ABSTRACT

Exploring the Beliefs of Preservice Teachers:
The Nature and Learning of Mathematics and Student Achievement in Mathematics

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The purpose of this study was to identify the beliefs of preservice mathematics teachers with regards to the nature of mathematics, the learning of mathematics, and student achievement in mathematics. Past research indicates that teacher beliefs affect the way they teach as well as influence the students they instruct (Schoenfeld 1992; Thompson 1992); therefore, educators in teacher education programs would be remiss to ignore addressing the beliefs their students hold.

The United States has jockeyed for a leading position respect to K-12 mathematical performance compared to other countries since the launching of Sputnik in 1957. After the launch, the United States responded to the deficits in the teaching and learning of mathematics in public schools. Efforts to increase student performance in mathematics in the United States has resulted in little to no change.

The purpose of this study is to compare the beliefs of preservice mathematics teachers from Baylor University, a private institution in central Texas, to three countries that all participated in Teacher Education Study in Mathematics (TEDS-M), Trends in

International Mathematics and Science Study (TIMSS), and Programme for International Student Assessment (PISA).

The TEDS-M was developed and carried out to identify key components and patterns within teacher preparation programs. Results from TEDS-M provided evidence to support research that students and teachers with conceptual belief systems have a better understanding of mathematics and higher self-esteem (Tatto, Ed., 2013).

Results offer insight into the means of Baylor compared to the three other countries. Baylor's results indicated little to no agreement with calculational belief systems and agreement overall with conceptual belief systems. Significance was found between Baylor and both Countries L and H on a frequent basis; those items having significance tend to support more calculational beliefs and less conceptual beliefs for those countries.

Utilizing the most recent Standards published in 2017 by the Association of Mathematics Teacher Educators (AMTE), the continuing need for early, intentional intervention in teacher education programs is discussed. These interventions would serve to strengthen the conceptual belief systems of students in teacher preparation programs encourage eradication of pre-existing bias among gender and ethnic groups.

Exploring the Beliefs of Preservice Teachers:
The Nature and Learning of Mathematics and Student Achievement in Mathematics

by

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A Dissertation

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DEDICATION

To my Dad and Mom:
I wouldn’t be where I am without you.

To my babies, Trey, Annie, Ty, Rylee, and Molly:
I wouldn’t want to be anywhere without you.

To my husband, Rusty:
I will go where you go.
You have been God’s gift to me and I thank Him daily for you!

I love you all! You are why I kept going.

CHAPTER ONE

Introduction

The launch of Sputnik by Russia in 1957 meant that the United States of America lost the Space Race. As a result, federal money was earmarked for science and mathematics education in public schools. A focused shift occurred in education: the learning of science and mathematics by K-12 students became a priority (Permuth & Dalzell, 2013). Despite state and federal legislation being implemented with the intent of raising scores in the United States via the National Defense Education Act in 1958, the knowledge of mathematics that students in the United States exhibit has maintained a level of mediocrity. The most recent results of Trends in International Mathematics and Science Study (TIMSS) in 2015 (TIMSS and PIRLS International Study Center, 2016) identify the performance of eighth-grade mathematics students in the United States at tenth place out of 39 participating countries. The Programme for International Student Assessment (PISA) was last administered to 15-year-olds in 2015 as well and the United States ranked number 41 out of 72 participating countries. The performance in mathematics of students in the United States can be summarized as mediocre at best. Based on the students' mean score on the 2015 TIMSS and PISA administrations, the top ten performing countries are listed in Table 1.1.

Table 1.1

Top Ten Countries' Mathematical Performance 2015

PISA	TIMSS
1. Singapore	1. Singapore
2. Hong Kong	2. Republic of Korea
3. Macao	3. Chinese Taipei
4. Taiwan	4. Hong Kong SAR
5. Japan	5. Japan
6. China*	6. Russian Federation
7. Korea	7. Kazakhstan
8. Switzerland	8. Canada
9. Estonia	9. Ireland
10. Canada	10. United States

**China is represented by the provinces of Beijing, Shanghai, Jiangsu, and Guangdong.*

As researchers explore different ways to improve student achievement on international tests, attention should be paid to the teachers that are being prepared to serve in the classrooms. Knowing that teacher beliefs do affect instruction (Thompson 1992), exploring the beliefs of preservice teachers of mathematics (hereinafter referred to as PSTMs) could prove beneficial. If their beliefs do not lend themselves to a more current, progressive belief system, intervention on the part of post-secondary educators could be warranted. To this end, utilizing the vast amount of data accumulated by the researchers who collaborated on the TEDS-M can provide insight on the beliefs of PSTMs.

In 2008, the International Association for the Evaluation of Educational Achievement (IEA) successfully implemented the TEDS-M which was conducted in 17 countries around the world: Botswana, Canada (four provinces), Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Oman (lower secondary teacher education only), the Philippines, Poland, the Russian Federation, Singapore, Spain (primary teacher education only), Switzerland (German-speaking cantons), Thailand, and the United States of America (public and private institutions) (Brese & Tatto, 2012, p. 10). The study was the first of its kind and its purpose was to create a benchmark by which post-secondary teacher preparatory institutions could compare their primary and secondary mathematics teacher education programs to other programs of study available at other higher education institutions, both nationally and internationally.

As Tatto (Ed.) itemized in the TEDS-M Technical Report (2013), TEDS-M had two specific goals: first, to investigate how different programs in different countries prepared their PSTMs in the field of mathematics; and second, to examine how the different preparation methods across the participating countries affected the teaching and learning of mathematics in their teacher preparation programs. The results of the study could be utilized by the following (Tatto, Ed., 2013):

- Educational policy makers – the data shows institutional programs which are effective in producing PSTMs who are confident both in mathematics and in mathematical pedagogy;
- Teacher educators – the design of PSTM curricula can be influenced by what has been empirically proven to work and not work;
- Current mathematics teachers already in the field – the study provides a better understanding of what qualified teachers of mathematics learn about content and pedagogy and how that learning takes place; and
- All other readers of the results of the study – a better understanding of how PSTMs learn content and pedagogy in their own matriculation.

Purpose of the Study

Research supports that the students of PSTMs and current teachers with a progressive belief system have a greater understanding of mathematics and a higher self-esteem concerning mathematics (Tatto, Ed., 2013). The purpose of this study was to examine the beliefs of PSTMs in a mathematics education program at Baylor University, a private central Texas University and how those beliefs compare to three other countries that participated in TEDS-M: the United States, a country performing on the higher end of PISA and TIMSS (Country H) and a country performing on the lower end of PISA and TIMSS (Country L). The TEDS-M study and this research study define a lower secondary mathematics program as a program preparing PSTMs to teach on the levels of sixth through tenth grades (Tatto, et al, 2012); PSTMs from Baylor planning to teach kindergarten through fifth grade or eleventh or twelfth grades are also addressed in this research study because of the overlaps in grade levels that occur in different teacher education programs. Therefore, all Baylor PSTMs preparing to teach in classrooms from kindergarten through grade twelve are included in this research study.

By studying the alignment of the beliefs of Baylor's PSTMs with these three countries, potential change in the construct of the mathematics education program could be warranted. The TEDS-M study specified three belief systems on which this proposed study will focus: the nature of mathematics, the learning of mathematics, and student achievement in mathematics (Tatto, Ed., 2013). Summaries of those belief systems follow.

The nature of mathematics within the context of TEDS-M specifically addresses how PSTMs view mathematics as a subject and the role of mathematics in daily life. One of the listed purposes (Brese & Tatto, Eds., 2012) of the TEDS-M study is to differentiate how people perceive the nature of mathematics. For example, is mathematics a set of rules and procedures that must be followed in order to obtain the one correct answer or is mathematics able to be explored using different methods to obtain more than one correct answer? Is mathematics comprised of creativity and new ideas or are algorithms and memorization the best way to learn? (Brese & Tatto, Eds., 2012).

The learning of mathematics in the context of the TEDS-M study addresses how mathematics should be taught for efficacy and understanding. Another purpose of the TEDS-M study is to differentiate how people perceive the learning of mathematics. For example, should only one method or procedure be taught or should students be allowed to explore several methods? Are speed and efficiency more important than exploration and queries? (Brese & Tatto, Eds., 2012).

Finally, student achievement in mathematics within TEDS-M addresses why PSTMs think that students “get” mathematics. Another purpose of the TEDS-M study is to differentiate how people perceive student achievement in mathematics. For example, is student achievement related to gender? To ethnicity? Do students reach and show their potential at an early age or is the understanding of mathematics attainable for everyone? (Brese & Tatto, Eds., 2012).

Problem Statement

After decades of educational reform, there have been no significant gains in mathematical understanding for students in the United States (PISA 2015 & TIMSS 2015). Instead of singularly focusing on current research concerning classroom content and methodologies, research should also address teachers being placed into those classrooms, particularly their beliefs regarding mathematics. The beliefs that teachers carry with them into the classroom have an effect on how much students learn and retain (Nespor 1987; Pajares 1992, Philipp 2007; Thompson 1992; Wilson and Cooney 2002, Beswick 2017, Heyd-Metzuyanim 2017). The beliefs of PSTMs could potentially have an effect, overtly or subconsciously, on the students' learning processes.

The purpose of this study was to determine if there is a significant difference in the beliefs of the PSTMs of a private central Texas institution, referred to hereafter as "Baylor," and three out of seventeen participants in the TEDS-M study:

- United States of America ("USA") whose students' performance on the most recent TIMMS and PISA tests (2015) could best be described as average;
- A country whose students performed exceptionally well on the most recent TIMSS and PISA tests (2015), hereinafter referred to as "Country H;" and
- A country whose students had a low median score on the most recent TIMSS and PISA (2015) tests, referred to hereafter as "Country L."

Research Questions

In order to further explore the beliefs of PSTMs at Baylor, the following research questions were developed. These questions are supported by the data collected for the TEDS-M study.

1. What beliefs do Baylor PSTMs hold regarding the nature of mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in TEDS-M?
2. What beliefs do Baylor PSTMs hold regarding the learning of mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?
3. What beliefs do Baylor PSTMs hold regarding student achievement in mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?

Limitations

In the TEDS-M User Guide (Brese & Tatto, Eds., 2012) several limitations should be noted since their occurrences may have an effect upon the results of data analyses. Because the format of the TEDS-M study was a survey, the responses from faculty and PSTMs were voluntary and not necessarily consistent. Therefore, the sampling of institutions and participants are not random and the beliefs of the PSTMs may or may not be indicative of the population as a whole. Within the United States, 604 PSTMs participated in TEDS-M but only 502 of those participants were able to complete the entire survey. Responses from Country H were culled so that small institutions were omitted; this removed about 5% of their data. Within the submissions themselves, some data were unreadable or missing. Caution should be used in projecting the results of this study to the population at large.

