarely any theoretical underpinning supporting quadratic growth. Nonetheless, quadratic and cubic functions are native to common structural equation modeling software and are easily implemented.

Linear

The most commonly described form of latent growth curve model is the linear growth curve model. The linear growth curve is described by the following equation:

$$y_{nt} = \alpha_{n1} + \alpha_{n2} t + \epsilon_{nt}. \tag{2.10}$$

It is a first degree polynomial meaning that in addition to the intercept, there is only one additional latent growth factor included in the model: the slope. All polynomial and structured latent curve models require a certain number of repeated measures for correct identification that is the number of latent growth factors + 1. In this case, a minimum of three repeated measures are necessary. Interpretation is straightffor-ward with the intercept being initial level and slope representing growth per repeated measure. These can either be fixed to a group mean, or allowed to individually vary. Linear growth is monotonic meaning the growth is always increasing or always decreasing but it is not bounded meaning it will trend towards negative or positive infinity given enough time. The matrix equivalent to Equation 2.10 for six evenly spaced time points is as follows:

$$\begin{bmatrix} y_{n1} \\ y_{n2} \\ y_{n3} \\ y_{n4} \\ y_{n5} \\ y_{n6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} \alpha_{n1} \\ \alpha_{n2} \end{bmatrix} + \begin{bmatrix} \epsilon_{n1} \\ \epsilon_{n2} \\ \epsilon_{n3} \\ \epsilon_{n4} \\ \epsilon_{n5} \\ \epsilon_{n6} \end{bmatrix}. \tag{2.11}$$

Quadratic

Another commonly used growth model is the quadratic, which is a second degree polynomial. It is shown in the equation below.

$$y_{nt} = \alpha_{n1} + \alpha_{n2}t + \alpha_{n3}t^2 + \epsilon_{nt} \tag{2.12}$$

It is typically used when data shows nonlinearity to aid in model fit. It is easily implemented in most structural equation modeling software. The quadratic function contains three growth factors which are the slope, intercept, and the quadratic component. Interpretation of intercept and slope are the same, with the quadratic component being interpreted as the rate of change of the change (slope) over time. Because there are three latent growth factors in the quadratic model, a total of four repeated measures are required to identify the model. Quadratic growth is not monotonic meaning it is either increasing or decreasing depending on the temporal location of the observation and it is also not bounded. The matrix equivalent to Equation 2.12 is shown below:

$$\begin{bmatrix} y_{n1} \\ y_{n2} \\ y_{n3} \\ y_{n4} \\ y_{n5} \\ y_{n6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix} \begin{bmatrix} \alpha_{n1} \\ \alpha_{n2} \\ \alpha_{n3} \end{bmatrix} + \begin{bmatrix} \epsilon_{n1} \\ \epsilon_{n2} \\ \epsilon_{n3} \\ \epsilon_{n4} \\ \epsilon_{n5} \\ \epsilon_{n6} \end{bmatrix}. \tag{2.13}$$

Cubic

The final polynomial model that will be considered is the cubic model which is a third degree polynomial. Its equation is as follows:

$$y_{nt} = \alpha_{n1} + \alpha_{n2}t + \alpha_{n3}t^2 + \alpha_{n4}t^3 + \epsilon_{nt}. \tag{2.14}$$

Like the quadratic model, it is typically used when data shows nonlinearity to aid in model fit. It too is easily implemented in most structural equation modeling software. The cubic function contains four growth factors which are the slope, intercept, quaratic, and cubic components. Interpretation of the first three growth factors are the same as the quadratic model, with the cubic component being interpreted as the rate of change quadratic component over time. Essentially, the interpretation of the model involves describing the rate of change of the rate of change of change over time. Because there are four latent growth factors in the cubic model, a total of five repeated measures are required to identify the model. Cubic growth, like quadratic, is not monotonic meaning it is either increasing or decreasing depending on the temporal location of the observation and it is also not bounded. The matrix equivalent to Equation 2.14 is shown below:

$$\begin{bmatrix} y_{n1} \\ y_{n2} \\ y_{n3} \\ y_{n4} \\ y_{n5} \\ y_{n6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 16 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix} \begin{bmatrix} \alpha_{n1} \\ \alpha_{n2} \\ \alpha_{n3} \\ \alpha_{n4} \end{bmatrix} + \begin{bmatrix} \epsilon_{n1} \\ \epsilon_{n2} \\ \epsilon_{n3} \\ \epsilon_{n4} \\ \epsilon_{n5} \\ \epsilon_{n6} \end{bmatrix} \tag{2.15}$$

Structured Latent Curve Models

Although originally described as a linear model with the only latent growth functions being intercept and slope, latent growth curves can take many forms. Polynomial functions are a natural extension of the linear model and are the most commonly utilized latent curve models. Structured latent curve models as introduced by Browne and du Toit (1991) further extend the types of models that can be used in longitudinal structural equation modeling by allowing for a broad range of differentiable functions to be used when modeling longitudinal growth. Of the functions most commonly modeled using structured latent curve models are the exponential function and sigmoidal or S-shaped functions. One of the most commonly used families of sigmoidal functions include the Gompertz curve (Gompertz, 1825) and the Logistic curve (Verhulst, 1845). Richards (1959) demonstrated that the Gompertz and Logistic curves could be subsumed by a new function referred to as the generalized logistic or Richards function. The principal difference between the Gompertz and Logistic curves was the relative asymmetry. That is, with the Logistic function, 50% of the growth happens by the point of inflection whereas, in the Gompertz curve, the proportion of growth is only $\frac{1}{e}$ or roughly 36.8%. The Richards curve includes an additional parameter that allows for varying levels of asymmetry to be modeled. The

Richards curve, like the Logistic and Gompertz curves, are all monotonic and they are asymptotically bounded. It is the Richards curve that is the focus of this simulation.

Sigmoidal

Using the first-order Taylor series approximation method detailed in Browne and du Toit (1991) and Browne (1993), the Richards curve is written as the mean of the latent variables $i, \beta, \delta, \rho, \gamma$ with its target function expressed as

$$y_{nt} = i + \frac{\beta_n}{(1 + \gamma \cdot exp(-\rho_n(t - \delta_n)))^{\frac{1}{\gamma}}} + \epsilon_{nt}. \tag{2.16}$$

Figure 2.1 depicts different levels of i which is the universal lower asymptote. β_n is the total amount of change for individual n. Holding everything else constant, Figure 2.2 shows how different levels of β change the curve. δ_n is the time of most rapid change for individual n. Figure 2.3 shows how changing δ can re-center the inflection point within the measurement window. ρ_n is the rate of approach to the upper asymptote for individual n. Figure 2.4 shows how the slope of the curve changes as the values of ρ change. Finally, γ is the parameter that controls the asymmetry of the curve. Under certain conditions, the Logistic curve and Gompertz curves are subsumed by the Richards curve equation. When $\gamma = 1$, the Richards curve equation collapses into the Logistic curve equation. Likewise, as γ approaches 0 from the positive, the asymmetry of the Richards curve approaches the asymmetry of the Gompertz curve. Figure 2.5 shows how different levels of γ change the shape of the curve.

The target function of the Richards curve re-expressed as a first-order Taylor series is as follows:

$$y_{nt} = \alpha_1 \frac{\partial y}{\partial i_1} + \alpha_{n2} \frac{\partial y}{\partial \beta_1} + \alpha_{n3} \frac{\partial y}{\partial \delta_1} + \alpha_{n4} \frac{\partial y}{\partial \rho_1} + \alpha_5 \frac{\partial y}{\partial \gamma_1} + \epsilon_{nt}. \tag{2.17}$$

The matrix equivalent for six evenly spaced repeated measures is shown in Equation 2.18 where repeated measures are represented by y_{nt}, the universal lower asymptote by α_1, and the partial derivative of the target function with respect to i by $\frac{\partial y}{\partial i_1}$.

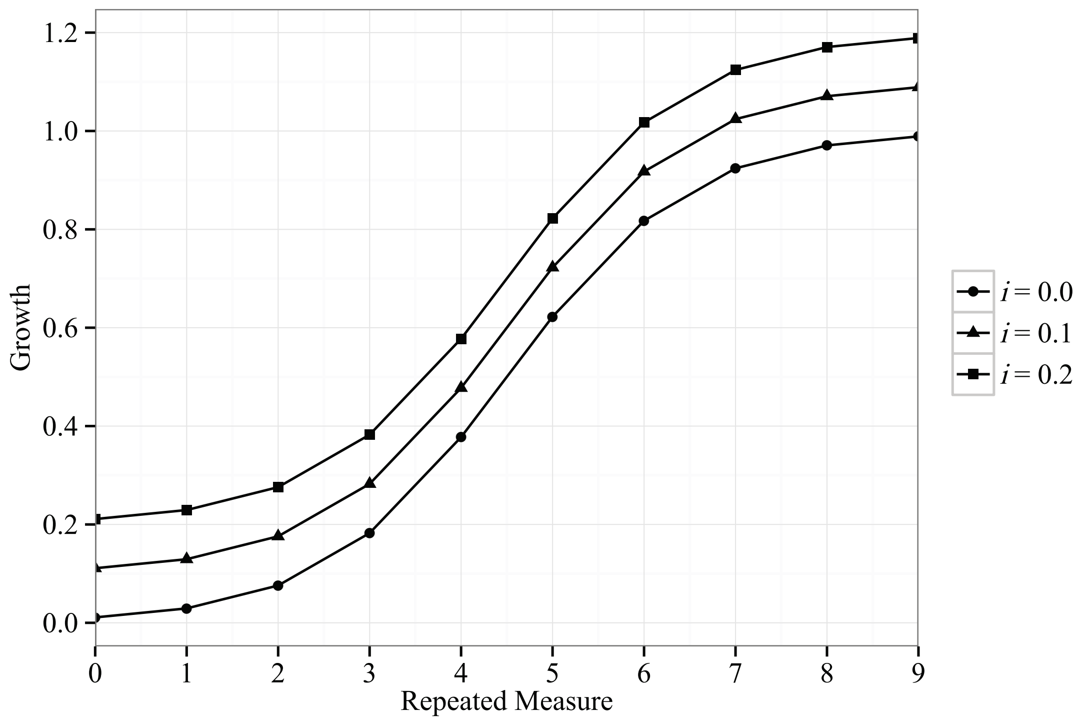

Figure 2.1. Sigmoidal growth with different lower asymptotes (i).

The individual amount of overall growth from lower to upper asymptote is represented by α_{n2} and the partial derivative of the target function with respect to β by $\frac{\partial y}{\partial \beta_1}$. The individual rate of approach to the asymptote is given by α_{n3} and the partial derivative of the target function with respect to δ by $\frac{\partial y}{\partial \delta_1}$. The individual timing coefficient is given by α_{n4} and the partial derivative of the target function with respect to ρ by $\frac{\partial y}{\partial \rho_1}$. The universal asymmetry of the curve is given by α_5, the partial derivative of

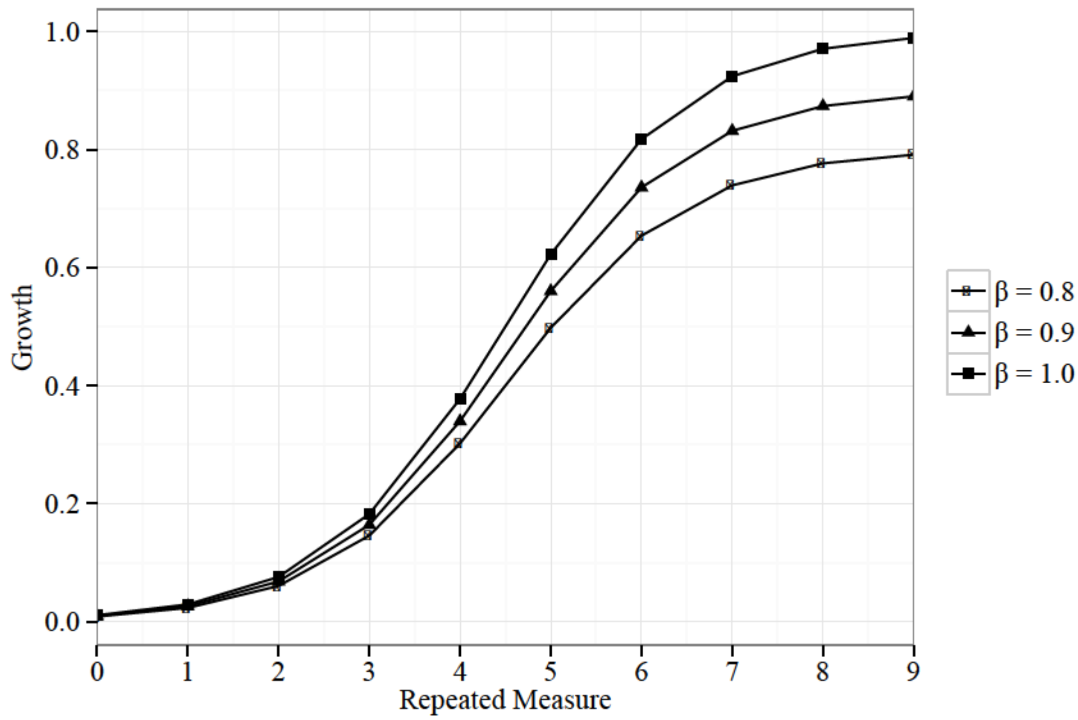

Figure 2.2. Sigmoidal growth with different amounts of overall growth (β).

the target function with respect to γ by $\frac{\partial y}{\partial \gamma_1}$, and ϵ_{nt} is the time dependent residual.

$$\begin{bmatrix} y_{n1} \\ y_{n2} \\ y_{n3} \\ y_{n4} \\ y_{n5} \\ y_{n6} \end{bmatrix} = \begin{bmatrix} \frac{\partial y}{\partial i_1} & \frac{\partial y}{\partial \beta_1} & \frac{\partial y}{\partial \delta_1} & \frac{\partial y}{\partial \rho_1} & \frac{\partial y}{\partial \gamma_1} \\ \frac{\partial y}{\partial i_2} & \frac{\partial y}{\partial \beta_2} & \frac{\partial y}{\partial \delta_2} & \frac{\partial y}{\partial \rho_2} & \frac{\partial y}{\partial \gamma_2} \\ \frac{\partial y}{\partial i_3} & \frac{\partial y}{\partial \beta_3} & \frac{\partial y}{\partial \delta_3} & \frac{\partial y}{\partial \rho_3} & \frac{\partial y}{\partial \gamma_3} \\ \frac{\partial y}{\partial i_4} & \frac{\partial y}{\partial \beta_4} & \frac{\partial y}{\partial \delta_4} & \frac{\partial y}{\partial \rho_4} & \frac{\partial y}{\partial \gamma_4} \\ \frac{\partial y}{\partial i_5} & \frac{\partial y}{\partial \beta_5} & \frac{\partial y}{\partial \delta_5} & \frac{\partial y}{\partial \rho_5} & \frac{\partial y}{\partial \gamma_5} \\ \frac{\partial y}{\partial i_6} & \frac{\partial y}{\partial \beta_6} & \frac{\partial y}{\partial \delta_6} & \frac{\partial y}{\partial \rho_6} & \frac{\partial y}{\partial \gamma_6} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_{n2} \\ \alpha_{n3} \\ \alpha_{n4} \\ \alpha_5 \end{bmatrix} + \begin{bmatrix} \epsilon_{n1} \\ \epsilon_{n2} \\ \epsilon_{n3} \\ \epsilon_{n4} \\ \epsilon_{n5} \\ \epsilon_{n6} \end{bmatrix} \tag{2.18}$$

The partial derivatives of the latent variables, $i, \beta, \delta, \rho, \gamma$, are:

$$\frac{\partial y}{\partial i} = 1 \tag{2.19}$$

$$\frac{\partial y}{\partial \beta} = \frac{e^{\frac{\rho\, t}{\gamma}}}{(\gamma\, e^{\delta\, \rho} + e^{\rho\, t})^{\frac{1}{\gamma}}} \tag{2.20}$$

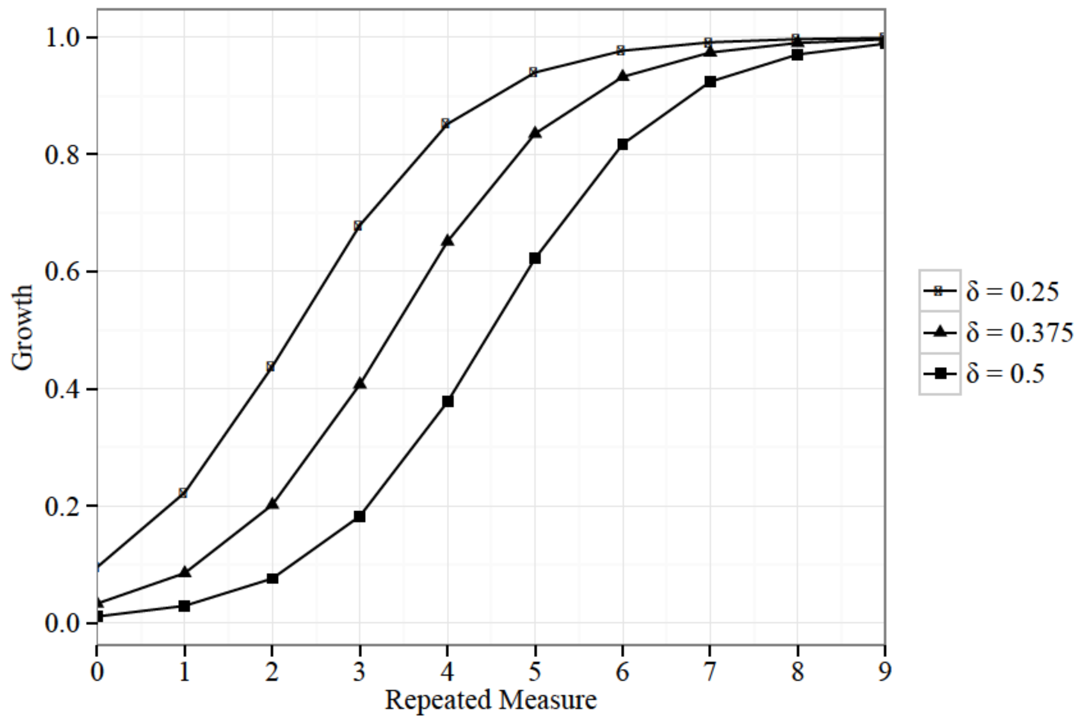

Figure 2.3. Sigmoidal growth with different locations of the inflection point (δ) within the measurement window.

$$\frac{\partial y}{\partial \delta} = -\frac{\beta\,\rho\,e^{\frac{\delta\,\gamma\,\rho+\rho\,t}{\gamma}}}{\left(\gamma\,e^{\delta\,\rho}+e^{\rho\,t}\right)^{\frac{1+\gamma}{\gamma}}} \tag{2.21}$$

$$\frac{\partial y}{\partial \rho} = \frac{\left(\beta\,e^{\delta\,\rho}\,t-\beta\,\delta\,e^{\delta\,\rho}\right)\,e^{\frac{\rho\,t}{\gamma}}}{\left(\gamma\,e^{\delta\,\rho}+e^{\rho\,t}\right)^{\frac{1+\gamma}{\gamma}}} \tag{2.22}$$

$$\frac{\partial y}{\partial \gamma} = \frac{\left(-\beta\,\rho\,t\,e^{\rho\,t}-\beta\,\gamma\,\rho\,e^{\delta\,\rho}\,t-\beta\,\gamma\,e^{\delta\,\rho}\right)\,e^{\frac{\rho\,t}{\gamma}}+\left(\beta\,e^{\rho\,t}+\beta\,\gamma\,e^{\delta\,\rho}\right)\,e^{\frac{\rho\,t}{\gamma}}\,log\left(\gamma\,e^{\delta\,\rho}+e^{\rho\,t}\right)}{\left(\gamma\,e^{\delta\,\rho}+e^{\rho\,t}\right)^{\frac{1}{\gamma}}\,\left(\gamma^2\,e^{\rho\,t}+\gamma^3\,e^{\delta\,\rho}\right)} \tag{2.23}$$

By using the Browne and du Toit (1991) method, the partial derivatives with respect to the latent random coefficients only depend on fixed-effect parameters. This makes the model linear with respect to the latent variables and thus able to be estimated using common structural equation modeling software. In other words, the partial derivatives become factor loadings in what is mathematically equivalent to

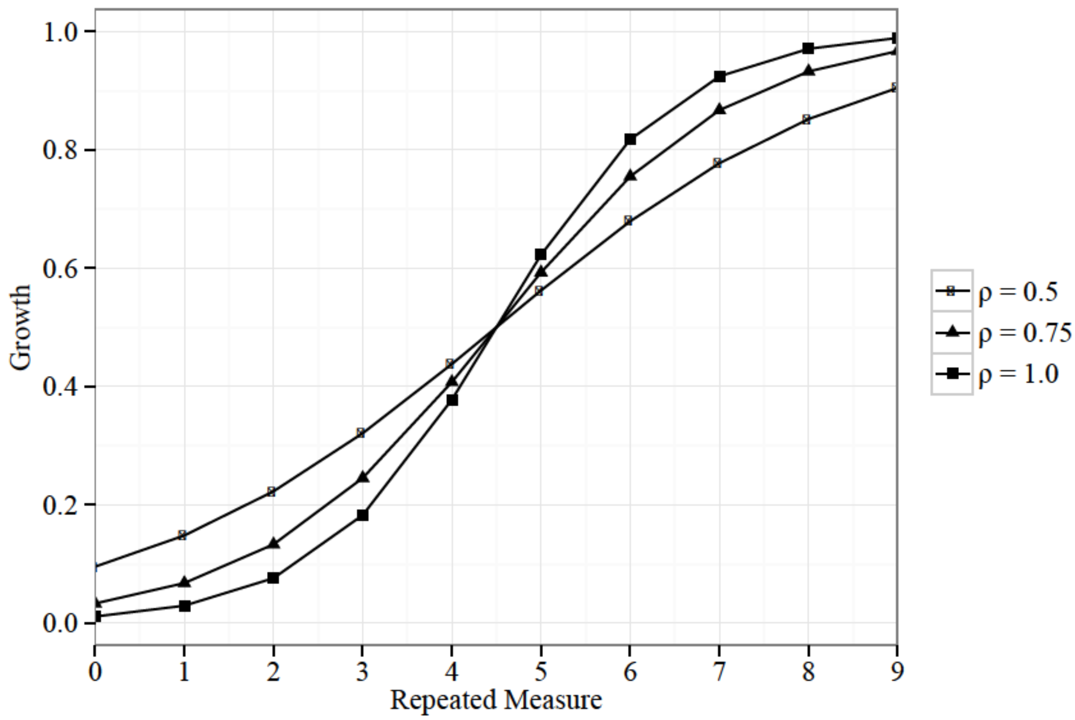

Figure 2.4. Sigmoidal growth with different rates of change at the inflection point (ρ).

a common factor model and is thus estimatable using commonly used structural equation modeling software. Figure 2.6 shows the curves of the partial derivatives under a set of sample conditions indicating the factor loadings that each parameter contributes to the model.

It should be noted that polynomial models are trivially a special case of structured latent curve models. Taking Equation 2.12 as an example, we can see that the partial derivatives of α_{n1} and $\alpha_{n2}t$ and $\alpha_{n3}t^2$ are:

$$\frac{\partial y}{\partial \alpha_{n1}} = 1 \tag{2.24}$$

$$\frac{\partial y}{\partial \alpha_{n2}} = t \tag{2.25}$$

$$\frac{\partial y}{\partial \alpha_{n3}} = t^2. \tag{2.26}$$

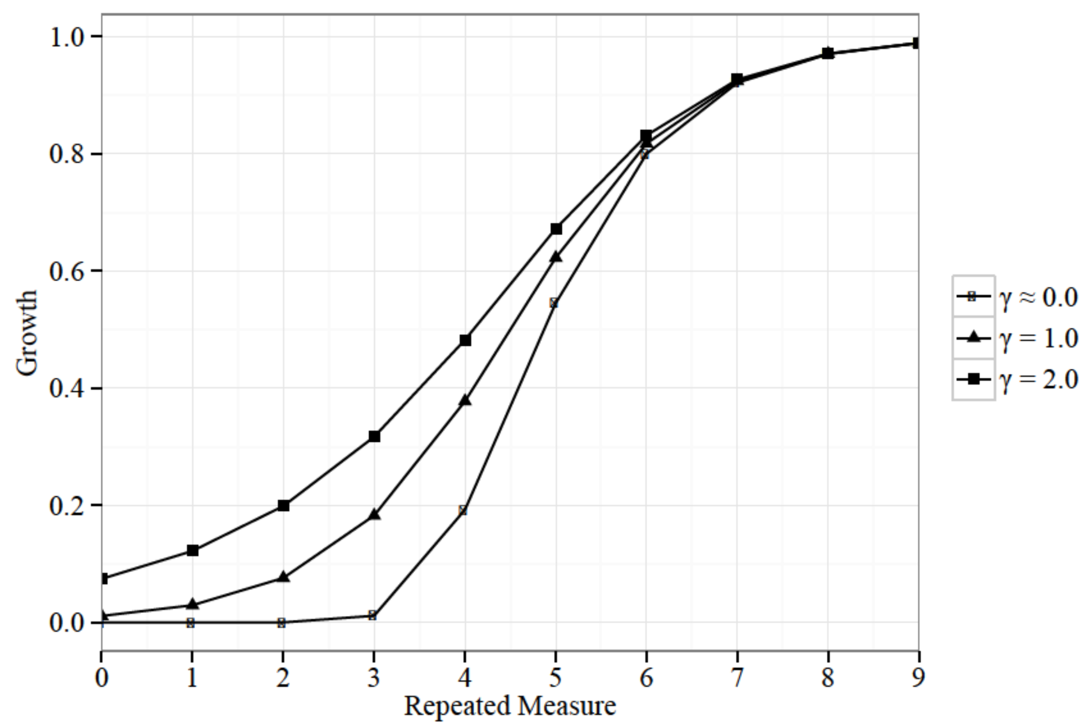

Figure 2.5. Sigmoidal growth with different levels of relative asymmetry (γ).

Which, assuming six evenly spaced repeated measures from 0 to 5, yields a factor loading matrix of:

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}. \tag{2.27}$$

Which is identical to the factor loading matrix depicted in Equation 2.13.

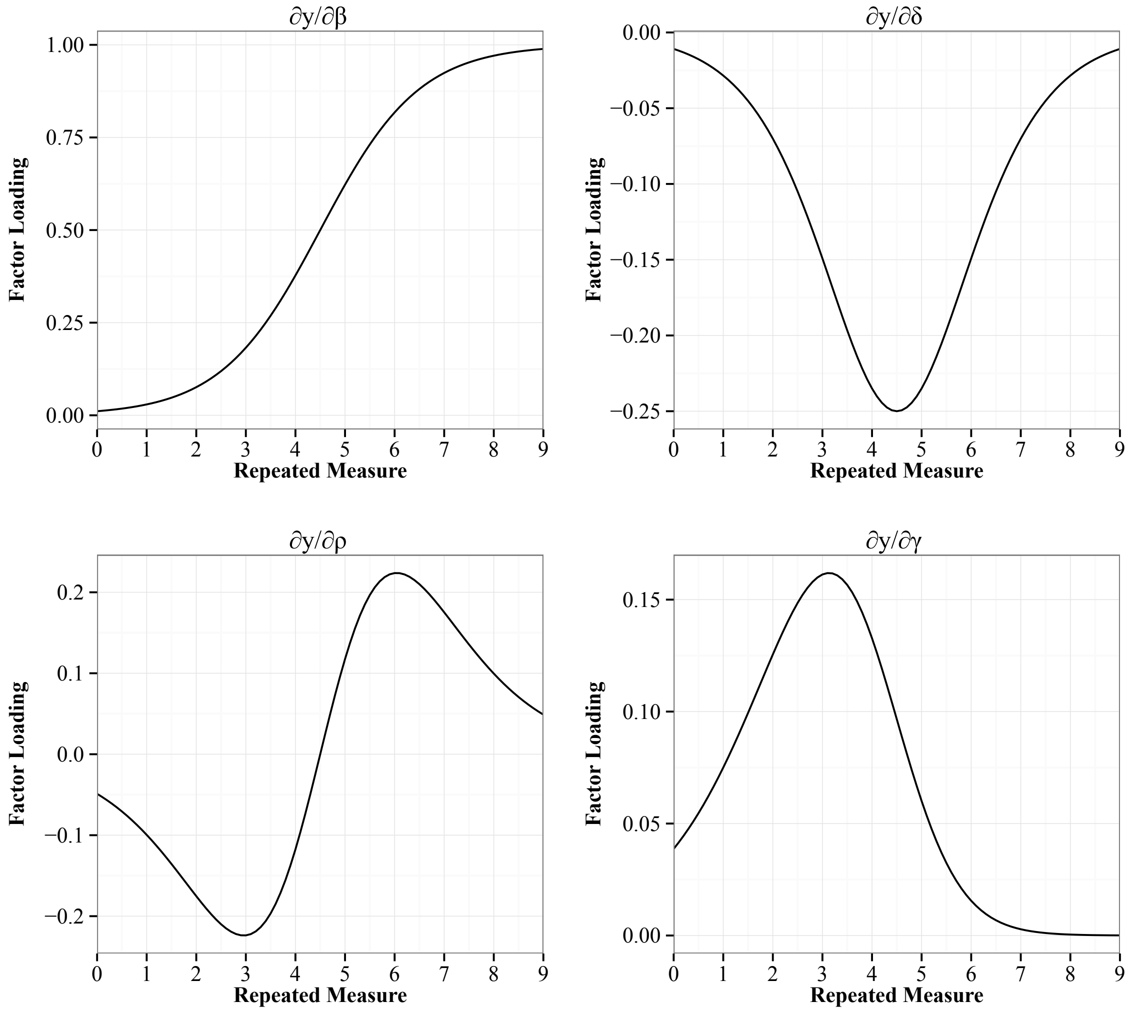

Figure 2.6. Partial derivative curves of β, δ, ρ, and γ.

Empirical Examples

Structured latent curve models have not been widely used in applied research. This may be due to several reasons including the lack of native support within common structural equation modeling software. Grimm and Ram (2009) and Grimm et al. (2010) demonstrated the use of sigmoidal latent curve models using Mplus (L. Muthén & Muthén, 2015) and OpenMX (Boker et al., 2011). Having shown it is possible to model nonlinear growth of various forms, there is little reason not to model growth more realistically. If theory suggests that latent growth follows the form of a specific function, that function should be modeled. Use of polynomial functions that do not map onto theoretical developmental trajectories simply because they are easy to implement and show adequate model fit should be discouraged. The onus is on the researcher to select an appropriate model as opposed to letting software dictate what models are available.

To identify previous studies that used or demonstrated structured latent curve models, *EBSCOhost* was used to search the *Educational Research Complete, Education Resources Information Center (ERIC), PsycINFO, Psychology and Behavioral Sciences Collection* and *PsycARTICLES* databases. Keywords used included: Gompertz, Richards, structured latent curve, structured latent, exponential, and various combinations of those terms. It must be noted that this search did not discover several relevant studies referenced within other studies. When dealing with large volumes of published research, searches such as this are not exhaustive.

A total of 16 published studies were found ranging in publication data from 1991 to 2015. Several of the studies used multiple datasets for a total of 20 datasets. Of the published studies, six were purely empirical with the remainder being demonstrations of structured latent curve models with included empirical examples. The sample size ranged from 103 (Dodonov & Dodonova, 2012) to 21,260 (Cameron, Grimm, Steele, Castro-Schilo, & Grissmer, 2015) with a median sample size of 275. Three of the datasets used had sample sizes in excess of 12,686 whereas thirteen of the studies had sample sizes of 383 or fewer. See Table 2.1 for the five number summaries for the sample sizes and repeated measures from the review of research. The number of repeated measures used in applied research ranged from four to 12 with the median amount of repeated measures being nine. In the majority of studies, multiple types of models were fit to the data. Fourteen of the studies made use of the exponential model, thirteen used sigmoidal models (Logistic, Gompertz, or Richards), four used polynomial models, two used hyperbolic models, and one used a Preece-Baines model.

Table 2.1

Summary of Conditions for 16 Structured Latent Curve Studies

Statistic	Sample Size	Repeated Measure
Min	103	4
Q1	145	5.5
Median	275	9
Q3	834	9.5
Max	21,260	12

Some of these models require as few as four repeated measures to be properly identified. For example, the exponential function required a minimum of four repeated measures, whereas the Richards curve requires a minimum of six repeated measures for proper identification.

Potential Applications

Throughout the lifespan of individuals, there are periods during which development (or decline) occurs significantly more rapidly than at other times (Blakemore & Mills, 2014; Lupien, McEwen, Gunnar, & Heim, 2009). These are called sensitive periods. Bornstein (1989) contends they are meaningful in that, among other reasons, they provide evidence of endogenous and exogenous forces exerting differential effects that profoundly influence development. It is theorized that these periods do not begin and end abruptly, but rather begin and end gradually (Knudsen, 2004). If plotted, they could look similar to probability density curves and exhibit differing amounts of skewness and kurtosis depending on the structure of the period. The integral of this type of function is the cumulative distribution function that represents the area under the probability density curve at any given point on the x axis. For non-uniform density functions, this is an asymptotic sigmoidal curve.