Limitations regarding Baylor include a small number of respondents in comparison to the USA, Country H, and Country L. Baylor is also a private institution given invitation to participate; however, participation in the TEDS-M study at large was restricted to public institutions in all countries but the USA (Center for Research in Mathematics and Science Education, 2010). Finally, the location of Baylor – a centrally located institution in Texas – could have an impact on the responses of the PSTMs since different areas of the USA have different cultural influences and, therefore, different beliefs.

Background

TEDS-M was completed in 2008. The study was comprised of three different questionnaires:

1. Institutional Program Questionnaire – Faculty from each participating institution provided information specific to their institution. Topics covered included but not limited to: length of program, hours in field, number of faculty, pedagogical courses (both general and mathematics); etc.
2. Educator Questionnaire – The background of each of the educators at the institutions was explored including but not limited to their qualifications, title, years of experience, etc.
3. Survey for Future Teachers of Mathematics – PSTMs were asked questions about their personal lives including but not limited to household income, number of books in their home, the highest level of education achieved by their parents or guardians, their perceptions of the program in which they were enrolled, and their beliefs about mathematics.

Because the scope of the TEDS-M study is exceedingly broad, this research study focused on one portion of one questionnaire, the Survey for Future Teachers of Mathematics. The fourth section of that questionnaire addressed the beliefs of the PSTMs. This section was broken down into five subsections; the first three will be analyzed in this proposed study: beliefs about the nature of mathematics (12 questions), beliefs about the learning of mathematics (14 questions), and beliefs about student achievement in mathematics (8 questions).

Conclusion

The efforts of the researchers behind the TEDS-M study have provided a plethora of data by which current and future mathematics educators can inform their classroom instruction. This research study was designed to find relationships between the PSTMs of Baylor and the three identified countries; reflection on the curriculum of Baylor could be warranted.

As stated, it is paramount to be firmly acquainted with the beliefs of PSTMs about the nature of mathematics, the learning of mathematics, and student achievement in mathematics. The opportunity to impact student learning in the classroom by identifying the beliefs of PSTMs affords researchers another opportunity to effectively guide student learning and comprehension of mathematics in the classroom (Thompson, 1992). Fostering a progressive and conceptual belief system through teacher education programs will potentially enable strides of positive progress in which student understanding and achievement is possible and probable.

Effectively identifying the belief systems of PSTMs can only be beneficial as a means to identify potential curricular improvements. Having the opportunity to mold

PSTMs' beliefs before they enter the classroom is one method that researchers need to explore to effectively impact student comprehension and understanding. Mathematics education in the USA has shown no significant change and cannot globally compete in its current state (TIMSS 2015, PISA 2015).

CHAPTER TWO

Literature Review

Introduction

Government legislation and funding was established to increase students' mathematics scores in the USA, after the USA lost the Space Race in 1957 with the launch of Sputnik. Despite the implementation of these measures, the USA still retains a mediocre ranking against other nations when observing student performance on international tests. According to Hargreaves and Evans (1997), one possible reason for this gap could have existed within the educational system itself: "(L)egislation only sets a framework for improvement; it is teachers who must make that improvement happen" (pg. 3). If a reform agenda is to be implemented, it will not be successful without the beliefs of teachers being aligned with that reform (Battista, 1994), especially the beliefs of PSTMs (Handal & Herrington, 2003). Research continues to push forward with the intent of seeking ways to improve student understanding and retention of mathematics.

In 2008, the Teacher Education and Development Study in Mathematics (TEDS-M) was conducted internationally in order to collect data for use by the TEDS-M researchers as well as making the data available for other researchers to use intra-nationally and internationally. TEDS-M was comprised of surveys that addressed institutional program requirements; the background and knowledge base of educators at those institutions; and background and knowledge of PSTMs participating in the programs. The survey administered to the PSTMs contained a section focusing on

PSTMs’ beliefs about the nature and learning of mathematics and student achievement in mathematics.

There is a lack of a definition of beliefs that is fully agreed upon. Beliefs could be formed from several perspectives and influences including but not limited to cultural, societal, cognitive, historical traditions, and educational identities (Tang & Hsieh, 2014). Adopting a working definition of “beliefs” is vital to understanding research regarding the beliefs of PSTMs. Philipp (2007), culling from a profusion of definitions and applications from other researchers, has created a concise definition of both “beliefs” and “beliefs systems.” Beliefs are “psychologically held understandings, premises, or propositions about the world that are thought to be true” (p. 259). Philipp reasons that beliefs are harder to change than attitudes or emotions; they are not felt intensely and exhibit themselves in a more cognitive fashion. Beliefs may be thought to be as lenses through which a person would view some topic or aspect or even incline one toward taking action. Beliefs can be embraced with varying degrees of certitude. A beliefs system is “a metaphor for describing the manner in which one’s beliefs are organized in a cluster, generally around a particular idea or object” (Philipp, 2007, p. 259). These clusters can be further organized by labeling them as primary or derivative and they can be related to a central theme or topic or be peripheral; however, beliefs systems cannot be isolated unto themselves and they must, at minimum, exist in clusters.

Beaudette (2012) remarks that, while knowledge and beliefs are certainly related, there is a distinction between the two. He cites Roulet’s (1998) summary of the difference, which follows:

> Knowledge is taken to be built up through intellectual activity: experimentation, debate and reasoning. It is stored in the form of propositions that are open to

> further evaluation and change. Beliefs, on the other hand, are not developed through rational thought, but are rather mental summaries of significant past episodes. (p. 3)

Current research supports that the beliefs of classroom teachers have an effect on student achievement. This should lead to institutions of higher education placing a level of importance upon challenging the beliefs of PSTMs to align with current research rather than outdated or "old-fashioned" methods of learning. Weldeana and Abraham (2014) state the following:

> Studies on teachers' beliefs reveal that teachers hold well-articulated beliefs that shape their instructional practice (e.g., Thompson 1992); each teacher holds a particular belief system, comprising a range of beliefs about learners, teachers, teaching, learning, schooling, resources, knowledge, and curriculum (Handal 2003; Leatham 2006); teachers rely on established beliefs to choose pedagogical content and curriculum guidelines (e.g., Clark and Peterson 1986); and teachers reflect their beliefs in their teaching, thus shaping their students' beliefs (e.g., Schoenfeld 1992; Thompson 1992). (p. 305).

Based on Weldeana's and Abraham's assertions, it becomes obvious that the beliefs of teachers' will impact students in a wide variety of ways. Leatham (2006) notes that research on teacher beliefs in and of itself is difficult; however, the end result is worth the effort because the results have enormous potential to inform future educational practice and research.

The TEDS-M study modified the "traditional" versus "constructivist" vocabulary by categorizing them as "calculational" versus "conceptual." In *The Second Handbook of Research on Mathematics Teaching* (Lester, Ed., 2007), Philipp describes what he refers to as "teachers' orientations." He describes a teacher orientation as a pattern of beliefs held by a teacher with regard to mathematics and the teaching of mathematics. Philipp describes two different constructs, conceptual and calculational, that describe important aspects upon which teachers are known to disagree. Philipp (2007) argues that teachers who have a conceptual orientation hold ideas that mathematics is a system of philosophies and methods of thinking. Conceptual teachers will work on materials, expositions, and activities that will engage students and lead to productive time in the mathematics classroom. Students are expected to be intellectually engaged in the task at hand and teachers expect and insist on that engagement.

Teachers with calculational orientations will be driven by the image of mathematics as a subject consisting of applications and procedures whose purpose is to obtain numerical results. This orientation will show itself in the classroom with teachers speaking in a language of numbers and numerical operations, a passivity towards context, and problem-solving as a method of producing numbers rather than as a critical thinking skill (Philipp 2007).

Because of the differences between the calculational and conceptual constructs, it is expected that teachers of one construct will have different practices in their classrooms than teachers ascribing to the other construct. Thompson and his colleagues offer evidence that this idea does hold true (Thompson, Philipp, Thompson, & Boyd, 1994). Research thus far includes quasi-experimental and naturalistic studies including those noted by Stone (Ed. 2007) in the anthology *The Best Practices for Teaching Mathematics: What Award-Winning Classroom Teachers Do*. Staub and Stern (2002) made comparisons with third grade students whose teachers either aligned more with a conceptual orientation or a calculational orientation. Students whose teachers followed the conceptual orientation showed a higher achievement in arduous contextual problems than those whose teachers had a calculational orientation. Staub and Stern also found that students whose teachers followed a conceptual orientation had an equivalent or higher achievement on routine undertakings involving mathematical procedures and facts than those students whose teachers ascribed to a calculational orientation.

This evidence suggests that the beliefs PSTMs align with when entering a classroom may influence their methods of teaching and, consequently, how their students learn (Tatto, Ed., 2003). One purpose of the TEDS-M study was to gather data reflective of PSTM beliefs for use in informing curriculum in teacher education programs. The authors of TEDS-M created their survey to state the beliefs as a modest reflection of the calculational and conceptual orientations previously elaborated upon but should not be seen as directly equivalent to them.

In the development phase of the TEDS-M study, research was conducted specific to PSTMs' beliefs in the nature of mathematics, learning of mathematics, and student achievement in mathematics. *The Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)* (Tatto, Ed., 2013) provides the rationale behind the decision to study the beliefs of PSTMs. Both content and pedagogical knowledge are understood to be essential for teachers to be successful in the classroom; however, it is also understood that the beliefs held by students and teachers also have an effect on learning and teaching. At the time of the genesis of the TEDS-M study, there was not much supportive research that the beliefs of PSTMs can be influenced by their teacher education programs or even that the beliefs are an inherent trait of those who become teachers (Tatto & Coupland, 2003).

A Brief History of Mathematics Education in the USA

Permuth and Dalzell (2013) offer a summary of the recent history of mathematics education in the United States by noting distinct moments that resulted in mathematics reform. Beginning with John Dewey and his progressive movement (1915-1952), mathematics was taught in schools using a democratic citizenry approach: students should be taught with an end result of becoming productive members of a democratic society. Dewey's ideal was for teachers to form a link between education in the classroom and daily life by connecting the two using foundations of health, vocation, and citizenship. His view of education was holistic in that he intended for education to include moral, intellectual, and emotional growth; as individuals grew in these three areas, the democratic society would, by natural consequence, benefit as well. Under Dewey's progressive education, instruction in mathematics took the form of exploration and guidance by teachers, moving from a system where students had to "toe the line" in order to make high marks in their subject. Students learning under Dewey's progressive education took ownership of their education and rigorously attacked problems given to them. Dewey's philosophy was two-fold: meaning was added to the educational experience, creating an environment of student interest and understanding of material, and, at the same time, it promoted the ideals of the democratic society.

Variants of Dewey's ideas continued to be upheld with minor changes in the purposes behind his strategies. After World War I, a desire to buoy the foundation of the USA military put an emphasis on learning mathematics in a more formal way, previous to Dewey's progressive educational reform. However, a call to honor humanity, culled from the loss of life from the war, also pushed for recognition of the arts. This push still affected the mathematics classroom as the creativity and "lovely logic" (pg. 238) of mathematics still appealed to the leaders of the progressive movement.

"New math" was introduced in the 1960s. Again, a desire to make sure the USA military was the best military in the world saw the implementation of tests before enlistment; many of those trying to enlist failed the exams. The technology being introduced also made clear the need for students to excel in mathematics. New math was introduced for a short period of time; it was full of abstract thought and strange symbols. Kindergartners were not even immune to the effects of new math; set theory using technical mathematics terms was used in their curriculum. In short order, new math was deigned to be too adult and abstract for children so more reform was sought.

Permuth and Dalzell (2013) begin their organization of the history of mathematics education just before this point, in the 1950s. They note clear points in history where an impetus for change in the mathematics classroom was created: not necessarily by educators but by historical events themselves.

The United States lost the Space Race to Russia when Russia launched the first satellite into orbit around the earth on October 4, 1957. The pursuit of space exploration was then and continues to be predominantly an endeavor of science and mathematics; attention was rapidly drawn to education as a possible reason for the USA's loss to Russia (Amrein & Berliner 2002). Conclusions were drawn that, if the USA could not win this academic challenge against Russia, then student education in mathematics and sciences in public schools must be being taught improperly. Mathematicians and scientists now came to be perceived as "essential elements" of American society (p. 240). Mathematics and sciences were introduced at an earlier stage in students' matriculation.

Bruner (1971) advocated for this earlier introduction since understanding concepts on a deeper level would allow for extrapolation to more detailed concepts later. Bruner advocated mathematicians working directly with teachers to create new curriculum; at this point, the importance of consulting mathematicians in the arena of educational reform was acknowledged. Bruner advocated that the teaching of the sciences and mathematics should occur in such a manner that the mind would be operating at its best: teaching should encourage the mind to be "active, extrapolative, innovative, [and go] from something firmly held to areas which were not so firmly known in order to have a basis for test" (Bruner, 1971, pg. 18).

Response to the loss of the Space Race as well as concerns over the philosophical nature of mathematics education brought about the National Defense Education Act (NDEA) in 1958. Education was no longer aligned with desires for supporting a democratic society per Dewey; it was no longer meant to support the already-powerful military of the USA. Now, the teaching of the sciences and mathematics were grouped with foreign language instruction addressing the intent of defending the USA against other countries. Public schools could receive federal funding for pursuing instruction in those three particular subjects.

Twelve countries came together to form the International Association for the Evaluation of Educational Achievement (IAEEA) in 1962. The purpose of the coalition was to conduct research in multicultural education. As the coalition grew in size, international assessments were administered in multiple subjects, including mathematics, with the intent of comparing student achievement among the different participating countries. The USA was noted to already have a gap in the achievement of its students in comparison to other countries at this point (Thomas 2005).

Permuth and Dalzell (2007) continue their abbreviated history of education in the USA by calling attention to the importance of the publication of *A Nation at Risk* (1983). They recall and reiterate the fact that educational reform is not always instigated at the behest of educators or even the need for reform; in this case, the report was written within the context and content of historical movements and change. *A Nation at Risk* was published as an indirect response to the preceding decades of the 1960s and 1970s. After government legislation and funding was introduced with the NDEA, the shift from the USA from military offense to military defense became embedded, overtly or not, in the curriculum of public schools. The unrest created by the Vietnam War along with the viewpoint of freedom from oppression of the flower children of the 1970s resulted in a disparaging attitude towards the institution of education itself. The Civil Rights Movement addressed the racism in the country (Bruner 1971).

The National Council of Teachers of Mathematics published recommendations in *An Agenda for Action* (1980). Responding to the stagnancy of mathematics curriculum and instruction from the 1960s and dependence on test scores in the 1970s, recommendations for the direction that mathematics programs should take during the 1980s. “Schools have responded to this concern in a variety of ways, but a clear-cut and carefully reasoned sense of direction that looks toward the future has been lacking” (NCTM i). Eight recommendations were offered to strengthen the mathematics curriculum in schools and the teaching of that curriculum. This publication culminated in the publishing of *Curriculum and Evaluation Standards for School Mathematics* (NCTM

1989). These Standards were developed to guide mathematical content and associated instruction to support a high-quality mathematics program and evaluate both the quality of curriculum in schools as well as student learning. The development of the Standards were a response to *A Nation at Risk*.

A Nation at Risk (1983) was a report authored by the National Commission on Excellence in Education under President Ronald Reagan. Results from the report included inflammatory statements that the current state of education in the USA would cause an inability to be competitive in the global market in the future. The authors remarked that the mathematics curriculum in public schools contained minimal levels of standards and content. The authors stated that the current condition of mathematics education in the USA was comparatively a criminal act:

> If an unfriendly power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in achievement made in the wake of the Sputnik challenge. Moreover, we have dismantled essential support systems, which helped make these gains possible. We have, in effect, been committing an act of unthinking, unilateral educational disarmament (pg. 5).

In the 1980s, a different approach to teaching mathematics was introduced into the mathematics classroom. Reminiscent of Dewey's progressive education, harkening the works of Schoenfeld (1985), Vygotsky (Galloway 2010), Bruner (1961, 1971), and most remarkably Piaget (Bhattacharya & Han 2010), constructivism called for students to explore mathematics and learn on a deeper level. Classroom discussions were encouraged to facilitate better understanding. Teachers were again asked to act as facilitators or mediators while students were encouraged to be active participants rather than passive recipients of knowledge (Clements & Battista 1990).

Summarized by Clements & Battista (1990), there are two major goals of constructivism which are supported by five main ideas. The two major goals follow: first, students need to motivate themselves in their mathematical activity as well as maintain autonomy; second, in order to be able to solve more complicated problems, students need to develop mathematical structures that are more complex, powerful, and abstract than those they currently possess. A summary of the five main ideas follow:

- Knowledge is actively created by children, not passively obtained by the environment around them.
- New mathematical knowledge is created by children when they reflect upon their physical and mental actions.
- There is not a singular true reality which exists; individual interpretations exist that are informed by experience and social interactions.
- Learning is social and contributes to the environment of the learner. Those around the learner become part of the learning process and contribute to discourse, discovery, and invention.
- Teachers demanding that students follow a set of procedures or rules seriously inhibits the learning of children. Children will naturally try to follow the example of the teacher and therefore not necessarily understand the sense-making process of mathematics.

This shift from other more traditional methods of instruction to constructivism has merit in research. By exploring what PSTMs believe about the nature and learning of mathematics and student achievement in mathematics, gains can possibly be made in their future students' understanding and comprehension of mathematics.

Research on the Beliefs of PSTMs

Beliefs Regarding the Nature of Mathematics

Based on the work of Ernest (1989), Carney and colleagues conducted research to determine if an intervention in the form of professional development would have an effect on teacher beliefs about the nature of mathematics. One construct addressed was problem solving: mathematics is interactive and ever-changing. Another construct was a Platonist perspective; mathematics is a static body of knowledge yet concepts are intertwined. Finally, the instrumentalist construct of the beliefs of the nature of mathematics was described. This viewpoint holds that mathematics is simply a useful set of rules, steps, and methods (2014). Research upholds that teacher beliefs have a significant effect on classroom instruction and the achievement of students; however, the difficulty of affecting change still exists (Carney, Brendefu, Thiede, Hughes, & Sutton 2014; Pajares 1992; Philipp 2007). However, statistical gains were made in the research study conducted by Carney and colleagues (2014). Dweck's (2006) research in the field of psychology is centered on the power of the beliefs people hold.

This portion of the TEDS-M study was based on previous work conducted by Grigutsch, Raatz, and Törner (1998). Two different scales were developed for this subsection of PSTM beliefs: mathematics is a set of rules and procedures to be followed and mathematics is a process of enquiry. Tatto, et al, (2012) provide statements that PSTMs would agree with if they hold that mathematics is solely rules and procedures, also termed as calculational (Philipp 2007) or direct-transmission (Staub & Stern 2002):

1. Mathematics is a collection of rules and procedures that prescribe how to solve a problem.
2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts, and procedures.
3. When solving mathematical tasks, you need to know the correct procedure else you would be lost.
4. Fundamental to mathematics is its logical rigor and precision.
5. To do mathematics requires much practice, correct application of routines, and problem solving strategies.
6. Mathematics means learning, remembering, and applying. (p. 154).

Tatto, et al, (2012) provide additional statements that PSTMs who believe in mathematics as a process of enquiry would agree with, also termed as conceptual (Philipp 2007) or cognitive-constructivist (Staub & Stern 2002):

1. Mathematics involves creativity and new ideas.
2. In mathematics many things can be discovered and tried out by oneself.
3. If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).
4. Mathematical problems can be solved correctly in many ways.
5. Many aspects of mathematics have practical relevance.
6. Mathematics helps solve everyday problems and tasks. (p. 155).

The authors of TEDS-M did not force PSTMs taking the survey to fully identify with only calculational or conceptual beliefs; PSTMs could very well agree with components of both types of thinking; e.g., memorization of multiplication facts is important but students should be able to discover the concept of repetitive addition on their own. However, the authors did believe that PSTMs would align more completely with one way of thinking or the other and that the two subcategories would have a negative correlation. In general, this was found to be true (Tatto, et al, 2012).

In Liljedahl's research, he employed the reasoning of PSTMs by requesting them to observe in-service teachers. Those teachers were asked to identify how much time they spent in a toolbox construct which would be more calculational; a systems construct, also calculational but with a heavy emphasis on language and proof; or a process construct, which would be conceptual. The PSTMs discussed and noted the espoused beliefs of the in-service teachers and then observed what the teachers actually implemented in their classrooms. The findings in this particular study contradict some other research which he discusses; that research supports espoused beliefs directly affecting the practice of teaching: Fosnot, 1989; Millsaps, 2000; Skott, 2001; Uusimaki & Nason, 2004. Liljedahl's PSTMs rationalized the in-service teachers' experiences as students in a classroom during their training would lead them to believe that the nature of mathematics was more conceptual; however, they tended to practice in their own teaching a more calculational approach. The PSTMs supposed this could be from their own experiences as students.