If we assume that these sensitive periods, when plotted, look similar to a normal probability distribution, we can make certain conjectures. There exists a developmental density function that represents an arbitrary amount of development that occurs at a particular point in time and when that function is integrated becomes a cumulative developmental function. The cumulative developmental function would represent the total amount of development that had occurred in a specific domain at a particular point in time. Domains with sensitive periods, therefore, should have developmental trajectories that can be modeled longitudinally as sigmoidal functions. Furthermore, the latent trajectory of these developmental constructs can be used not only descriptively but predictively. Conversely, latent growth curves that demonstrate sigmoidal trajectories when differentiated can be used to identify the timing of sensitive periods within the measured construct. To my knowledge, this connection has not appeared in the literature.

It should be noted that L. L. Thurstone (1955) used the Gompertz curve to model growth on subscale scores of the Primary Mental Abilities battery (T. G. Thurstone & Thurstone, 1947). The subscales were Perceptual Speed, Space, Reasoning, Number, Memory, Verbal, and Word Fluency. Thurstone used longitudinal and cross-sectional data to fit Gompertz curves to subscale means to predict asymptotic adult values on these subscales. Although this was prior to latent curve modeling and the advent of the generalized logistic function, it shows that sigmoidal curves have demonstrated utility in educational and psychological research.

Model Selection

Statistically based model selection via model fit measures falls into three overarching categories (Kenny, 2015). First are comparative fit measures. These measures are typically based on information theory and are only useful when comparing two or more models with the best model having the lowest score. The second category of fit measures is incremental fit indices. These fit measures compare the hypothetical model to a baseline model. For these models, lower scores indicate worse fit. Finally, there are the absolute fit measures. For these measures, a value of zero indicates perfect fit with higher values indicating worse fit.

Comparative Fit Measures

Fit measures based on information theory were originally introduced by Akaike (1973) with what is now call the Akaike information criterion (AIC) as a way of comparing competing statistical models. Shortly afterward, Schwarz (1978) introduced the Bayesian information criterion (BIC) that followed a similar mathematical form:

$$-2\, log\; L(\theta) + penalty\; term \tag{2.28}$$

where -2 times the log likelihood of the model is modified by a penalty term based on sample size and/or number of parameters in the model. After the AIC and BIC, there have been several other information criteria based fit measures that use different penalty terms to improve performance in selecting the best model. Although these models appear on the surface to be very similar, the theory behind their development is fundamentally different. Essentially there are two families of models: those based on the AIC and those based on the BIC. For a technical discussion of the conceptual differences of AIC based and BIC based criteria, see Burnham and Anderson (2003).

AIC. The Akaike information criterion (AIC), Akaike (1973), was among the first information criterion. Akaike formalized the relationship between Kullback-Leibler information (Kullback & Leibler, 1951) and maximum likelihood. This allowed the use of maximized log-likelihood as an approximation for the distance between the true model and the hypothesized model. Akaike acknowledged that researchers can only approximate truth and that out of a series of candidate models, none were true and the goal was to find the one that came closest to truth. Thus, AIC will attempt to find the model closest to the truth for any given sample size. It is expressed by the equation:

$$AIC = -2 \, log \, L(\theta) + 2p \tag{2.29}$$

where p is the number of free parameters in the model. Although AIC is still one of the most frequently used comparative fit measures, Woodroofe (1982) showed that AIC is not consistent. That is, as sample size approaches infinity, AIC does not converge on a true model. This is not necessarily a problem considering the AIC is not attempting to find the true model, only the one closest to the truth.

AICc. The corrected AIC (AICc), is a corrected version of the AIC give by:

$$AICc = -2 \, log \, L(\theta) + 2p + \frac{2p(p+1)}{N-p-1}. \tag{2.30}$$

where N is the sample size and p is the number of free parameters to be estimated. The AICc is based on work by Sugiura (1978) and further explored by Hurvich and Tsai (1989). Although the AIC is an unbiased estimator of Kullback-Leibler information, it performed poorly in practice when the number of parameters in relation to the sample size was large (Burnham & Anderson, 2003; Sugiura, 1978). Because AIC and AICc converge when sample size is large, AICc is recommended over AIC especially in cases where the ratio of sample size to the number of parameters in the model is less than 40 (Burnham & Anderson, 2003).

CAIC. To address the inconsistency found in AIC, Bozdogan (1987) introduced the CAIC, a consistent version of AIC:

$$CAIC = -2 \, log \, L(\theta) + p(log \, N + 1) \tag{2.31}$$

where N is the sample size and p is the number of free parameters to be estimated. The CAICs penalty term allows for consistency by modeling sample size into the criterion. Additionally, the penalty term is larger than AIC or BIC and therefore will tend to favor more parsimonious models. The CAIC departs conceptually from AIC in that AIC tends to select less parsimonious models as being closer to "truth" and thus minimizing Kullback-Leibler divergence.

HQ. The Hannan and Quinn (1979) information criterion (HQ), was originally developed as a consistent version of AIC that would be useful for autoregressive and time series models. The equation for the HQ is given as:

$$HQ = -2\, log\, L(\theta) + 2p\, log\, (log(N)) \tag{2.32}$$

where N is the sample size and p is the number of free parameters to be estimated.

BIC. The Bayesian information criterion (BIC) was proposed by Schwarz (1978). The BIC is based on the assumption that a true model exists, it is one of the models being considered, the goal is to select the true model, and that the true model exists independently of sample size (Burnham & Anderson, 2003). The BIC equation is:

$$BIC = -2\, log\, L(\theta) + p\, log\, N \tag{2.33}$$

where N is the sample size and p is the number of free parameters to be estimated. BIC has an advantage over AIC in that it is consistent and performs better as sample size increases. That is, it will converge on the "true" model with a probability of 1.

aBIC. Sclove (1987) suggested a sample size adjustment to the penalty term of BIC. The penalty term comes from Rissanen (1978) who looked at model selection for autoregressive time series models. The Sclove adjustment replaces the original N with N* in the penalty term where

$$N^{*} = \frac{N+2}{24}. \tag{2.34}$$

Thus, the Bayesian information criterion (aBIC) with the Sclove (1987) adjustment is

$$aBIC = -2\, log\, L(\theta) + p\, log\left(\frac{N+2}{24}\right). \tag{2.35}$$

where N is the sample size and p is the number of free parameters to be estimated.

DIC. In another attempt to improve small sample performance, Draper (1995) adjusted the penalty of the BIC and although this penalty term is negligible as sample size tends towards infinity, Draper indicates that it can improve correct model selection in small to moderate sample sizes. The DIC is given as:

$$DIC = -2\, log\, L(\theta) + p\, log\left(\frac{N}{2\pi}\right). \tag{2.36}$$

where N is the sample size and p is the number of free parameters to be estimated and π is the mathematical constant.

Incremental Fit Measures

Correct specification of a baseline model is necessary for the estimation and interpretation of the comparative fit index (CFI) and the Tucker-Lewis Index (TLI). Preacher (2010) points out that the majority of structural equation modeling software uses a baseline model where the means and variances of the observed variables are estimated and covariances are constrained to zero. While this is appropriate for structural equation models that do not incorporate a mean structure, it is an inappropriate baseline model for latent growth curve models. Widaman and Thompson (2003) describe in detail what is necessary to select a correct baseline model. In the case of polynomial functions as well as the sigmoidal functions discussed in this study, they share the same baseline model which is a no-growth (intercept only) model where only the mean of the intercept and the residual variances of the observed variables are estimated. It is necessary to generate this model independently and then use the resulting χ^2 values to recalculate the CFI and TLI rather than relying on the values provided by the software. Doing this will also generate a baseline RMSEA, that Kenny (2015) suggests needs to be above 0.158 for CFI and TLI to be informative. Kenny (2015) further indicates that CFI and TLI are analogous to R^2 and thus the worst possible model would have a fit of zero, and the best possible would have a fit of one. Both the TLI and CLI depend on the size of the correlation in the data. If the correlations are low, the TLI and CFI will be low.

CFI. The comparative fit index (Bentler, 1990) is an incremental fit index based on the non-centrality measure. It was proposed to summarize the relative reduction in the non-centrality parameter of a tested model over a baseline model. The CFI describes the relative improvement in fit on a scale of 0 to 1 with 1 higher scores being better. The equation for the CFI is:

$$CFI = 1 - \frac{max(x_H^2 - df_H, 0)}{max(x_H^2 - df_H, x_N^2 - df_N, 0)}. \tag{2.37}$$

TLI. The Tucker-Lewis index was proposed by Bentler and Bonett (1980) as a non-normed fit index based on earlier work by Tucker and Lewis (1973) to evaluate the fit of exploratory factor analysis models (Bentler, 1990). The TLI differed from the Normed Fit Index (Bentler & Bonett, 1980) in that it added a penalty for model complexity by including degrees of freedom in the equation. The TLI compares the proposed model against a baseline model to determine how well a model fits. Higher scores are better than low scores. The equation for the TLI is:

$$TLI = \frac{\frac{x_N^2}{df_N} - \frac{x_H^2}{df_N}}{\frac{x_N^2}{df_N} - 1}. \tag{2.38}$$

Absolute Fit Measures

These measures can be used to assess how well a model fits compared to a theoretical best fitting model that would have a fit of zero.

RMSEA. The root mean square error of approximation (RMSEA) (Steiger & Lind, 1980) is a parsimony corrected index that is based on the non-centrality parameter. According to Steiger (2000), the rationale behind the RMSEA was to develop a way to determine how good model fit is in the population and how well it has been described by the sample. One advantage RMSEA enjoys over other fit measures is that it is possible to construct confidence intervals around the estimate. The equation for RMSEA is:

$$RMSEA = \frac{\sqrt{x^2 - df}}{\sqrt{df(N-1)}}. \tag{2.39}$$

SRMR. The standardized root mean square residual (SRMR) (Jöreskog & Sörbom, 1981) is defined as the standardized mean difference between non-duplicated elements of the actual correlation matrix and the model-implied correlation matrix:

$$SRMR = \sqrt{\frac{2\sum_{i=1}^{p}\sum_{j=1}^{1}\left(\frac{s_{ij}-\hat{\sigma_{ij}}}{s_{ii}s_{jj}}\right)^2}{p(p+1)}}. \tag{2.40}$$

Unlike some structural equation models, latent growth curves make use of a mean structure in their estimation. The SRMR only uses the actual and model-implied correlation matrices in its estimation and not the mean structure of the model. As such, it is possible to have a poorly specified mean structure and still have good fit of the covariance structure. Although Wu, West, and Taylor (2009) caution against its use in structural equation models that include a mean structure, an SRMR value indicating good fit combined with other measures that indicate poor fit may point to a problem in the mean structure of the model, so it may have use as a diagnostic tool.

Summary

Structured latent curve models allow a flexible framework for modeling longitudinal data within a structural equation modeling framework where there are both theory about the developmental trajectory of the construct and a differentiable function that describes the growth. Introduced by Browne and du Toit (1991), the method saw little empirical use until Grimm et al. (2010) demonstrated the technique using Mplus and OpenMX. To date, there have been no published simulation studies of structured latent curve models that examine either model selection or parameter bias. In spite of this, applied researchers have begun using the technique. Without guidance for sample size, number of repeated measures, or other parameters of interest, it is unclear if these researchers have selected the correct model to describe the data, or how biased the recovered parameter estimates are. Simulation studies are needed to provide support for the decisions these and future researchers will make on the structure of their data collection if these types of models are being used.

The subsequent chapters describe the research design and methodology of a simulation study examining sigmoidal structured latent curve models, provide the statistics related to model selection and parameter estimate bias, discussion of the findings, and recommendations for applied researchers.

CHAPTER THREE

Methodology

Structured Latent Curve Models

Latent growth curve models are one of several common methods used in educational, psychological, and social science research to analyze longitudinal data. Typically, longitudinal data have been modeled as first order (linear), second order (quadratic), or third order (cubic) polynomials. One possible reason is that linear, quadratic, and cubic models can be easily fit with readily available statistical software. More recently, however, several researchers have demonstrated that nonlinear structured latent curve models (SLCMs) can be fit using R, Mplus, and SAS (e.g., Grimm & Ram, 2009; Grimm et al., 2010; Grimm, Ram, & Hamagami, 2011). SLCMs allow additional flexibility for the researcher because they allow for modeling of sigmoidal (S-shaped) and other nonlinear growth patterns. As more researchers explore SLCMs in applied research, caution is recommended because there have been no published methodological studies investigating how model selection or parameter recovery. The purpose of this study was to fill the gap in the literature. It does so by examining fit measure performance in selecting the sigmoidal model over competing incorrect models when the true model is sigmoidal and assessing the quality of recovered parameter estimates under simulated conditions.

Model Estimation

Two different families of models were estimated: polynomial and sigmoidal. The polynomial models were linear, quadratic, and cubic. As an example, the third order polynomial (cubic) model is expressed as:

$$y_{nt} = \alpha_{n1} + \alpha_{n2}t + \alpha_{n3}t^2 + \alpha_{n4}t^3 + \epsilon_{nt} \tag{3.1}$$

where α_{n1} is the predicted value of the intercept for individual n when $t = 0$, α_{n2}, α_{n3}, and α_{n4} are the linear, quadratic, and cubic change components, and ϵ_{nt} is the residual for individual n at time t. In the case of the quadratic function, α_{n4} is set equal to zero. For the linear, both α_{n3} and α_{n4} are set equal to zero. The sigmoidal models in this study were estimated using the first-order Taylor series approximation method detailed in Browne and du Toit (1991) and Browne (1993), the Richards curve is written as the mean of the latent variables $i, \rho, \beta, \delta, \gamma$ and is expressed as:

$$y_{nt} = i + \frac{\beta_n}{(1 + \gamma \cdot exp(-\rho_n(t - \delta_n)))^{\frac{1}{\gamma}}} + \epsilon_{nt} \tag{3.2}$$

where i is the universal lower asymptote, β_n is the total amount of change for individual n, δ_n is the point of inflection which is the time of most rapid change for the individual n, ρ_n is the maximum rate of change which occurs at the inflection point also known as the rate of approach to the upper asymptote for individual n, γ is the parameter that controls the asymmetry of the curve, and ϵ_{nt} is the residual for individual n at time t.

Fit Measures

All models were estimated using Mplus (version 7.4, L. Muthén & Muthén, 2015) which provides some fit measures natively. These measures fell into two different categories: relative and absolute fit measures. Beyond the native Mplus fit measures, there were several additional fit measures that were calculated based on summary information generated in R (version 3.3.2, R Development Core Team, 2016) with the MplusAutomation package (Hallquist, 2014).

Comparative Fit Measures

All of the comparative fit measures, also known as information criteria, work in the same general manner. There is a value based on the log likelihood with a penalty term which differs from criterion to criterion. The penalty term corrects for parsimony and/or sample size. When comparing models, the one with the lowest value on a particular criterion is considered better fitting than competing models with higher values. Mplus reports Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Bayesian Information Criterion with the Sclove sample size penalty adjustment (aBIC). While these are the fit indices most typically used and reported, several other information criteria were reported as well: Consistent AIC (CAIC), Draper Information Criterion (DIC), the Hannan and Quinn adjustment of AIC (HQ), and Sugiura's adjustment of the AIC (AICc). While most of these are not commonly used, they are easily calculated and are assessed in this study due to their potential to perform well in the simulated conditions.

Absolute and Incremental Fit Measures

In addition to the three information criteria that Mplus natively generates, it also generates several measures of absolute and incremental fit. The comparative fit index (CFI) is an incremental fit index and describes the relative improvement in fit between the null model and a hypothesized model while compensating for sample size (Bentler, 1990). Values should range from 0 to 1 with values above .90 indicating good fit (Hu & Bentler, 1999) in models without a mean structure. The Tucker-Lewis Index (TLI), also called the non-normed fit index, is a parsimony corrected incremental fit index that includes a penalty for complex models (Tucker & Lewis, 1973). Like the CFI, values on the TLI should range from 0 to 1 with values above .95 indicating good fit (Hu & Bentler, 1999) in models without a mean structure. The Root Mean Square Error of Approximation (RMSEA) is a parsimony corrected absolute fit index that is based on the non-centrality parameter (Steiger, 1990). Values range from 0 to 1 with .06 or lower considered acceptable (Hu & Bentler, 1999) in models without a mean structure. Finally, the Standardized Root Mean Residual (SRMR) is an absolute fit index where the average difference between non-duplicated elements of the actual correlation matrix and the model-implied correlation matrix (Jöreskog & Sörbom, 1981). Values range from 0 to 1 with .08 or lower considered adequate fit of the covariance structure (Hu & Bentler, 1999). It should be noted that recommended values from Hu and Bentler (1999) were not based on models with a mean structure, such as growth models, therefore these cutoff values are not realistic or appropriate for the models simulated in this study.

Data Generation

When testing new methodologies, it is important to have data with known parameterization. It is usually not possible to do this with non-simulated data. Monte Carlo simulations allow data to be generated using known values and structures and then attempt to recover those structures with the methodological procedure of interest. For this study, SLCMs were generated with known parameters for the following: relative asymmetry, inflection point, rate of change at inflection point, lower asymptote, and overall growth. All parameters were allowed to have some randomness and thus variance. Once generated, several competing models were fitted to determine if the true model was the best-fitting. That is, sigmoidal data with known parameters were simulated and then analyzed using linear, quadratic, cubic, and Richards models to examine how well the correct model (sigmoidal) was preferred over competing models. Finally, parameter estimates were recovered from converged true models to assess parameter estimate bias and standard error bias.

Manipulated Factors

Repeated Measures

It is important to have an idea of the minimum number of repeated measures necessary to correctly select any particular model. This number varies depending on the number of parameters that are estimated when fitting the model. With only two time points, a linear model can be perfectly identified every time. Therefore, to test for goodness of fit, the minimum number of repeated measures for correct identification of a linear model is three. When the model is quadratic, this number increases to four due to the additional parameter that needs to be estimated. Cubic models require five repeated measures to be correctly identified. Likewise, five repeated measures are required for Logistic and Gompertz sigmoidal models due to the fact that the relative asymmetry parameter (γ) is not estimated, but explicitly included in the model. When $\gamma = 1$, the Richards model is equivalent to the Logistic model and as γ approaches zero, the Richards function approaches the Gompertz function. For the Richards model, relative asymmetry (γ) is estimated along with lower asymptote (i), total amount of growth (β), maximum rate of change (ρ), and timing of the inflection point (δ). The Richards function, therefore, requires a minimum of six repeated measures for proper identification. Applied use of the sigmoidal SLCMs ranged from five (Rast, 2011) to ten repeated measures(Cameron et al., 2015). This study simulated six, eight, and ten equally spaced repeated measures which captured a range from the minimum required repeated measures to a number not exceeding the upper limit seen in applied research.

Sample Size

Like other structural equation models, SLCMs have a lower bound on sample size required to correctly select a model depending on the parameters. Applied studies investigating SLCMs are sparse. At the low end of the sample size range, Dodonov and Dodonova (2012) used a sample size of 103 fit to modified exponential and logistic models. At the high end of the sample size range ($n = 20,000+$), two studies have examined sigmoidal growth patterns using data from large public datasets such as the Early Childhood Longitudinal Survey Kindergarten Cohort (ECLS-K) and the National Longitudinal Survey of Youth Children and Young Adults (NLSY-CYA) (see

Cameron et al., 2015; Grimm et al., 2010). A lower bound on sample size required for model selection has not been established, therefore, it was essential to investigate minimum sample sizes required for correct selection over competing models. Likewise, it was important to select a range of sample sizes from small to relatively large to understand the levels where correct model selection fails. For this study, six different sample sizes will be examined: 50, 100, 200, 500, 1,000, and 1,500.

Timing of the Inflection Point

The point of inflection is defined as the point on a curve where the concavity changes, and is usually tested mathematically by taking the second derivative of the function. The point where $f''(x) = 0$, and $f''(x) < 0$ on one side and $f''(x) > 0$ on the other side is the point of inflection. Because this study examined different numbers of repeated measures, δ^* was used as a proportion as opposed to a raw repeated measure value where

$$\delta^* = \frac{\delta}{RM - 1}. \tag{3.3}$$

For example, when using six repeated measures, the mid-point of the SLCM δ equal to 2.5 which is the midpoint between the $t = 2$ and $t = 3$ repeated measure. This point is $\delta^* = .50$ and it will remain stable for different repeated measures. Changes in the inflection point will result in changes in the shape of the curve as it appears within the measurement window which can increase the chances of an incorrect model being selected. By examining different levels, the way timing affects correct model selection over a similar but incorrect competing model can be better understood. On this matter, applied research provides little practical guidance. Cameron et al. (2015) as well as Grimm et al. (2010) examine early childhood reading or mathematics achievement and therefore their results indicate similar inflection points. To compensate for the lack of variability in applied studies, this study examined three different values for the timing of the inflection point: $\delta^* = .25$, .375, and .50 which correspond to $\delta = 1.25$, 1.875, 2.5 for six repeated measures, 1.75, 2.625, 3.5 for eight repeated measures, and 2.25, 3.375, 4.5 for 10 repeated measures.

Maximum Rate of Change

In the Richards function, as well as the Logistic and Gompertz functions, the maximum rate of change occurs at the point of inflection. Also called the rate of approach, the maximum rate of change has the effect of flattening out the sigmoidal curve so that it begins to look similar to a linear model when using certain sets of conditions. As with the choice of the inflection point, the sparsity of applied research means there is, again, little guidance on selection of realistic values. In this study, three values for the maximum rate of change were simulated: .125, .1875, and .25. The instantaneous rate of change at the inflection point is obtained by taking the partial derivative of the Richards target function with respect to time, then setting time equal to the simulated δ parameter. For the Richards function, this rate of change is

$$\rho^* = \frac{\beta * \rho}{(\gamma + 1)(\gamma + 1)^{\frac{1}{\gamma}}}. \tag{3.4}$$

The maximum rate of change is dependent on three different parameters, β (overall growth), ρ (rate of approach), and γ (relative asymmetry). In this study, the values of β and γ are non-manipulated factors that were both set to 1.0. As such, to simulate maximum rates of change values where ρ^* is equal to .125, .1875, and .25 , the value of ρ was simulated as 0.5, 0.75, and 1.0 respectively. Lower values of ρ are flatter than higher values.

Model Simulation Summary

In addition to sample size, the factors this study examined are the number of repeated measures (6, 8, 10), maximum rate of change ($\rho = .5, .75, 1.0$), and the timing of the inflection point ($\delta = .25, .375, .50$). The overall shape of the curves being simulated for six repeated measures are shown in Figure 3.1, for eight repeated measures in Figure 3.2, and for 10 repeated measures in Figure 3.3. Each individual curve will be simulated with sample sizes of 50, 100, 200, 500, 1,000, and 1,500.

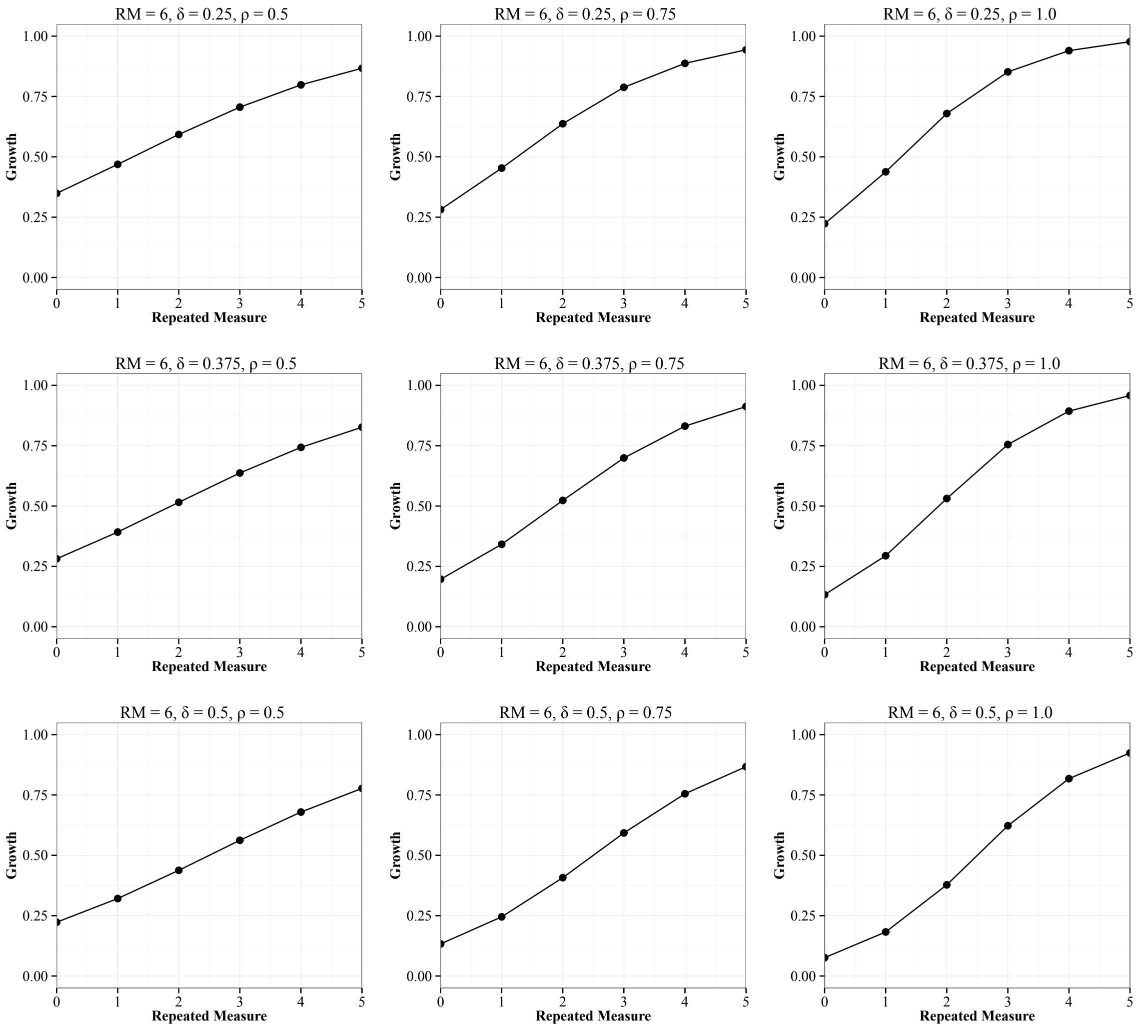

Figure 3.1. Simulated growth curves for six repeated measures.

Non-Manipulated Factors

Relative Asymmetry

The Richards function contains an additional parameter (γ) over the Logistic and Gompertz functions that controls the relative asymmetry of the function. That is, the value of γ determines which asymptote, upper or lower, the inflection point

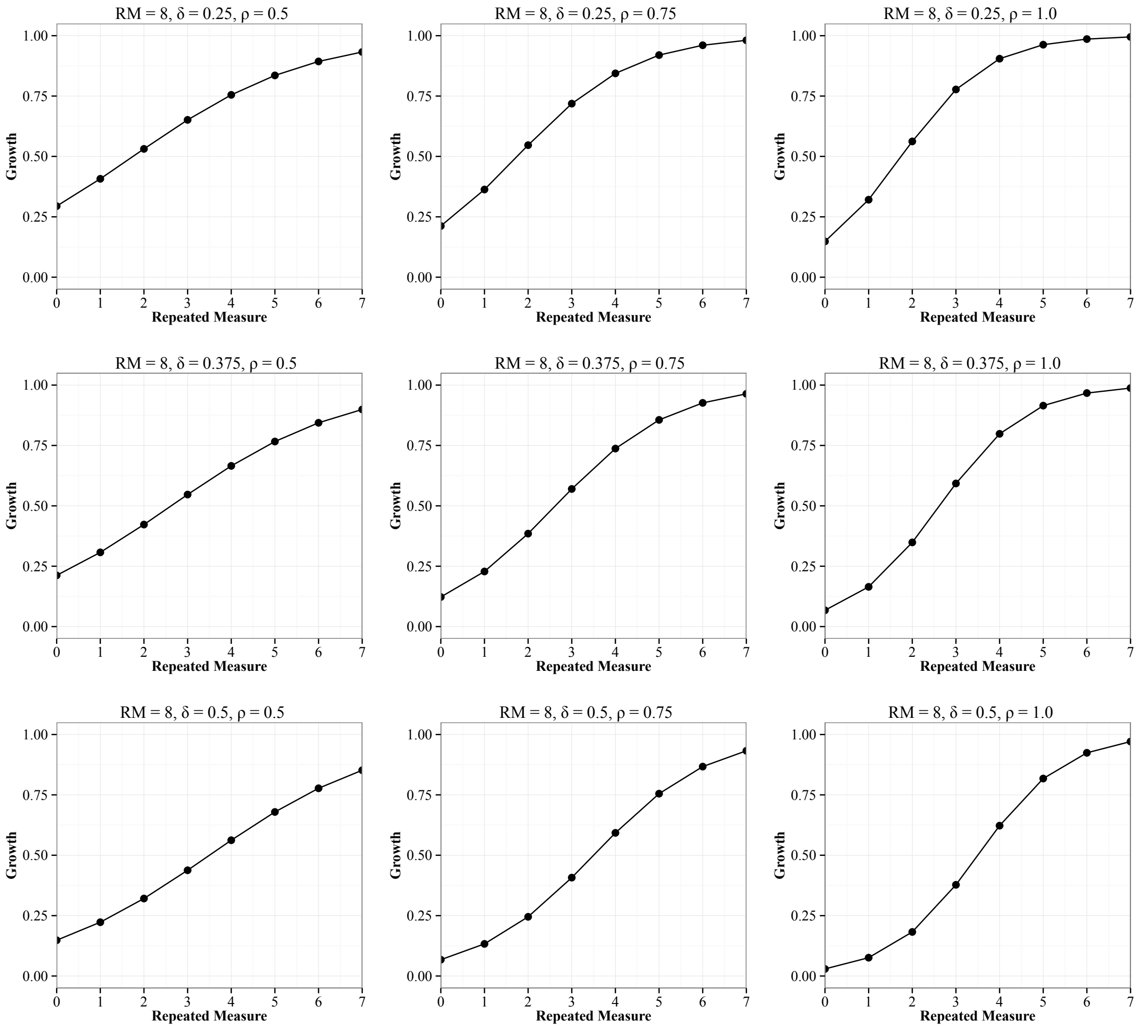

Figure 3.2. Simulated growth curves for eight repeated measures.

is nearest. When $\gamma = 1$, the Richards function is equivalent to the Logistic function which represents symmetrical growth where 50% of growth occurring prior to the inflection point. As γ approaches zero, the Richards curve approximates the shape of the Gompertz curve which is an asymmetrical curve with approximately 36.8% $(\frac{1}{e})$ of growth occurring prior to the inflection point. This is calculated by taking the partial derivative of the Richards function with respect to β and setting time equal to the simulated parameter δ. The proportion of growth occurring prior to the inflection

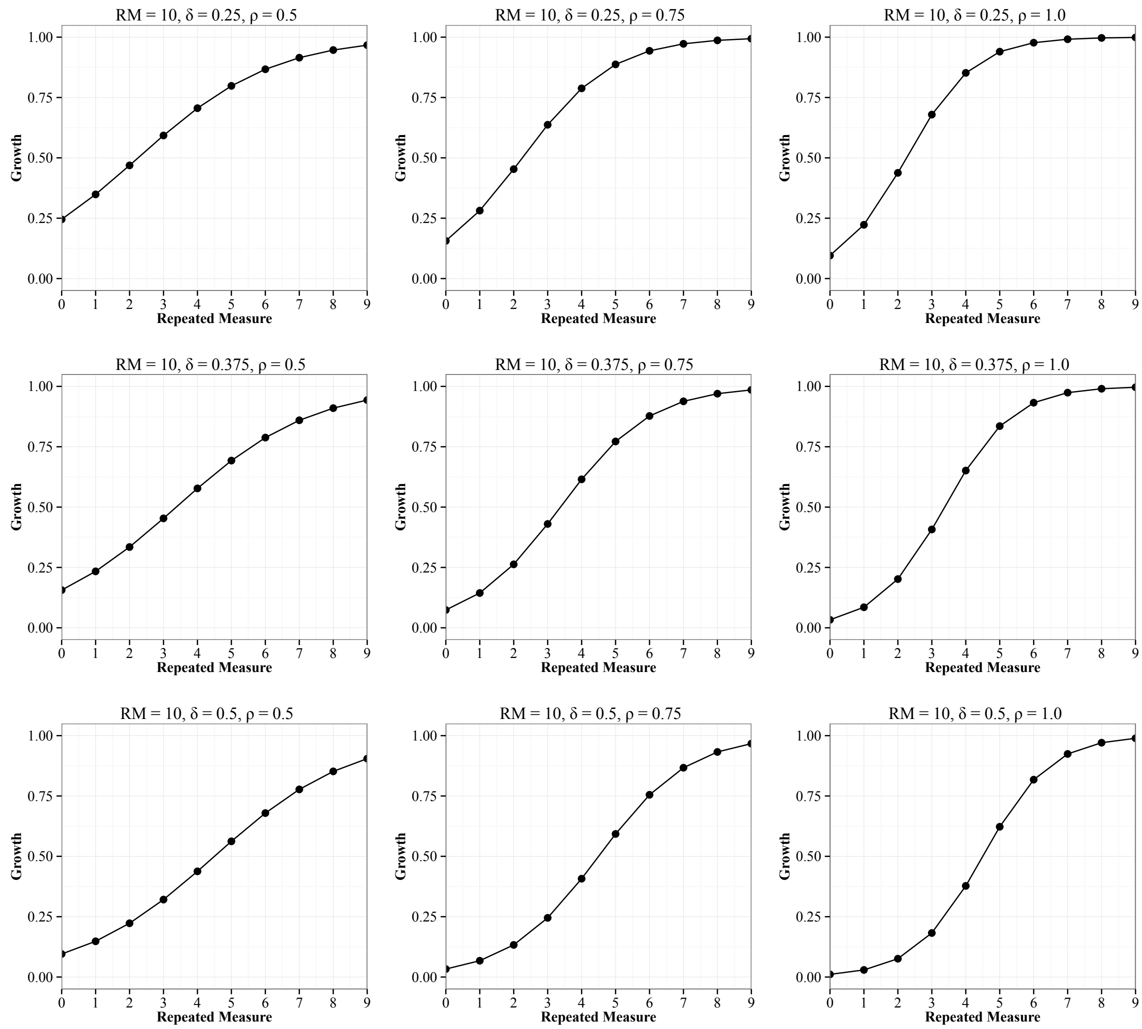

Figure 3.3. Simulated growth curves for ten repeated measures.

point is given by

$$\gamma^* = \frac{1}{(\gamma + 1)^{\frac{1}{\gamma}}}. \tag{3.5}$$

Due to the limited scope of this research, comparisons between different sigmoidal functions were not included. The percentage of growth occurring prior to the inflection point (γ^*) was set to 50%, therefore the value of γ was fixed to 1 when generating data.