Another possibility broached by Liljedahl's PSTMs was the idea of two different types of mathematics: mathematics as school mathematics or mathematics as real mathematics. School mathematics would have a calculational perspective since the teaching would focus on rules and procedures. However, real mathematics could very much be about discovery and proof. The PSTMs reasoned that in-service teachers may belief that mathematics is more conceptual but the teaching of it reverts to calculational.

Beliefs Regarding the Learning of Mathematics

The authors of the survey followed two lines of thinking, calculational and conceptual, in their preparation of the survey questions covering the learning of mathematics. This section covers how appropriate certain instructional activities are in the classroom; the purposes of mathematics as a school subject, and the reasoning behind students' cognitive processes. One scale portrays students learning directly from teacher instruction (calculational) while the other scale contains statements about student learning through active involvement (conceptual). The statements regarding students learning mathematics through teacher instruction follow (Tatto, et al, 2012):

1. The best way to do well in mathematics is to memorize all the formulas.
2. Pupils need to be taught exact procedures for solving mathematical problems.
3. It doesn't really matter if you understand a mathematical problem, if you can get the right answer.
4. To be good in mathematics, you must be able to solve problems quickly.
5. Pupils learn mathematics best by attending to the teacher's explanations.
6. When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.
7. Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.
8. Hands-on mathematics experiments aren't worth the time and expense. (p. 155).

According to a conceptual type of belief system, students should conduct their own enquiries and develop their own methods of problem solving if their learning is to be effective (Tatto, et al, 2012). PSTMs who agree with the following statements will be more likely to view the learning of mathematics as an active, conceptual process rather than a passive, calculational process:

1. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
2. Teachers should allow pupils to figure out their own ways to solve mathematical problems.
3. Time used to investigate why a solution to a mathematical problem works is time well spent.
4. Pupils can figure out a way to solve mathematical problems without a teacher's help.
5. Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.
6. It is helpful for pupils to discuss different ways to solve particular problems. (p. 156).

Weldeana and Abraham (2014) offer research to support a progressive view of learning mathematics. Citing previous studies by Boaler (1997), Madden (2008), McLeod (1992), Renga and Dalla (1993), and Stevens (2005), Weldeana and Abraham uphold that traditional (calculational orientation) beliefs about learning mathematics will cause students to suffer from low self-perception, helplessness in learning mathematics, low self-esteem, and low self-confidence. These effects will cause students to lack understanding and lack motivation as well as underperform in mathematics. Conversely, if students are taught using a progressive (conceptual orientation) view of mathematics, student performance is enhanced.

In 2016, Boaler published an entire book on developing a mathematical mindset, which strongly aligns with a conceptual belief system. She notes that Finland has consistently scored very well on PISA studies but their students do not even get exposed to algorithms or memorization until the age of seven. Students in many other countries, including the USA, expose children to mathematical methods at much earlier ages than that.

Ly and Brew (2010) also offer support in their research for a more progressive approach in the learning of mathematics. Their findings show that PSTMs show support for progressive beliefs but their own teaching efficacy is tied to using a more traditional approach. Conner, Edenfield, Gleason, & Ersoz (2011) also found in their research that PSTMs entered a two-semester consecutive program – a content course followed by methods course – with a more traditional belief system. However, at the end of the second semester, the PSTMs beliefs about the learning of mathematics had a significant shift to a progressive approach while their beliefs about the nature of mathematics remained the same.

Goldin, Rösken, and Törner provide additional evidence supporting the adoption and implementation of a conceptual view of learning mathematics. Learning needs to be viewed as creating meaning, developing or obtaining understanding, or making sense of mathematics. They present Toeplitz' (1927) example using topics in calculus: the methods of deriving the Mean Value Theorem, the definite integral, convergence and divergence, and even the difference quotient itself must have been “objects of a fascinating search, an exciting action, namely at that time they were created” (p. 92ff).

Goldin, Rösken, and Törner (2009) mention researchers, Halmos and Rav, who view problem-solving to be at the very heart of mathematics which necessitates developing the trappings necessary to promote problem solving such as different tools, methods, concepts, and strategies. Problem solving could be interpreted in the more calculational context of presenting an activity that would lead a student toward a specific goal, while the student may or may not understand how to reach that goal. However, the authors promote a wider, more conceptual, view of problem solving as a comprehensive methodology of not only what mathematics is but what it means to learn and teach.

Beliefs Regarding Achievement in Mathematics

Unlike the subsets of the nature of mathematics and the learning of mathematics, the subset of beliefs regarding student achievement in mathematics is not broken down into the two philosophies of calculational and conceptual. The authors of the TEDS-M survey believed that PSTMs that agreed with the statements in this section would believe that students either do or do not have mathematical ability. Effort should be made to identify those students in order to teach more effectively. PSTMs who believe this line of thinking would agree with the following statements (Tatto, Ed., 2013):

1. Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.
2. To be good at mathematics, you need to have a kind of “mathematical mind.”
3. Mathematics is a subject in which natural ability matters a lot more than effort.

4. Only the more able pupils can participate in multi-step problem-solving activities.
5. In general, boys tend to be naturally better at mathematics than girls.
6. Mathematical ability is something that remains relatively fixed throughout a person's life.
7. Some people are good at mathematics and some aren't.
8. Some ethnic groups are better at mathematics than others. (p. 156).

Research by Tang and Hsieh (2014) concludes that there exists cultural differences in the beliefs of PSTMs in the five subgroups identified for their research purposes (2014). Countries that they classified as Developing Asia and East Europe were more prone to agree with the TEDS-M statements above while the American and Developed Europe groups tended towards disapproving of a natural, stable mathematical ability. Confucian Asia was neutral.

These beliefs about student achievement in mathematics do not fall into categories as the nature and learning of mathematics did (conceptual and calculational). It does, however, reflect "a view of mathematics learning that, if evident in teachers' actions, is likely to result in lower expectations for many students. This view is, therefore, one that experts in mathematics education discourage" (Tatto, et al, 2012).

Upadyaya and Eccles (2014) conducted research with three cohort groups of children along with their teachers. The longitudinal study took place from their kindergarten year through sixth grade. The beliefs that teachers had about the children's efforts and their potential as learners of math showed a positive trend, increasing their interest in mathematics. If teachers had beliefs about the children's mathematic ability, those children seemed to be affected only early in their primary school years. It is not unreasonable to suspect correlation between the beliefs of teachers about student achievement and the actual achievement of those students.

Holder and Kessels (2017) utilized both objective and subjective scales to explore two stereotypes: girls in mathematics and immigrant students with certain ethnic backgrounds will have lower achievement levels in mathematics. The objective scales required PSTMs to evaluate students based on “educational standards” while the subjective scales required PSTMs to evaluate students based on “inclusion.” When the more objective scale was used, biases tended to be masked well. However, when the more subjective scale was used stereotypes were found to be upheld in PSTMs and did have an effect on the children in the study. This supports previous research by Parks and Kennedy (2007) showing that teachers judgements and perceptions are affected by racial stereotypes. Because these stereotypes can and have a consequence on how teachers interact with their students, it is important to evaluate how current curriculum in teacher preparation programs addresses these concerns.

Conclusion

Understanding and exploring the beliefs of PSTMs regarding the nature of mathematics, the learning of mathematics, and student achievement in mathematics has a great deal of benefit. Noting that the beliefs of PSTMs and current teachers can and do have an effect on student learning offers researchers another opportunity to affect student learning and comprehension of mathematics in the classroom. If the beliefs of PSTMs and current teachers can be swayed towards a more progressive, conceptual, belief system, there could potentially be gains in student understanding and achievement. As Leatham (2006), states,

> One goal of mathematics teacher education, however, might be to affect teachers' beliefs about mathematics such that those beliefs move high on the list of those beliefs that most influence teaching. In order to have this impact, however, teacher educators and the teachers themselves need to become aware of the beliefs that are currently filling those ''most influential'' roles. From this perspective, teachers' belief systems are not simply ''fixed'' through a process of replacing certain beliefs with more desirable beliefs. Rather, teachers' beliefs must be challenged in such a way that ''desirable'' beliefs are seen by teachers as the most sensible beliefs with which to cohere. (p. 100)

Bahr, Monroe, and Shaha (2013) discuss that preservice teachers' experiences in the classrooms as students greatly influence their own chosen methods of instruction. They further point out that, distinctive to the field of teaching, PSTMs will enter their career with many years of experience – as students – in the same field in which they will work. Recognizing that a conceptual method of beliefs benefits students, matriculation in higher education should reflect this belief system in their own PSTM preparation.

Acknowledging that mathematics education reform through recent USA history has not resulted in significant change in the international competitiveness of public school students means that researchers need to explore other methods of affecting change in student comprehension and understanding. One method that might bear that effect is evaluating and potentially changing the beliefs of PSTMs during their education in mathematics education programs before they begin their service.

Philipp, et al, (2007) developed a list of seven beliefs that characterize a conceptual belief framework.

Beliefs About Mathematics

1. Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).

Beliefs About Learning or Knowing Mathematics, or Both

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's (Students') Learning and Doing Mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible. (p. 21)

CHAPTER THREE

Methodology

The purpose of this study was to determine if there are significant differences in the beliefs of the PSTMs in a mathematics education program of a private central Texas institution, the United States, and two other countries that participated in TEDS-M. The private university will be referred to hereafter as "Baylor;" the United States, referred to hereafter as "USA;" a country with high scores on the most recent TIMSS and PISA tests, referred to hereafter as "Country H;" and finally a country with low scores on the most recent TIMSS and PISA tests, referred to hereafter as "Country L." Results of this study could impact training of PSTMs to reform their beliefs in order to better prepare them to instruct their future students in mathematics. This chapter contains the following components of the study: (1) Research Questions, (2) Description of the TEDS-M Study, (3) Participants in Context, and (4) Statistical Analysis.

Research Questions

This study was designed to determine how closely the beliefs of PSTMs from Baylor align with the USA, Country H, and Country L. The following research questions were formulated to guide the research study:

1. What beliefs do Baylor PSTMs hold regarding the nature of mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in TEDS-M?

2. What beliefs do Baylor PSTMs hold regarding the learning of mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?
3. What beliefs do Baylor PSTMs hold regarding student achievement in mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?

Description of the TEDS-M Study

After decades of research, the International Association for the Evaluation of Educational Achievement (IEA) undertook a cross-national survey study of post-secondary mathematics programs, institutions, educators, and students. A 98-page IEA report, hereinafter referred to as *Framework*, details the research design and methodology of the TEDS-M study (Tatto et al., 2008).

TEDS-M seeks answers to the following research questions (Tatto et al, 2008, p. 14):

1. What is the impact of mathematics-related policies for quality assurance and the accreditation of teacher education programs on teacher education institutions, programs, and outcomes?
2. How do national or program policies influence the recruitment, preparation, graduation, and retention of teachers of mathematics?
3. What are the characteristics of teacher education policies, institutions, and programs that lead to high levels of mathematics knowledge and knowledge of mathematics pedagogy in future teachers?

4. What relationship is there between the beliefs about mathematics of teacher educators and those of future teachers?
5. What kinds of practicum arrangements and school experiences are most effective in preparing future mathematics teachers?
6. What are the costs of programs in different settings?

Seventeen countries participated in the TEDS-M study: Botswana, Canada, Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Oman, Philippines, Poland, Russian Federation, Singapore, Spain, Switzerland (German-speaking cantons), Thailand, and United States. Participating institutions in those countries received three different surveys to administer to faculty and students in the mathematics teacher education programs. A synopsis of those surveys follow.

One survey was administered to the PSTMs in the participating institution and was entitled the Survey for Future Teachers of Elementary/Secondary Mathematics. This survey was broken down into four different parts.

- The first section of this survey addressed the personal background of the future teacher; e.g., parents' education, financial position, when the decision was made to become a teacher, etc.
- The second section of this survey was a mathematics content test to determine the mathematical knowledge of the future teachers.
- The third section of this survey addressed the college/university program from the future teachers' points of view.
- The fourth section required information regarding the beliefs of the future teachers on the teaching of mathematics.

Another survey, entitled the Institutional Program Questionnaire, was completed by the current faculty in the participating institution. The faculty of the institution participating in the TEDS-M study provided minutely detailed information of the structure of their program including but not limited to such topics as the method of credentialing future teachers (eligibility to teach is granted by graduation, licensure, testing, etc.), duration of the program, and the type of program (consecutive or concurrent).

The countries identified for inclusion in this proposed study had both concurrent and consecutive program types at the time of the TEDS-M administration. The TEDS-M technical report supplied definitions for these types of programs. A concurrent program, implemented by most participating countries, offers a single credential at the end of a single-phase teacher education program. That single phase includes matriculation in subject matter courses, pedagogical courses, and other general education courses. Consecutive programs, on the other hand, have a two-phase approach to credentialing PSTMs for entering the classroom. A post-secondary degree in the subject matter is first conferred followed by a second phase that includes mostly pedagogical courses and experience in practicum. This second phase culminates in a second credential being awarded. The USA has both concurrent and consecutive program types; Country L has concurrent program types; and Country H has consecutive program types (Tatto, Ed., 2013).

Finally, the Survey for Mathematics, Mathematics Pedagogy, and General Pedagogy Educators, is the third survey distributed to the institutions for completion. The personal and professional backgrounds of the current faculty members were explored in this survey.

The TEDS-M survey at large is categorized into three components. The first component served to provide data on the policy and context of mathematics teacher education. TEDS-M focused on the two types of policies: those that influence primary and lower secondary teachers' achieved level and depth of mathematics and related pedagogical knowledge; and the policies that influence the structure of primary and lower secondary mathematics teachers' opportunities to learn. Data on policy and context are collected at the national level in this component.

The second component provides data on the processes of mathematics teacher education. In this component, TEDS-M focused on the opportunities to learn available to future primary and lower secondary mathematics teachers that enable them to attain the knowledge they need to teach mathematics. It also determined the structure of the opportunities to learn, the content taught in teacher education programs, and the organization of instruction. These data were collected at national, institutional, teacher educator, and future teacher levels in this component.

The third component was designed to provide data on the outcomes of mathematics teacher education. TEDS-M focused on the level and depth of the mathematics and related pedagogical knowledge attained by future primary and lower secondary teachers and how this knowledge varied across programs, routes, and countries. This was achieved through collection of data from representative samples of future teachers in the last year of their program.

For the purposes of this study, only a subset of the third component was utilized. Of the three major components of the TEDS-M study, only the third component contained responses from PSTMs. Therefore, the other two major components of TEDS-M were not used or referenced in this study. Of the third component of TEDS-M, only a small portion references the beliefs of PSTMs; only that portion of the third component analyzed.

Validity

The Teacher Education and Development Study in Mathematics (TEDS-M) policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Technical report (Tatto, Ed., 2013) is a report published alongside several other reports within the TEDS-M study and contains detailed information on the development of TEDS-M including sampling, question development, instruments, and scoring guides. Summaries of the relevant portions of the technical report follow, including page numbers where additional details can be found for those desiring a more comprehensive overview.

Sampling

A two-stage sampling design was used in TEDS-M. The IEA Data Processing and Research Center (DPC) in Hamburg, Germany, coordinated with national research centers in each participating country to identify institutions with programs that were representative of the national population of teacher preparation programs and including only those with programs for primary and lower secondary mathematics education programs. Once these institutions had been identified, the national center in each participating country utilized software provided by IEA DPC, WinW3S, to select educators and PSTMs from within the institutions. The identified individuals were then asked to complete the appropriate questionnaire(s) (pg. 14).

Language

The instruments utilized in the TEDS-M study were prepared in English. If a country participating in the TEDS-M study required translation of the documents into a native language, as 12 countries did, procedures were put into place to ensure proper translation so reliability of the content would not be compromised (pg. 71). A document providing the guidelines used in translation is available: *TEDS-M 2008 Survey Operations Procedures, Unit 3: Translation/Verification* (IEA, 2007).

Beliefs Questionnaire Items

The Teaching and Learning to Teach Study at Michigan State University (Deng, 1995; Tatto, 1996, 1998, 2003) contributed to the development of the TEDS-M questionnaire scales. These five belief scales include the three focused areas addressed in this proposed study: the nature of mathematics, the learning of mathematics, and student achievement in mathematics. The other two belief scales, beliefs about preparedness to teach mathematics and beliefs about program effectiveness, will not be addressed in this proposed study. The items on the questionnaires used to measure PSTM beliefs were culled from several studies, including research by Deng (1995), the feasibility study for TEDS-M (Schmidt et al., 2007), and research studies by Tatto (1996, 1998, 1999, 2003).

In the *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice and readiness to teach primary and secondary mathematics in 17* *countries: Findings from the IEA Teacher Education and Development Study in Mathematics* (TEDS-M), Tatto, et al, (2012), provide additional information behind the development of the questionnaire items for their study. Each of the three subsets presents statements that lead to one way of thinking versus another way of thinking; however, the PSTMs could support both ways of thinking. The creators of the questionnaire believed that there would exist a negative correlation between the two tracks and this proved to be the case (pg. 154).

The first subset of the beliefs portion of the PSTM questionnaire that will be addressed regards the nature of mathematics. Specifically, the twelve items can be categorized as PSTMs viewing mathematics as a set of rules and procedures and mathematics as a process of enquiry which corresponds with the two tracks mentioned in the previous paragraph. These questions also addressed the PSTMs beliefs of mathematics as a subject: “mathematics as formal, structural, procedural, or applied” (pg. 154). These items were based on the previous work of Grigutsch, et al, (1998) and Ingvarson, et al. (2005, 2007).

The second subset of the beliefs portion of the PSTM questionnaire addresses the learning of mathematics with fourteen questionnaire items. In this section, the PSTMs were presented statements on methods of instruction. Students’ cognitive processes, suitability of instructional activity, and the purpose behind requiring mathematics as a school subject were addressed in the questionnaire items. The two tracks in this subset are learning through following teacher instruction and learning mathematics through active involvement (p. 156).

Finally, the third subset addresses student achievement in mathematics with eight questionnaire items. The two tracks in this case are mathematics comes as a fixed ability based on gender, genetics, or ethnicity versus the idea that mathematics is accessible to everyone with a high work ethic (p. 156)

Participants in Context

Seventeen countries participated in the TEDS-M research study. This study was focused on both the primary and lower secondary information gathered by the TEDS-M researchers (Tatto, et al, 2012). Table 3.1 summarizes the demographics of PSTMs in the primary mathematics teacher education programs. Table 3.2 summarizes the demographics of PSTMs in the lower secondary teacher education programs.

Table 3.1

Demographics of PSTMs in Participating Countries – Primary

Country	Number in Program	Mean Age	Percent Female
Botswana	86	26.0	59.5%
Canada*	---	---	---
Chile	636	23.6	85.4%
Chinese Taipei	921	23.2	71.8%
Georgia	502	21.3	100%
Germany	1,020	27.4	92.2%
Malaysia	568	25.9	63.2%
Norway	548	25.5	73.8%
Oman	---	---	---
Philippines	591	20.9	81.1%
Poland	2,110	25.2	95.2%
Russian Federation	2,232	24.2	93.9%
Singapore	379	26.7	74.1%
Spain**	1,093	23.6	80.5%
Switzerland	934	23.9	85.0%
Thailand	659	22.3	74.9%
United States	1,499	25.4	88.9%

*Canada was not included in the primary data because of lack of responses.
**Oman was not included in the primary data because of non-participating at this level.

Table 3.2 summarizes the demographics of PSTMs in the lower secondary teacher education programs.

Table 3.2

Demographics of PSTMs in Participating Countries – Lower Secondary

Country	Number in Program	Mean Age	Percent Female
Botswana	52	24.2	37.4%
Canada*	---	---	---
Chile	725	23.9	83.7%
Chinese Taipei	365	24.0	38.3%
Georgia	74	21.3	84.3%
Germany	763	29.8	64.1%
Malaysia	383	22.6	81.6%
Norway	569	26.2	51.5%
Oman	267	21.9	59.8%
Philippines	731	21.0	65.4%
Poland	298	23.2	75.5%
Russian Federation	2,133	22.0	70.7%
Singapore	392	26.8	48.3%
Spain**	---	---	---
Switzerland	141	26.3	42.4%
Thailand	651	22.4	75.0%
United States	606	26.1	68.5%

*Canada was not included in the lower secondary data because of lack of responses.
**Spain was not included in the lower secondary data because only primary were surveyed.

Demographics

The TEDS-M survey was administered to PSTMs in the final year of matriculation in their mathematics teacher education program. Information specific to the countries included in this study and Baylor University follow.