Asymptotes

Unlike linear models where an intercept and slope are specified, the Richards function has parameters for a lower asymptote and overall amount of growth. With guidance from L. K. Muthén and Muthén (2002) as well as Whittaker and Khojasteh (2017), the simulated data have the lower asymptote set to zero and the overall growth set to .2 per repeated measure. This provided an overall growth for a six repeated measure model of 1.0 which is equivalent to the overall slope of .2 recommend for linear longitudinal data simulations. Bounded sigmoidal models, however, are conceptually different from unbounded linear models and the overall growth can be treated as a scaling constant. As such, the overall growth was held at 1 for numbers of repeated measures to aid in interpretation of results. All simulated outcome variables were continuous. Again, using guidance from L. K. Muthén and Muthén (2002) as well as Whittaker and Khojasteh (2017), the residual variances of the manifest variables at each measurement occasion were set to .5 which represents constant variance over time. Variances of the latent variables β, δ, and ρ were also set to .5 and the variances of i and γ were constrained to zero because these represent universal intercept and relative asymmetry respectively.

Condition Summarization

Non-Convergence

Non-convergence is a routine problem when estimating Latent Growth Curve models (Preacher, 2010) as well as other structural equation models. It was, therefore, reasonable to expect that non-convergence would be observed when fitting SLCMs. Those models that did not converge, or had other estimation errors, were excluded from the fit measure summary and parameter estimate bias calculations as has been done in previous studies (e.g., Flora & Curran, 2004; Morgan, 2015; Yang-Wallentin, Jöreskog, & Luo, 2010). The rate of non-convergence was tracked and the proportions were reported. In an attempt to minimize non-convergence while being mindful of the time-consuming nature of simulations of this nature, plausible starting values were used. It was beyond the scope of this study to individually investigate and attempt to rectify convergence issues within each simulated dataset.

Model Selection

A model was considered correctly selected when the sigmoidal model was selected as the best-fitting for each fit measure within each replication. Results were scored as zero (an incorrect model was selected as the best-fitting) or one (the true sigmoidal model was selected as the best-fitting). The mean of the results for each condition was the proportion of correctly selected models with higher means indicating better accuracy than lower means. Finally, a logistic regression was run on the results to determine how the individual manipulated factors contributed to fit measure performance.

Comparative Fit Measures

For all converged solutions, fit measures were collected from Mplus or calculated from the model summaries extracted from the Mplus output for each individual condition. The fit measures collected or calculated were: 1) Akaike Information Criterion (AIC), 2) Bayesian Information Criterion (BIC), 3) Bayesian Information Criterion with sample size adjustment (aBIC), 4) Consistent AIC (CAIC), 5) Draper Information Criterion (DIC), 6) Hannan and Quinns adjustment of AIC (HQ), and 7) Sugiuras adjustment of AIC (AICc).

Absolute and Incremental Fit Measures

For all converged solutions, fit measures were extracted from Mplus output for each individual condition. The fit measures examined were: 1) the comparative fit index (CFI), 2) the Tucker-Lewis Index (TLI), 3) the Root Mean Square Error of

Approximation (RMSEA), and 4) the Standardized Root Mean Residual (SRMR). Although these are stand-alone measures that can be used without model comparison if there are pre-established standards for what constitutes acceptable fit, the Hu and Bentler (1999) guidelines for use of these measures were developed on confirmatory factor models that did not include a mean structure. In the absence of appropriate guidance, these measures were used in a manner similar to the comparative fit measures where the model with the best score (higher score for CFI and TLI or lower score for RMSEA and SRMR) was selected. If the true model and a competing incorrect model had the same best score, the true model was scored as correctly selected. In either case, the mean of the results for each condition was the proportion of correctly selected models with higher means indicating better accuracy than lower means. A logistic regression was run on the results to determine how the individual manipulated factors contribute to fit measure performance.

Parameter Recovery

After data were generated and convergence tested to ensure that there were 1,000 converged solutions for each model analyzed as a Richards model, parameter estimates were extracted from the Mplus output for each replication. The parameter estimates that were recovered were the unique latent parameter estimates from the Richards model. These parameters were the universal lower asymptote (i), the calculated upper asymptote (β^*), and the timing of most rapid change within the window of measurement (δ^*). Also included were the instantaneous maximum rate of change ($\rho*$) which was calculated from Equation 3.4 using the ρ parameter estimated in Mplus and the proportion of growth that occurred prior to the inflection point (γ^*) which was calculated using Equation 3.5 and was based on the γ parameter estimated in Mplus.

Bias. The parameter estimate bias of each individual parameter was assessed to determine the extent of bias under differing conditions. The relative bias of a parameter estimate is given by the equation:

$$relative\ Bias(\hat{\theta}) = \sum_{j=1}^{n} \left(\frac{\hat{\theta}_j - \theta}{\theta} \right) /n \tag{3.6}$$

where $\hat{\theta}_j$ is the recovered parameter estimate from each sample j of a given parameter, θ is the true value of the parameter, and n is the number of replications within a particular condition. In the case of i, the relative bias cannot be calculated because the value of the true parameter is zero. In this case, raw bias was estimated:

$$raw\ Bias(\hat{\theta}) = \sum_{j=1}^{n} \frac{(\hat{\theta}_j - \theta)}{n}. \tag{3.7}$$

Once bias was calculated on a per-parameter basis, the results were evaluated to determine the conditions under which the amount of bias became problematic using guidance from B. Muthén, Kaplan, and Hollis (1987) and Hoogland and Boomsma (1998).

Coverage. Coverage is the proportion of times that the simulated true population value is contained within a confidence interval constructed around the recovered parameter estimate (Hancock & Mueller, 2013). Coverage is used to assess the adequacy of confidence intervals and is affected by bias that is present in either recovered parameter estimates or recovered standard errors. As an example, if coverage for a particular parameter was determined to be .90, that would mean that 90% of the simulated parameter estimates would fall within a 95% confidence interval constructed around the simulated population parameter. Collins, Schafer, and Kam (2001) indicate that coverage is the empirical type I error rate which should be compared to the nominal type I error rate, which in this case is .95. They suggest that coverage rates should be no less than .90, below which point they are problematic. L. K. Muthén and Muthén (2002) indicate that coverage proportions should fall between .91 and

.98 to be considered acceptable. Both Bandalos and Leite (2013) and Enders (2001) use the Collins et al. (2001) cutoff, and that will be the cutoff used in this study.

Standard Error Bias. Standard error bias was calculated similar to the method of calculating paramater bias. The main difference between them is that rather than explicitly simulating the true values, as in the case of the parameters, standard errors must be calculated after the simulation. The equation for standard error bias is

$$relative\ Bias(\widehat{SE}(\hat{\theta})) = \sum_{j=1}^{n} \left(\frac{\widehat{SE}(\hat{\theta_j}) - SE(\hat{\theta})}{SE(\hat{\theta})} \right) / n \tag{3.8}$$

where $SE(\hat{\theta})$ is the estimated population standard error and $\widehat{SE}(\hat{\theta}_j)$ is the recovered standard error estimate from each sample j of a given parameter.

Empirical Example

As an example of the use of structured latent curve models in applied research, math achievement scores from the Early Childhood Longitudinal Study, Kindergarten Class of 1998-1999 (ECLS-K) (Tourangeau, Nord, Lê, Sorongon, & Najarian, 2009) were fit with linear, quadratic, cubic, and Richards models. Using the previously discussed fit measures, the best fitting model was selected and the parameters interpreted. Although this data have been previously analyzed using structured latent curve models (Cameron et al., 2015), the partial derivatives for the latent basis curves in the models presented in the Cameron et al. (2015) paper are different from the ones derived here and should be treated as different models.

Summary

The purpose of this study was to examine sigmoidal structured latent curve models under varying conditions to provide recommendations on how to properly structure longitudinal research. This is necessary to ensure adequate sample size and number of repeated measures when there is an *a priori* hypothesis of sigmoidal growth.

Recommendations are provided for model selection when completing *post hoc* analysis of existing data as well as for which fit measures work best under those conditions. The manipulated factors in this simulation included sample size (six conditions), repeated measures (three conditions), point of inflection (three conditions), and rate of change at the inflection point (three conditions) for a total of 162 conditions. Each condition was replicated 1,000 times for a total of 162,000 datasets.

Once the data were generated, each dataset was analyzed in Mplus (L. Muthén & Muthén, 2015) as linear, quadratic, cubic, and sigmoidal models for a total of 648,000 analyses. Upon completion of the analysis, the conditions were summarized using the MplusAutomation package (Hallquist, 2014) within R (R Development Core Team, 2016). Fit measures not natively generated within Mplus were calculated and saved with the summary data. For each replication, the best-fitting model was selected for each fit measure. For each condition, the proportion of times the true model was selected for each fit measure was recorded. Higher proportions indicated better fit measure performance. A logistic regression was also run on the summary results to determine how the individual manipulated factors contributed to fit measure performance. Parameter estimates were then recovered from each converged Richards solution and the quality of those estimates was assessed. An ANOVA with two-way interactions and main effects was run to determine the extent to which the manipulated variables contributed to biased parameter estimates.

CHAPTER FOUR

Results

Convergence Rates

To ensure that there were 1,000 converged replications for each of the 162 conditions of this study, 1,000 datasets for each cell were generated to test convergence. Once the 162,000 data sets were generated, the Richards model was used to analyze the data. Models that converged were scored a one and those that did not were scored a zero. These convergence rates were used to calculate how many replications were needed for each condition to ensure 1,000 converged replications for parameter estimate bias and model selection evaluations. To judge the impact of the various manipulated factors on convergence rates, logistic regressions were run and odds ratios (OR) were calculated to be used as a measure of effect size. Odds ratios are an indication of the relative measure of effect between a reference and an intervention group. In this case, the reference group was: a sample size of 50, the number of repeated measures of six, the location within the measurement window (δ^*) of 0.25, and the maximum rate of change (ρ^*) of 0.125. Because of the large number of data sets used in this simulation, all p values, unless explicitly stated, are assumed to be $< .001$. In an effort to make sure that the highest possible convergence rates were achieved while being mindful of the time involved in analyzing a large number of data sets, plausible starting values were used.

Convergence rates varied drastically on a cell by cell basis with many of the conditions having unacceptably low rates of convergence. The overall convergence rate was 70.4% with individual conditions ranging from 6.9% to 100% (see Table 4.1). For conditions with six repeated measures, the rates of convergence were poorest with a mean of 52.0%. The condition with the lowest rate of convergence (6.9%) was the

Table 4.1

Convergence Rate by Condition

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	9.3	19.5	25.2	9.2	19.6	28.0	6.9	15.9	19.7
	100	18.8	29.9	43.6	13.7	33.1	45.3	11.8	24.2	36.0
	200	24.9	47.6	59.9	21.0	44.8	62.4	15.9	37.3	50.3
	500	38.5	62.8	75.1	31.1	64.7	83.2	22.6	57.0	80.6
	1,000	41.9	70.9	81.9	36.3	81.0	92.6	32.8	71.7	91.8
	1,500	47.7	73.7	84.8	45.7	84.5	96.0	37.0	81.0	96.7
8	50	26.0	36.5	37.9	23.8	37.2	42.4	21.2	29.5	38.6
	100	34.9	49.8	55.5	33.9	54.2	61.5	33.1	51.6	55.3
	200	47.2	64.5	74.0	45.2	72.2	80.2	46.1	71.9	76.5
	500	60.3	80.6	87.3	64.5	90.2	93.4	67.2	91.1	93.2
	1,000	71.9	86.4	95.8	84.3	96.5	98.4	84.5	97.7	98.1
	1,500	78.8	91.9	97.8	88.3	99.0	99.3	90.9	99.4	99.4
10	50	33.6	44.3	47.2	39.1	49.1	49.5	39.1	48.8	43.6
	100	50.4	60.6	63.7	57.5	68.1	66.0	57.9	66.3	65.7
	200	64.4	78.6	80.6	75.4	84.7	85.3	81.1	85.6	81.7
	500	78.9	90.2	93.5	92.7	96.6	96.3	95.7	98.6	97.0
	1,000	87.5	95.9	98.8	96.9	99.2	99.7	98.2	99.8	99.5
	1,500	93.1	98.2	99.3	99.1	99.9	100.0	99.6	100.0	100.0

Note. Numbers are percentages. RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

condition with a sample size of 50, the lowest maximum rate of change, and the most centered timing of the inflection point. The highest convergence rate, for six repeated measures, was 96.7% in the condition with a sample size of 1,500 and the highest maximum rate of change and most centered timing of the inflection point. For eight repeated measures, the mean convergence rate was 68.2% ranging from 21.2% to 99.4% and observed under the same conditions as six repeated measures. For 10 repeated measures the mean convergence rate was 74.2%. The lowest convergence rate for 10 repeated measures was 33.6% and occurred in the condition with a sample size of 50, the lowest maximum rate of change and the most off-centered timing of the inflection point. The highest convergence rates were 100% and occurred under several different conditions where the sample size was 1,500. A logistic regression was run to understand how the different levels of the manipulated parameters impacted convergence rates (see Table 4.2). Increasing the number of repeated measures from six to eight with all other conditions held equal, increased the odds of convergence roughly 3.4 times (OR = 3.44, 95% CI: 3.34-3.54). Increasing from six to 10 repeated measures increased odds of convergence by roughly 7.3 times (OR = 7.31, 95% CI: 7.07-7.55).

The location of the inflection point within the measurement window (δ^*) had relatively little impact on the rate of convergence. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 30% increase in the odds of convergence (OR = 1.30, 95% CI: 1.26-1.34) and an increase to $\delta^* = .5$ resulted in a 13% increase in the odds of convergence (OR = 1.13, 95% CI: 1.09-1.16).

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on convergence rates, but not as large as the impact of increasing the number of repeated measures. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of convergence by 2.7 times (OR = 2.67, 95% CI: 2.59-2.75) and an increase to $\rho^* = .5$ increased odds of convergence by roughly 4 times (OR = 4.01, 95% CI: 3.88-4.14).

Table 4.2

Logistic Regression for Convergence Rates

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-2.97	0.02	-125.96	< 0.001	0.05	0.05	0.05
RM = 8	1.23	0.02	80.89	< 0.001	3.44	3.34	3.54
RM = 10	1.99	0.02	119.11	< 0.001	7.31	7.07	7.55
n = 100	0.77	0.02	38.79	< 0.001	2.15	2.07	2.24
n = 200	1.54	0.02	75.94	< 0.001	4.66	4.48	4.85
n = 500	2.45	0.02	110.90	< 0.001	11.54	11.05	12.05
n = 1,000	3.03	0.02	125.86	< 0.001	20.76	19.80	21.76
n = 1,500	3.36	0.03	131.08	< 0.001	28.83	27.41	30.31
$\delta^* = 0.375$	0.26	0.02	16.97	< 0.001	1.30	1.26	1.34
$\delta^* = 0.5$	0.12	0.02	7.74	< 0.001	1.13	1.09	1.16
$\rho^* = 0.1875$	0.98	0.02	63.96	< 0.001	2.67	2.59	2.75
$\rho^* = 0.25$	1.39	0.02	86.81	< 0.001	4.01	3.88	4.14

Note. n = sample size. δ^* = location of the inflection point within the measurement window expressed as a proportion. ρ^* = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Of all manipulated conditions, sample size had the largest impact on the rates of convergence. The mean convergence rate for n=50 was 31.1% across all conditions. An increase in sample size from 50 to 100 increased the rate of convergence to 46.0%. The odds of convergence increased by just over 2 times (OR = 2.15, 95% CI: 2.07-2.24). Increasing to 200 brought the convergence rate to 61.5% and increased the odds by roughly 4.5 times (OR = 4.66, 95% CI: 4.48-4.85). When the sample size was 500, the convergence rate was 77.1%. The increase in sample size from 50 to 500 improved the odds of convergence by over 11.5 times (OR = 11.54, 95% CI: 11.05-12.05). Increasing from 50 to 1,000 increased the convergence rate to 84.8% and made the odds of convergence 20.8 times more likely (OR = 20.76, 95% CI: 19.80-21.76). Moving from a sample size of 50 to 1,500, the convergence rate was 88.2% and the odds of convergence increased by 28.8 times (OR = 28.83, 95% CI: 27.41-30.31).

Coverage

Coverage is the proportion of times that the simulated true population value is contained within a confidence interval constructed around the recovered parameter estimate. For this study, the Collins et al. (2001) guideline that coverage values under .90 are problematic were used. Coverage rates were examined for each parameter used in the Richards model: lower asymptote (i), upper asymptote (β^*), maximum rate of change (ρ^*), location of the inflection point (δ^*), and the relative asymmetry (γ^*). To better understand the impact of the various manipulated factors on coverage rates, logistic regressions were run and odds ratios were calculated to be used as a measure of effect size. Odds ratios are a relative measure of effect between a reference and an intervention group. As with convergence, the reference group was: a sample size of 50, the number of repeated measures of 6, the location within the measurement window (δ^*) of 0.25, and the maximum rate of change (ρ^*) of 0.125. Because of the large number of data sets used in this simulation, all p values, unless explicitly stated, are assumed to be $< .001$. Individual parameters are discussed below.

Lower Asymptote (i)

Under the conditions simulated in this study, coverage rates for the lower asymptote were for the most part unacceptably low. This may be due in part to the direction of the offset of the location of the inflection point. Because of the direction of the offset, more information was available for the upper portion of the sigmoidal curve than for the lower portion. Had the location of the inflection point been offset in such a way as to provide more information on the lower part of the curve, it is possible that the coverage for the lower asymptote would have improved. The overall coverage proportion was .82 across all conditions with individual cells ranging from .37 to .97 (see Table 4.3). The conditions where coverage was .90 or greater tended to be those conditions with eight or 10 repeated measure and the location of the inflection point was centered within the measurement window.

Logistic regression results were used to determine which of the manipulated conditions had the largest impact on the rate of coverage for the lower asymptote parameter (see Table 4.4). The location of the inflection point within the measurement window (δ^*) had a larger impact on the proportion of coverage for the lower asymptote parameter than any manipulated factor except the number of repeated measures. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 77% increase in the odds of coverage (OR = 1.77, 95% CI: 1.72-1.83) and an increase to $\delta^* = .5$ resulted in a 3 fold increase in the odds of coverage (OR = 3.02, 95% CI: 2.92-3.13).

The maximum rate of change parameter (ρ^*) had a larger impact than sample size on coverage proportions, but not as large as the impact of increasing the number of repeated measures. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of coverage by 65% (OR = 1.65, 95% CI: 1.60-1.71) and an increase to $\rho^* = .5$ increased odds of coverage by roughly 2.8 times (OR = 2.79, 95% CI: 2.69-2.88).

Increasing the number of repeated measures from six to eight with all other conditions held equal, increased the odds of coverage by just over 2 times (OR = 2.09, 95% CI: 2.03-2.16). Increasing from six to 10 repeated measures increased those odds by a little under 3.6 times (OR = 3.59, 95% CI: 3.47-3.72).

Of all manipulated conditions, sample size had the smallest impact on the rates of coverage. An increase in sample size to 100 decreased the odds of coverage by 11% (OR = 0.89, 95% CI: 0.86-0.93). An increase in sample size to 200 was not statistically significant ($p = .788$, OR = 1.01, 95% CI: 0.96-1.05). The increase in sample size to 500 improved the odds of coverage by 35% (OR = 1.35, 95% CI: 1.29-1.41). Increasing sample size to 1,000 increased the odds of coverage 64% (OR = 20.76, 95% CI: 19.80-21.76). Finally, when sample size was 1,500, the odds of coverage increased by 92% over when sample size was 50 (OR = 1.92, 95% CI: 1.83-2.02).

Table 4.3

Coverage Proportion for Lower Asymptote (i) Parameter

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.49	0.60	0.71	0.61	0.74	0.85	0.75	0.84	0.91
	100	0.37	0.53	0.69	0.49	0.68	0.80	0.67	0.77	0.87
	200	0.40	0.57	0.70	0.53	0.67	0.80	0.64	0.80	0.90
	500	0.55	0.70	0.80	0.60	0.75	0.84	0.75	0.83	0.89
	1,000	0.58	0.75	0.81	0.67	0.78	0.86	0.78	0.84	0.89
	1,500	0.61	0.74	0.84	0.71	0.86	0.88	0.86	0.86	0.91
8	50	0.57	0.72	0.82	0.69	0.82	0.89	0.84	0.87	0.93
	100	0.56	0.67	0.81	0.74	0.83	0.91	0.86	0.90	0.94
	200	0.60	0.73	0.85	0.78	0.84	0.91	0.86	0.91	0.95
	500	0.70	0.77	0.86	0.83	0.89	0.91	0.92	0.91	0.94
	1,000	0.75	0.84	0.88	0.86	0.88	0.94	0.90	0.93	0.94
	1,500	0.78	0.85	0.90	0.88	0.91	0.92	0.91	0.92	0.94
10	50	0.68	0.80	0.87	0.85	0.89	0.94	0.92	0.93	0.96
	100	0.70	0.79	0.88	0.86	0.90	0.93	0.94	0.94	0.96
	200	0.76	0.84	0.87	0.87	0.89	0.95	0.94	0.94	0.97
	500	0.80	0.86	0.91	0.89	0.91	0.94	0.93	0.94	0.95
	1,000	0.87	0.91	0.92	0.91	0.94	0.94	0.93	0.95	0.95
	1,500	0.88	0.92	0.92	0.92	0.94	0.94	0.95	0.95	0.94

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

Table 4.4

Logistic Regression Results for Lower Asymptote (i) Coverage

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-0.21	0.02	-9.82	< 0.001	0.81	0.78	0.85
RM = 8	0.74	0.02	46.71	< 0.001	2.09	2.03	2.16
RM = 10	1.28	0.02	71.80	< 0.001	3.59	3.47	3.72
n = 100	-0.11	0.02	-5.05	< 0.001	0.89	0.86	0.93
n = 200	0.01	0.02	0.27	0.788	1.01	0.96	1.05
n = 500	0.30	0.02	12.75	< 0.001	1.35	1.29	1.41
n = 1,000	0.50	0.02	20.57	< 0.001	1.64	1.57	1.72
n = 1,500	0.65	0.02	26.30	< 0.001	1.92	1.83	2.02
$\delta^* = 0.375$	0.57	0.02	36.20	< 0.001	1.77	1.72	1.83
$\delta^* = 0.5$	1.11	0.02	62.78	< 0.001	3.02	2.92	3.13
$\rho^* = 0.1875$	0.50	0.02	31.76	< 0.001	1.65	1.60	1.71
$\rho^* = 0.25$	1.02	0.02	58.41	< 0.001	2.79	2.69	2.88

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Upper Asymptote (β^*)

Contrary to coverage rates for the lower asymptote, coverage proportions for the upper asymptote parameter were, for the most part, above the Collins et al. (2001) .90 threshold. The overall coverage proportion was .90 across all conditions with individual cells ranging from .65 to .97 (see Table 4.5). With six repeated measures, those conditions where the maximum rate of change was highest tended to have acceptable coverage across all sample sizes. With eight repeated measures, only those conditions with the lowest maximum rate of change had values below .90. When the number of repeated measures was 10, all conditions were .90 or above.

The results of a logistic regression were used to determine which of the manipulated conditions had the largest impact on the rate of coverage for the upper asymptote parameter (see Table 4.6). The location of the inflection point within the measurement window (δ^*) had a small impact on the proportion of coverage for the

Table 4.5

Coverage Proportion for Upper Asymptote (β^*) Parameter

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.85	0.95	0.97	0.78	0.91	0.95	0.68	0.84	0.94
	100	0.72	0.90	0.96	0.64	0.88	0.93	0.60	0.80	0.90
	200	0.65	0.88	0.95	0.63	0.86	0.93	0.61	0.81	0.89
	500	0.69	0.91	0.95	0.71	0.87	0.93	0.66	0.82	0.91
	1,000	0.72	0.92	0.94	0.75	0.88	0.95	0.78	0.84	0.92
	1,500	0.80	0.92	0.95	0.81	0.92	0.94	0.80	0.87	0.92
8	50	0.90	0.95	0.97	0.85	0.93	0.96	0.81	0.90	0.95
	100	0.88	0.94	0.96	0.85	0.93	0.96	0.80	0.91	0.94
	200	0.87	0.94	0.95	0.89	0.94	0.95	0.86	0.92	0.95
	500	0.92	0.95	0.96	0.91	0.95	0.95	0.88	0.94	0.96
	1,000	0.93	0.95	0.95	0.94	0.95	0.95	0.88	0.96	0.96
	1,500	0.95	0.96	0.95	0.92	0.96	0.96	0.90	0.94	0.93
10	50	0.94	0.95	0.94	0.92	0.94	0.96	0.90	0.96	0.93
	100	0.94	0.95	0.94	0.93	0.95	0.95	0.90	0.94	0.95
	200	0.95	0.94	0.94	0.95	0.97	0.96	0.91	0.96	0.94
	500	0.95	0.95	0.95	0.94	0.96	0.96	0.94	0.96	0.96
	1,000	0.96	0.96	0.96	0.95	0.95	0.95	0.94	0.95	0.96
	1,500	0.95	0.96	0.95	0.96	0.95	0.95	0.93	0.94	0.96

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

lower asymptote parameter. Contrary to the lower asymptote results, the most offset location of the inflection point allowed for better estimation of the upper asymptote with the reference level performing better than the other levels of this parameter. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 10% decrease in the odds of coverage (OR = 0.90, 95% CI: 0.86-0.94) and an increase to $\delta^* = .5$ resulted in a 31% decrease in the odds of coverage (OR = 0.69, 95% CI: 0.67-0.72).

The maximum rate of change parameter (ρ^*) had the second largest impact on coverage proportions, second only to the number of repeated measures. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of coverage by 2.3 times (OR = 2.25, 95% CI: 2.16-2.34) and an increase to $\rho^* = .5$ increased odds of coverage by roughly 3.3 times (OR = 3.26, 95% CI: 3.12-3.41).

The number of repeated measures parameter had the largest impact on coverage proportions. An increase in the number of repeated measures from six to eight, with all other conditions held equal, increased the odds of coverage by 2.4 times (OR = 2.40, 95% CI: 2.31-2.50). Increasing from six to 10 repeated measures increased those odds by a little under 3.4 times (OR = 3.38, 95% CI: 3.23-3.53).

As with the lower asymptote parameter, sample size had the smallest impact on the rates of coverage. An increase in sample size from 50 to 100 decreased the odds of coverage by 21% (OR = 0.79, 95% CI: 0.75-0.84). The increase in sample size to 200 diminished the odds of coverage by 18% (OR = 0.82, 95% CI: 0.77-0.86). An increase in sample size to 500 was not statistically significant ($p = .132$, OR = 0.96, 95% CI: 0.90-1.01). An increase in sample size to 1,000 increased the odds of coverage 11% (OR = 1.11, 95% CI: 1.05-1.18). When sample size was increased from 50 to 1,500, the odds of coverage increased by only 26% (OR = 1.26, 95% CI: 1.18-1.34).

Table 4.6

Logistic Regression Results for Upper Asymptote (β^*) Coverage

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	1.29	0.03	46.05	< 0.001	0.64	3.44	3.84
RM = 8	0.88	0.02	42.76	< 0.001	2.40	2.31	2.50
RM = 10	1.22	0.02	53.78	< 0.001	3.38	3.23	3.53
n = 100	-0.23	0.03	-7.91	< 0.001	0.79	0.75	0.84
n = 200	-0.20	0.03	-6.97	< 0.001	0.82	0.77	0.86
n = 500	-0.05	0.03	-1.51	0.132	0.96	0.90	1.01
n = 1,000	0.11	0.03	3.39	0.001	1.11	1.05	1.18
n = 1,500	0.23	0.03	7.25	< 0.001	1.26	1.18	1.34
δ^* = 0.375	-0.11	0.02	-4.96	< 0.001	0.90	0.86	0.94
δ^* = 0.5	-0.37	0.02	-17.24	< 0.001	0.69	0.67	0.72
ρ^* = 0.1875	0.81	0.02	39.70	< 0.001	2.25	2.16	2.34
ρ^* = 0.25	1.18	0.02	52.19	< 0.001	3.26	3.12	3.41

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Maximum Rate of Change (ρ^*)

Coverage rates were excellent for the maximum rate of change parameter. The overall proportion of coverage .96 across all conditions with individual cells ranging from .91 to .99 (see Table 4.7).

Logistic regression used to determine the impact of the manipulated conditions on the rate of coverage for the maximum rate of change parameter. Because of the uniformity of the results, odds ratios were very low with many of the results being not statistically significant so all p values were reported for this parameter (see Table 4.8).

The location of the inflection point within the measurement window (δ^*) had little impact on the proportion of coverage for the maximum rate of change parameter. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 6% decrease in the odds of coverage ($p = .028$, OR = 0.94, 95% CI: 0.89-0.99) and an increase to $\delta^* = .5$ resulted in a 6% increase in the odds of coverage ($p = .045$, OR = 1.06, 95% CI: 1.00-1.13).

Table 4.7

Coverage Proportion for Maximum Rate of Change (ρ^*) Parameter

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.96	0.97	0.99	0.96	0.97	0.99	0.95	0.97	0.99
	100	0.96	0.97	0.99	0.96	0.97	0.99	0.95	0.98	0.99
	200	0.94	0.97	0.98	0.95	0.97	0.98	0.94	0.98	0.99
	500	0.95	0.97	0.96	0.94	0.98	0.98	0.96	0.99	0.98
	1,000	0.94	0.97	0.96	0.95	0.96	0.94	0.96	0.97	0.96
	1,500	0.95	0.96	0.95	0.95	0.96	0.93	0.96	0.97	0.94
8	50	0.94	0.96	0.96	0.95	0.94	0.97	0.92	0.96	0.96
	100	0.94	0.96	0.97	0.95	0.96	0.95	0.97	0.97	0.94
	200	0.95	0.95	0.96	0.98	0.96	0.94	0.96	0.97	0.94
	500	0.96	0.95	0.96	0.96	0.94	0.94	0.97	0.96	0.95
	1,000	0.96	0.96	0.95	0.96	0.94	0.94	0.96	0.95	0.95
	1,500	0.95	0.96	0.94	0.95	0.96	0.95	0.95	0.95	0.95
10	50	0.93	0.94	0.94	0.94	0.94	0.92	0.96	0.96	0.92
	100	0.95	0.95	0.94	0.95	0.94	0.91	0.96	0.95	0.91
	200	0.97	0.95	0.94	0.96	0.95	0.94	0.97	0.95	0.93
	500	0.96	0.95	0.94	0.95	0.94	0.94	0.95	0.96	0.96
	1,000	0.96	0.96	0.95	0.95	0.95	0.95	0.96	0.95	0.95
	1,500	0.96	0.94	0.95	0.94	0.96	0.96	0.95	0.96	0.94

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

The maximum rate of change parameter (ρ^*) also had little impact on coverage proportions. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of coverage by 15% ($p < .001$, OR = 1.15, 95% CI: 1.09-1.22) and an increase to $\rho^* = .5$ increased odds of coverage by 1% ($p = .709$, OR = 1.01, 95% CI: 0.96-1.07).

The number of repeated measures parameter had the largest, although still small, impact on coverage proportions. An increase in the number of repeated measures from six to eight decreased the odds of coverage by 24% ($p < .001$, OR = 0.76, 95% CI: 0.72-0.81). Increasing from six to 10 repeated measures decreased those odds by 35% ($p < .001$,OR 0.65, 95% CI: 0.61-0.69).