Country H

The *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)* (Tatto, et al, 2012), provides a summary of the lower secondary teacher education programs available in Country H. A very limited number of institutions are available in Country H to provide education for PSTMs. Because of the existence of this, a higher degree of control over the teacher education programs and certification in the country has been maintained. The Ministry of Education recruits future teachers and sends them to the institutions for their training. Upon graduation, PSTMs are automatically qualified to teach in Country H's public schools.

Lower secondary education in Country H is considered to be grades seven through ten; grades eleven and twelve are considered post-secondary. The lower secondary teacher education program generally takes five years to complete and is a consecutive program: four years of coursework followed by one year of the study of teacher education.

Country H was chosen for inclusion in this study because it participated in both the TEDS-M study as well as the most recent TIMSS and PISA studies.

Country L

The *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)* (Tatto, et al, 2012, pp. 61-62), provides a summary of the lower secondary teacher education programs available in Country L. Country L provides training to PSTMs at both the university and teacher education colleges. At the time of the administration of the TEDS-M surveys, Country L had only one university preparing lower secondary PSTMs.

Primary school in Country L extends from first grade through seventh grade. Lower secondary schools, called “junior secondary schools” include eighth through tenth grades. At the time of the TEDS-M study, only 56% of the appropriate age-group population was enrolled in lower secondary school.

The PSTMs enroll in teacher education programs that align with the organization of the public school systems. PSTMs enrolled in lower secondary mathematics education programs can pursue their education through four program types: one at either of the two colleges for teachers and three available at the university. However, only two of these options were included in the TEDS-M study: the Diploma of Secondary Education, available at the teacher colleges, and the Bachelor of Secondary Education (Science) at the university. One of the courses of qualification was not included for lack of enrollment while the other was not included because it was a non-traditional method of qualification to teach.

Programs are either three or four years in length. The teacher colleges require five weeks of practicum while the university requires fourteen weeks, seven weeks for each of the last two years.

Country L was chosen for inclusion in this study because it participated in both the TEDS-M study as well as the most recent TIMSS and PISA studies.

USA

The *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)* (Tatto, et al, 2012, pp. 91-93), provides a summary of the lower secondary teacher education programs available in the USA. There are more than 1300 colleges and universities, both public and private, as well as private institutions, school districts, and state agencies that provide teacher education for future teachers. With the passing of the federal No Child Left Behind legislation, teachers would need to show that they are “highly qualified” to teach; they must show competency in the subject area in which they are teaching. However, there is no national curriculum by which to measure their qualifications.

While there are multiple pathways to obtain licensure in teaching, those pathways can be summarized into six basic ways: primary, lower secondary, and secondary education programs can each be completed by either concurrent program or consecutive programs.

TEDS-M and this study both focused on primary and lower secondary programs but secondary programs bear mentioning. Different states classify grade levels into different categories. For instance, a primary teacher in one state may be licensed in grades K-3 while in another state may be licensed in grades 1-6. The content studied by these PSTMs can vary greatly.

There are also alternative methods of certification for PSTMs available in the USA. In 2004-2005, approximately 33% of all teachers hired (more than 50,000) followed the alternative certification route. These routes are determined on the state level and are implemented through school districts, colleges and universities, state agencies, and private organizations. Personnel would be hired then, while working in the classroom, follow the guidelines to achieve certification. Because these alternative routes do not align with the purpose of TEDS-M, these routes were excluded from TEDS-M and, consequently, were excluded from this study.

As of 2007-2008, three states did not require PSTMs to pass any type of exam to begin teaching. Forty-one states required PSTMs to pass a content test. Thirty-eight states required PSTMs to pass tests on basic literacy and numeracy. Thirty-nine of the fifty states required anywhere from five to eighteen weeks of student teaching.

The USA was chosen for inclusion in this study because of its participation in the TEDS-M study, the most recent TIMSS and PISA studies, as well as being the home country of Baylor.

Baylor

This teacher education program at Baylor, a major university in the southern part of the United States, is unique among the universities sharing the same setting. The primary attribute of the teacher education program at this institution is the amount of field experience in actual classrooms during the program. Students are involved in classrooms for six semesters in which they will experience direct contact with students and teachers in dynamic classroom environments.

The four-year teacher education program is developmental in design allowing PSTMs to incrementally increase their knowledge base and instructional skills. All field experiences take place in Professional Development Schools (PDSs) that are part of a cooperative endeavor with the School of Education to provide excellent and innovative classroom environments for all teacher education candidates. The PDS and Baylor personnel plan, implement, and evaluate the teacher education program so it is designed to provide students with quality teacher training.

The teacher education program is rigorous and is designed to develop teachers who are knowledgeable, adaptable, reflective, competent, and morally-prepared professionals who are ready to meet the challenges of dynamic and changing learning environments. These programs are child-centered, focusing on guiding all children to achieve their best academic and personal potential.

The preceding information is a summary of the teacher preparation program of Baylor, summarized from the website of the School of Education (*http://www.baylor.edu/soe/index.php?id=65702*).

Fifty-five PSTMs participated in the TEDS-M study. Their age range was from 21 years old through 32 years old; the average age was 22.2 years and the median age was 22. Nine males participated (16.4%)and 46 females participated (83.6%).

Statistical Analyses

The 34 line-items on which the analyses was conducted are found in Appendix A. The first twelve questions address beliefs about the nature of mathematics; the next fourteen questions address beliefs about the learning of mathematics; and the final eight questions address student achievement in mathematics.

The beliefs questionnaire items administered to the PSTMs were categorized into five different subsets. Three of those subsets will be utilized for this proposed study and the other two subsets will not be addressed. The three subsets are as follows:

- The beliefs of PSTMs regarding the nature of mathematics.
- The beliefs of PSTMs regarding the learning of mathematics.
- The beliefs of PSTMs regarding student achievement in mathematics.

Table 3.3

Summary of Hypotheses

Research Question	Null Hypotheses	Alternative Hypotheses
What beliefs do Baylor PSTMs hold regarding the nature of mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?	• H_o: $\mu_b = \mu_{ch}$ • H_o: $\mu_b = \mu_{cl}$ • H_o: $\mu_b = \mu_{usa}$	• H_a: $\mu_b \neq \mu_{ch}$ • H_a: $\mu_b \neq \mu_{cl}$ • H_a: $\mu_b \neq \mu_{usa}$
What beliefs do Baylor PSTMs hold regarding the learning of mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?	• H_o: $\mu_b = \mu_{ch}$ • H_o: $\mu_b = \mu_{cl}$ • H_o: $\mu_b = \mu_{usa}$	• H_a: $\mu_b \neq \mu_{ch}$ • H_a: $\mu_b \neq \mu_{cl}$ • H_a: $\mu_b \neq \mu_{usa}$
What beliefs do Baylor PSTMs hold regarding student achievement in mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?	• H_o: $\mu_b = \mu_{ch}$ • H_o: $\mu_b = \mu_{cl}$ • H_o: $\mu_b = \mu_{usa}$	• H_a: $\mu_b \neq \mu_{ch}$ • H_a: $\mu_b \neq \mu_{cl}$ • H_a: $\mu_b \neq \mu_{usa}$

Statistical analyses of the 34 line-item questions and the three broad subsets were conducted using the following procedures.

An analysis of variance (ANOVA) was conducted on the data with a common level of significance of $\alpha = 0.05$. ANOVA is a comparison of the means of populations using independent samples of quantitative response variables (Utts & Heckard, 2015). There were three groups of data used for analysis in this part of the proposed study:

Country H, Country L, and USA. The null hypothesis was that there was no difference in the means of the three groups for any of the variables. The alternative hypothesis was understood to be that the means were different. An *F*-statistic was also determined: the larger the result of the *F*-statistic, the greater the variance among the means. The *p*-value, the probability that the resultant *F*-statistic is as large as it is if the null hypothesis is true, was found using an *F*-distribution. The null hypothesis was rejected if the *p*-value generated was smaller than the stated level of significance.

A *post hoc* test also was performed on the data. An ANOVA only addresses that there exists statistical significance within the data; it does not indicate where the significance occurs. A *post-hoc* test identified where the significance existed. In this situation, Tukey's Studentized Honest Significant Difference (HSD) Range (Lowry, R., 2017) was employed to determine among which variables the significance existed. In the tables, it appears as though the results are reported twice. This simply indicates the direction of the different of the means. When the difference is computed, a positive result will indicate the referent has the greater mean than the object of comparison while a negative result will indicate the referent has a lesser mean than the object of comparison. The duplication provides ease of interpretation: the referent subject will be correlated with the directional mean.

In some cases within the results of this study, the ANOVA did not indicate rejection of the null hypothesis but the *post hoc* test did indicate significance. In these instances, the data as a whole, on the group level; was saturated enough to not indicate significance. Once the *post hoc* test was carried out, there was significance between two groups, on an individual level of comparison.

A sum of squares for each of the three research questions was also calculated. This calculation is "the total variation in the data from all samples combined [and] is measured by computing the sum of squared deviations between data values and the mean of all the data" (Lowry, p. 646).

A mean square term for the model was figured. This was conducted by dividing the sum of squares by the degrees of freedom (Reliasoft, 2017). A mean square error also was calculated. This value is the numerator of the ratio for computing the *F*-statistic and estimates the variance of the population based on variability within the given measures. In this research study, the measures were the results from the surveys of the PSTMs.

The coefficient of variation, R^2, and pooled standard deviation (RMSE) were generated. The R^2 statistic measures how well the data fits the linear regression model. In this research study, the treatment was the origin of the PSTMs: Country L, Country H, USA, and Baylor. Cohen's method for interpretation of the R^2 statistic will be applied (Gravetter & Wallnau, 2008):

Table 3.4

Percentage of Variance as Proposed by Cohen

Statistic	Effect Size
$R^2 = 0.01$	Small effect
$R^2 = 0.09$	Medium effect
$R^2 = 0.25$	Large effect

The RMSE is found by taking the square root of the mean square error. As stated previously, the populations were assumed to have equal standard deviations (Utts & Heckard, 2015). The RMSE is an estimate of this common value.

Conclusion

By analyzing the TEDS-M data from Baylor against institutions in the USA, Country H, and Country L, information was extrapolated that could have benefits in reforming education in PSTM programs. By studying what beliefs PSTMs have in their final year of learning, teacher educators could modify their curriculum to affect change in the PSTMs' beliefs, thus affecting their future students and their achievement in mathematics.

CHAPTER FOUR

Results

Existing data from the 2008 TEDS-M database was procured from International Association for the Evaluation of Educational Achievement and permission for use in this research study was obtained. For the purposes of this study, only some of the results from the portion answered by the PSTMs of the surveys was utilized; while a great deal of information was obtained by TEDS-M, only the questions pertaining directly to the beliefs of PSTMs was analyzed for the purposes of this study. Survey respondents indicated their agreement with each statement by use of a six-point Likert scale: 1 denoting *strongly disagree* and 6 corresponding with *strongly agree*.

The following research questions were used to identify appropriate methodologies and apply focus to the research study:

1. What beliefs do Baylor PSTMs hold regarding the nature of mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in TEDS-M?
2. What beliefs do Baylor PSTMs hold regarding the learning of mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?
3. What beliefs do Baylor PSTMs hold regarding student achievement in mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?

The contents of this chapter will provide details of the results of the statistical analyses. With the abundance of information gleaned, an emphasis on the organization of the results was made. The following is a summary of the organization to aid the reader. All survey statements and their corresponding bullet points are presented to exhaustion.

- Research question
    - Survey statement
        - Summary of results for survey statement
        - Details of results
            - First table – Results of the ANOVA procedure
            - Second table – Results for goodness of fit
            - Third table – Tukey's Studentized Range

Significance within the first table, The ANOVA Procedure, is indicated comparing the critical *F*-value ("*F*-Crit") to the *F* statistic ("*F*"). If the probability of obtaining the critical *F*-value is less than $\alpha=0.05$, then there exists statistical significance.

The second table under each survey statement offers more descriptors of the data: Goodness of Fit. The R^2 value indicates how volatile the data is. Extreme values of -1 or 1 indicate data that is well-aligned; however, values closer to 0 indicate very little if any relationship among the data. The Root MSE is another indicator of variability; it measures how much the data varies about the projected values. The coefficient of variance (CV) indicates how much of the data correspond with the mean of the data.

Finally, the third table associated with each survey question presents the findings of Tukey's Studentized Honest Significant Difference (HSD) Range. In this table, the analysis further examines where the significance occurs that was indicated in the

ANOVA. If the signs are the same in the third and fourth columns, there was not a statistically significant result. If there was a change in signs from the third to the fourth columns, Tukey's HSD identifies the exact nature of the significance indicated by the ANOVA. Because this test identifies where all significance occurs, all significance has been noted with a single asterisk: whether the significance included Baylor or not, the significance indicated by the test was noted for clarity.

At the end of the chapter, five additional tables are provided: four provide a summary of the calculational survey statements versus the conceptual survey statements while the fifth is an overview of the statistical significance of each survey statement.

Analysis

The following tables and narrative reflect the line-item analyses corresponding to each survey question administered to the PSTMs. The results are grouped by research question and, within each research question, by survey question.

Research Question One

> *What beliefs do Baylor PSTMs hold regarding the nature of mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in TEDS-M?*

The 36 tables that follow are grouped into 12 subgroups, three tables each, corresponding to the survey item indicated by the subheadings. These twelve survey items are each listed and correspond to the beliefs about the nature of mathematics.

Survey Item 1: Mathematics is a collection of rules and procedures that prescribe how to solve a problem.

The mean of Baylor's data is 4.5874. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.1, 4.2, and 4.3 provide details of the statistical analyses.

Table 4.1

The ANOVA Procedure for Nature of Mathematics Survey Item 1

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	69.8669	23.2890	19.22	3.6355	<0.0001*
Error	2594	3143.7990	1.2120			
Corrected Total	2597	3213.6659				
Note α=0.05						

Because the *F*-value is 19.22, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.2

Goodness of Fit for Nature of Mathematics Survey Item 1

R^2	CV	Root MSE	Mean
0.0217	23.9982	1.1009	4.5874

The R^2 value shows an effect on the smaller side. The CV shows that 23.9982% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.3

Tukey's HSD Test for Nature of Mathematics Survey Item 1

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.3244	0.0765	0.5752	*
Country L – Baylor	0.4401	-0.0112	0.8913	
Country L – Country H	0.6232	0.3616	0.8848	*
USA – Country L	-0.3244	-0.5752	-0.0735	*
USA – Baylor	0.1157	-0.2723	0.5037	
USA – Country H	0.2988	0.1751	0.4225	*
Baylor – Country L	-0.4401	-0.8913	0.0112	
Baylor – USA	-0.1157	-0.5037	0.2723	
Baylor – Country H	0.1831	-0.2119	0.5781	
Country H – Country L	-0.6232	-0.8848	-0.3616	*
Country H – USA	-0.2988	-0.4225	-0.1751	*
Country H – Baylor	-0.1831	-0.5781	0.2119	

The variance among the means is not significant between Baylor and any of the three countries. Baylor's respondents were not significantly different than the mean.

Survey Item 2: Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.

The mean of Baylor's data is 4.6636. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.4, 4.5, and 4.6 provide details of the statistical analyses.

Table 4.4

The ANOVA Procedure for Nature of Mathematics Survey Item 2

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	47.6509	15.8836	15.17	3.6355	<0.0001*
Error	2597	2718.9911	1.0470			
Corrected Total	2600	2766.6451				
Note α=0.05						

Because the *F*-value is 15.17, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.5

Goodness of Fit for Nature of Mathematics Survey Item 2

R^2	CV	Root MSE	Mean
0.0172	21.9410	1.0232	4.6636

The R^2 value shows an effect on the smaller side. The CV shows that 21.9410% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.6

Tukey's HSD Test for Nature of Mathematics Survey Item 2

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.4355	0.2032	0.6679	*
Country L – Baylor	0.5659	0.1469	0.9849	*
Country L – Country H	0.6074	0.3650	0.8498	*
USA – Country L	-0.4355	-0.6679	-0.2032	*
USA – Baylor	0.1303	-0.2303	0.4909	
USA – Country H	0.1719	0.0569	0.2868	*
Baylor – Country L	-0.5659	-0.9849	-0.1469	*
Baylor – USA	-0.1303	-0.4909	0.2303	
Baylor – Country H	0.04156	-0.3256	0.4067	
Country H – Country L	-0.6074	-0.8498	-0.3650	*
Country H – USA	-0.1719	-0.2868	-0.0569	*
Country H – Baylor	-0.04156	-0.4087	0.32557	

The variance among the means is significant between Baylor and Country L.

Survey Item 3: Mathematics involves creativity and new ideas.

The mean of Baylor's data is 4.7524. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.7, 4.8, and 4.9 provide details of the statistical analyses.

Table 4.7

The ANOVA Procedure for Nature of Mathematics Survey Item 3

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	43.6963	14.5654	13.23	3.6355	<0.0001*
Error	2589	2851.3511	1.1013			
Corrected Total	2592	2895.0474				
Note α=0.05						

Because the *F*-value is 13.23, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.8

Goodness of Fit for Nature of Mathematics Survey Item 3

R^2	CV	Root MSE	Mean
0.01509	22.0824	1.0494	4.7524

The R^2 value shows an effect on the smaller side. The CV shows that 22.0824% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.9

Tukey's HSD Test for Nature of Mathematics Survey Item 3

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5304	0.2913	0.7696	*
Country L – Baylor	0.1809	-0.2493	0.6111	
Country L – Country H	0.5522	0.3027	0.8017	*
USA – Country L	-0.5304	-0.7696	-0.2913	*
USA – Baylor	-0.3495	-0.7194	0.0203	
USA – Country H	0.0218	-0.0963	0.1399	
Baylor – Country L	-0.1809	-0.6111	0.2493	
Baylor – USA	0.3495	-0.0203	0.7194	
Baylor – Country H	0.3713	-0.0053	0.7479	
Country H – Country L	-0.5522	-0.8017	-0.3027	*
Country H – USA	-0.0218	-0.1399	0.0963	
Country H – Baylor	-0.3713	-0.7479	0.0053	

The variance among the means is not significant among Baylor and the three countries.

Survey Item 4: In mathematics many things can be discovered and tried out by oneself.

The mean of Baylor's data is 4.9441. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.10, 4.11, and 4.12 provide details of the statistical analyses.

Table 4.10

The ANOVA Procedure for Nature of Mathematics Survey Item 4

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	30.5298	10.1766	10.95	3.6355	<0.0001*
Error	2593	2410.3743	0.9296			
Corrected Total	2596	2440.9041				
Note α=0.05						

Because the *F*-value is 10.95, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.11

Goodness of Fit for Nature of Mathematics Survey Item 4

R^2	CV	Root MSE	Mean
0.0125	19.5006	0.9641	4.9441

The R^2 value shows an effect on the smaller side. The CV shows that 19.5006% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.12

Tukey's HSD Test for Nature of Mathematics Survey Item 4

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.2008	-0.0196	0.4213	
Country L – Baylor	-0.0539	-0.4495	0.3418	
Country L – Country H	0.4353	0.0894	0.7813	*
USA – Country L	-0.2008	-0.4213	0.0196	
USA – Baylor	-0.2547	-0.5945	0.0851	
USA – Country H	0.1806	0.0723	0.2890	*
Baylor – Country L	0.0539	-0.3418	0.4495	
Baylor – USA	0.2547	-0.0851	0.5945	
Baylor – Country H	0.4353	0.0894	0.7813	*
Country H – Country L	-0.3814	-0.6113	-0.1516	*
Country H – USA	-0.1806	-0.2890	-0.0723	*
Country H – Baylor	-0.4353	-0.7813	-0.0894	*

The variance among the means is significant between Baylor and Country H.

Survey Item 5: When solving mathematical tasks you need to know the correct procedures else you would be lost.

The mean of Baylor's data is 4.0231. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.13, 4.14, and 4.15 provide details of the statistical analyses.

Table 4.13

The ANOVA Procedure for Nature of Mathematics Survey Item 5

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	120.2471	40.0824	25.05	3.6355	<0.0001*
Error	2596	4154.3683	1.6003			
Corrected Total	2599	4274.6154				
Note α=0.05						

Because the *F*-value is 40.0824, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.14

Goodness of Fit for Nature of Mathematics Survey Item 5

R^2	CV	Root MSE	Mean
0.02813	31.4443	1.2650	4.0231

The R^2 value shows an effect on the smaller side. The CV shows that 31.4443% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.15

Tukey's HSD Test for Nature of Mathematics Survey Item 5

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.9160	0.6287	1.2033	*
Country L – Baylor	0.9728	0.4548	1.4908	*
Country L – Country H	0.7001	0.4004	0.9998	*
USA – Country L	-0.9156	-1.2033	-0.6287	*
USA – Baylor	0.0568	-0.3890	0.5026	
USA – Country H	-0.2159	-0.3580	-0.0738	*
Baylor – Country L	-0.9728	-1.4908	-0.4548	*
Baylor – USA	-0.0568	-0.5026	0.3890	
Baylor – Country H	-0.2727	-0.7266	0.1812	
Country H – Country L	-0.7001	-0.9998	-0.4004	*
Country H – USA	0.2159	0.0738	0.3580	*
Country H – Baylor	0.2727	-0.1812	0.7266	

The variance among the means is significant between Baylor and Country L.