There were no statistically significant differences between any of the sample sizes and the reference group. An increase in sample size from 50 to 100 increased the odds of coverage by 5% ($p = .255$, OR = 1.05, 95% CI: 0.97-1.14). The increase in sample size to 200 increased the odds of coverage by 8% ($p = .064$, OR = 1.08, 95% CI: 1.00-1.17) .An increase in sample size to 500 increased odds of coverage by 6% ($p = .171$, OR = 1.06, 95% CI: 0.98-1.15). An increase in sample size to 1,000 increased had no effect on the odds of coverage ($p = .967$, OR = 1.00, 95% CI: 0.92-1.09). When sample size was increased from 50 to 1,500, the odds of coverage decreased by 4% ($p = .291$, OR = 0.96, 95% CI: 0.89-1.04).

Location of the Inflection Point (δ^*)

Although not as good as the maximum rate of change parameter, coverage proportions for the location of the inflection point parameter were mostly above the Collins et al. (2001) .90 threshold. The overall coverage proportion was .92 across all conditions with individual cells ranging from .69 to .98 (see Table 4.9). With six repeated measures, those conditions where the maximum rate of change was highest tended to have acceptable coverage across all sample sizes. With eight repeated measures, conditions with sample sizes 500 or more had acceptable convergence rates.

Table 4.8

Logistic Regression Results for Maximum Rate of Change (ρ^*) Coverage

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	3.23	0.04	76.30	< 0.001	25.16	23.16	27.33
RM = 8	-0.27	0.03	-8.76	< 0.001	0.76	0.72	0.81
RM = 10	-0.43	0.03	-14.28	< 0.001	0.65	0.61	0.69
n = 100	0.05	0.04	1.14	0.255	1.05	0.97	1.14
n = 200	0.08	0.04	1.86	0.064	1.08	1.00	1.17
n = 500	0.06	0.04	1.37	0.171	1.06	0.98	1.15
n = 1,000	0.00	0.04	0.04	0.967	1.00	0.92	1.09
n = 1,500	-0.04	0.04	-1.06	0.291	0.96	0.89	1.04
δ^* = 0.375	-0.06	0.03	-2.20	0.028	0.94	0.89	0.99
δ^* = 0.5	0.06	0.03	2.00	0.045	1.06	1.00	1.13
ρ^* = 0.1875	0.14	0.03	4.77	< 0.001	1.15	1.09	1.22
ρ^* = 0.25	0.01	0.03	0.37	0.709	1.01	0.96	1.07

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, δ^* = 0.25, and ρ^* = 0.125.

When the number of repeated measures was 10, conditions with sample sizes 200 or more had proportions of .90 or above.

The results of a logistic regression were used to understand which of the manipulated conditions had the largest impact on the rate of coverage for the location of the inflection point parameter (see Table 4.10). The location of the inflection point within the measurement window (δ^*) had the second largest, although still small, impact on the proportion of coverage for the location of the inflection point parameter. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in an 81% increase in the odds of coverage (OR = 1.81, 95% CI: 1.73-1.89) and an increase to $\delta^* = .5$ resulted in a 2.3 times increase in the odds of coverage (OR = 2.26, 95% CI: 2.16-2.36).

The maximum rate of change parameter (ρ^*) had the second smallest impact on coverage proportions. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of

Table 4.9

Coverage Proportion for Location of Inflection Point (δ^*) Parameter

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.71	0.75	0.81	0.90	0.88	0.91	0.95	0.95	0.95
	100	0.69	0.77	0.85	0.87	0.88	0.94	0.96	0.95	0.94
	200	0.71	0.80	0.87	0.90	0.92	0.94	0.96	0.95	0.96
	500	0.79	0.89	0.93	0.94	0.95	0.96	0.98	0.97	0.96
	1,000	0.81	0.91	0.94	0.97	0.96	0.95	0.97	0.97	0.95
	1,500	0.82	0.91	0.96	0.97	0.97	0.96	0.98	0.97	0.96
8	50	0.71	0.78	0.85	0.82	0.84	0.88	0.88	0.85	0.89
	100	0.75	0.85	0.91	0.88	0.89	0.91	0.92	0.92	0.93
	200	0.81	0.89	0.95	0.93	0.93	0.93	0.95	0.94	0.95
	500	0.90	0.94	0.96	0.95	0.95	0.96	0.96	0.94	0.94
	1,000	0.92	0.94	0.97	0.95	0.96	0.96	0.96	0.93	0.96
	1,500	0.94	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.94
10	50	0.76	0.81	0.87	0.83	0.87	0.87	0.90	0.88	0.87
	100	0.86	0.89	0.90	0.90	0.89	0.91	0.93	0.92	0.91
	200	0.91	0.95	0.93	0.93	0.93	0.94	0.94	0.92	0.95
	500	0.96	0.95	0.95	0.96	0.95	0.97	0.95	0.95	0.95
	1,000	0.96	0.96	0.96	0.95	0.95	0.95	0.95	0.95	0.96
	1,500	0.95	0.95	0.95	0.95	0.96	0.95	0.94	0.96	0.94

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

coverage by 21% (OR = 1.21, 95% CI: 1.16-1.26) and an increase to $\rho^* = .5$ increased odds of coverage by 53% (OR = 1.53, 95% CI: 1.46-1.60).

The number of repeated measures parameter had the smallest impact on coverage proportions. An increase in the number of repeated measures from six to eight, with all other conditions held equal, increased the odds of coverage by only 10% (OR = 1.10, 95% CI: 1.06-1.15). Increasing from six to 10 repeated measures increased those odds by a 25% (OR = 1.25, 95% CI: 1.19-1.30).

Unlike the coverage proportions of some of the other manipulated parameters, sample size had the largest impact on the rates of coverage showing a monotonic increase in proportion as sample size increased. An increase in sample size from 50 to 100 increased the odds of coverage by 36% (OR = 1.36, 95% CI: 1.29-1.43). The increase in sample size to 200 diminished the odds of coverage by 90% (OR = 1.90, 95% CI: 1.80-2.00) .An increase in sample size to 500 increased the odds of coverage by nearly 3 times (OR = 2.95, 95% CI: 2.77-3.13). An increase in sample size to 1,000 increased the odds of coverage 3.2 times (OR = 3.18, 95% CI: 2.98-3.38). When sample size was increased from 50 to 1,500, the odds of coverage increased by about 3.5 times (OR = 3.45, 95% CI: 3.23-3.68).

Relative Asymmetry (γ^*)

Coverage rates, although not as good as for the relative asymmetry parameter, were mostly good for the relative asymmetry parameter. Of the 162 individual conditions simulated for this study, coverage was .90 or above in 150 of them. The remaining 12 conditions had coverage rates of .88 or .89. The overall proportion of coverage was .93 across all conditions with individual cells ranging from .88 to .99 (see Table 4.11). Like the maximum rate of change parameter, there were numerous p-values > .001 so they were reported for this parameter.

Table 4.10

Logistic Regression Results for Location of the Inflection Point (δ^*) Coverage

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	1.01	0.03	38.75	< 0.001	2.75	2.62	2.90
RM = 8	0.10	0.02	4.52	< 0.001	1.10	1.06	1.15
RM = 10	0.22	0.02	9.86	< 0.001	1.25	1.19	1.30
n = 100	0.31	0.03	11.91	< 0.001	1.36	1.29	1.43
n = 200	0.64	0.03	22.97	< 0.001	1.90	1.80	2.00
n = 500	1.08	0.03	34.33	< 0.001	2.95	2.77	3.13
n = 1,000	1.16	0.03	35.86	< 0.001	3.18	2.98	3.38
n = 1,500	1.24	0.03	37.43	< 0.001	3.45	3.23	3.68
δ^* = 0.375	0.59	0.02	27.70	< 0.001	1.81	1.73	1.89
δ^* = 0.5	0.81	0.02	35.91	< 0.001	2.26	2.16	2.36
ρ^* = 0.1875	0.19	0.02	8.99	< 0.001	1.21	1.16	1.26
ρ^* = 0.25	0.42	0.02	18.84	< 0.001	1.53	1.46	1.60

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

The location of the inflection point within the measurement window (δ^*) had a small impact on the proportion of coverage for the relative asymmetry parameter. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 30% decrease in the odds of coverage ($p < .001$, OR = 0.70, 95% CI: 0.67-0.74) and an increase to $\delta^* = .5$ resulted in a 26% decrease in the odds of coverage ($p < .001$, OR = 0.74, 95% CI: 0.71-0.78).

The maximum rate of change parameter (ρ^*) had little impact on coverage proportions. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of coverage by 6% ($p = .026$, OR = 1.06, 95% CI: 1.01-1.11) and an increase to $\rho^* = .5$ increased odds of coverage by 8% ($p = .002$, OR = 1.08, 95% CI: 1.03-1.13).

The number of repeated measures parameter had the largest, although still small, impact on coverage proportions. An increase in the number of repeated measures from six to eight decreased the odds of coverage by 34% ($p < .001$, OR = 0.67,

Table 4.11

Coverage Proportion for Relative Asymmetry ($\gamma *$) Parameter

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.99	0.99	0.98	0.98	0.97	0.97	0.98	0.98	0.98
	100	0.98	0.98	0.97	0.98	0.97	0.95	0.98	0.97	0.95
	200	0.97	0.97	0.96	0.98	0.95	0.93	0.97	0.96	0.92
	500	0.97	0.97	0.95	0.95	0.92	0.90	0.95	0.92	0.91
	1,000	0.96	0.97	0.96	0.90	0.90	0.91	0.92	0.90	0.91
	1,500	0.96	0.96	0.95	0.89	0.91	0.92	0.91	0.90	0.93
8	50	0.95	0.95	0.94	0.94	0.91	0.92	0.93	0.93	0.92
	100	0.93	0.94	0.93	0.94	0.91	0.90	0.91	0.90	0.89
	200	0.93	0.93	0.91	0.93	0.92	0.90	0.92	0.89	0.93
	500	0.92	0.94	0.95	0.89	0.91	0.94	0.92	0.93	0.94
	1,000	0.91	0.95	0.95	0.89	0.95	0.94	0.89	0.93	0.95
	1,500	0.91	0.95	0.95	0.91	0.94	0.95	0.92	0.95	0.94
10	50	0.93	0.92	0.93	0.89	0.88	0.90	0.89	0.90	0.90
	100	0.93	0.90	0.90	0.88	0.89	0.91	0.90	0.92	0.92
	200	0.92	0.92	0.92	0.91	0.88	0.93	0.90	0.91	0.95
	500	0.93	0.95	0.95	0.91	0.94	0.97	0.93	0.95	0.95
	1,000	0.95	0.97	0.95	0.93	0.95	0.96	0.94	0.95	0.96
	1,500	0.95	0.96	0.95	0.94	0.95	0.95	0.94	0.94	0.95

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

95% CI: 0.63-0.70). Increasing from six to 10 repeated measures decreased those odds by 33% ($p < .001$,OR 0.67, 95% CI: 0.64-0.71).

There were no statistically significant differences between sample sizes of 500, 1,000, or 1,500 and the reference group. Smaller sample sizes had very small effect sizes. An increase in sample size from 50 to 100 decreased the odds of coverage by 10% ($p = .004$, OR = 0.90, 95% CI: 0.84-0.97). The increase in sample size to 200 decreased the odds of coverage by 12% ($p < .001$, OR = 0.88, 95% CI: 0.82-0.94). An increase in sample size to 500 decreased odds of coverage by 4% ($p = .203$, OR = 0.96, 95% CI: 0.89-1.13). Increasing to 1,000 decreased the odds of coverage by 5% ($p = .158$, OR = 0.95, 95% CI: 0.89-1.02). When sample size was increased to 1,500, the odds of coverage decreased by 3% ($p = .594$, OR = 0.98, 95% CI: 0.92-1.05).

Table 4.12

Logistic Regression Results for Relative Asymmetry (γ^*) Coverage

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	3.18	0.04	85.49	< 0.001	24.10	22.40	25.92
RM = 8	-0.41	0.03	-15.97	< 0.001	0.66	0.63	0.70
RM = 10	-0.40	0.03	-15.45	< 0.001	0.67	0.64	0.71
n = 100	-0.10	0.03	-2.91	0.004	0.90	0.84	0.97
n = 200	-0.13	0.03	-3.71	< 0.001	0.88	0.82	0.94
n = 500	-0.04	0.04	-1.27	0.203	0.96	0.89	1.03
n = 1,000	-0.05	0.04	-1.41	0.158	0.95	0.89	1.02
n = 1,500	-0.02	0.04	-0.53	0.594	0.98	0.92	1.05
$\delta^* = 0.375$	-0.35	0.03	-13.91	< 0.001	0.70	0.67	0.74
$\delta^* = 0.5$	-0.30	0.03	-11.63	< 0.001	0.74	0.71	0.78
$\rho^* = 0.1875$	0.05	0.02	2.22	0.026	1.06	1.01	1.11
$\rho^* = 0.25$	0.07	0.02	3.04	0.002	1.08	1.03	1.13

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Parameter Recovery

The generalized logistic function, also known as the Richards function, has five parameters in its equation: the lower asymptote (i), the total amount of growth (β), the maximum rate of change (ρ), the timing of the inflection point within the measurement window (δ), and the relative asymmetry (γ) of the curve. Because structured latent curves use the Taylor series approximation method (Browne, 1993; Browne & du Toit, 1991), a partial derivative or each parameter was taken and put into an additive form similar to polynomial models. Each partial derivative is a latent growth factor in the same way that intercept and slope are latent growth factors in linear growth models. However, the partial derivatives of the Richards curve are complex nonlinear equations and so additional consideration was required to understand implications of the parameters simulated and recovered.

Parameter Estimate Bias

1,000 converged solutions for each model were analyzed using the Richards model and parameter estimates were extracted from the Mplus output for each replication. The parameter estimates to be recovered were: the universal lower asymptote (i), the location of the upper asymptote (β^*), the timing of most rapid change within the window of measurement as a proportion (δ^*), the instantaneous maximum rate of change (ρ^*), and the proportion of growth that occurs prior to the inflection point (γ^*). Published guidelines for how much parameter bias is acceptable ranges from .05 (Hoogland & Boomsma, 1998) to .10-.15 (B. Muthén et al., 1987). For this simulation, .10 was considered the upper limit on acceptable parameter estimate bias. Summary statistics for parameter estimate bias are presented in Table 4.13. Results are reported for each parameter individually. Due to the non-normality of the parameter bias, non-parametric aligned rank transform ANOVA were used to generate effect sizes for comparison. Partial omega squared (ω_p^2) was reported as a measure of effect size.

Table 4.13

Descriptive Statistics of Overall Parameter Bias

Paramater	Mean Bias	sd	Median Bias	Skewness	Kurtosis
i	-0.07	0.51	0.02	-10.83	246.13
β^*	-0.01	0.12	-0.01	1.11	8.06
ρ^*	0.16	0.35	0.06	3.2	16.77
δ^*	0.06	0.33	0.03	0.79	5.09
γ^*	0.06	0.29	0.03	0.26	0.41

Note. Sample size = 162000. i = lower asymptote. β^* = upper asymptote. δ^* = location of the inflection point within the measurement window expressed as a proportion. ρ^* = rate of change at the inflection point. γ^* = relative asymmetry.

Lower Asymptote (i)

Because the simulated true parameter value for the lower asymptote was zero, it was not possible to calculate relative bias for this parameter. As such, raw or absolute bias was used. Because raw bias was used, the results for this parameter cannot necessarily be interpreted in the same context as the other parameters that use relative bias (Bandalos & Leite, 2013). Overall, for the lower asymptote, parameter estimation bias was non-uniform. Most of the bias was negative and ranged in absolute measures from 0 to .31. In non-absolute measures the range was -0.31 to 0.22 (see Table 4.14). An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.15).

Parameter estimate bias across the different sample sizes was fairly consistent. When sample size was 50, the mean parameter estimate bias was -.05. At 100, bias increased to -.06 and from 200 to 1,000 held steady at -.06 before falling again to -.06 when sample size was 1,500. The effect size for sample size was small (ω_p^2 = 0.021, $p < .001$).

Table 4.14

Absolute Bias of Estimates of the Lower Asymptote (i)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.20	0.10	-0.02	0.12	0.02	-0.11	0.05	-0.01	-0.08
	100	0.22	0.08	-0.08	0.16	-0.05	-0.11	0.07	-0.02	-0.06
	200	0.19	0.01	-0.10	0.11	-0.05	-0.12	0.07	-0.03	-0.08
	500	0.07	-0.08	-0.17	0.03	-0.15	-0.19	0.01	-0.05	-0.07
	1,000	0.00	-0.08	-0.13	-0.06	-0.19	-0.18	-0.02	-0.06	-0.04
	1,500	-0.01	-0.08	-0.16	-0.12	-0.24	-0.11	-0.04	-0.08	-0.03
8	50	0.08	-0.09	-0.15	0.04	-0.10	-0.18	-0.01	-0.04	-0.11
	100	0.02	-0.08	-0.19	-0.02	-0.12	-0.17	-0.04	-0.06	-0.09
	200	-0.01	-0.14	-0.31	-0.10	-0.19	-0.15	-0.05	-0.08	-0.06
	500	-0.06	-0.13	-0.16	-0.23	-0.15	-0.06	-0.08	-0.03	-0.02
	1,000	-0.06	-0.13	-0.10	-0.23	-0.06	-0.03	-0.07	-0.02	-0.01
	1,500	-0.09	-0.11	-0.08	-0.16	-0.06	-0.01	-0.04	-0.01	-0.01
10	50	-0.02	-0.17	-0.22	-0.10	-0.15	-0.21	-0.08	-0.09	-0.10
	100	-0.08	-0.21	-0.23	-0.13	-0.15	-0.11	-0.06	-0.06	-0.05
	200	-0.15	-0.18	-0.14	-0.15	-0.11	-0.05	-0.04	-0.04	-0.03
	500	-0.12	-0.13	-0.11	-0.10	-0.05	-0.02	-0.03	-0.01	-0.01
	1,000	-0.14	-0.08	-0.05	-0.07	-0.04	-0.01	-0.01	0.00	0.00
	1,500	-0.11	-0.06	-0.04	-0.03	-0.01	0.00	-0.01	0.00	0.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

Counterintuitively, bias for the lower asymptote parameter estimate increased in absolute terms as the number of repeated measures increased. When the number of repeated measures was six, mean parameter estimate bias was -.03. At eight repeated measures, mean parameter estimate bias had risen to -.08. At 10 repeated measures, mean bias remained at -.08. The effect size for this manipulated parameter although small, was the largest effect for the lower asymptote parameter ($\omega_p^2 = 0.044$, $p < .001$).

Again, counterintuitively, as the maximum rate of change increased, the amount of parameter estimate bias also increased in absolute terms. When $\rho^* = .125$, bias was -.03. At $\rho^* = .1875$, bias increased to -.07 and when the maximum rate of change was highest, $\rho^* = .5$, bias was -.09. The effect size for this parameter was small (ω_p^2 $= 0.040$, $p < .001$).

When $\delta^* = .25$, parameter estimate bias was .-.07, which increased to -.08 when $\delta^* = .375$. When $\delta^* = .5$, bias decreased to -.04. The effect size for the location of the inflection point within the measurement window was small (($\omega_p^2 = 0.026$, $p < .001$)

The only two-way interaction that had an effect size .01 or larger was the interaction between the number of repeated measures and the maximum rate of change. For this interaction, the effect size was small ($\omega_p^2 = 0.011$, $p < .001$).

Upper Asymptote (β^*)

Because the true value for the upper asymptote was simulated as non-zero, relative bias could be calculated for this parameter and provided more interpretability than the raw bias presented for the lower asymptote. Overall parameter estimate bias for the upper asymptote was very good (see Table 4.16). Generally speaking, parameter estimate bias was acceptable (.10 or less) under all conditions when the number of repeated measures was eight or 10. When the number of repeated measures was six, only in conditions with the lowest maximum rate of change and sample sizes 500 or fewer had an unacceptable bias. Parameter estimate bias ranged, in absolute

Table 4.15

Results of Aligned Rank Transformed ANOVA for Estimation of Lower Asymptote (i)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	6.81E+12	3.11E+14	709.25	< 0.001	0.021
RM	2	1.40E+13	3.04E+14	3727.38	< 0.001	0.044
ρ^*	2	1.25E+13	3.03E+14	3342.20	< 0.001	0.040
δ^*	2	8.65E+12	3.21E+14	2182.13	< 0.001	0.026
SS × RM	10	2.66E+12	3.13E+14	137.44	< 0.001	0.008
SS × ρ^*	10	2.28E+12	3.12E+14	118.43	< 0.001	0.007
RM × ρ^*	4	3.41E+12	3.11E+14	443.95	< 0.001	0.011
SS × δ^*	10	7.65E+11	3.13E+14	39.64	< 0.001	0.002
RM × δ^*	4	1.31E+12	3.13E+14	169.64	< 0.001	0.004
ρ^* × δ^*	4	1.60E+12	3.12E+14	207.52	< 0.001	0.005

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

terms, from 0 to -.17 and in non-absolute terms from -.17 to .02 with a mean bias of -.01. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.17) to assess the practical effects of the manipulated parameters.

For all sample sizes, parameter estimate bias was acceptable. When sample size was 50, the mean parameter estimate bias for the upper asymptote was -.02. An increase in sample size to 100 increased bias slightly to -.03. As sample size increased from 200 to 1,500, parameter estimate bias decreased from -.02 to 0.0. The effect size for sample size was small ($\omega_p^2 = 0.021$, $p < .001$).

Likewise, bias was acceptable for all numbers of repeated measures. When the number of repeated measures was six, mean parameter estimate bias was -.04. At eight repeated measures, mean parameter estimate bias had fallen 0.0 where it remained

Table 4.16

Relative Bias of Estimates of the Epper Asymptote (β^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	-0.09	-0.01	0.01	-0.13	-0.04	0.00	-0.17	-0.09	-0.01
	100	-0.11	-0.03	0.00	-0.15	-0.04	0.00	-0.17	-0.07	-0.02
	200	-0.11	-0.03	-0.01	-0.12	-0.03	0.01	-0.16	-0.06	-0.01
	500	-0.09	-0.01	0.00	-0.09	-0.01	0.00	-0.12	-0.03	0.01
	1,000	-0.06	0.00	0.00	-0.05	0.00	0.00	-0.06	0.00	0.01
	1,500	-0.04	0.00	0.00	-0.04	0.00	0.00	-0.05	0.00	0.00
8	50	-0.05	0.00	0.01	-0.06	-0.01	0.02	-0.08	-0.02	0.01
	100	-0.05	0.00	0.00	-0.05	0.01	0.01	-0.07	0.00	0.01
	200	-0.03	0.00	0.00	-0.02	0.01	0.00	-0.03	0.01	0.01
	500	-0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.01
	1,000	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00
	1,500	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
10	50	-0.02	0.00	0.01	-0.01	0.01	0.02	0.00	0.01	0.01
	100	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.02	0.01
	200	0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.00
	500	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00
	1,000	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	1,500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

for 10 repeated measures. This manipulated parameter had a medium practical effect, and was the largest effect for the upper asymptote parameter ($\omega_p^2 = 0.063$, $p < .001$).

As the maximum rate of change increased, the amount of parameter estimate bias decreased. When $\rho^* = .125$, bias was -.04. At $\rho^* = .1875$, bias was -.01 and when the maximum rate of change was highest, $\rho^* = .5$, bias was 0.0. The effect size for this parameter was small to medum ($\omega_p^2 = 0.059$, $p < .001$).

Unlike the other parameters, as delta increased, so did the parameter estimate bias. When $\delta^* = .25$, parameter estimate bias was -.01, remaining steady when $\delta^* = .375$. As $\delta^* = .5$, bias increased slightly to -.02. The effect size for this parameter was very small, although statistically significant ($\omega_p^2 = 0.005$, $p < .001$).

There were three two-way interaction effects that had effect sizes .01 or larger. The first was the interaction between the number of repeated measures and the maximum rate of change parameter ($\omega_p^2 = 0.034$, $p < .001$). The second interaction was between sample size and the maximum rate of change ($\omega_p^2 = 0.011$, $p < .001$). The third interaction was between sample size and the number of repeated measures ($\omega_p^2 = 0.010$, $p < .001$).

Maximum Rate of Change (ρ^*)

The maximum rate of change is essentially the instantaneous slope at the inflection point. Like the upper asymptote, the true simulated values for this parameter were non-zero so relative bias was calculated. Overall, bias was worse for this parameter than for the upper asymptote (see Table 4.18). Parameter estimate bias ranged from 0 to 1.03. Mean parameter estimate bias under all conditions was positive with the highest mean bias of any parameter at .16. When the number of repeated measures was six or eight, acceptable levels of bias were found in where the sample size was 500 or greater. When the number of repeated measures was 10, acceptable levels of bias were found in conditions where the sample size was between 200 and 500 de-

Table 4.17

Results of Aligned Rank Transformed ANOVA for Estimation of Upper Asymptote (β^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	7.31E+12	3.45E+14	686.61	< 0.001	0.021
RM	2	2.22E+13	3.29E+14	5468.42	< 0.001	0.063
ρ^*	2	2.08E+13	3.31E+14	5092.32	< 0.001	0.059
δ^*	2	1.77E+12	3.50E+14	409.01	< 0.001	0.005
SS × RM	10	3.42E+12	3.48E+14	158.89	< 0.001	0.010
SS × ρ^*	10	3.97E+12	3.48E+14	184.68	< 0.001	0.011
RM × ρ^*	4	1.19E+13	3.40E+14	1422.78	< 0.001	0.034
SS × δ^*	10	5.95E+11	3.51E+14	27.48	< 0.001	0.002
RM × δ^*	4	1.52E+12	3.50E+14	175.86	< 0.001	0.004
ρ^* × δ^*	4	3.32E+11	3.51E+14	38.26	< 0.001	0.001

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.

[a] Type III sum of squares.

pending on the maximum rate of change value. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.19).

As sample size increased, parameter estimate bias monotonically decreased. When sample size was 50, the mean parameter estimate bias for the upper asymptote was .45. As sample size increased to 100, bias dropped to .26. When sample size was 200 biased fell to .14. Once sample size reached 500, parameter estimate bias fell to acceptable levels, in this case, .05. When the sample size was 1,000 or greater, bias was .02. Sample size had a very large practical effect on parameter estimate bias for this parameter ($\omega_p^2 = 0.215$, $p < .001$).

When the number of repeated measures was six, mean parameter estimate bias was .26. At eight repeated measures, mean parameter estimate bias had fallen to .013. At 10 repeated measures, mean bias was acceptable at .08. The effect size for this manipulated parameter was medium to large ($\omega_p^2 = 0.137$, $p < .001$).

Table 4.18

Relative Bias of Estimates of the Maximum Rate of Change (ρ^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.97	0.59	0.39	1.03	0.59	0.40	0.97	0.66	0.41
	100	0.68	0.41	0.26	0.67	0.38	0.27	0.61	0.42	0.27
	200	0.46	0.23	0.15	0.39	0.22	0.12	0.38	0.21	0.13
	500	0.20	0.09	0.07	0.18	0.08	0.04	0.16	0.09	0.04
	1,000	0.11	0.05	0.03	0.09	0.03	0.01	0.08	0.04	0.02
	1,500	0.08	0.04	0.02	0.06	0.01	0.01	0.06	0.02	0.01
8	50	0.66	0.38	0.31	0.58	0.40	0.25	0.54	0.42	0.23
	100	0.40	0.24	0.17	0.30	0.19	0.14	0.26	0.18	0.14
	200	0.21	0.13	0.08	0.13	0.08	0.06	0.12	0.07	0.06
	500	0.07	0.04	0.03	0.04	0.01	0.01	0.04	0.02	0.01
	1,000	0.03	0.02	0.01	0.02	0.01	0.00	0.02	0.01	0.01
	1,500	0.02	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
10	50	0.43	0.29	0.23	0.36	0.25	0.16	0.29	0.23	0.19
	100	0.20	0.14	0.10	0.13	0.10	0.08	0.13	0.12	0.10
	200	0.09	0.06	0.06	0.05	0.04	0.04	0.04	0.04	0.04
	500	0.03	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.02
	1,000	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01
	1,500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

As the maximum rate of change increased, the amount of parameter estimate bias decreased. When $\rho^* = .125$, bias was .23. At $\rho^* = .1875$, bias was .14 and when the maximum rate of change was highest, $\rho^* = .5$, bias was finally just acceptable at .10. The effect size for this parameter was medium ($\omega_p^2 = 0.091$, $p < .001$) and had the third largest practical effect on the estimation of this parameter.

The location of the inflection point parameter remained above .10 bias across its three levels. When $\delta^* = .25$, parameter estimate bias was .17, decreasing to .15 when $\delta^* = .375$ or $\delta^* = .5$. The effect size for this parameter was very small, although statistically significant ($\omega_p^2 = 0.005$, $p < .001$).

There were three two-way interaction effect that had an effect size .01 or larger for this parameter. The first was the interaction between sample size and the number of repeated measures ($\omega_p^2 = 0.078$, $p < .001$). Next was the interaction between sample size and the maximum rate of change ($\omega_p^2 = 0.056$, $p < .001$). Finally, there was the interaction between the number of repeated measures and the maximum rate of change ($\omega_p^2 = 0.040$, $p < .001$).

Location of the Inflection Point (δ^*)

The location of the inflection point within the measurement window was a manipulated condition that had non-zero simulated true values. As such, relative bias was calculated and reported. Parameter estimate bias for this parameter ranged, in absolute terms from 0 to .60 and non-absolute terms from -.08 to .60 with a mean bias of .06. (see Table 4.20). The majority of cells had a positive bias with only a few exhibiting negative bias. The majority of conditions had acceptable levels of bias across all numbers of repeated measures. The conditions with unacceptable levels tended to have fewer repeated measures, smaller maximum rates of change, and more offset locations of the inflection point. An aligned rank transform ANOVA with two-

Table 4.19

Results of Aligned Rank Transformed ANOVA for Estimation of the Maximum Rate of Change (ρ^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	7.55E+13	2.76E+14	8855.83	< 0.001	0.215
RM	2	4.68E+13	2.95E+14	12838.65	< 0.001	0.137
ρ^*	2	3.13E+13	3.12E+14	8126.23	< 0.001	0.091
δ^*	2	1.56E+12	3.45E+14	366.14	< 0.001	0.005
SS × RM	10	2.67E+13	3.15E+14	1373.18	< 0.001	0.078
SS × ρ^*	10	1.92E+13	3.25E+14	955.31	< 0.001	0.056
RM × ρ^*	4	1.38E+13	3.33E+14	1678.56	< 0.001	0.040
SS × δ^*	10	5.87E+11	3.46E+14	27.46	< 0.001	0.002
RM × δ^*	4	2.49E+11	3.47E+14	29.03	< 0.001	0.001
ρ^* × δ^*	4	5.80E+11	3.46E+14	67.83	< 0.001	0.002

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.

[a] Type III sum of squares.

way interactions and main effects was run on the results (see Table 4.21) to assess the practical effects of the manipulated parameters.

Increases in the sample size resulted in a monotonic decrease in parameter estimate bias for this parameter. When sample size was 50, the mean parameter estimate bias for the location of the inflection point was .13. As sample size increased to 100 or greater, bias became acceptable. At 100, bias was .10, falling to .07 at a sample size of 200. When sample size was 500 biased decreased to .04 and then fell to .02 when sample size was 1,000. When sample size increased to 1,500, parameter estimate bias increased to .01. The effect size for sample size had small practical significance ($\omega_p^2 = 0.03$, $p < .001$).

As the number of repeated measures increased, parameter estimate bias fell. When the number of repeated measures was six, mean parameter estimate bias was .11. At eight repeated measures, mean parameter estimate bias had fallen to an ac-

Table 4.20

Relative Bias of Estimates of the Location of the Inflection Point (δ^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.60	0.51	0.32	0.13	0.10	0.06	-0.08	-0.07	-0.02
	100	0.58	0.41	0.24	0.12	0.10	0.05	-0.04	-0.01	0.02
	200	0.52	0.30	0.16	0.11	0.07	0.04	-0.05	0.00	0.03
	500	0.41	0.16	0.03	0.09	0.06	0.03	-0.02	0.02	0.03
	1,000	0.29	0.06	0.02	0.05	0.03	0.03	-0.01	0.02	0.02
	1,500	0.22	0.05	0.00	0.04	0.01	0.01	-0.01	0.03	0.01
8	50	0.43	0.28	0.15	0.14	0.09	0.05	-0.01	0.05	0.05
	100	0.38	0.16	0.05	0.08	0.04	0.03	0.00	0.02	0.05
	200	0.26	0.09	0.03	0.05	0.03	0.04	0.00	0.03	0.04
	500	0.11	0.01	0.00	0.03	0.02	0.02	0.03	0.01	0.02
	1,000	0.05	0.00	0.00	0.01	0.00	0.01	0.02	0.01	0.00
	1,500	0.00	-0.02	0.00	0.02	0.01	0.00	0.02	0.00	0.00
10	50	0.27	0.13	0.03	0.07	0.05	0.06	0.03	0.06	0.06
	100	0.13	0.06	0.03	0.03	0.03	0.05	0.02	0.04	0.04
	200	0.07	0.00	0.02	0.03	0.03	0.02	0.02	0.03	0.03
	500	-0.01	0.01	0.01	0.02	0.01	0.01	0.02	0.01	0.01
	1,000	-0.02	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00
	1,500	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

ceptable level of .05. At 10 repeated measures, mean bias fell further to 0.03. The effect size for this manipulated parameter was small ($\omega_p^2 = 0.025$, $p < .001$).

As the maximum rate of change increased, the amount of parameter estimate bias decreased. When $\rho^* = .125$, bias was .10. At $\rho^* = .1875$, bias was .06 and when the maximum rate of change was highest, $\rho^* = .5$, bias remained acceptable at .04. The effect size for this parameter was small ($\omega_p^2 = 0.013$, $p < .001$).

For the location of the inflection point parameter, the initial level, $\delta^* = .25$, had an unacceptable amount of parameter estimate bias at .14. This decreased to .04 when $\delta^* = .375$ and .01 when $\delta^* = .5$. The effect size for this parameter was small to medium ($\omega_p^2 = 0.054$, $p < .001$).

There were three two-way interaction effects that had an effect size .01 or larger for this parameter. The first was the interaction between sample size and the location of the inflection point ($\omega_p^2 = 0.025$, $p < .001$). Next was the interaction between the number of repeated measures and the location of the inflection point ($\omega_p^2 = 0.035$, $p < .001$). Finally, there was the interaction between the maximum rate of change and the location of the inflection point ($\omega_p^2 = 0.022$, $p < .001$).

Relative Asymmetry (γ^*)

The relative asymmetry parameter represents the amount of growth that occurs prior to the point of inflection. This was a non-manipulated condition that was set at $\gamma^* = 0.5$ which represents a symmetrical growth curve. Because the simulated true value was non-zero, relative bias was used for this parameter. Overall, parameter estimate bias was very good with very few conditions exceeding the 0.10 threshold for acceptable parameter estimate bias (see Table 4.22). The conditions where bias was unacceptable were in conditions with sample sizes 100 or fewer where the location of the inflection point was not the most offset level. Parameter estimate bias for relative asymmetry ranged, in absolute measures from 0 to .16 and in non-absolute measures

Table 4.21

Results of Aligned Rank Transformed ANOVA for Estimation of the Location of the Inflection Point (δ^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	1.04E+13	3.43E+14	985.19	< 0.001	0.030
RM	2	8.72E+12	3.45E+14	2046.53	< 0.001	0.025
ρ^*	2	4.75E+12	3.49E+14	1102.10	< 0.001	0.013
δ^*	2	1.91E+13	3.33E+14	4630.67	< 0.001	0.054
SS $\times$ RM	10	1.27E+12	3.53E+14	58.36	< 0.001	0.004
SS $\times$ ρ^*	10	7.08E+11	3.53E+14	32.46	< 0.001	0.002
RM $\times$ ρ^*	4	1.87E+12	3.52E+14	215.53	< 0.001	0.005
SS $\times$ δ^*	10	8.79E+12	3.45E+14	412.88	< 0.001	0.025
RM $\times$ δ^*	4	1.23E+13	3.42E+14	1461.14	< 0.001	0.035
ρ^* $\times$ δ^*	4	7.73E+12	3.46E+14	904.19	< 0.001	0.022

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

from -.03 to .16 with a mean bias of .05. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.23).

For the relative asymmetry parameter, parameter estimate bias was acceptable across all sample sizes. When sample size was 50, the mean parameter estimate bias was .10. As sample size increased, bias decreased, finally reaching a bias of .02 when sample size was 1,500. The effect size for sample size on parameter estimate bias for the relative asymmetry parameter was very small ($\omega_p^2 = 0.008$, $p < .001$).

Parameter estimate bias was relatively stable across the different numbers of repeated measures. When the number of repeated measures was six, mean parameter estimate bias was .06. At eight repeated measures, mean parameter estimate bias had fallen to .05. At 10 repeated measures, mean bias remained an acceptable .05. The effect size for the number of repeated measures was effectively zero, however it was still statistically significant ($\omega_p^2 = 0.000$, $p < .001$).

Table 4.22

Relative Bias of Estimates of the Relative Asymmetry (γ^*) of the Latent Curve.

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.08	0.09	0.06	0.08	0.07	0.09	0.10	0.06	0.09
	100	0.07	0.07	0.08	0.06	0.07	0.06	0.11	0.09	0.10
	200	0.06	0.05	0.06	0.04	0.04	0.06	0.09	0.07	0.10
	500	0.05	0.04	0.04	0.02	0.07	0.08	0.08	0.07	0.07
	1,000	0.01	0.01	0.04	0.01	0.07	0.08	0.04	0.06	0.04
	1,500	-0.01	0.01	0.04	0.03	0.07	0.04	0.05	0.07	0.03
8	50	0.08	0.10	0.08	0.09	0.11	0.10	0.10	0.13	0.14
	100	0.06	0.05	0.05	0.06	0.07	0.09	0.10	0.08	0.13
	200	0.02	0.06	0.09	0.05	0.06	0.09	0.06	0.08	0.09
	500	-0.03	0.01	0.04	0.06	0.07	0.04	0.08	0.03	0.04
	1,000	-0.02	0.04	0.03	0.05	0.03	0.03	0.05	0.02	0.01
	1,500	-0.01	0.03	0.03	0.06	0.03	0.01	0.04	0.01	0.01
10	50	0.07	0.09	0.08	0.08	0.10	0.14	0.11	0.14	0.16
	100	0.02	0.06	0.09	0.05	0.08	0.11	0.08	0.10	0.11
	200	0.02	0.04	0.06	0.05	0.06	0.06	0.05	0.06	0.08
	500	0.00	0.05	0.05	0.04	0.03	0.02	0.03	0.01	0.02
	1,000	0.04	0.04	0.03	0.03	0.02	0.02	0.01	0.00	0.00
	1,500	0.03	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

As with the number of repeated measures, the maximum rate of change had little impact on the amount of parameter estimate bias which remained acceptable at all levels. When $\rho^* = .125$, bias was .05. At $\rho^* = .1875$, bias remained at .05 and when the maximum rate of change was highest, $\rho^* = .5$, bias increased slightly to .06. The effect size for the maximum rate of change condition was effectively zero, however it was still statistically significant ($\omega_p^2 = 0.000$, $p < .001$).

For the relative asymmetry parameter, the initial level, $\delta^* = .25$, had the smallest parameter estimate bias at .04. This increased to a still acceptable .05 when $\delta^* = .375$ and to .07 when $\delta^* = .5$. The effect size for the location of the inflection point was effectively zero, however it was still statistically significant ($\omega_p^2 = 0.000$, $p < .001$).

Table 4.23

Results of Aligned Rank Transformed ANOVA for Estimation of Relative Asymmetry (γ^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	2.86E+12	3.51E+14	263.42	< 0.001	0.008
RM	2	5.02E+10	3.54E+14	11.48	< 0.001	0.000
ρ^*	2	1.29E+11	3.54E+14	29.62	< 0.001	0.000
δ^*	2	3.92E+11	3.53E+14	89.77	< 0.001	0.001
SS × RM	10	1.15E+11	3.54E+14	5.28	< 0.001	0.000
SS × ρ^*	10	6.51E+10	3.54E+14	2.98	0.001	0.000
RM × ρ^*	4	4.89E+10	3.54E+14	5.59	< 0.001	0.000
SS × δ^*	10	9.89E+10	3.54E+14	4.52	< 0.001	0.000
RM × δ^*	4	6.91E+10	3.54E+14	7.91	< 0.001	0.000
ρ^* × δ^*	4	8.88E+10	3.54E+14	10.16	< 0.001	0.000

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

Standard Error Bias

Analysis of standard error bias was done in a manner similar to how parameter estimate bias was analyzed. The standard deviations of the parameter estimates were used as the empirical standard errors in this simulation (Bandalos & Leite, 2013). The standard error estimates recovered were: the universal lower asymptote (i), the location of the upper asymptote (β^*), the timing of most rapid change within the window of measurement as a proportion (δ^*), the instantaneous maximum rate of change (ρ^*), and the proportion of growth that occurred prior to the inflection point (γ^*). Results are reported for each parameter individually. Hoogland and Boomsma (1998) suggest that standard error bias should not exceed $\pm 10\%$. Due to the non-normality of the standard error bias, non-parametric aligned rank transform ANOVA were used to generate effect sizes for comparison. Partial omega squared (ω_p^2) was reported as a measure of effect size.

Lower Asymptote (i)

Much like parameter estimate bias for the lower asymptote parameter, standard error bias for the lower asymptote was non-uniform. Table 4.24 shows that individual conditions where bias was within acceptable limits are not necessarily adjacent to other cells that are also within acceptable limits. Standard error bias for this parameter ranged, in absolute measures from 0 to 1.61 with a mean bias of .22. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.25).

When sample size was 50, the mean standard error bias was .63. As sample size increased, bias decreased, finally reaching .09 bias when sample size was 500. When sample size was 1,000 or 1,500, standard error bias was -.02. The effect size for sample size was, however, very small ($\omega_p^2 = 0.007$, $p < .001$).

Table 4.24

Standard Error Estimate Bias for the Lower Asymptote (i)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	1.22	1.06	1.24	1.61	1.09	1.19	1.30	0.83	0.49
	100	1.22	1.42	1.20	0.60	0.98	0.68	0.66	0.18	0.25
	200	0.85	1.08	0.92	0.47	0.35	0.29	0.68	0.28	0.11
	500	0.75	0.89	0.61	0.49	0.36	0.01	0.36	-0.06	-0.08
	1,000	0.56	0.69	0.31	0.43	0.16	-0.21	0.42	-0.04	-0.16
	1,500	0.64	0.66	0.25	0.36	0.01	-0.38	0.31	-0.18	-0.18
8	50	1.23	1.08	0.89	0.54	0.46	0.20	0.32	0.02	0.04
	100	0.59	0.86	0.75	0.32	0.33	0.05	0.01	0.07	-0.25
	200	0.37	1.11	0.44	0.49	0.03	-0.25	-0.17	-0.23	-0.20
	500	0.31	0.43	0.04	0.19	-0.14	-0.35	-0.15	-0.21	-0.07
	1,000	0.55	0.14	-0.20	-0.11	-0.19	-0.32	-0.15	-0.23	-0.12
	1,500	0.77	0.08	-0.32	-0.17	-0.38	-0.22	-0.19	-0.12	-0.12
10	50	0.64	0.96	0.85	0.40	0.06	0.00	-0.15	-0.24	-0.44
	100	0.78	0.54	0.44	0.14	-0.14	-0.20	-0.19	-0.05	-0.22
	200	0.63	0.43	0.11	-0.11	-0.20	-0.17	-0.03	-0.26	-0.23
	500	0.45	-0.06	-0.30	-0.22	-0.37	-0.16	-0.14	-0.07	-0.09
	1,000	0.28	-0.16	-0.54	-0.48	-0.79	-0.17	-0.08	-0.02	0.03
	1,500	0.15	-0.27	-0.74	-0.17	-0.05	-0.06	-0.08	-0.02	-0.05

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

Likewise, an increase in the number of repeated measures reduced bias. When the number of repeated measures was six, mean standard error bias was .54. At eight repeated measures, mean standard error bias had fallen to .15. At 10 repeated measures, mean bias was an acceptable -.02. Again, as with sample size, the effect size for this manipulated parameter was very small although statistically significant ($\omega_p^2 = 0.001$, $p < .001$).

As the maximum rate of change increased, the amount of standard error bias decreased. When $\rho^* = .125$, bias was .36. At $\rho^* = .1875$, bias fell to .22 and when the maximum rate of change was highest, $\rho^* = .5$, bias fell to an acceptable level of .09. The effect size for this parameter was effectively zero ($\omega_p^2 = 0.000$, $p < .001$).

The location of the inflection point within the measurement window, on the other hand, had the highest effect size for the lower asymptote standard error bias (($\omega_p^2 = 0.016$, $p < .001$), although still considered a small effect. When $\delta^* = .25$, standard error bias was .54, falling to .12 when $\delta^* = .375$. When $\delta^* = .5$, bias was an acceptable .02. The only two-way interaction effect that had an effect size .01 or larger was the interaction between sample size and the location of the inflection point within the measurement window ($\omega_p^2 = 0.010$, $p < .001$).

Upper Asymptote (β^*)

Unlike the lower asymptote parameter, the upper asymptote parameter seemed to have more distinct patterns of standard error bias (Table 4.26). Generally speaking, standard error bias was within acceptable limits when the number of repeated measures was eight or 10 and the maximum rate of change was not at its lowest level. The same held true for six repeated measures, adding in the provision that sample size was 1,000 or greater. Standard error bias for this parameter ranged, in absolute terms from 0 to .61 with a mean bias of .12. An aligned rank transform ANOVA with

Table 4.25

Results of Aligned Rank Transformed ANOVA for Standard Error Estimate Bias of Lower Asymptote (i)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	1.86E+12	2.55E+14	236.84	< 0.001	0.007
RM	2	2.13E+11	2.47E+14	70.02	< 0.001	0.001
ρ^*	2	5.22E+10	2.41E+14	17.51	< 0.001	0.000
δ^*	2	4.20E+12	2.52E+14	1350.17	< 0.001	0.016
SS × RM	10	1.92E+12	2.37E+14	131.32	< 0.001	0.008
SS × ρ^*	10	2.69E+11	2.40E+14	18.13	< 0.001	0.001
RM × ρ^*	4	1.41E+11	2.40E+14	23.81	< 0.001	0.001
SS × δ^*	10	2.47E+12	2.36E+14	169.33	< 0.001	0.010
RM × δ^*	4	1.75E+12	2.39E+14	296.11	< 0.001	0.007
ρ^* × δ^*	4	7.64E+11	2.39E+14	129.44	< 0.001	0.003

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

two-way interactions and main effects was run on the results (see Table 4.27) to assess the practical effects of the manipulated parameters.

When sample size was 50, the mean standard error bias for the upper asymptote was .19. As sample size increased, bias decreased, again, reaching .09 bias when sample size was 500. When sample size was 1,000 or 1,500, standard error bias was .06. The effect size for sample size was, however, very small ($\omega_p^2 = 0.005$, $p < .001$).

Likewise, an increase in the number of repeated measures reduced bias. When the number of repeated measures was six, mean standard error bias was .26. At eight repeated measures, mean standard error bias had fallen to an acceptable level of .08. At 10 repeated measures, mean bias remained acceptable at .02. The effect size for this manipulated parameter was zero, although statistically significant ($\omega_p^2 = 0.000$, $p < .001$).

Table 4.26

Standard Error Estimate Bias for the Upper Asymptote (β^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.30	0.26	0.23	0.38	0.36	0.29	0.43	0.46	0.39
	100	0.36	0.18	0.20	0.38	0.34	0.18	0.63	0.43	0.39
	200	0.35	0.28	0.13	0.37	0.27	0.14	0.66	0.50	0.26
	500	0.40	0.19	0.01	0.47	0.19	0.00	0.60	0.22	-0.01
	1,000	0.32	0.06	-0.01	0.35	0.07	-0.04	0.68	0.11	-0.01
	1,500	0.33	0.06	0.04	0.38	0.07	-0.01	0.67	0.01	-0.04
8	50	0.18	0.06	0.07	0.21	0.11	0.09	0.40	0.25	0.12
	100	0.21	0.09	0.02	0.34	0.09	-0.01	0.41	0.20	0.00
	200	0.18	0.03	-0.01	0.29	0.04	0.04	0.40	0.09	0.07
	500	0.12	-0.04	0.01	0.15	-0.04	-0.02	0.20	0.01	-0.02
	1,000	0.03	-0.01	-0.04	0.07	-0.01	0.00	0.00	0.05	-0.01
	1,500	0.05	0.01	0.01	-0.04	0.02	0.03	-0.01	-0.01	-0.05
10	50	0.10	0.00	-0.04	0.15	0.05	-0.02	0.14	0.11	-0.03
	100	0.05	0.02	-0.04	0.15	0.05	-0.04	0.12	-0.01	-0.03
	200	0.09	-0.04	-0.02	0.11	-0.02	0.01	0.09	0.02	-0.07
	500	-0.01	-0.01	-0.01	0.00	0.00	0.02	0.07	0.04	0.00
	1,000	0.02	0.04	0.02	0.02	0.00	-0.02	-0.01	0.00	-0.01
	1,500	0.01	0.03	0.01	-0.01	0.02	0.00	-0.02	0.00	-0.01

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

As the maximum rate of change increased, the amount of standard error bias decreased. When $\rho^* = .125$, bias was .23. At $\rho^* = .1875$, bias was .10 and when the maximum rate of change was highest, $\rho^* = .5$, bias remained acceptable at .09. The effect size for this parameter was negligible (($\omega_p^2 = 0.001$, $p < .001$).

Unlike the other parameters, as delta increased, so did the standard error bias. When $\delta^* = .25$, standard error bias was acceptable at .09, increasing to .11 when $\delta^* = .375$. At $\delta^* = .5$, bias increased more to .16. Like the number of repeated measures, the effect size for this parameter was zero, although statistically significant ($\omega_p^2 = 0.000$, $p < .001$).

The only two-way interaction that had an effect size .01 or larger was the interaction between the number of repeated measures and the maximum rate of change parameter ($\omega_p^2 = 0.011$, $p < .001$). This was the largest effect for the standard error bias of the upper asymptote. The second largest effect for this parameter was the interaction between sample size and the number of repeated measures ($\omega_p^2 = 0.008$, $p < .001$) and while significant the practical effect was less than .01.

Maximum Rate of Change (ρ^*)

Similar to the upper asymptote parameter, the maximum rate of change parameter has distinct patterns of standard error bias (Table 4.28). When the number of repeated measures was six, acceptable levels of bias were found in conditions with the highest maximum rate of change, the location of the inflection point at either of the highest two levels, and the sample size was 100 or greater. When the number of repeated measures was eight or 10 and the sample size was 200 or greater, standard error bias was acceptable in most conditions. Standard error bias for this parameter ranged, in absolute terms from 0 to .49 with a mean bias of .03. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.29) to assess the practical effects of the manipulated parameters.

Table 4.27

Results of Aligned Rank Transformed ANOVA for Standard Error Estimate Bias of Upper Asymptote (β^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	1.52E+12	3.16E+14	156.18	< 0.001	0.005
RM	2	6.00E+10	3.17E+14	15.35	< 0.001	0.000
ρ^*	2	2.86E+11	3.11E+14	74.50	< 0.001	0.001
δ^*	2	1.24E+11	3.09E+14	32.44	< 0.001	0.000
SS $\times$ RM	10	2.52E+12	3.05E+14	133.93	< 0.001	0.008
SS $\times$ ρ^*	10	2.47E+11	3.08E+14	13.02	< 0.001	0.001
RM $\times$ ρ^*	4	3.49E+12	3.03E+14	466.58	< 0.001	0.011
SS $\times$ δ^*	10	3.51E+11	3.07E+14	18.53	< 0.001	0.001
RM $\times$ δ^*	4	1.07E+12	3.06E+14	141.65	< 0.001	0.003
ρ^* $\times$ δ^*	4	1.02E+12	3.07E+14	134.90	< 0.001	0.003

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

When sample size was 50, the mean standard error bias for the maximum rate of change was .04. As sample size increased, bias decreased to .01 at 100, then increased slightly to .02 at a sample size of 200. When sample size was 500 biased increased to .05 before decreasing to .04 when sample size was 1,000 or larger. The effect size for sample size was the largest for the maximum rate of change parameter (ω_p^2 = 0.085, $p < .001$).

Unlike sample size, where all levels had acceptable standard error bias, when the number of repeated measures was six, mean standard error bias was .16. At eight repeated measures, mean standard error bias had fallen to an acceptable level of -.01. At 10 repeated measures, mean bias increased but remained acceptable at -.06. The effect size for this manipulated parameter was the second largest although small (ω_p^2 = 0.056, $p < .001$).

Table 4.28

Standard Error Estimate Bias for the Maximum Rate of Change (ρ^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.37	0.25	0.39	0.49	0.24	0.22	0.43	0.26	0.20
	100	0.31	0.38	0.14	0.28	0.20	0.02	0.29	0.11	0.01
	200	0.12	0.19	0.23	0.22	0.10	-0.02	0.14	0.12	0.07
	500	0.17	0.20	0.13	0.14	0.12	0.04	0.29	0.17	0.07
	1,000	0.14	0.20	0.08	0.16	0.10	-0.02	0.19	0.08	-0.06
	1,500	0.25	0.22	0.01	0.17	0.07	-0.03	0.21	0.08	-0.05
8	50	0.02	0.07	-0.04	-0.01	-0.11	-0.11	-0.07	-0.08	-0.11
	100	-0.02	-0.04	0.00	0.00	-0.12	-0.18	0.11	-0.14	-0.17
	200	0.00	0.01	-0.06	0.09	-0.11	-0.15	0.01	0.00	-0.07
	500	0.08	0.01	0.03	0.06	0.00	-0.05	0.06	0.01	-0.06
	1,000	0.09	0.08	0.00	0.06	-0.04	-0.06	0.04	0.04	-0.06
	1,500	0.06	0.08	0.00	0.05	0.01	0.00	0.02	-0.01	0.01
10	50	-0.09	-0.12	-0.10	-0.13	-0.25	-0.24	-0.09	-0.18	-0.24
	100	-0.03	-0.05	-0.10	-0.06	-0.16	-0.21	0.01	-0.12	-0.20
	200	0.01	-0.08	-0.13	0.05	-0.06	-0.09	0.05	-0.04	-0.11
	500	0.04	0.00	-0.08	0.03	-0.06	-0.07	-0.05	-0.02	-0.02
	1,000	0.01	0.06	-0.03	-0.01	-0.01	-0.01	0.03	0.00	-0.02
	1,500	0.02	-0.02	-0.01	-0.01	-0.01	0.03	0.02	-0.01	-0.03

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

As the maximum rate of change increased, the amount of standard error bias decreased. When $\rho^* = .125$, bias was .09. At $\rho^* = .1875$, bias was .03 and when the maximum rate of change was highest, $\rho^* = .5$, bias remained acceptable at -.02. The effect size for this parameter was ($\omega_p^2 = 0.046$, $p < .001$).

The location of the inflection point parameter remained below .10 bias across its three levels. When $\delta^* = .25$, standard error bias was .06, decreasing to .01 when $\delta^* = .375$. When $\delta^* = .5$, bias increased to .02. The effect size for this parameter was negligible, although statistically significant ($\omega_p^2 = 0.002$, $p < .001$).

The only two-way interaction effect that had an effect size .01 or larger was the interaction between sample size and the number of repeated measures ($\omega_p^2 = 0.033$, $p < .001$).

Table 4.29

Results of Aligned Rank Transformed ANOVA for Standard Error Estimate Bias of Maximum Rate of Change (ρ^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	2.90E+13	3.14E+14	2996.03	< 0.001	0.085
RM	2	1.78E+13	3.02E+14	4774.21	< 0.001	0.056
ρ^*	2	1.47E+13	3.06E+14	3877.59	< 0.001	0.046
δ^*	2	5.07E+11	3.19E+14	128.68	< 0.001	0.002
SS × RM	10	1.05E+13	3.02E+14	561.83	< 0.001	0.033
SS × ρ^*	10	1.47E+12	3.18E+14	74.59	< 0.001	0.005
RM × ρ^*	4	6.99E+11	3.18E+14	88.96	< 0.001	0.002
SS × δ^*	10	9.43E+11	3.18E+14	48.02	< 0.001	0.003
RM × δ^*	4	7.55E+10	3.19E+14	9.58	< 0.001	0.000
ρ^* × δ^*	4	1.26E+12	3.18E+14	160.74	< 0.001	0.004

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.

[a] Type III sum of squares.

Location of the Inflection Point (δ^*)

The location of the inflection point parameter had distinct patterns of standard error bias (Table 4.30). When the number of repeated measures was six, acceptable levels of standard error bias were sparse, primarily occurring where the location of the inflection point was centered and the maximum rate of change was at the highest level. When the number of repeated measures was eight and the location of the inflection point was not at the lowest level, or when it was, the maximum rate of change was at its highest level, bias levels were acceptable. At 10 repeated measures, bias levels were acceptable except when the maximum rate of change was lowest and the location of the inflection point was most offset. Standard error bias for this parameter ranged, in absolute terms from 0 to .46 with a mean bias of .08. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.31) to assess the practical effects of the manipulated parameters.

Changes in the sample size did not result in a monotonic change in parameter estimate bias for this parameter. When sample size was 50, the mean standard error bias for the location of the inflection point was .05. As sample size increased, bias increased to .10 at 100, then increased slightly to .11 at a sample size of 200. When sample size was 500 biased decreased to .09 and then fell to .06 when sample size was 1,000. When sample size increased to 1,500, standard error bias increased to .08. The effect size for sample size had small practical significance ($\omega_p^2 = 0.023$, $p < .001$).

As the number of repeated measures increased, standard error bias fell. When the number of repeated measures was six, mean standard error bias was .20. At eight repeated measures, mean standard error bias had fallen to an acceptable level of .05. At 10 repeated measures, mean bias fell further to 0.00. The effect size for this manipulated parameter was very small ($\omega_p^2 = 0.003$, $p < .001$).

As the maximum rate of change increased, the amount of standard error bias decreased. When $\rho^* = .125$, bias was unacceptable at .12. At $\rho^* = .1875$, bias was .08

Table 4.30

Standard Error Estimate Bias for the Location of the Inflection Point (δ^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.18	0.17	0.31	0.30	0.19	0.30	0.26	0.08	0.05
	100	0.25	0.34	0.39	0.13	0.17	0.15	0.20	0.13	0.15
	200	0.21	0.40	0.43	0.20	0.13	0.14	0.20	0.17	0.14
	500	0.33	0.40	0.25	0.29	0.14	0.02	0.28	0.07	0.05
	1,000	0.34	0.36	0.18	0.21	0.10	-0.08	0.28	0.08	-0.02
	1,500	0.32	0.46	0.23	0.20	0.11	-0.09	0.30	0.06	-0.03
8	50	0.03	0.16	0.03	-0.08	-0.10	-0.07	-0.03	-0.10	-0.10
	100	0.05	0.19	0.26	0.04	0.00	-0.01	0.04	-0.02	0.03
	200	0.17	0.35	0.17	0.07	0.01	0.01	0.11	0.01	0.02
	500	0.29	0.22	0.06	0.03	-0.04	-0.04	0.08	-0.03	-0.03
	1,000	0.32	0.14	0.01	0.01	0.04	-0.02	0.03	-0.09	-0.03
	1,500	0.46	0.15	0.03	0.02	0.01	-0.02	0.00	0.00	-0.06
10	50	-0.01	0.08	0.09	-0.09	-0.09	-0.09	-0.02	-0.09	-0.10
	100	0.13	0.10	0.04	-0.06	-0.10	-0.01	0.02	-0.03	-0.01
	200	0.15	0.09	0.02	-0.02	-0.10	-0.02	-0.01	-0.09	-0.02
	500	0.21	0.06	-0.05	-0.02	-0.02	0.04	-0.01	-0.03	-0.02
	1,000	0.18	-0.01	-0.05	-0.05	-0.06	-0.05	-0.03	-0.04	0.00
	1,500	0.14	0.00	-0.02	-0.03	0.00	-0.02	-0.02	0.00	-0.04

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

and when the maximum rate of change was highest, $\rho^* = .5$, bias remained acceptable at .05. The effect size for this parameter was negligible, although statistically significant ($\omega_p^2 = 0.001$, $p < .001$).

For the location of the inflection point parameter, the initial level, $\delta^* = .25$, had an unacceptable amount of standard error bias at .18. This decreased to .03 when $\delta^* = .375$ and when $\delta^* = .5$. The effect size for this parameter was negligible, although statistically significant ($\omega_p^2 = 0.002$, $p < .001$).

There were no two-way interaction effects that had an effect size .01 or larger. The largest practical effect was the interaction between sample size and the number of repeated measures ($\omega_p^2 = 0.006$, $p < .001$).

Table 4.31

Results of Aligned Rank Transformed ANOVA for Standard Error Estimate Bias of Location of the Inflection Point (δ^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	7.37E+12	3.16E+14	756.09	< 0.001	0.023
RM	2	8.23E+11	3.22E+14	206.59	< 0.001	0.003
ρ^*	2	1.90E+11	3.17E+14	48.38	< 0.001	0.001
δ^*	2	7.57E+11	3.28E+14	186.96	< 0.001	0.002
SS × RM	10	1.92E+12	3.16E+14	98.52	< 0.001	0.006
SS × ρ^*	10	5.22E+11	3.17E+14	26.70	< 0.001	0.002
RM × ρ^*	4	4.17E+11	3.16E+14	53.42	< 0.001	0.001
SS × δ^*	10	1.43E+11	3.17E+14	7.28	< 0.001	0.000
RM × δ^*	4	2.59E+11	3.16E+14	33.17	< 0.001	0.001
ρ^* × δ^*	4	3.76E+11	3.16E+14	48.16	< 0.001	0.001

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

Relative Asymmetry (γ^*)

Standard error bias for the relative asymmetry parameter was generally unacceptably high. When the number of repeated measures was 10, the majority of individual cells had an acceptable amount of bias. When the number of repeated measures was eight, fewer than half of the cells were acceptable. At six repeated measures, only 1 in 5 had acceptable levels of bias (see Table 4.32). Standard error bias for this parameter ranged, in absolute measures from 0 to 1.12 with a mean bias of .23. An aligned rank transform ANOVA with two-way interactions and main effects was run on the results (see Table 4.33).

For the relative asymmetry parameter, standard error bias only became acceptable when sample size was 1,000 or larger. When sample size was 50, the mean standard error bias was .45. As sample size increased, bias decreased, finally reaching .09 bias when sample size was 1,000. Bias remained at .09 when sample size was 1,500. The effect size for sample size on standard error bias for the relative asymmetry parameter was small ($\omega_p^2 = 0.022$, $p < .001$).

An increase in the number of repeated measures reduced bias. When the number of repeated measures was six, mean standard error bias was .47. At eight repeated measures, mean standard error bias had fallen to .17. At 10 repeated measures, mean bias was an acceptable .07. The effect size for the number of repeated measures was the largest for this parameter, however, it was still small ($\omega_p^2 = 0.034$, $p < .001$).

As the maximum rate of change increased, the amount of standard error bias decreased but was unacceptably high at all levels. When $\rho^* = .125$, bias was .34. At $\rho^* = .1875$, bias fell to .22 and when the maximum rate of change was highest, $\rho^* = .5$, bias fell to .15. The effect size for this parameter was small ($\omega_p^2 = 0.010$, $p < .001$).

For the relative asymmetry parameter, the initial level, $\delta^* = .25$, had an unacceptable amount of standard error bias at .34. This improved to a still unacceptable

Table 4.32

Standard Error Estimate Bias for Relative Asymmetry (γ^*)

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.83	0.75	0.78	1.12	0.83	0.86	0.96	0.62	0.60
	100	0.92	0.72	0.67	0.71	0.68	0.43	0.82	0.49	0.52
	200	0.67	0.69	0.51	0.60	0.45	0.29	0.69	0.45	0.29
	500	0.63	0.51	0.40	0.53	0.19	0.04	0.49	0.18	0.07
	1,000	0.51	0.50	0.25	0.30	0.11	-0.07	0.52	0.08	-0.02
	1,500	0.51	0.51	0.19	0.41	0.10	-0.08	0.69	0.01	-0.03
8	50	0.48	0.51	0.37	0.34	0.28	0.19	0.34	0.30	0.18
	100	0.37	0.40	0.43	0.35	0.25	0.22	0.24	0.20	0.11
	200	0.37	0.49	0.20	0.30	0.10	0.05	0.23	0.08	0.10
	500	0.31	0.21	0.14	0.16	-0.02	-0.05	0.16	0.01	-0.01
	1,000	0.29	0.16	-0.01	0.05	0.05	-0.06	0.00	-0.04	-0.03
	1,500	0.36	0.13	-0.02	0.00	-0.09	-0.06	-0.01	0.01	-0.08
10	50	0.36	0.31	0.37	0.25	0.11	0.10	0.13	0.14	0.09
	100	0.33	0.25	0.22	0.09	0.05	0.05	0.14	0.12	0.07
	200	0.26	0.18	0.11	0.08	-0.05	0.01	0.05	-0.06	0.01
	500	0.19	0.08	-0.07	0.01	-0.06	0.03	-0.01	-0.02	-0.02
	1,000	0.20	0.02	-0.13	-0.07	-0.10	-0.08	-0.04	-0.02	0.02
	1,500	0.10	-0.07	-0.05	-0.04	-0.02	-0.02	-0.03	-0.02	-0.03

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

.18 when $\delta^* = .375$ and $\delta^* = .5$. The effect size for this parameter was negligible, although statistically significant ($\omega_p^2 = 0.002$, $p < .001$).

The only two-way interaction effect that had an effect size .01 or larger was the interaction between sample size and the number of repeated measures ($\omega_p^2 = 0.013$, $p < .001$).

Table 4.33

Results of Aligned Rank Transformed ANOVA for Standard Error Estimate Bias of Relative Asymmetry (γ^*)

Effect	df	Sum Sq.[a]	Sum Sq. Res.[a]	F value	Pr (>F)	ω_p^2
SS	5	7.35E+12	3.26E+14	730.90	< 0.001	0.022
RM	2	1.12E+13	3.23E+14	2812.07	< 0.001	0.034
ρ^*	2	3.24E+12	3.25E+14	806.19	< 0.001	0.010
δ^*	2	6.86E+11	3.32E+14	167.42	< 0.001	0.002
SS × RM	10	4.38E+12	3.22E+14	219.97	< 0.001	0.013
SS × ρ^*	10	8.32E+10	3.28E+14	4.11	< 0.001	0.000
RM × ρ^*	4	1.58E+12	3.26E+14	196.81	< 0.001	0.005
SS × δ^*	10	3.54E+11	3.28E+14	17.49	< 0.001	0.001
RM × δ^*	4	2.61E+11	3.27E+14	32.29	< 0.001	0.001
ρ^* × δ^*	4	1.63E+11	3.28E+14	20.18	< 0.001	0.000

Note. Residual df = 161946. SS = sample size. RM = number of repeated measures. ρ = rate of change at the inflection point. δ = location of the inflection point within the measurement window.
[a] Type III sum of squares.

Summary of Fit Measure Results

The overall accuracy of each fit measure, as well as the accuracy by each condition, is shown in Table 4.34. To assess the influence of the manipulated factors on fit measure performance, logistic regressions were run and odds ratios (OR) were calculated to be used as a measure of effect size. Odds ratios are an indication of the relative measure of effect between a reference and an intervention group. In this case, the reference group was: a sample size of 50, the number of repeated measures of 6, the location within the measurement window (δ^*) of 0.25, and the maximum rate of change (ρ^*) of 0.125. Each fit measure is discussed in detail below. Because of the large number of data sets used in this simulation, all p values, unless explicitly stated, are assumed to be $< .001$.

Akaikes Information Criterion

Under the conditions simulated in this study, AIC had the overall best performance of the information criteria. Across all conditions, it selected the correct model 62.6% of the time (see Table 4.34). The accuracy ranged from 9.0% when the sample size was the smallest, the number of repeated measures was lowest, the maximum rate of change was lowest, and the location of the inflection point was most offset to 100% under a variety of less adverse conditions (see Table 4.35). AIC was the best performing information criterion when the number of repeated measures was six: the minimum number required identify the Richards model. Under conditions with eight or 10 repeated measures, it was the second most accurate of the information criterion. When sample size was 200 or larger, AIC was the best performer of the information criterion and the second best when sample size was below 200. AIC performed best when the location of the inflection point was most offset, and second best when under the other two levels of this condition. When the maximum rate of change was the lowest of the three conditions, AIC performed better than the other information criterion and second best when it was the highest rate of change.

A logistic regression was run to determine which of the manipulated conditions had the largest impact on the performance of the AIC in selecting the correct model (see Table 4.36). Of the manipulated conditions, the location of the inflection point within the measurement window (δ^*) had the smallest impact on the performance of AIC. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 91% increase in the odds of correct model selection (OR = 1.91, 95% CI: 1.85-1.97) and an increase to $\delta^* = .5$ resulted in a 35% increase in the odds of correct model selection (OR = 1.35, 95% CI:

Table 4.34

Proportion of Correctly Selected Sigmoidal Models by Condition

Condition	AIC	BIC	aBIC	AICc	CAIC	DIC	HQ	RMSEA	SRMR	CFI	TLI
Overall	0.63	0.34	0.62	0.54	0.30	0.49	0.50	0.83	0.51	0.81	0.65
RM = 6	0.38	0.06	0.33	0.32	0.04	0.15	0.18	0.76	0.27	0.75	0.47
RM = 8	0.67	0.35	0.70	0.58	0.29	0.54	0.54	0.83	0.52	0.81	0.67
RM = 10	0.83	0.63	0.84	0.72	0.56	0.79	0.77	0.89	0.72	0.88	0.81
n = 50	0.35	0.08	0.51	0.03	0.03	0.34	0.22	0.76	0.39	0.75	0.52
n = 100	0.44	0.12	0.52	0.29	0.07	0.32	0.28	0.78	0.39	0.77	0.55
n = 200	0.56	0.23	0.55	0.52	0.17	0.39	0.40	0.80	0.44	0.79	0.59
n = 500	0.72	0.42	0.64	0.72	0.37	0.53	0.59	0.85	0.53	0.83	0.69
n = 1,000	0.82	0.56	0.73	0.82	0.52	0.65	0.73	0.88	0.62	0.86	0.75
n = 1,500	0.86	0.65	0.78	0.86	0.61	0.72	0.79	0.90	0.66	0.87	0.78
$\delta^* = 0.25$	0.58	0.25	0.55	0.49	0.21	0.40	0.42	0.83	0.61	0.82	0.66
$\delta^* = 0.375$	0.68	0.43	0.68	0.59	0.39	0.58	0.58	0.84	0.51	0.82	0.67
$\delta^* = 0.5$	0.62	0.34	0.63	0.54	0.29	0.50	0.50	0.81	0.40	0.79	0.61
$\rho^* = 0.125$	0.51	0.21	0.50	0.43	0.17	0.35	0.36	0.79	0.43	0.77	0.56
$\rho^* = 0.1875$	0.63	0.33	0.62	0.54	0.29	0.48	0.49	0.83	0.50	0.81	0.65
$\rho^* = 0.25$	0.74	0.49	0.75	0.65	0.43	0.64	0.64	0.87	0.59	0.85	0.73

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point. AIC = Akaikes Information Criterion. BIC = Bayesian Information Criterion. aBIC = Sample Size Adjusted BIC. AICc = Corrected AIC. CAIC = Consistent AIC. DIC = Draper Information Criterion. HQ = Hannan and Quinn adjustment of AIC. RMSEA = Root Mean Square Error of Approximation. SRMR = Standardized Root Mean Residual. CFI = Comparative Fit Index. TLI = Tucker-Lewis Index.

Table 4.35

Proportion of Correctly Selected Sigmoidal Models Using AIC

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.09	0.14	0.16	0.11	0.12	0.19	0.10	0.14	0.17
	100	0.14	0.21	0.25	0.12	0.16	0.31	0.09	0.15	0.27
	200	0.15	0.22	0.33	0.13	0.24	0.45	0.13	0.20	0.39
	500	0.26	0.35	0.50	0.22	0.51	0.82	0.16	0.39	0.72
	1,000	0.35	0.48	0.70	0.38	0.74	0.89	0.27	0.64	0.87
	1,500	0.42	0.60	0.82	0.47	0.82	0.91	0.33	0.76	0.88
8	50	0.28	0.31	0.43	0.27	0.38	0.51	0.24	0.33	0.45
	100	0.36	0.38	0.48	0.32	0.51	0.71	0.30	0.42	0.65
	200	0.40	0.48	0.64	0.47	0.72	0.90	0.40	0.67	0.81
	500	0.49	0.70	0.87	0.79	0.91	0.94	0.72	0.85	0.90
	1,000	0.71	0.90	0.99	0.89	0.94	0.98	0.85	0.87	0.93
	1,500	0.80	0.95	1.00	0.90	0.95	0.99	0.88	0.89	0.96
10	50	0.42	0.50	0.57	0.48	0.62	0.74	0.41	0.56	0.70
	100	0.41	0.55	0.70	0.67	0.78	0.91	0.59	0.75	0.85
	200	0.56	0.69	0.88	0.85	0.92	0.97	0.81	0.89	0.90
	500	0.81	0.96	0.99	0.93	0.98	1.00	0.87	0.94	0.97
	1,000	0.94	0.99	1.00	0.97	0.99	1.00	0.90	0.96	1.00
	1,500	0.99	1.00	1.00	0.98	1.00	1.00	0.91	0.98	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

1.31-1.39). It is interesting to note that all of the information criteria performed better when δ^* was equal to .375, which was the middle level of this condition indicating a slightly off-centered location of the inflection point.

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on AIC performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 2.2 times (OR = 2.16, 95% CI: 2.09-2.22) and an increase to $\rho^* = .5$ increased the odds by roughly 4.6 times (OR = 4.62, 95% CI: 4.47-4.77).

Increasing the number of repeated measures from six to eight with all other conditions held equal, increased the odds of AIC selecting the true sigmoidal model by 5.5 times (OR = 5.50, 95% CI: 5.33-5.67). Increasing from six to 10 repeated measures increased those odds by a little over 16.5 times (OR = 16.65, 95% CI: 16.07-17.26).

Of all manipulated conditions, sample size had the largest impact on the performance of AIC. An increase in sample size from 50 to 100 increased the odds of AIC correctly selecting the true model by just over 1.7 times (OR = 1.72, 95% CI: 1.65-1.79). Increasing to 200 increased the odds by roughly 3.3 times (OR = 3.33, 95% CI: 3.20-3.47). When the sample size was 500 the odds of correct model selection improved by almost 9 times (OR = 8.80, 95% CI: 8.42-9.20). Increasing from 50 to 1,000 made the odds of correct model selection 18 times more likely (OR = 18.02, 95% CI: 17.18-18.91). When the sample size was 1,500, the odds of correct model selection increased by almost 26 times (OR = 25.96, 95% CI: 24.67-27.31).

Corrected AIC

Overall, the AICc performed worse than AIC with an accuracy rate of 54.4%. Generally speaking, the AICc uniformly underperformed the AIC with the exception of conditions with a sample size of 1,500 in which case, the two criteria performed equally well with an 86% accuracy rate (see Table 4.34). Accuracy on a per cell

Table 4.36

Logistic Regression Results for AIC

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-3.46	0.03	-137.85	< 0.001	0.03	0.03	0.03
RM = 8	1.70	0.02	107.89	< 0.001	5.50	5.33	5.67
RM = 10	2.81	0.02	154.67	< 0.001	16.65	16.07	17.26
n = 100	0.54	0.02	26.17	< 0.001	1.72	1.65	1.79
n = 200	1.20	0.02	57.48	< 0.001	3.33	3.20	3.47
n = 500	2.18	0.02	96.79	< 0.001	8.80	8.42	9.20
n = 1,000	2.89	0.02	118.16	< 0.001	18.02	17.18	18.91
n = 1,500	3.26	0.03	125.95	< 0.001	25.96	24.67	27.31
$\delta^* = 0.375$	0.64	0.02	40.44	< 0.001	1.91	1.85	1.97
$\delta^* = 0.5$	0.30	0.02	18.98	< 0.001	1.35	1.31	1.39
$\rho^* = 0.1875$	0.77	0.02	49.56	< 0.001	2.16	2.09	2.22
$\rho^* = 0.25$	1.53	0.02	92.17	< 0.001	4.62	4.47	4.77

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

basis ranged from 0% under many conditions when sample size was the smallest to 100% under a number of conditions where the sample size was at least 1,500 or the maximum rate of change was at its highest (see Table 4.37). AICc tied AIC as the best performing information criterion when the sample size was 1,500. Under conditions where the sample size was 500 or 1,000, the AICc performed second best of the information criteria. However, when sample size was 50, it was the worst performing fit measure with an accuracy rate of only 2.8%.

A logistic regression was run to determine which of the manipulated conditions had the largest impact on the performance of the AICc in selecting the correct model (see Table 4.38). Much like the AIC, the location of the inflection point within the measurement window (δ^*) had the smallest impact on the performance of AICc. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 2.5 times increase in the odds of correct model selection (OR = 2.53, 95% CI: 2.44-2.63) and an increase to $\delta^* = .5$

Table 4.37

Proportion of Correctly Selected Sigmoidal Models Using AICc

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	100	0.05	0.08	0.12	0.03	0.05	0.13	0.02	0.04	0.10
	200	0.10	0.17	0.28	0.07	0.17	0.38	0.08	0.14	0.30
	500	0.23	0.33	0.47	0.19	0.47	0.83	0.14	0.36	0.70
	1,000	0.34	0.47	0.69	0.36	0.73	0.89	0.25	0.63	0.88
	1,500	0.41	0.59	0.82	0.46	0.82	0.92	0.32	0.77	0.89
8	50	0.02	0.02	0.05	0.00	0.02	0.05	0.00	0.01	0.03
	100	0.15	0.20	0.29	0.14	0.28	0.56	0.11	0.19	0.45
	200	0.32	0.42	0.57	0.40	0.68	0.90	0.29	0.61	0.85
	500	0.47	0.67	0.87	0.80	0.92	0.95	0.71	0.87	0.92
	1,000	0.71	0.90	0.99	0.89	0.95	0.99	0.86	0.89	0.94
	1,500	0.80	0.95	1.00	0.91	0.95	0.99	0.89	0.89	0.96
10	50	0.02	0.05	0.11	0.02	0.05	0.18	0.01	0.02	0.08
	100	0.23	0.35	0.52	0.48	0.72	0.91	0.35	0.56	0.80
	200	0.49	0.62	0.84	0.87	0.94	0.98	0.81	0.93	0.94
	500	0.80	0.95	0.99	0.95	0.98	1.00	0.89	0.94	0.97
	1,000	0.94	0.99	1.00	0.98	0.99	1.00	0.91	0.96	1.00
	1,500	0.98	1.00	1.00	0.98	1.00	1.00	0.92	0.98	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

resulted in a 57% increase in the odds of correct model selection (OR = 1.57, 95% CI: 1.52-1.63).

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on AICc performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 2.6 times (OR = 2.64, 95% CI: 2.55-2.74) and an increase to $\rho^* = .5$ increased the odds by 7.3 times (OR = 7.31, 95% CI: 7.03-7.61).

An increase in the number of repeated measures from six to eight increased the odds of AICc selecting the true sigmoidal model by nearly 7.5 times (OR = 7.45, 95% CI: 7.18-7.73). Increasing from six to 10 repeated measures increased those odds over 31 times (OR = 31.64, 95% CI: 30.24-33.10).

Much like the AIC, sample size had the largest impact on the performance of AICc. However, the odds ratios of the AICc were as much as 50 times larger than the AIC under similar conditions. This does not indicate better performance; it just reflects a large improvement over the poor performance of AICc under the reference conditions. An increase in sample size from 50 to 100 increased the odds of AIC correctly selecting the true model by just over 26 times (OR = 26.37, 95% CI: 24.28-28.65). Increasing to 200 increased the odds by roughly 117 times (OR = 116.57, 95% CI: 107.12-126.86). When the sample size was 500 the odds of correct model selection improved by over 400 times (OR = 423.53, 95% CI: 387.75-462.61). Increasing the sample size to 1,000 made the odds of correct model selection almost 1,000 times more likely (OR = 987.56, 95% CI: 900.95-1082.50). Finally, when the sample size was 1,500, the odds of correct model selection increased by nearly 1,500 times (OR = 1487.35, 95% CI: 1354.01-1633.82).

Consistent AIC

Overall, the CAIC was the worst performing information criteria with an accuracy rate of only 30%. The lone condition where CAIC did not perform worse than

Table 4.38

Logistic Regression Results for AICc

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-7.91	0.05	-156.77	< 0.001	0.00	0.00	0.00
RM = 8	2.01	0.02	106.30	< 0.001	7.45	7.18	7.73
RM = 10	3.45	0.02	149.50	< 0.001	31.64	30.24	33.10
n = 100	3.27	0.04	77.47	< 0.001	26.37	24.28	28.65
n = 200	4.76	0.04	110.30	< 0.001	116.57	107.12	126.86
n = 500	6.05	0.05	134.32	< 0.001	423.53	387.75	462.61
n = 1,000	6.90	0.05	147.23	< 0.001	987.56	900.95	1082.50
n = 1,500	7.30	0.05	152.43	< 0.001	1487.35	1354.01	1633.82
$\delta^* = 0.375$	0.93	0.02	49.55	< 0.001	2.53	2.44	2.63
$\delta^* = 0.5$	0.45	0.02	24.76	< 0.001	1.57	1.52	1.63
$\rho^* = 0.1875$	0.97	0.02	53.39	< 0.001	2.64	2.55	2.74
$\rho^* = 0.25$	1.99	0.02	99.27	< 0.001	7.31	7.03	7.61

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

any other fit measure was when sample size was 50 (see Table 4.34) and in that case, it was the second worst performing. Accuracy on a per cell basis ranged from 0% under a large number of conditions where the number of repeated measures was six or when the maximum rate of change was at its lowest to 100% when sample size was high and/or the number of repeated measures was greatest (see Table 4.39). While it is possible that CAIC would continue to perform better as sample size increases, those conditions were not simulated.

Results of a logistic regression were used to determine which of the manipulated conditions had the largest impact on the performance of the CAIC when selecting the correct model (see Table 4.40). Much like other fit measures, the location of the inflection point within the measurement window (δ^*) had the smallest impact on the performance of CAIC. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in an increase in the odds of correct model selection by 15.5 times (OR = 15.59, 95% CI:

Table 4.39

Proportion of Correctly Selected Sigmoidal Models Using CAIC

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	200	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00
	500	0.00	0.01	0.03	0.00	0.00	0.06	0.00	0.00	0.01
	1,000	0.00	0.02	0.08	0.00	0.02	0.46	0.00	0.00	0.07
	1,500	0.00	0.03	0.13	0.00	0.07	0.81	0.00	0.00	0.28
8	50	0.01	0.01	0.04	0.00	0.01	0.03	0.00	0.01	0.02
	100	0.01	0.02	0.05	0.00	0.02	0.10	0.00	0.01	0.02
	200	0.01	0.04	0.11	0.01	0.06	0.37	0.00	0.01	0.13
	500	0.03	0.09	0.29	0.07	0.52	0.97	0.01	0.14	0.79
	1,000	0.07	0.24	0.67	0.42	0.97	1.00	0.05	0.73	1.00
	1,500	0.12	0.44	0.90	0.82	1.00	1.00	0.26	0.98	1.00
10	50	0.03	0.08	0.16	0.03	0.10	0.23	0.01	0.03	0.10
	100	0.04	0.11	0.22	0.08	0.22	0.55	0.03	0.08	0.31
	200	0.07	0.18	0.42	0.30	0.66	0.96	0.08	0.36	0.80
	500	0.17	0.52	0.88	0.90	1.00	1.00	0.62	0.96	1.00
	1,000	0.44	0.87	1.00	1.00	1.00	1.00	0.99	1.00	1.00
	1,500	0.69	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

14.75-16.48) and an increase to $\delta^* = .5$ resulted in a 4.1 times increase in the odds of correct model selection (OR = 4.13, 95% CI: 3.93-4.35).

While still relatively small for CAIC, the maximum rate of change parameter (ρ^*) had a larger impact than δ^* on correct model selection. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by just over 7 times (OR = 7.18, 95% CI: 6.80-7.58) and an increase to $\rho^* = .5$ increased the odds by over 52 times (OR = 52.07, 95% CI: 48.85-55.51).

Increasing the number of repeated measures had a very large impact on the performance on the CAIC mainly due to poor performance at the six repeated measures reference level. An increase in the number of repeated measures from six to eight increased the odds of CAIC selecting the correct model by 87 times (OR = 87.08, 95% CI: 80.94-93.69). Increasing from six to 10 repeated measures increased those odds nearly 2100 times (OR = 2097.28, 95% CI: 1908.44-2304.81).

Similar to the performance gains by increasing the number of repeated measures, sample size increases resulted in very large odds ratios due to the poor performance of this fit measure with small sample sizes. An increase in sample size from 50 to 100 increased the odds of CAIC correctly selecting the true model 3.1 times (OR = 3.14, 95% CI: 2.85-3.46). Increasing sample size to 200 increased the odds by roughly 20 times (OR = 20.08, 95% CI: 18.29-22.05). Further increasing the sample size to 500 increased the odds of correct model selection 246 times (OR = 245.72, 95% CI: 222.14-271.80). Increasing the sample size to 1,000 made the odds of correct model selection 1390 times more likely (OR = 1390.30, 95% CI: 1244.63-1553.01). Finally, when the sample size was 1,500, the odds of correct model selection increased by over 4000 times (OR = 4028.03, 95% CI: 4531.43).

Table 4.40

Logistic Regression Results for CAIC

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-14.52	0.09	-161.87	< 0.001	0.00	0.00	0.00
RM = 8	4.47	0.04	119.67	< 0.001	87.08	80.94	93.69
RM = 10	7.65	0.05	158.88	< 0.001	2097.28	1908.44	2304.81
n = 100	1.14	0.05	22.85	< 0.001	3.14	2.85	3.46
n = 200	3.00	0.05	62.93	< 0.001	20.08	18.29	22.05
n = 500	5.50	0.05	106.94	< 0.001	245.72	222.14	271.80
n = 1,000	7.24	0.06	128.16	< 0.001	1390.30	1244.63	1553.01
n = 1,500	8.30	0.06	138.16	< 0.001	4028.03	3580.55	4531.43
$\delta^* = 0.375$	2.75	0.03	96.74	< 0.001	15.59	14.75	16.48
$\delta^* = 0.5$	1.42	0.03	54.29	< 0.001	4.13	3.93	4.35
$\rho^* = 0.1875$	1.97	0.03	71.27	< 0.001	7.18	6.80	7.58
$\rho^* = 0.25$	3.95	0.03	121.23	< 0.001	52.07	48.85	55.51

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Hannan and Quinn adjustment of AIC

The HQ, much like the DIC discussed below, was neither one of the best information criterion, nor one of the worst. It performed worse than the AIC, the aBIC, and the AICc, but better than the CAIC or the BIC and about the same as the DIC. Its overall accuracy rate was 49.9% (see Table 4.34). Accuracy on a per cell basis ranged from 1% under conditions with six repeated measures and the smallest maximum rate of change up to 100% under more optimal conditions where the number of repeated measures was eight or 10 (see Table 4.41).

Logistic regression results were used to determine which of the manipulated conditions had the largest impact on the performance of the HQ in correct model selection (see Table 4.42). Much like the other information criterion, the HQ showed a preference when selecting models where δ^* was .375. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 3.8 times increase in the odds of correct model selection (OR =

Table 4.41

Proportion of Correctly Selected Sigmoidal Models Using HQ

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.02	0.05	0.06	0.03	0.04	0.07	0.02	0.04	0.06
	100	0.03	0.04	0.09	0.01	0.03	0.10	0.01	0.02	0.06
	200	0.01	0.07	0.14	0.01	0.04	0.17	0.01	0.03	0.09
	500	0.04	0.11	0.24	0.02	0.15	0.58	0.01	0.06	0.31
	1,000	0.09	0.22	0.42	0.05	0.44	0.91	0.02	0.17	0.76
	1,500	0.13	0.30	0.56	0.09	0.67	0.98	0.02	0.33	0.94
8	50	0.13	0.17	0.29	0.12	0.21	0.32	0.09	0.17	0.24
	100	0.13	0.19	0.28	0.12	0.25	0.52	0.08	0.16	0.41
	200	0.17	0.26	0.41	0.20	0.48	0.84	0.11	0.30	0.70
	500	0.25	0.44	0.71	0.57	0.93	0.99	0.33	0.80	0.98
	1,000	0.43	0.72	0.95	0.90	0.99	1.00	0.72	0.98	0.99
	1,500	0.56	0.88	0.99	0.99	0.99	1.00	0.92	0.99	1.00
10	50	0.27	0.37	0.45	0.32	0.48	0.66	0.23	0.36	0.57
	100	0.25	0.37	0.54	0.48	0.72	0.90	0.33	0.55	0.79
	200	0.35	0.50	0.75	0.80	0.93	0.99	0.60	0.89	0.97
	500	0.59	0.87	0.98	0.98	1.00	1.00	0.97	0.99	0.99
	1,000	0.86	0.99	1.00	1.00	1.00	1.00	0.98	0.99	1.00
	1,500	0.95	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

3.78, 95% CI: 3.65-3.92) whereas an increase from $\delta^* = .25$ to $\delta^* = .5$ resulted in a 2 times increase in the odds of correct model selection (OR = 2.02, 95% CI: 1.95-2.09).

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on HQ performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ resulted in the odds of correct model selection increasing by 3.2 times (OR = 3.15, 95% CI: 3.04-3.26). An increase to $\rho^* = .5$ increased the odds by 11 times (OR = 10.92, 95% CI: 10.50-11.36).

Increasing the number of repeated measures from six to eight resulted in a corresponding increase in the odds of HQ selecting the true model by 17.4 times (OR = 17.40, 95% CI: 16.70-18.13). Increasing from six to 10 repeated measures increased those odds over 100 times (OR = 100.79, 95% CI: 95.93-105.90).

Although the HQ performed much like the DIC, unlike the DIC, sample size increase resulted in a monotonic increase in the odds of correct model selection. An increase in sample size from 50 to 100 increased the odds of correctly selecting the true model by 67% (OR = 1.67, 95% CI: 1.59-1.76). Increasing sample size to 200 increased the odds by 4.4 times (OR = 4.40, 95% CI: 4.19-4.62). When the sample size was 500 the odds of correct model selection improved by 18.3 times (OR = 18.30, 95% CI: 17.35-19.30). Increasing the sample size to 1,000 made the odds of correct model selection 59 times more likely (OR = 58.73, 95% CI: 55.38-62.30). Finally, when the sample size was 1,500, the odds of correct model selection increased by over 108 times (OR = 108.46, 95% CI: 101.87-115.47).

Bayesian Information Criterion

Much like the CAIC, the BIC performed very poorly overall with the second worst accuracy rate of any fit measure at 34.4%. When sample size was 50, the BIC performed better than 2 other information criteria, but that accuracy rate was still only 8.3% (see Table 4.34). Aside from that lone condition, BIC only performed better than CAIC for the remaining conditions. Accuracy on a per cell basis ranged from

Table 4.42

Logistic Regression Results for HQ

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-6.66	0.04	-175.86	< 0.001	0.00	0.00	0.00
RM = 8	2.86	0.02	136.30	< 0.001	17.40	16.70	18.13
RM = 10	4.61	0.03	182.96	< 0.001	100.79	95.93	105.90
n = 100	0.52	0.03	20.32	< 0.001	1.67	1.59	1.76
n = 200	1.48	0.03	58.48	< 0.001	4.40	4.19	4.62
n = 500	2.91	0.03	106.88	< 0.001	18.30	17.35	19.30
n = 1,000	4.07	0.03	135.55	< 0.001	58.73	55.38	62.30
n = 1,500	4.69	0.03	146.60	< 0.001	108.46	101.87	115.47
$\delta^* = 0.375$	1.33	0.02	71.51	< 0.001	3.78	3.65	3.92
$\delta^* = 0.5$	0.70	0.02	38.98	< 0.001	2.02	1.95	2.09
$\rho^* = 0.1875$	1.15	0.02	62.60	< 0.001	3.15	3.04	3.26
$\rho^* = 0.25$	2.39	0.02	119.25	< 0.001	10.92	10.50	11.36

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

0% under a large number of conditions where the number of repeated measures was six to 100% when sample size was high and/or the number of repeated measures was greatest (see Table 4.43). BICs preference for more parsimonious models was likely the cause of its poor performance when selecting complex models such as the Richards model in this study.

Logistic regression results were used to determine the impact of the manipulated conditions on the performance of the BIC when selecting the correct model (see Table 4.44). The location of the inflection point within the measurement window (δ^*) had the smallest impact on the performance of the BIC. An increase in δ^* from .25 to .375 resulted in an increase in the odds of correct model selection by 9.3 times (OR = 9.25, 95% CI: 8.83-9.70) and an increase to $\delta^* = .5$ resulted in a 3.2 times increase in the odds of correct model selection (OR = 3.18, 95% CI: 3.04-3.32).

Table 4.43

Proportion of Correctly Selected Sigmoidal Models Using BIC

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01
	100	0.00	0.00	0.02	0.00	0.00	0.01	0.00	0.00	0.00
	200	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00
	500	0.00	0.02	0.06	0.00	0.01	0.14	0.00	0.00	0.02
	1,000	0.01	0.03	0.13	0.00	0.05	0.59	0.00	0.00	0.15
	1,500	0.01	0.04	0.18	0.00	0.13	0.88	0.00	0.02	0.47
8	50	0.04	0.05	0.11	0.02	0.04	0.10	0.02	0.03	0.07
	100	0.02	0.05	0.12	0.01	0.05	0.22	0.01	0.03	0.09
	200	0.03	0.08	0.17	0.03	0.14	0.55	0.01	0.03	0.28
	500	0.07	0.14	0.39	0.14	0.69	0.99	0.03	0.29	0.90
	1,000	0.12	0.33	0.75	0.58	0.98	1.00	0.14	0.87	1.00
	1,500	0.19	0.56	0.94	0.89	1.00	1.00	0.42	0.99	1.00
10	50	0.10	0.17	0.28	0.10	0.21	0.43	0.05	0.11	0.28
	100	0.08	0.17	0.32	0.18	0.38	0.73	0.09	0.21	0.52
	200	0.12	0.26	0.54	0.48	0.79	0.99	0.19	0.56	0.90
	500	0.24	0.63	0.92	0.94	1.00	1.00	0.78	0.99	1.00
	1,000	0.57	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1,500	0.78	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

Like other fit measures, the maximum rate of change parameter (ρ^*) had a larger impact than δ^* on correct model selection. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 5 times (OR = 5.03, 95% CI: 4.80-5.27) and an increase to $\rho^* = .5$ increased the odds by 28.6 times (OR = 28.56, 95% CI: 27.09-30.11).

Increasing the number of repeated measures had a relatively large impact on the performance on the BIC. An increase in the number of repeated measures from six to eight increased the odds of selecting the correct model by 53 times (OR = 53.11, 95% CI: 49.98-56.43). Increasing from six to 10 repeated measures increased those odds 773 times (OR = 772.63, 95% CI: 716.20-833.52).

Like other fit measures, sample size increases resulted in increases in performance. In the case of BIC, an increase in sample size from 50 to 100 increased the odds correctly selecting the true model 2 times (OR = 2.02, 95% CI: 1.88-2.17). Increasing sample size to 200 increased the odds 8 times (OR = 8.08, 95% CI: 7.54-8.66). Increasing the sample size to 500 increased the odds of correct model selection 58.5 times (OR = 58.53, 95% CI: 54.36-63.02). Increasing the sample size to 1,000 brought the odds of correct model selection to 251 times more likely (OR = 251.14, 95% CI: 231.57-272.35). When the sample size was 1,500, the odds of correct model selection increased to 630 times (OR = 630.15, 95% CI: 577.75-687.30) over when the sample size was 50.

Sample Size Adjusted BIC

Sample size adjusted BIC had the second best overall performance of the information criteria studied in this simulation. Across all conditions, it selected the correct model 62.3% of the time (see Table 4.34). The accuracy ranged from 2.0% when the sample size was large, the number of repeated measures was lowest, the maximum rate of change was lowest, and the location of the inflection point was centered to

Table 4.44

Logistic Regression Results for BIC

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-11.34	0.07	-174.15	< 0.001	0.00	0.00	0.00
RM = 8	3.97	0.03	128.17	< 0.001	53.11	49.98	56.43
RM = 10	6.65	0.04	171.83	< 0.001	772.63	716.20	833.52
n = 100	0.70	0.04	19.24	< 0.001	2.02	1.88	2.17
n = 200	2.09	0.04	59.31	< 0.001	8.08	7.54	8.66
n = 500	4.07	0.04	107.98	< 0.001	58.53	54.36	63.02
n = 1,000	5.53	0.04	133.54	< 0.001	251.14	231.57	272.35
n = 1,500	6.45	0.04	145.53	< 0.001	630.15	577.75	687.30
$\delta^* = 0.375$	2.22	0.02	92.80	< 0.001	9.25	8.83	9.70
$\delta^* = 0.5$	1.16	0.02	51.08	< 0.001	3.18	3.04	3.32
$\rho^* = 0.1875$	1.62	0.02	68.12	< 0.001	5.03	4.80	5.27
$\rho^* = 0.25$	3.35	0.03	124.28	< 0.001	28.56	27.09	30.11

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

100% under a variety of less adverse conditions (see Table 4.45). aBIC was the best performing information criterion when the number of repeated measures was eight or 10 and second best when the number of repeated measures was six. When sample size was under 200, aBIC was the best performer of the information criterion and the second best when sample size was 200. aBIC performed second best when the location of the inflection point was most offset, and best when under the other two levels of this condition. When the maximum rate of change was the highest of the three conditions, aBIC performed better than the other information criterion and second best under the other rate of change levels.

Results of a logistic regression show which of the manipulated conditions had the largest impact on the performance of the aBIC in selecting the correct model (see Table 4.46). As with other fit measures, the location of the inflection point within the measurement window (δ^*) had the smallest impact on the performance of the aBIC.

Table 4.45

Proportion of Correctly Selected Sigmoidal Models Using aBIC

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.39	0.40	0.41	0.42	0.40	0.40	0.39	0.46	0.42
	100	0.27	0.32	0.35	0.25	0.28	0.44	0.24	0.31	0.40
	200	0.13	0.20	0.32	0.10	0.21	0.42	0.11	0.18	0.36
	500	0.08	0.18	0.31	0.06	0.25	0.70	0.02	0.14	0.47
	1,000	0.11	0.25	0.44	0.06	0.47	0.92	0.02	0.19	0.79
	1,500	0.12	0.28	0.54	0.08	0.64	0.98	0.02	0.31	0.94
8	50	0.51	0.50	0.55	0.51	0.52	0.58	0.50	0.57	0.54
	100	0.46	0.47	0.56	0.45	0.59	0.72	0.43	0.55	0.68
	200	0.38	0.46	0.61	0.45	0.71	0.90	0.36	0.65	0.83
	500	0.34	0.53	0.79	0.69	0.95	0.98	0.49	0.87	0.97
	1,000	0.45	0.75	0.95	0.91	0.99	0.99	0.74	0.98	0.99
	1,500	0.54	0.87	0.99	0.98	1.00	1.00	0.91	0.99	1.00
10	50	0.56	0.59	0.65	0.53	0.60	0.65	0.51	0.58	0.63
	100	0.51	0.63	0.76	0.69	0.76	0.87	0.67	0.73	0.77
	200	0.54	0.68	0.87	0.86	0.93	0.97	0.82	0.89	0.91
	500	0.68	0.91	0.99	0.98	0.99	1.00	0.96	0.98	0.99
	1,000	0.87	0.99	1.00	1.00	1.00	1.00	0.98	0.99	1.00
	1,500	0.94	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 2.2 times increase in the odds of correct model selection (OR = 2.17, 95% CI: 2.10-2.23) and an increase to $\delta^* = .5$ resulted in a 1.6 times increase in the odds of correct model selection (OR = 1.59, 95% CI: 1.55-1.64).

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on aBIC performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 2 times (OR = 2.01, 95% CI: 1.95-2.07) and an increase to $\rho^* = .5$ increased the odds by roughly 4.4 times (OR = 4.42, 95% CI: 4.28-4.55).

Increasing the number of repeated measures from six to eight with all other conditions held equal, increased the odds of aBIC selecting the true sigmoidal model by 6.2 times (OR = 6.21, 95% CI: 6.03-6.39). Increasing from six to 10 repeated measures increased those odds by just under 16 times (OR = 15.84, 95% CI: 15.33-16.37).

Unlike previously reported information criteria, sample size did not have the largest impact on the performance of aBIC. An increase in sample size from 50 to 100 increased the odds of correctly selecting the true model by a mere 8% (OR = 1.08, 95% CI: 1.04-1.12). Increasing to 200 increased the odds by 25% (OR = 1.25, 95% CI: 1.20-1.30). When the sample size was 500 the odds of correct model selection improved by almost 2.1 times (OR = 2.13, 95% CI: 2.05-2.22). Increasing to 1,000 made the odds of correct model selection almost 4 times more likely (OR = 3.86, 95% CI: 3.70-4.03). When the sample size was 1,500, the odds of correct model selection increased by almost 5.5 times (OR = 5.46, 95% CI: 24.67-27.31).

Draper Information Criterion

The DIC, much like the HQ above, was neither one of the best information criterion, nor one of the worst. It performed worse than the AIC, the aBIC, and the AICc, but better than the CAIC or the BIC and about the same as the HQ. Its overall

Table 4.46

Logistic Regression Results for aBIC

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-2.65	0.02	-117.15	< 0.001	0.07	0.07	0.07
RM = 8	1.83	0.01	123.88	< 0.001	6.21	6.03	6.39
RM = 10	2.76	0.02	164.36	< 0.001	15.84	15.33	16.37
n = 100	0.08	0.02	3.73	< 0.001	1.08	1.04	1.12
n = 200	0.22	0.02	11.04	< 0.001	1.25	1.20	1.30
n = 500	0.76	0.02	36.43	< 0.001	2.13	2.05	2.22
n = 1,000	1.35	0.02	61.97	< 0.001	3.86	3.70	4.03
n = 1,500	1.70	0.02	75.04	< 0.001	5.46	5.22	5.71
$\delta^* = 0.375$	0.77	0.02	50.67	< 0.001	2.17	2.10	2.23
$\delta^* = 0.5$	0.47	0.01	31.20	< 0.001	1.59	1.55	1.64
$\rho^* = 0.1875$	0.70	0.01	47.35	< 0.001	2.01	1.95	2.07
$\rho^* = 0.25$	1.49	0.02	93.71	< 0.001	4.42	4.28	4.55

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

accuracy rate was 49.2%. Generally speaking, the DIC uniformly underperformed the more well-known AIC and aBIC (see Table 4.34). Accuracy on a per cell basis ranged from 0% under conditions with six repeated measures, the smallest maximum rate of change, centered inflection point and sample sizes 500 or greater to 100% under more optimal conditions where the number of repeated measures was eight or 10 (see Table 4.47).

Logistic regression results were used to determine which of the manipulated conditions had the largest impact on the performance of the DIC in selecting the correct model (see Table 4.48). Much like the other information criterion, the DIC performed better when selecting models where δ^* was .375. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 4 times increase in the odds of correct model selection (OR = 3.98, 95% CI: 3.84-4.12) whereas an increase from $\delta^* = .25$ to $\delta^* = .5$ resulted in a 2 times increase in the odds of correct model selection (OR = 2.16, 95% CI: 2.09-2.24).

Table 4.47

Proportion of Correctly Selected Sigmoidal Models Using DIC

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.08	0.13	0.15	0.10	0.11	0.17	0.09	0.12	0.16
	100	0.05	0.08	0.12	0.03	0.05	0.13	0.02	0.04	0.09
	200	0.01	0.06	0.13	0.01	0.04	0.15	0.01	0.02	0.07
	500	0.01	0.07	0.16	0.01	0.07	0.45	0.00	0.02	0.16
	1,000	0.02	0.10	0.27	0.01	0.20	0.84	0.00	0.05	0.48
	1,500	0.04	0.16	0.35	0.02	0.41	0.95	0.00	0.11	0.80
8	50	0.26	0.30	0.42	0.24	0.37	0.50	0.23	0.31	0.42
	100	0.17	0.23	0.34	0.16	0.31	0.57	0.13	0.22	0.48
	200	0.15	0.24	0.39	0.18	0.46	0.84	0.10	0.27	0.68
	500	0.18	0.33	0.63	0.42	0.88	1.00	0.19	0.67	0.98
	1,000	0.27	0.57	0.88	0.82	0.99	1.00	0.45	0.97	1.00
	1,500	0.35	0.75	0.98	0.96	1.00	1.00	0.75	1.00	1.00
10	50	0.40	0.49	0.55	0.46	0.61	0.75	0.40	0.54	0.69
	100	0.29	0.43	0.58	0.56	0.75	0.91	0.40	0.63	0.83
	200	0.33	0.47	0.74	0.78	0.93	0.99	0.57	0.88	0.97
	500	0.47	0.81	0.97	0.98	1.00	1.00	0.96	1.00	1.00
	1,000	0.77	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1,500	0.90	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on DIC performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ resulted in the odds of correct model selection increasing by 2.8 times (OR = 2.83, 95% CI: 2.74-2.93). An increase to $\rho^* = .5$ increased the odds by 9.7 times (OR = 9.65, 95% CI: 9.30-10.02).

The number of repeated measures had the largest impact on the performance of DIC. Increasing the number of repeated measures from six to eight resulted in a corresponding increase in the odds of DIC selecting the true model by 15.9 times (OR = 15.89, 95% CI: 15.29-16.51). Increasing from six to 10 repeated measures increased those odds over 87 times (OR = 87.40, 95% CI: 83.52-91.46).

Unlike the other information criterion, sample size increase did not result in a monotonic increase in the odds of correct model selection. An increase in sample size from 50 to 100 decreased the odds of DIC correctly selecting the true model by 12% (OR = 0.88, 95% CI: 0.84-0.92). Increasing sample size to 200 increased the odds by roughly 47% (OR = 1.47, 95% CI: 1.41-1.54). When the sample size was 500 the odds of correct model selection improved by 4.3 times (OR = 4.30, 95% CI: 4.10-4.50). Increasing the sample size to 1,000 made the odds of correct model selection 11 times more likely (OR = 11.06, 95% CI: 10.52-11.63). Finally, when the sample size was 1,500, the odds of correct model selection increased by nearly 20 times (OR = 19.94, 95% CI: 18.91-21.02).

Root Mean Square Error of Approximation

Unlike the information criteria discussed previously, the RMSEA can be used as a stand-alone fit measure that can be used to assess how well a model fits based on certain pre-established cut-offs. Unfortunately, the established cut-offs (Hu & Bentler, 1999) are not applicable to the type of models being investigated, and without guidance for appropriate cut-off values, RMSEA, SRMR, CFI, and TLI were used comparatively much like the information criteria. That is, the values for the true model

Table 4.48

Logistic Regression Results for DIC

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-5.53	0.03	-167.68	< 0.001	0.00	0.00	0.00
RM = 8	2.77	0.02	140.87	< 0.001	15.89	15.29	16.51
RM = 10	4.47	0.02	192.87	< 0.001	87.40	83.52	91.46
n = 100	-0.13	0.02	-5.51	< 0.001	0.88	0.84	0.92
n = 200	0.39	0.02	16.40	< 0.001	1.47	1.41	1.54
n = 500	1.46	0.02	60.32	< 0.001	4.30	4.10	4.50
n = 1,000	2.40	0.03	93.57	< 0.001	11.06	10.52	11.63
n = 1,500	2.99	0.03	110.55	< 0.001	19.94	18.91	21.02
$\delta^* = 0.375$	1.38	0.02	77.64	< 0.001	3.98	3.84	4.12
$\delta^* = 0.5$	0.77	0.02	44.82	< 0.001	2.16	2.09	2.24
$\rho^* = 0.1875$	1.04	0.02	60.40	< 0.001	2.83	2.74	2.93
$\rho^* = 0.25$	2.27	0.02	119.50	< 0.001	9.65	9.30	10.02

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

were compared against the values for the competing polynomial models and the best score indicates the model selected.

Under the conditions simulated in this study, the RMSEA had the overall best performance of any fit measure. Across all conditions, RMSEA selected the correct model 82.8% of the time. RMSEA was the best performing information criterion for all numbers of repeated measures, all sample sizes, all maximum rates of change, and all inflection point locations (see Table 4.34). On a per-cell basis, the accuracy was no less than 70.0

Results of a logistic regression were used to determine which of the manipulated conditions had the largest impact on the performance of the RMSEA when selecting the correct model (see Table 4.50). Unlike many of the information criteria, the odds ratios for the RMSEA were relatively small due to its consistent performance across the conditions.

Table 4.49

Proportion of Correctly Selected Sigmoidal Models Using RMSEA

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.76	0.76	0.74	0.78	0.76	0.73	0.75	0.77	0.77
	100	0.76	0.72	0.72	0.76	0.75	0.77	0.77	0.75	0.77
	200	0.73	0.71	0.75	0.71	0.72	0.75	0.72	0.73	0.78
	500	0.70	0.72	0.80	0.71	0.76	0.83	0.70	0.75	0.79
	1,000	0.73	0.77	0.86	0.74	0.78	0.85	0.71	0.78	0.80
	1,500	0.74	0.80	0.87	0.75	0.81	0.85	0.75	0.79	0.81
8	50	0.73	0.72	0.76	0.73	0.75	0.81	0.72	0.77	0.78
	100	0.75	0.76	0.78	0.73	0.77	0.82	0.72	0.77	0.80
	200	0.76	0.79	0.85	0.76	0.79	0.87	0.74	0.81	0.82
	500	0.79	0.90	0.93	0.83	0.88	0.92	0.81	0.82	0.86
	1,000	0.84	0.93	0.98	0.84	0.92	0.98	0.81	0.85	0.91
	1,500	0.89	0.96	0.99	0.85	0.93	0.99	0.80	0.86	0.94
10	50	0.74	0.74	0.77	0.72	0.77	0.82	0.72	0.77	0.80
	100	0.76	0.80	0.87	0.81	0.84	0.90	0.78	0.80	0.83
	200	0.82	0.89	0.94	0.85	0.90	0.96	0.83	0.85	0.87
	500	0.92	0.97	0.99	0.91	0.98	1.00	0.82	0.90	0.96
	1,000	0.96	0.99	1.00	0.96	0.99	1.00	0.88	0.95	0.99
	1,500	0.98	1.00	1.00	0.98	1.00	1.00	0.90	0.97	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

Of the manipulated conditions, the location of the inflection point within the measurement window (δ^*) had smallest impact on the performance of RMSEA. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 7% increase in the odds of correct model selection (OR = 1.07, 95% CI: 1.03-1.10) and an increase to $\delta^* = .5$ resulted in a 12% decrease in the odds of correct model selection (OR = 0.88, 95% CI: 0.85-0.91). Like the information criterion, RMSEA performed better when δ^* was equal to .375, which was the middle level of this condition indicating a slightly off-centered location of the inflection point.

Much like with the information criterion, the maximum rate of change parameter (ρ^*) had a larger impact than δ^* on correct model selection. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 29% (OR = 1.29, 95% CI: 1.25-1.33) and an increase to $\rho^* = .5$ increased the odds by 75% (OR = 1.75, 95% CI: 1.69-1.81).

Increasing the number of repeated measures from six to eight with all other conditions held equal, increased the odds of RMSEA selecting the true sigmoidal model by 56% (OR = 1.56, 95% CI: 1.51-1.61). Increasing from six to 10 repeated measures increased those odds by a little over 2.6 times (OR = 2.63, 95% CI: 2.64-2.72).

Of the manipulated conditions, sample size had a slightly larger impact on the performance of RMSEA than other conditions. An increase in sample size from 50 to 100 increased the odds of RMSEA correctly selecting the true model by 15% (OR = 1.15, 95% CI: 1.10-1.20). Increasing to 200 increased the odds by 32% (OR = 1.32, 95% CI: 1.27-1.38). When the sample size was 500 the odds of correct model selection improved by 86% (OR = 1.86, 95% CI: 1.78-1.95). Increasing from 50 to 1,000 made the odds of correct model selection roughly 2.5 times more likely (OR = 2.48, 95% CI: 2.36-2.60). When the sample size was 1,500, the odds of correct model selection increased by almost 2.9 times (OR = 2.89, 95% CI: 2.75-3.03).

Table 4.50

Logistic Regression Results for RMSEA

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	0.47	0.02	22.24	< 0.001	1.60	1.53	1.66
RM = 8	0.44	0.02	28.46	< 0.001	1.56	1.51	1.61
RM = 10	0.97	0.02	55.70	< 0.001	2.63	2.54	2.72
n = 100	0.14	0.02	6.65	< 0.001	1.15	1.10	1.20
n = 200	0.28	0.02	13.18	< 0.001	1.32	1.27	1.38
n = 500	0.62	0.02	27.62	< 0.001	1.86	1.78	1.95
n = 1,000	0.91	0.02	37.87	< 0.001	2.48	2.36	2.60
n = 1,500	1.06	0.02	42.65	< 0.001	2.89	2.75	3.03
$\delta^* = 0.375$	0.06	0.02	3.84	< 0.001	1.07	1.03	1.10
$\delta^* = 0.5$	-0.13	0.02	-8.02	< 0.001	0.88	0.85	0.91
$\rho^* = 0.1875$	0.26	0.02	16.03	< 0.001	1.29	1.25	1.33
$\rho^* = 0.25$	0.56	0.02	33.19	< 0.001	1.75	1.69	1.81

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Standardized Root Mean Residual

The SRMR is the standardized mean difference between non-duplicated elements of the actual correlation matrix and the model-implied correlation matrix. As such, SRMR does not explicitly model the mean structure of the growth model. As such, this fit measure was not appropriate for use with the types of models this simulation study examined. However, as mentioned previously, the measure may serve diagnostically, when used with other absolute or incremental fit measures, to indicate problems in the mean structure of a model. Overall, the SRMR was the worst performing of the incremental and absolute fit measures with an accuracy rate of 50.5% and was outperformed by the best information criterion. Generally speaking, the SRMR uniformly underperformed the RMSEA, TLI, and CFI under all conditions (see Table 4.34). Accuracy on a per cell basis ranged from the teens under many conditions when there were six repeated measures and the location of the inflection point was centered to 100% under a conditions with 10 repeated measures, sample sizes of 1,000 or higher, the highest maximum rate of change, and all but the centered location of the inflection point (see Table 4.51).

Logistic regression was run to determine which of the manipulated conditions had the largest impact on the performance of the SRMR in selecting the correct model (see Table 4.52). Unlike every other fit measure, the location of the inflection point within the measurement window (δ^*) did not have the smallest impact on the performance of SRMR. Also unlike every other fit measure, as the location of the inflection point became more centered, the odds ratios of correct model selection monotonically decreased. An increase from $\delta^* = .25$ to $\delta^* = .375$ resulted in a 39% reduction in the odds of correct model selection (OR = 0.61, 95% CI: 0.60-0.63) and an increase to $\delta^* = .5$ resulted in a 66% decrease in the odds of correct model selection (OR = 0.34, 95% CI: 0.33-0.35).

The maximum rate of change parameter (ρ^*) had a smaller impact than δ^* on SRMR performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 1.5 times (OR = 1.45, 95% CI: 1.41-1.49) and an increase to $\rho^* = .5$ increased the odds by 2.3 times (OR = 2.25, 95% CI: 2.18-2.31).

Increasing the number of repeated measures from six to eight increased the odds of SRMR selecting the true sigmoidal model by nearly 3.5 times (OR = 3.44, 95% CI: 3.35-3.53). Increasing from six to 10 repeated measures increased those odds nearly 9 times (OR = 8.93, 95% CI: 8.67-9.19).

Much like the other absolute and incremental fit measures, effect sizes were small in comparison with those of the information criteria. This does not necessarily indicate better or worse performance; in this case, it was just reflective of the relative inconsistency of the SRMR. An increase in sample size from 50 to 100 was not statistically significant (p = .073, OR = 1.04, 95% CI 1.00-1.08). Increasing to 200 increased the odds correct model selection by roughly 27% (OR = 1.27, 95% CI: 1.22-

Table 4.51

Proportion of Correctly Selected Sigmoidal Models Using SRMR

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.32	0.33	0.32	0.29	0.28	0.24	0.25	0.28	0.22
	100	0.31	0.30	0.31	0.26	0.23	0.27	0.21	0.21	0.19
	200	0.29	0.30	0.36	0.19	0.19	0.24	0.18	0.16	0.18
	500	0.24	0.30	0.43	0.19	0.19	0.30	0.17	0.18	0.19
	1,000	0.27	0.39	0.59	0.20	0.23	0.36	0.16	0.20	0.22
	1,500	0.28	0.41	0.64	0.17	0.27	0.49	0.18	0.20	0.26
8	50	0.43	0.43	0.50	0.39	0.39	0.42	0.33	0.37	0.31
	100	0.45	0.48	0.55	0.32	0.38	0.44	0.25	0.27	0.33
	200	0.45	0.56	0.69	0.34	0.39	0.54	0.25	0.30	0.36
	500	0.53	0.72	0.85	0.39	0.55	0.73	0.30	0.33	0.48
	1,000	0.68	0.86	0.96	0.48	0.71	0.90	0.35	0.43	0.62
	1,500	0.72	0.92	0.99	0.57	0.79	0.95	0.38	0.53	0.74
10	50	0.53	0.59	0.64	0.43	0.46	0.54	0.37	0.41	0.44
	100	0.56	0.65	0.77	0.49	0.56	0.64	0.36	0.40	0.46
	200	0.67	0.76	0.89	0.56	0.65	0.80	0.39	0.49	0.58
	500	0.84	0.92	0.98	0.71	0.86	0.96	0.49	0.64	0.80
	1,000	0.93	0.99	1.00	0.88	0.97	1.00	0.64	0.82	0.94
	1,500	0.94	0.99	1.00	0.93	0.99	1.00	0.72	0.88	0.98

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

1.32). When the sample size was 500 the odds of correct model selection improved by 2 times (OR = 2.03, 95% CI: 1.95-2.11). Increasing the sample size to 1,000 made the odds of correct model selection nearly 3.3 times more likely (OR = 3.26, 95% CI: 3.14-3.39). Finally, when the sample size was 1,500, the odds of correct model selection increased by just over 4 times (OR = 4.11, 95% CI: 3.95-4.27).

Table 4.52

Logistic Regression Results for SRMR

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	-1.58	0.02	-78.22	< 0.001	0.21	0.20	0.21
RM = 8	1.23	0.01	88.61	< 0.001	3.44	3.35	3.53
RM = 10	2.19	0.01	147.07	< 0.001	8.93	8.67	9.19
n = 100	0.04	0.02	1.79	0.073	1.04	1.00	1.08
n = 200	0.24	0.02	12.32	< 0.001	1.27	1.22	1.32
n = 500	0.71	0.02	36.19	< 0.001	2.03	1.95	2.11
n = 1,000	1.18	0.02	59.52	< 0.001	3.26	3.14	3.39
n = 1,500	1.41	0.02	70.04	< 0.001	4.11	3.95	4.27
$\delta^* = 0.375$	-0.49	0.01	-35.35	< 0.001	0.61	0.60	0.63
$\delta^* = 0.5$	-1.09	0.01	-76.60	< 0.001	0.34	0.33	0.35
$\rho^* = 0.1875$	0.37	0.01	26.91	< 0.001	1.45	1.41	1.49
$\rho^* = 0.25$	0.81	0.01	57.54	< 0.001	2.25	2.18	2.31

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Comparative Fit Index

Under the conditions simulated in this study, the CFI had the second best overall performance of any fit measure, performing only slightly worse than the RMSEA. Across all conditions, it selected the correct model 81.1% of the time (see Table 4.34). The accuracy ranged from 69% when the sample size was 1,000, the number of repeated measures was lowest, maximum rate of change was lowest, and the location of the inflection point was centered to 100% under a variety of conditions where the number of repeated measures was 10 and sample size was 500 or larger(see Table 4.53).

CFI was the second best performing fit measure when the number regardless of sample size, the number of repeated measures, the location of the inflection point, or the maximum rate of change.

Results of a logistic regression show which of the manipulated conditions had the largest impact on the performance of CFI in selecting the correct model (see Table 4.54). Much like the RMSEA, and unlike the information criterion, the odds ratios for the CFI were relatively small due to its consistent and relatively good performance across the conditions.

As with other fit measures, the location of the inflection point within the measurement window (δ^*) had smallest impact on the performance of CFI. An increase from $\delta^* = .25$ to $\delta^* = .375$ was not statistically significant ($p = 0.356$, OR = 1.02, 95% CI: 0.98-1.05) whereas an increase to $\delta^* = .5$ resulted in a 17% decrease in the odds of correct model selection (OR = 0.83, 95% CI: 0.80-0.86).

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on CFI performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 27% (OR = 1.27, 95% CI: 1.23-1.31) and an increase to $\rho^* = .5$ increased the odds by roughly 68% (OR = 1.68, 95% CI: 1.63-1.73).

Increasing the number of repeated measures from six to eight with all other conditions held equal, increased the odds of CFI selecting the true sigmoidal model by 47% (OR = 1.47, 95% CI: 1.42-1.51). Increasing from six to 10 repeated measures increased those odds by just under 2.5 times (OR = 2.46, 95% CI: 2.38-2.54).

Unlike many other of the fit measures examined in this study, sample size did not have the largest impact on the performance of CFI. An increase in sample size from 50 to 100 increased the odds of correctly selecting the true model by 11% (OR = 1.11, 95% CI: 1.07-1.16). Increasing to 200 increased the odds by 23% (OR = 1.23, 95% CI: 1.18-1.28). When the sample size was 500 the odds of correct model selection improved by 58% (OR = 1.58, 95% CI: 1.51-1.65). Increasing to 1,000 made the odds

Table 4.53

Proportion of Correctly Selected Sigmoidal Models Using CFI

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.77	0.77	0.73	0.79	0.76	0.73	0.77	0.78	0.78
	100	0.77	0.72	0.72	0.77	0.75	0.77	0.78	0.76	0.76
	200	0.72	0.71	0.74	0.72	0.72	0.73	0.72	0.74	0.77
	500	0.70	0.71	0.78	0.70	0.73	0.79	0.71	0.73	0.73
	1,000	0.71	0.74	0.83	0.72	0.73	0.81	0.69	0.74	0.74
	1,500	0.71	0.76	0.84	0.72	0.76	0.81	0.74	0.75	0.75
8	50	0.72	0.72	0.76	0.74	0.74	0.80	0.72	0.77	0.78
	100	0.75	0.76	0.78	0.72	0.75	0.80	0.72	0.77	0.79
	200	0.74	0.78	0.83	0.74	0.75	0.85	0.74	0.78	0.78
	500	0.76	0.87	0.92	0.79	0.85	0.90	0.77	0.76	0.82
	1,000	0.81	0.92	0.98	0.80	0.89	0.97	0.74	0.81	0.88
	1,500	0.86	0.94	0.99	0.82	0.91	0.98	0.75	0.83	0.92
10	50	0.73	0.73	0.78	0.70	0.76	0.80	0.71	0.76	0.77
	100	0.75	0.79	0.86	0.78	0.82	0.88	0.75	0.77	0.80
	200	0.80	0.87	0.93	0.82	0.88	0.95	0.79	0.82	0.85
	500	0.89	0.97	0.99	0.88	0.97	1.00	0.78	0.87	0.95
	1,000	0.95	0.99	1.00	0.95	0.99	1.00	0.84	0.94	0.99
	1,500	0.97	1.00	1.00	0.97	1.00	1.00	0.87	0.96	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

of correct model selection 2 times more likely (OR = 2.02, 95% CI: 1.93-2.11). When the sample size was 1,500, the odds of correct model selection increased by 2.3 times (OR = 2.31, 95% CI: 2.21-2.42).

Table 4.54

Logistic Regression Results for CFI

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	0.54	0.02	26.05	< 0.001	1.71	1.65	1.78
RM = 8	0.38	0.01	25.58	< 0.001	1.47	1.42	1.51
RM = 10	0.90	0.02	54.17	< 0.001	2.46	2.38	2.54
n = 100	0.10	0.02	5.09	< 0.001	1.11	1.07	1.16
n = 200	0.20	0.02	9.74	< 0.001	1.23	1.18	1.28
n = 500	0.46	0.02	21.01	< 0.001	1.58	1.51	1.65
n = 1,000	0.70	0.02	30.97	< 0.001	2.02	1.93	2.11
n = 1,500	0.84	0.02	35.83	< 0.001	2.31	2.21	2.42
$\delta^* = 0.375$	0.01	0.02	0.92	0.356	1.02	0.98	1.05
$\delta^* = 0.5$	-0.19	0.02	-11.90	< 0.001	0.83	0.80	0.86
$\rho^* = 0.1875$	0.24	0.02	15.68	< 0.001	1.27	1.23	1.31
$\rho^* = 0.25$	0.52	0.02	32.23	< 0.001	1.68	1.63	1.73

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Tucker-Lewis Index

Under the conditions simulated in this study, the TLI consistently performed worse than the CFI due to the increased penalty placed on complex models. It was the third best, out of four, absolute and incremental fit measures in performance. In spite of its weak performance, it still better overall than any of the information criteria. Across all conditions, it selected the correct model 64.7% of the time (see Table 4.34). The accuracy ranged from 36% when the sample size was 200, the number of repeated measures was lowest, maximum rate of change was lowest, and the location of the inflection point was least offset from center to 100% under multiple conditions where the number of repeated measures was 10 and sample size was 1,000 or larger (see Table 4.55). TLI did not perform as well as CFI under any condition.

Logistic regression results show which of the manipulated conditions had the largest impact on the performance of TLI in selecting the correct model (see Table 4.56). Unlike the information criterion, the odds ratios for the TLI were relatively small due to its consistent performance across the conditions.

As with other fit measures, the location of the inflection point within the measurement window (δ^*) had smallest impact on the performance of TLI. An increase from $\delta^* = .25$ to $\delta^* = .375$ was increased the odds of correct model selection by a mere, but statistically significant 4% (OR = 1.04, 95% CI: 1.01-1.07) whereas an increase to $\delta^* = .5$ resulted in a 23% decrease in the odds of correct model selection (OR = 0.77, 95% CI: 0.75-0.79).

The maximum rate of change parameter (ρ^*) had a larger impact than δ^* on TLI performance. An increase from $\rho^* = .125$ to $\rho^* = .1875$ increased the odds of correct model selection by 53% (OR = 1.53, 95% CI: 1.49-1.57) and an increase to $\rho^* = .5$ increased the odds over 2.3 times (OR = 2.34, 95% CI: 2.27-2.40).

Holding all other conditions equal, an increase in the number of repeated measures from six to eight increased the odds of TLI selecting the true sigmoidal model by 2.5 times (OR = 2.45, 95% CI: 2.39-2.51). Increasing from six to 10 repeated measures increased those odds by just under 5.4 times (OR = 5.38, 95% CI: 5.23-5.54).

Like CFI, sample size did not have as large of an impact as the number of repeated measures did on the performance of TLI. An increase in sample size from 50 to 100 increased the odds of correctly selecting the true model by 14% (OR = 1.14, 95% CI: 1.10-1.18). Increasing to 200 increased the odds by 41% (OR = 1.41, 95% CI: 1.36-1.46). When the sample size was 500 the odds of correct model selection improved by 2.3 times (OR = 2.25, 95% CI: 2.16-2.33). Increasing to 1,000 made the odds of correct model selection 3.3 times more likely (OR = 3.26, 95% CI: 3.14-3.39).

Table 4.55

Proportion of Correctly Selected Sigmoidal Models Using TLI

RM	n	$\delta^* = 0.25$			$\delta^* = 0.375$			$\delta^* = 0.5$		
		$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$	$\rho^* = 0.125$	$\rho^* = 0.1875$	$\rho^* = 0.25$
6	50	0.39	0.41	0.41	0.41	0.40	0.39	0.38	0.44	0.40
	100	0.39	0.39	0.42	0.39	0.40	0.46	0.41	0.41	0.44
	200	0.39	0.39	0.46	0.36	0.40	0.49	0.38	0.38	0.48
	500	0.39	0.44	0.59	0.39	0.49	0.61	0.40	0.49	0.55
	1,000	0.41	0.54	0.73	0.44	0.54	0.67	0.42	0.52	0.58
	1,500	0.43	0.58	0.77	0.48	0.56	0.71	0.43	0.53	0.57
8	50	0.51	0.51	0.56	0.49	0.52	0.61	0.49	0.58	0.55
	100	0.50	0.53	0.59	0.48	0.58	0.66	0.46	0.55	0.61
	200	0.50	0.57	0.70	0.52	0.59	0.74	0.49	0.61	0.66
	500	0.57	0.77	0.88	0.60	0.75	0.87	0.56	0.61	0.72
	1,000	0.70	0.88	0.98	0.65	0.84	0.96	0.57	0.67	0.81
	1,500	0.76	0.93	0.99	0.71	0.88	0.98	0.57	0.72	0.91
10	50	0.56	0.60	0.65	0.57	0.63	0.71	0.55	0.61	0.68
	100	0.56	0.67	0.78	0.64	0.71	0.82	0.59	0.65	0.69
	200	0.67	0.78	0.90	0.70	0.81	0.92	0.63	0.71	0.78
	500	0.83	0.95	0.99	0.81	0.94	0.99	0.64	0.81	0.93
	1,000	0.92	0.99	1.00	0.93	0.99	1.00	0.72	0.92	0.99
	1,500	0.97	1.00	1.00	0.96	1.00	1.00	0.78	0.95	1.00

*Note.*RM = repeated measures. n = sample size. δ^* = location of the inflection point within the measurement window as a proportion. ρ^* = rate of change at the inflection point.

When the sample size was 1,500, the odds of correct model selection increased by nearly 4 times (OR = 3.93, 95% CI: 3.78-4.09).

Table 4.56

Logistic Regression Results for TLI

Condition	Beta	Std. Error	z value	Pr(>|z|)	OR	2.5%	97.5%
(Intercept)	1.04	0.02	53.15	< 0.001	2.83	2.73	2.94
(Intercept)	-1.13	0.02	-58.81	< 0.001	0.32	0.31	0.34
RM = 8	0.90	0.01	67.87	< 0.001	2.45	2.39	2.51
RM = 10	1.68	0.01	115.33	< 0.001	5.38	5.23	5.54
n = 100	0.13	0.02	6.98	< 0.001	1.14	1.10	1.18
n = 200	0.34	0.02	18.55	< 0.001	1.41	1.36	1.46
n = 500	0.81	0.02	42.31	< 0.001	2.25	2.16	2.33
n = 1,000	1.18	0.02	59.41	< 0.001	3.26	3.14	3.39
n = 1,500	1.37	0.02	67.13	< 0.001	3.93	3.78	4.09
$\delta^* = 0.375$	0.04	0.01	2.78	0.006	1.04	1.01	1.07
$\delta^* = 0.5$	-0.26	0.01	-18.89	< 0.001	0.77	0.75	0.79
$\rho^* = 0.1875$	0.42	0.01	31.33	< 0.001	1.53	1.49	1.57
$\rho^* = 0.25$	0.85	0.01	60.31	< 0.001	2.34	2.27	2.40

Note. n = sample size. δ = location of the inflection point within the measurement window. ρ = rate of change at the inflection point. RM = repeated measures. OR = odds ratio. Reference level sample size = 50, repeated measures = 6, $\delta^* = 0.25$, and $\rho^* = 0.125$.

Empirical Example

The Early Childhood Longitudinal Study, Kindergarten Class of 1998-1999 (ECLS-K) data set (Tourangeau et al., 2009) is a longitudinal survey that collected data on students cognitive, physical, and social-emotional development. The ECLS-K used multistage cluster sampling to obtain a representative sample of roughly 22,000 kindergarteners beginning in the 1998-1999 school year. These students, their parents, and their teachers were surveyed and assessed longitudinally until the 8th grade. For this study, math achievement was examined. In the ECLS-K, math achievement is represented by vertically linked item response theory (IRT) scaled scores for 7 time periods: fall kindergarten, spring kindergarten, fall first grade, spring first grade, spring third grade, spring fifth grade, and spring eighth grade. See Table 4.57 for summary statistics. Missing data was handled using full information maximum likelihood because the missingness was assumed to be missing at random. Figure 4.1 shows a random sample of individual trajectories for the ECLS-K mathematics IRT scores. These trajectories indicate the possibility of nonlinear growth over time. To assess this growth, a total of four models were fit to the ECLS-K math achievement data: linear, quadratic, cubic, and Richards models. Additionally, an appropriate null model was run to establish a baseline χ^2 for calculating CFI and TLI and to determine a baseline RMSEA. All of the comparative, absolute, and incremental fit measures discussed previously were used in model selection.

Table 4.57

Summary Statistics for ECLS-K Math IRT Scores

Grade	n	Mean	sd	Skewness	Kurtosis
Fall Kindergarten	18,636	25.91	9.10	1.41	4.19
Spring Kindergarten	19,649	36.27	12.00	1.06	2.35
Fall 1st grade	5,223	43.26	14.39	0.91	1.60
Spring 1st grade	16,635	61.26	18.09	0.52	0.39
Spring 3rd grade	14,374	98.72	24.71	-0.03	-0.70
Spring 5th grade	11,274	123.69	24.79	-0.60	-0.22
Spring 8th grade	9,285	142.22	22.01	-0.89	0.29

Note. n= sample size.

Of the four models, the Richards model was selected as having the best overall fit (see Table 4.58). For the Richards model, the baseline RMSEA was above 0.158 indicating that the CFI and TLI should be interpreted for this model (Kenny, 2015). Both the CFI (.981) and TLI (.971) indicate the model fit well. The Richards model RMSEA was .106 indicating poor model fit according to Hu and Bentler (1999) however it should be noted that those cutoffs were established for three-factor CFA models, and may not be appropriate for structured latent curve models or any struc-

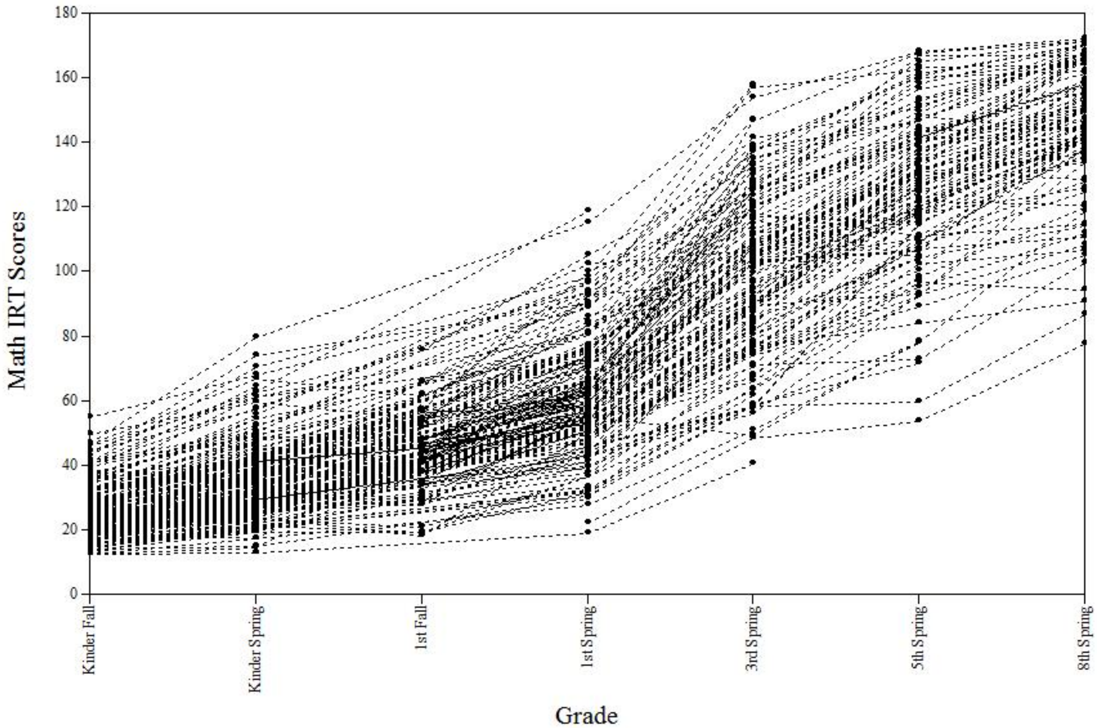

Figure 4.1. Longitudinal plot of math IRT scores of a 2% random sample from the Early Childhood Longitudinal Study - Kindergarten.

tural equation model incorporating a mean structure. Of the models considered, The Richards model had the lowest (best) RMSEA value. As far as the information criterion, again, the Richards model had the lowest scores on all of them versus competing models which indicated that it was the best fitting.

A brief interpretation of the parameters of the Richards model follows. Relative asymmetry (γ) is the defining feature of the Richards curve in comparison to the Logistic or Gompertz curves. In this case, γ was equal to -.074 which means that math achievement growth was asymmetrical with 35.4% of total growth occurring prior to the inflection point. The lower asymptote (i) was -1.7 and grew a total of 148.9 points to an upper asymptote (β) of 147.2. The rate of approach parameter (ρ) was equal to .397 which means that the maximum rate of change was 22.6 at the point of inflection (δ). In the case of the ECLS-K, the timing parameter, or point

Table 4.58

Model Fit Statistics for Growth Models, ECLS-K Math Achievement

Fit Measure	Linear	Quadratic	Cubic	Richards
χ^2	46,765.1	7,219.8	5,518.7	4,014.7
Parameters	12	16	21	18
CFI	0.778	0.966	0.974	0.981
TLI	0.500	0.942	0.966	0.971
RMSEA	0.311	0.134	0.137	0.106
RMSEA 5%	0.308	0.134	0.132	0.103
RMSEA 95%	0.313	0.140	0.137	0.108
SRMR	1.030	0.076	0.051	0.066
AIC	754,426	714,888	713,197	711,687
BIC	754,521	715,016	713,364	711,831
aBIC	754,483	714,964	713,298	711,773
CAIC	754,533	715,032	713,385	711,849
DIC	754,499	714,986	713,326	711,798
HQ	754,457	714,930	713,252	711,734
AICc	754,426	714,889	713,197	711,687

Note. Baseline RMSEA was 0.608.

of inflection, corresponds to a grade level and indicates that on average, a student in this study had achieved approximately 35.4% of his or her growth by second six weeks of the first grade ($\delta = 1.17$) and it was at this time that growth was most rapid. Additional information about the characteristics of the students is given by examining the correlations between freely estimated parameters. The total change component was significantly correlated with the rate of approach ($p < .001$, $r = .297$). This indicates that those students who had higher overall growth tended to grow at a faster rate than those students with lower overall growth. Total change was negatively correlated ($p < .001$, $r = -.235$) with the timing parameter. This indicates that students who had higher overall growth started earlier than those who grew less. Finally, the rate of approach negatively correlated ($p < .001$, $r = -.728$) with the timing parameter indicating that students who grew at a faster rate tended to change earlier than those who grew at a slower rate.

The Richards model provides an easily interpreted set of parameters that are useful in describing ECLS-K math development over time. An interpretation of the next closest fitting model would involve discussing the rate of change of the rate of change of change over time. It is often difficult to map that onto an interpretable developmental theory. Gompertz and Logistic models were not considered in this case, however, if they were, it is possible that the Gompertz model would have been selected due to the relative asymmetry parameter being very close to zero. The Gompertz model lacks the relative asymmetry parameter and therefore would have been more parsimonious to an equivalent Richards model.

CHAPTER FIVE

Discussion

This simulation study was the first methodological investigation of sigmoidal structured latent curve models. Although parameter estimate bias and fit measure performance were of primary concern, convergence rates, coverage proportions, and standard error bias were also investigated. Manipulated conditions in this study included three different numbers of repeated measures, six sample sizes, three maximum rates of change, and three different locations of the inflection point of the curve within the measurement window. Parameters included in the model included the lower asymptote, the upper asymptote, the maximum rate of change, the location of the inflection point, and the relative asymmetry of the latent curve. There were 162 individual conditions, with 1,000 converged data sets examined per condition for 162,000 total data sets. To evaluate fit measure performance, 11 different fit measures were examined (AIC, BIC, aBIC, AICc, CAIC, DIC, HQ, RMSEA, SRMR, CFI, and TLI).

Convergence

Rates of convergence are a real issue for applied researchers who are modeling structured latent curve models. Under some of the conditions in the present study, convergence rates were less than 10%. In a simulation study, non-converged replications are easily taken into consideration, and if necessary a new dataset can be generated. In an applied study, where it could potentially take years to collect the necessary data, minimization of non-convergence should be taken into consideration. Although beyond the scope of this study, there are steps that can be taken to get non-converging models to converge. It is possible to increase the number of iterations, the number of random starts, or provide appropriate starting values for maximum likelihood estimation. In some cases, this will allow the model to converge. Because of this, the rates of convergence in this study should be considered "worst case" under the conditions that were simulated.

For example, an initial run of the full data set resulted in an overall convergence rate of 64.8%. In an effort to judge the impact of altering the number of iterations for the estimation algorithm in Mplus, this was increased from the default of 1,000 to 50,000. This increased the overall convergence rate by only 5.6% yet it took over four times longer to run the models. Although the convergence rates did increase, without fully understanding if these models are finding local maxima or global maxima, the impact on parameter estimate bias is unknown. As such, the final models used in this study were estimated using only plausible starting values. Increasing numbers of random starts for a small sample of conditions was also investigated; however, the increased length of time to run the full set of conditions would have increased the computational time by an estimated eight months. For applied researchers working with a single or small number of data sets, using high numbers of iterations, multiple random starts, plausible starting values, or using numerical integration may allow convergence when models analyzed using the default Mplus values fail.

Examination of the odds ratios in Table 4.2 make it clear that sample size played the largest role in convergence, followed by the number of repeated measures. The location of the inflection point within the measurement window played relatively little role in convergence, and as long as it falls somewhere within the measurement interval it probably will not adversely impact convergence enough to matter if sample size and the number of repeated measures are adequate. The rate of maximum change, although not discusses in previous literature, acts as a scaling constant along the x-axis, similar to how overall growth is a scaling constant for the y-axis. That is, having a more rapid rate of change allows more of the sigmoidal curve to be measured in the same amount of time. For example, if ρ^* were set to 1, 90% of the growth curve may be captured within 10 repeated measures. If that ρ^* were equal to 0.1, the amount of growth captured in the same 10 repeated measures may only be 30%. It is important for the applied researcher to have an idea of the length of time involved in the growth of the measured construct to ensure than an adequate amount of the growth curve is being captured.

Consider convergence to be analogous to power in the sense that convergence of the data would be the null hypothesis, and power would be the ability to reject a false null which would be a nonconvergent model. It is a common rule of thumb to use 80% power as a lower bound in acceptability. In the case of the Richards curve, six repeated measures are the bare minimum to identify the model. Based on the results of this simulation, convergence rates when there were six repeated measures never met or exceeded 80% under any sample size used in this simulation. Therefore, where convergence is a primary concern in data analysis, either sample sizes exceeding 1,500 or numbers of repeated measures exceeding six are recommended. When eight repeated measures are used, a sample size of 500 exceeded the 80% threshold. Likewise, with 10 repeated measures with a sample size of 200 would have average convergence rates of 80%. Although a threshold lower than 80% will allow more flexibility in designing a study, the tradeoff is a higher risk of non-convergence. Regardless of recommendations, it is up to the applied researcher to assess the risk of non-convergence and plan data collection accordingly. If the applied researcher is doing *post hoc* analysis of existing data and the model converges, it is possible to extract relatively unbiased parameter estimates at lower sample sizes or numbers of repeated measures than were recommended above.

Coverage

Coverage is the proportion of times that a simulated true population value is contained within a confidence interval constructed around the parameter estimate. As such, coverage can be impacted by parameter estimate bias. Much like parameter estimate bias, and standard error bias, coverage is examined on a per-parameter basis, and although some of the parameters had good coverage across most conditions, other parameters displayed problematic coverage.

For the lower asymptote (i) parameter, coverage tended to be poor except when the location of the inflection point was centered. This is due to the fact that differing inflection point locations caused the curve to be shifted in a way that allowed more information about the upper portion of the curve to be available the more the inflection point was offset. Because the lower asymptote is retrospective, it may not necessarily be as important to applied researchers as predicting future outcomes. Therefore having a problematic lower asymptote may not be an issue, especially if the benefit is having reduced bias when predicting an upper asymptote.

The upper asymptote (β^*), unlike the lower asymptote, benefits from an offset inflection point. This is most apparent when there are only six repeated measures. Coverage, although higher, is still generally low except when the maximum rate of change is at its highest level. For eight and 10 repeated measures, most conditions are acceptable except those with the lowest maximum rate of change. Sample size had little effect on this parameter.

Coverage of the maximum rate of change parameter was uniformly good across all conditions as was the relative asymmetry parameter. Juxtaposed with those, was the location of the inflection point parameter where larger sample sizes (100 or greater) and more centered locations of the inflection point ($\delta^* = 0.375$ or 0.5) contributed to higher rates of coverage.

Overall, coverage is a tradeoff between the upper and lower asymptotes unless more of the latent growth curve can be captured within the measurement window.

Parameter Estimate Bias

Effect sizes ranged from a high of $\omega_p^2 = .22$ for the effect of sample size on the maximum rate of change to effectively $\omega_p^2 = .00$ for most of the main effects and two-way interactions for relative asymmetry. In the case of relative asymmetry, the small effect sizes were the result of uniformly good parameter estimate bias in almost all cases. This contrasts with the lower asymptote parameter where the small effect sizes were the result of a lack of pattern in improvement across conditions and uniformly marginal bias. Ultimately, the effect sizes merely pointed out where there were uniform improvements in bias across conditions. Their use, while informative to an extent, do not play a large part in providing guidance for structuring longitudinal research where there is an expectation of sigmoidal growth.

Parameter estimate bias varied greatly across parameters and although bias should be estimated on a per-parameter basis, decisions and recommendations must be made holistically by taking into account all the available evidence. The goal is to suggest minimum acceptable guidelines for the number of repeated measures and sample size when structuring a study using the Richards model. Furthermore, the issue of convergence must be included in any decision. Guidelines for what is considered acceptable parameter bias have been published by Hoogland and Boomsma (1998) as well as B. Muthén et al. (1987). This amount ranges from 0.5 in the case of Hoogland to 0.10 to 0.15 for Muthén. For this study, 0.10 was the cutoff used for acceptable bias.

For the lower asymptote (i) parameter, bias was seemingly random which could lead to excessive and unexpected bias when using any number of repeated measures or any sample size. However, when the timing of the inflection point was centered within the measurement window, bias was generally 0.10 or less for all sample sizes and all repeated measures with one exception. Bear in mind that raw bias was calculated for this parameter, and therefore cannot be interpreted the same way as the parameters calculated with relative bias.

Upper asymptote (β^*) parameter estimate bias, on the other hand, was uniformly unbiased across all conditions except where ρ^* had the lowest simulated maximum rate of change and there were only six repeated measures. For this parameter, any sample size was acceptable when using eight or 10 repeated measures, and when there was six repeated measures caution is only warranted when the maximum rate of change is at its lowest.

For the maximum rate of change (ρ^*) parameter bias, sample size and number of repeated measures had the strongest effects on the outcome. Examination of ρ^* parameter estimate bias, it was possible to achieve relatively unbiased estimates for six repeated measures with a sample size of 1000, for eight repeated measures, a sample size of 500, and for 10 repeated measures with a sample size of 200.

For the location of the inflection point (δ^*) parameter, as the simulated value of δ^* becomes more centered within the measurement window, the estimates became less biased. Likewise, as sample size and number of repeated measures increases, bias diminished. As long as the location of the inflection point was relatively centered within the measurement window and the maximum rate of change is not at its lowest, bias was acceptable.

Relative asymmetry (γ^*) had generally unbiased estimates across nearly all conditions. Even when sample size was small, bias only became problematic when the location of the inflection point was centered for small sample sizes (100 or less) and there were eight or 10 repeated measures.

Standard Error Bias

Standard error bias was, overall, problematic within the context of this simulation. Maximum likelihood estimation was used for this study because it was the default method for longitudinal models with continuous manifest variables in Mplus L. Muthén and Muthén (2015) and it had been used in applied use of structured latent curve models (e.g., Cameron et al., 2015; Grimm & Ram, 2009; Grimm et al., 2010). Maximum likelihood may not be appropriate for estimating standard errors for these models, however. There are several other options available that estimate standard errors differently that may be more robust to violations of the assumptions of maximum likelihood. Future research should be done to determine which estimator works best for recovering standard errors within structured latent curve models. Because maximum likelihood estimation of standard errors may not have been appropriate for these types of models, the only recommendation regarding standard errors is a reminder that larger sample sizes and more repeated measures will be less biased than those with fewer repeated measures and smaller sample sizes.

Model Selection

The overall accuracy of 11 different fit measures was assessed to determine how well they performed at selecting the true sigmoidal model over competing incorrect polynomial models. For this, seven different information criteria were examined: 1) Akaike Information Criterion (AIC), 2) Bayesian Information Criterion (BIC), 3) Bayesian Information Criterion with sample size adjustment (aBIC), 4) Consistent AIC (CAIC), 5) Draper Information Criterion (DIC), 6) Hannan and Quinns adjustment of AIC (HQ), and 7) Sugiuras adjustment of AIC (AICc). Three of them, the AIC, BIC, and aBIC are natively provided by Mplus. The remaining four were calculated externally.

Of the information criterion, AIC and aBIC worked the best with overall accuracy rates of .63 and .62 respectively. AIC performed better than any other information criterion when sample size was 200 or larger. aBIC performed better when sample size was below 200. AIC worked best with six repeated measures, whereas aBIC performed best when there were eight or 10 repeated measures. aBIC worked best when the maximum rate of change was at its highest, AIC performed best under the other conditions. aBIC worked best when the location of the inflection point was centered, AIC worked best under the other conditions. None of the information criteria that were calculated externally to Mplus performed well enough under any condition to consider using. Of note is that BIC was the second worst fit measure examined for identifying the true sigmoidal model. Perhaps this was due to BIC's known preference for parsimonious models.

In addition to the information criteria, four absolute or incremental fit measures were examined: 1) the comparative fit index (CFI), 2) the Tucker-Lewis Index (TLI), 3) the Root Mean Square Error of Approximation (RMSEA), and 4) the Standardized Root Mean Residual (SRMR). Typically these fit measures are used to judge how well a model fits versus a pre-established cut-off value. Because the (Hu & Bentler, 1999) guidelines for use of these measures were developed on confirmatory factor models that did not include a mean structure, commonly used cutoffs are not appropriate. As such, these fit measures were used comparatively against the measures generated by the polynomial models. It should be noted that Mplus does not use an appropriate null model for growth models. As such, an appropriate null model was run to establish a baseline χ^2 for calculating CFI and TLI.

RMSEA was the best performing fit measures with an overall accuracy rate of 83% and had the best performance across all sets of conditions. Coming in a close second to the RMSEA was the CFI which had an overall accuracy rate of 81%. The TLI had an accuracy rate, in some cases, nearly 30% lower than the CFI. It is sometimes recommended to use either CFI or TLI when reporting results. In the case of these models, CFI was more accurate. Finally, there was the SRMR. It performed poorly as was expected. SRMR only uses the covariance structure of the model and does not consider a mean structure making it generally inappropriate for growth models.

Recommendation

Convergence

As previously stated, for the Richards curve, six repeated measures are the bare minimum to identify the model. In light of the low convergence that is prevalent in conditions when there are six repeated measures, sample size would have to exceed the highest sample in this simulation, 1,500. For eight repeated measures with a sample size of 500 or 10 repeated measures with a sample size of 200 average convergence rates are at or over 80%. Future research on convergence rates when using high numbers of iterations, random starts, or numeric integration may relax the large sample sizes needed to achieve a high probability of convergence.

Parameter Estimate Bias

For the applied researcher using these models, it may possible to design a perfect study. It will have lots of repeated measures, a large sample size, and will encompass as close to 100% of the developmental curve as possible. This scenario, however, is unrealistic. It is important to, therefore, be mindful of what the parameters of interest are when using these types of models. For example, it may not be possible to measure 100% of the developmental curve. In that case, the applied researcher should decide if predicting an upper asymptote is more important than retrospectively finding the lower asymptote because it is going to be a trade-off. If the upper asymptote is of importance, the data collection must be designed so that there is more information available for the upper part of the sigmoidal curve by ensuring the inflection point is offset to the left.

If understanding the maximum rate of growth is the primary parameter of interest, it is paramount to have sample sizes of at least 1,000 for six repeated measures, 500 for eight repeated measures and 200 for 10 repeated measures. The location of the inflection point within the measurement window seems like a trivial parameter beyond assuring you are capturing the correct portion of the developmental curve. If this is the parameter of interest, bias seems to be less the more centered this parameter is within the measurement window. Unfortunately, theoretical expectations of when this growth occurs will be the main guidance for structuring data collection. The relative asymmetry parameter is the defining feature of the Richards model, and there is relatively little bias for this parameter across the range of conditions. Only 11 of the 162 conditions were over the .10 bias threshold, with the largest bias being .16 and that is with a sample size of 50.

Fit Measures

Selection of appropriate fit measures when selecting among competing models is an important consideration. Based on the results of this simulation, the RMSEA and CFI are recommended when selecting among competing models that includes at least one Richards model. Although there have been no guidelines established for stand-alone use, when used comparatively, they are the best at selecting the true sigmoidal model over competing polynomial models.

Among the information criteria, the AIC and aBIC performed the best and are recommended. Although under certain sets of conditions, their performance was unacceptably low, they did perform better than the alternatives that were native to Mplus as well as those calculated externally. When sample size was under 200, the aBIC performed better than AIC as well as when the number of repeated measures was above the minimum number necessary to identify the model.

Limitations and Future Research

This study provides a first step towards understanding the conditions under which sigmoidal structured latent curve models exhibit excessive parameter bias and provides guidance on how to structure longitudinal studies where there is the expectation of sigmoidal growth. However, like any Monte Carlo simulation, the findings are generalizable only to the conditions that are simulated in the study. Typically, a simulation of this nature would draw upon prior applied studies to guide the direction of the research and inform the decisions made regarding the conditions examined within the study. Unfortunately, published use of structured latent curve models have primarily focused on methodological demonstrations (e.g., Blozis, 2004; Browne, 1993; Browne & du Toit, 1991; Grimm & Ram, 2009; Neale & McArdle, 2000, etc.) as opposed to examinations of data in applied research. The applied data that have been examined have typically involved either small datasets (e.g., Dodonov & Dodonova, 2012; Grimm et al., 2011; Rast & Zimprich, 2009) or large longitudinal datasets such as the ECLS-K and the National Longitudinal Survey of Youth (NLSY) (e.g., Cameron et al., 2015; Grimm et al., 2010). It was, therefore, necessary to select a range of sample sizes from small to relatively large to understand the levels where parameter estimate bias becomes excessive.

All growth models require a minimum number of repeated measures for model fit based on the parameterization of the model. A minimum of six repeated measures was required to estimate a sigmoidal SLCM when relative asymmetry is estimated rather than being explicitly included in the model. The applied use of the SLCMs ranged from four to 12 repeated measures. Future studies with additional repeated measures may aid in SLCM model selection if some of the other parameterization are sub-optimal. Likewise, reconceptualizing repeated measures intervals may provide further insight on whether the number of repeated measures is more important or if the amount of growth that occurs between the baseline and the final repeated measure. For example, adding two additional measurement occasions can either be done by extending the length of a study or by spacing the measurement occasions closer together.

Due to the limited scope of this study, all true models were generated as sigmoidal models with relative asymmetry (γ) set to 1, which is the equivalent of the Logistic model. Future studies should look at the effects of different values of (γ) on model selection. Likewise, the family of sigmoidal models that include the Richards model also includes the Gompertz and Logistic where the relative asymmetry is implicitly fixed by their equations. It would be more parsimonious to use one of these models over the Richards model, where the relative asymmetry must be estimated, if they align with theoretical expectations. In his book on nonlinear regression modeling, Ratkowsky (1983) examined parameter estimate bias for a number of sigmoidal models using multiple parameterizations for each model. Using his research as guidance, it may be possible to find alternatives to the Richards model that are much less biased and yet still informative.

Other design issues that can be addressed in future studies include the effects of variance on correct model selection, the effects of non-equally spaced repeated measures, the effects of different levels of overall growth, and the effects of missing data to name a few. Clearly, there is work to be done beyond the scope of this study regarding sigmoidal structured latent curve model selection. Additionally, information regarding sigmoidal structured latent curve models can be extended into a line of simulation research involving model performance and selection within growth mixture modeling framework. The goal is to use these and future results to provide recommendations on how to appropriately structure longitudinal research to ensure adequate sample size and number of repeated measures when there is an *a priori* hypothesis of sigmoidal growth and to provide recommendations for model selection when completing *post hoc* analysis of existing data. Ultimately, by testing structured latent curve models under a variety of conditions, applied researchers can use realistic models that align with theoretical expectations of growth and have confidence in the results.

In summary, this dissertation provides the first methodological investigation of parameter estimate bias and model selection of sigmoidal structured latent growth models. The flexibility of these models allows applied researchers to use nonlinear growth models that are more grounded in developmental theory than polynomial models. When used appropriately, these models can provide new insight into the types of trajectories that realistically model growth in educational and social science research.

APPENDICES

APPENDIX A

Sample *MPlus* Data Generation Code

```
montecarlo:
names = y11-y16;
nobs = 1500;
seed = 6541500;
nreps = 1000;
repsave = all;
save = O:\DisHolding\Condition 54\Condition54_rep*.dat
Model montecarlo:
l BY  y11@1
y12@1
y13@1
y14@1
y15@1
y16@1 ;
a BY  y11@0.0758581800212435
y12@0.182425523806356
y13@0.377540668798145
y14@0.622459331201854
y15@0.817574476193644
y16@0.924141819978756 ;
d BY  y11@-0.0701037165451082
y12@-0.149146452070333
y13@-0.235003712201594
y14@-0.235003712201594
y15@-0.149146452070333
y16@-0.0701037165451081 ;
r BY  y11@-0.17525929136277
y12@-0.223719678105499
y13@-0.117501856100797
y14@0.117501856100797
y15@0.223719678105499
y16@0.17525929136277 ;
g BY  y11@0.125526165173793
y12@0.16123475637676
y13@0.132749963866643
y14@0.0600899303093475
y15@0.0155239031748601
y16@0.00280158608165023 ;
y11-y16@.5;
[a@1] (a);
```

```
[l@0] (l);
[r@0];
[d@0];
[g@0];
l@0 a*.5 d*.5 r*.5 g@0;
l WITH a@0 r@0 d@0 g@0;
a WITH r*0 d*0 g@0;
r WITH d*0 g@0;
d WITH g@0;
OUTPUT: Tech9;
```

APPENDIX B
Sample *MPlus* Command File for Richards Analysis

```
TITLE: condition54_rep1_test4
DATA: file is condition54_rep1.dat;
VARIABLE: names are y11-y16;
Analysis: Processors = 8;
MODEL:
l BY y11* (RL1)
y12 (RL2)
y13 (RL3)
y14 (RL4)
y15 (RL5)
y16 (RL6);
a BY y11* (RA1)
y12 (RA2)
y13 (RA3)
y14 (RA4)
y15 (RA5)
y16 (RA6);
d BY y11* (RD1)
y12 (RD2)
y13 (RD3)
y14 (RD4)
y15 (RD5)
y16 (RD6);
r BY y11* (RR1)
y12 (RR2)
y13 (RR3)
y14 (RR4)
y15 (RR5)
y16 (RR6);
g BY y11* (RG1)
y12 (RG2)
y13 (RG3)
y14 (RG4)
y15 (RG5)
y16 (RG6);
y11-y16*;
[y11-y16@0];
[l*] (l);
[a*] (a);
[r@0];
[d@0];
[g@0];
l@0 a* d* r* g@0;
```

```
l WITH a@0 r@0 d@0 g@0;
a WITH r* d* g@0;
r WITH d* g@0;
d WITH g@0;
MODEL CONSTRAINT:
new(r*0.5 d*1.25 g*1 as ds rs gs);
as = a+l;
ds = d/5;
rs = (a*r)/((g+1)*(g+1)^(1/g));
gs = 1/((g+1)^(1/g));
RL1 = 1;
RL2 = 1;
RL3 = 1;
RL4 = 1;
RL5 = 1;
RL6 = 1;
RA1 = exp((r*0)/g)/(g*exp(d*r)+exp(r*0))^(1/g);
RA2 = exp((r*1)/g)/(g*exp(d*r)+exp(r*1))^(1/g);
RA3 = exp((r*2)/g)/(g*exp(d*r)+exp(r*2))^(1/g);
RA4 = exp((r*3)/g)/(g*exp(d*r)+exp(r*3))^(1/g);
RA5 = exp((r*4)/g)/(g*exp(d*r)+exp(r*4))^(1/g);
RA6 = exp((r*5)/g)/(g*exp(d*r)+exp(r*5))^(1/g);
RD1 = (-1)*(a*r*exp((r*0+d*g*r)/g))/(g*exp(d*r)+exp(r*0))^((g
   +1)/g);
RD2 = (-1)*(a*r*exp((r*1+d*g*r)/g))/(g*exp(d*r)+exp(r*1))^((g
   +1)/g);
RD3 = (-1)*(a*r*exp((r*2+d*g*r)/g))/(g*exp(d*r)+exp(r*2))^((g
   +1)/g);
RD4 = (-1)*(a*r*exp((r*3+d*g*r)/g))/(g*exp(d*r)+exp(r*3))^((g
   +1)/g);
RD5 = (-1)*(a*r*exp((r*4+d*g*r)/g))/(g*exp(d*r)+exp(r*4))^((g
   +1)/g);
RD6 = (-1)*(a*r*exp((r*5+d*g*r)/g))/(g*exp(d*r)+exp(r*5))^((g
   +1)/g);
RR1 = ((a*exp(d*r)*0-a*d*exp(d*r))*exp((r*0)/g))/(g*exp(d*r)+
   exp(r*0))^((g+1)/g);
RR2 = ((a*exp(d*r)*1-a*d*exp(d*r))*exp((r*1)/g))/(g*exp(d*r)+
   exp(r*1))^((g+1)/g);
RR3 = ((a*exp(d*r)*2-a*d*exp(d*r))*exp((r*2)/g))/(g*exp(d*r)+
   exp(r*2))^((g+1)/g);
RR4 = ((a*exp(d*r)*3-a*d*exp(d*r))*exp((r*3)/g))/(g*exp(d*r)+
   exp(r*3))^((g+1)/g);
RR5 = ((a*exp(d*r)*4-a*d*exp(d*r))*exp((r*4)/g))/(g*exp(d*r)+
   exp(r*4))^((g+1)/g);
RR6 = ((a*exp(d*r)*5-a*d*exp(d*r))*exp((r*5)/g))/(g*exp(d*r)+
   exp(r*5))^((g+1)/g);
```

```
RG1 = ((a*g*exp(d*r)+a*exp(r*0))*exp((r*0)/g)*log(exp(r*0)+g*
   exp(d*r))+((-a*g*exp(d*r))-a*g*r*exp(d*r)*0-a*r*0*exp(r*0))
   *exp((r*0)/g))/((g*exp(d*r)+exp(r*0))^(1/g)*(g^3*exp (d*r)+
   g^2*exp(r*0)));
RG2 = ((a*g*exp(d*r)+a*exp(r*1))*exp((r*1)/g)*log(exp(r*1)+g*
   exp(d*r))+((-a*g*exp(d*r))-a*g*r*exp(d*r)*1-a*r*1*exp(r*1))
   *exp((r*1)/g))/((g*exp(d*r)+exp(r*1))^(1/g)*(g^3*exp(d*r)+g
   ^2*exp(r*1)));
RG3 = ((a*g*exp(d*r)+a*exp(r*2))*exp((r*2)/g)*log(exp(r*2)+g*
   exp(d*r))+((-a*g*exp(d*r))-a*g*r*exp(d*r)*2-a*r*2*exp(r*2))
   *exp((r*2)/g))/((g*exp(d*r)+exp(r*2))^(1/g)*(g^3*exp (d*r)+
   g^2*exp(r*2)));
RG4 = ((a*g*exp(d*r)+a*exp(r*3))*exp((r*3)/g)*log(exp(r*3)+g*
   exp(d*r))+((-a*g*exp(d*r))-a*g*r*exp(d*r)*3-a*r*3*exp(r*3))
   *exp((r*3)/g))/((g*exp(d*r)+exp(r*3))^(1/g)*(g^3*exp(d*r)+g
   ^2*exp(r*3)));
RG5 = ((a*g*exp(d*r)+a*exp(r*4))*exp((r*4)/g)*log(exp(r*4)+g*
   exp(d*r))+((-a*g*exp(d*r))-a*g*r*exp(d*r)*4-a*r*4*exp(r*4))
   *exp((r*4)/g))/((g*exp(d*r)+exp(r*4))^(1/g)*(g^3*exp(d*r)+g
   ^2*exp(r*4)));
RG6 = ((a*g*exp(d*r)+a*exp(r*5))*exp((r*5)/g)*log(exp(r*5)+g*
   exp(d*r))+((-a*g*exp(d*r))-a*g*r*exp(d*r)*5-a*r*5*exp(r*5))
   *exp((r*5)/g))/((g*exp(d*r)+exp(r*5))^(1/g)*(g^3*exp(d*r)+g
   ^2*exp(r*5)));
OUTPUT: TECH1 TECH9;
```

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