Survey Item 6: If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).

The mean of Baylor's data is 5.0581. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.16, 4.17, and 4.18 provide details of the statistical analyses.

Table 4.16

The ANOVA Procedure for Nature of Mathematics Survey Item 6

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	73.6673	24.5558	34.90	3.6355	<0.0001*
Error	2593	1824.5530	0.7036			
Corrected Total	2596	1898.2203				
Note α=0.05						

Because the *F*-value is 34.90, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.17

Goodness of Fit for Nature of Mathematics Survey Item 6

R^2	CV	Root MSE	Mean
0.0388	16.5839	0.8388	5.0581

The R^2 value shows an effect on the smaller side. The CV shows that 16.5839% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.18

Tukey's HSD Test for Nature of Mathematics Survey Item 6

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.3680	0.1762	0.5598	*
Country L – Baylor	0.2163	-0.1279	0.5605	
Country L – Country H	0.6566	0.4566	0.8565	*
USA – Country L	-0.3680	-0.5598	-0.1762	*
USA – Baylor	-0.1516	-0.4472	0.1440	
USA – Country H	0.2886	0.1944	0.3839	*
Baylor – Country L	-0.2163	-0.5605	0.1279	
Baylor – USA	0.1516	-0.1440	0.4472	
Baylor – Country H	0.4403	0.1393	0.7412	*
Country H – Country L	-0.6566	-0.8565	-0.4566	*
Country H – USA	-0.2886	-0.3829	-0.1944	*
Country H – Baylor	-0.4403	-0.7412	-0.1393	*

The variance among the means is significant between Baylor and Country H.

Survey Item 7: Fundamental to mathematics is its logical rigor and preciseness.

The mean of Baylor's data is 4.5706. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.19, 4.20, and 4.21 provide details of the statistical analyses.

Table 4.19

The ANOVA Procedure for Nature of Mathematics Survey Item 7

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	141.7323	47.2441	50.52	3.6355	<0.0001*
Error	2581	2413.6333	0.9352			
Corrected Total	2584	2555.3656				
Note α=0.05						

Because the *F*-value is 50.52, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.20

Goodness of Fit for Nature of Mathematics Survey Item 7

R^2	CV	Root MSE	Mean
0.0555	21.1577	0.9670	4.5706

The R^2 value shows an effect on the smaller side. The CV shows that 21.1577% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.21

Tukey's HSD Test for Nature of Mathematics Survey Item 7

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.1157	-0.1109	0.3422	
Country L – Baylor	0.2399	-0.1600	0.6397	
Country L – Country H	-0.3964	-0.6322	-0.1607	*
USA – Country L	-0.1157	-0.3422	0.1109	
USA – Baylor	0.1242	-0.2166	0.4650	
USA – Country H	-0.5121	-0.6208	-0.4034	*
Baylor – Country L	-0.2399	-0.6397	0.1600	
Baylor – USA	-0.1242	-0.4650	0.2166	
Baylor – Country H	-0.6363	-0.9834	-0.2893	*
Country H – Country L	0.3964	0.1607	0.6322	*
Country H – USA	0.5121	0.4034	0.62084	*
Country H – Baylor	0.6363	0.2893	0.9833	*

The variance among the means is significant between Baylor and Country H.

Survey Item 8: Mathematical problems can be solved correctly in many ways.

The mean of Baylor's data is 5.3638. Because there is not a statistical significance indicated, none of the means of any of the countries is significantly different than Baylor's, indicating an agreement with the Baylor PSTMs. Tables 4.22, 4.23, and 4.24 provide details of the statistical analyses.

Table 4.22

The ANOVA Procedure for Nature of Mathematics Survey Item 8

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	5.8802	1.9601	3.18	3.6355	0.0230
Error	2588	1594.0453	0.6159			
Corrected Total	2591	1599.9255				
Note α=0.05						

Because the *F*-value is 3.18, the null hypothesis is not rejected: there is not statistical significance in the variation among the means. The means of the respondents of all four entities are not significantly different.

Table 4.23

Goodness of Fit for Nature of Mathematics Survey Item 8

R^2	CV	Root MSE	Mean
0.0037	14.6317	0.7848	5.3638

The R^2 value shows an almost negligible effect. The CV shows that 14.6317% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.24

Tukey's HSD Test for Nature of Mathematics Survey Item 8

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals	
Country L – USA	0.1450	-0.0344	0.3245
Country L – Baylor	-0.1037	-0.4257	0.2184
Country L – Country H	0.1466	-0.0416	0.3336
USA – Country L	-0.1450	-0.3245	0.0344
USA – Baylor	-0.2487	-0.5253	0.0279
USA – Country H	0.0015	-0.0867	0.0898
Baylor – Country L	0.1037	-0.2184	0.4257
Baylor – USA	0.2487	-0.0279	0.5253
Baylor – Country H	0.2502	-0.0314	0.5318
Country H – Country L	-0.1466	-0.3336	0.0406
Country H – USA	-0.0015	-0.0898	0.0867
Country H – Baylor	-0.2502	-0.5318	0.0314

The variance among the means is not significant among Baylor and the three countries.

Survey Item 9: Many aspects of mathematics have practical relevance.

The mean of Baylor's data is 5.2126. Because there is not a statistical significance indicated in the ANOVA, it appears as though none of the means of any of the countries is significantly different than Baylor's, indicating an agreement with the Baylor PSTMs. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is statistical significance between Baylor and Country H. Tables 4.25, 4.26, and 4.27 provide details of the statistical analyses.

Table 4.25

The ANOVA Procedure for Nature of Mathematics Survey Item 9

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	10.5891	3.5297	5.02	3.6355	0.0018*
Error	2592	1824.0365	0.7037			
Corrected Total	2595	1834.6256				
Note α=0.05						

Because the *F*-value is 5.02, the null hypothesis is not rejected: there is not statistical significance in the variation among the means. The means of the respondents of all four entities are not significantly different.

Table 4.26

Goodness of Fit for Nature of Mathematics Survey Item 9

R^2	CV	Root MSE	Mean
0.0058	16.0932	0.8389	5.2126

The R^2 value shows an almost negligible effect. The CV shows that 14.6317% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.27

Tukey's HSD Test for Nature of Mathematics Survey Item 9

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	-0.0579	-0.2497	0.1339	
Country L – Baylor	-0.2466	-0.5908	0.0977	
Country L – Country H	0.0636	-0.1363	0.2636	
USA – Country L	0.0579	-0.1339	0.2497	
USA – Baylor	-0.1887	-0.4843	0.1070	
USA – Country H	0.1216	0.0273	0.2159	*
Baylor – Country L	0.2466	-0.0977	0.5908	
Baylor – USA	0.1887	-0.1070	0.4843	
Baylor – Country H	0.3102	0.0092	0.6112	*
Country H – Country L	-0.0636	-0.2637	0.1363	
Country H – USA	-0.1216	-0.2159	-0.0273	*
Country H – Baylor	-0.3102	-0.6112	-0.0092	*

Although the ANOVA showed no significance (relationships among all levels), Tukey's *post hoc* showed that there is significance between Baylor and Country H (relationship on an individual level). Therefore, the variation of means between Baylor and Country H is statistically significant.

Survey Item 10: Mathematics helps solve everyday problems and tasks.

The mean of Baylor's data is 5.3394. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.28, 4.29, and 4.30 provide details of the statistical analyses.

Table 4.28

The ANOVA Procedure for Nature of Mathematics Survey Item 10

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	51.2194	17.0731	28.78	3.6355	<0.0001*
Error	2595	1539.4640	0.5932			
Corrected Total	2598	1590.6833				
Note α=0.05						

Because the *F*-value is 28.78, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.29

Goodness of Fit for Nature of Mathematics Survey Item 10

R^2	CV	Root MSE	Mean
0.0322	14.4254	0.7702	5.3394

The R^2 value shows a small effect. The CV shows that 14.42547% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.30

Tukey's HSD Test for Nature of Mathematics Survey Item 10

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.1466	-0.0289	0.3221	
Country L – Baylor	-0.0057	-0.3214	0.3101	
Country L – Country H	0.4268	0.2438	0.6098	*
USA – Country L	-0.1466	-0.3221	0.0289	
USA – Baylor	-0.1523	-0.4237	0.1192	
USA – Country H	0.2802	0.1937	0.3667	*
Baylor – Country L	0.0057	-0.3101	0.3214	
Baylor – USA	0.1523	-0.1192	0.4237	
Baylor – Country H	0.4325	0.1561	0.7088	*
Country H – Country L	-0.4268	-0.6098	-0.2438	*
Country H – USA	-0.2802	-0.3667	-0.1937	*
Country H – Baylor	-0.4325	-0.7088	-0.1561	*

The variance among the means is significant between Baylor and Country H.

Survey Item 11: To do mathematics requires much practice, correct application of routines, and problem-solving strategies.

The mean of Baylor's data is 5.0000. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.31, 4.32, and 4.33 provide details of the statistical analyses.

Table 4.31

The ANOVA Procedure for Nature of Mathematics Survey Item 11

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	61.7325	20.5775	25.49	3.6355	<0.0001*
Error	2594	2094.2675	0.8074			
Corrected Total	2597	2156.0000				
Note α=0.05						

Because the *F*-value is 25.49, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.32

Goodness of Fit for Nature of Mathematics Survey Item 11

R^2	CV	Root MSE	Mean
0.0286	17.9705	0.8985	5.0000

The R^2 value shows a small effect. The CV shows that 17.9705% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.33

Tukey's HSD Test for Nature of Mathematics Survey Item 11

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5649	0.3609	0.7690	*
Country L – Baylor	0.6494	0.2815	1.0174	*
Country L – Country H	0.3326	0.1197	0.5454	*
USA – Country L	-0.5649	-0.7690	-0.3609	*
USA – Baylor	0.0845	-0.2321	0.4012	
USA – Country H	-0.2324	-0.3333	-0.1314	*
Baylor – Country L	-0.6494	-1.0174	-0.2815	*
Baylor – USA	-0.0845	-0.4012	0.2321	
Baylor – Country H	-0.3169	-0.6393	0.0055	
Country H – Country L	-0.3326	-0.5454	-0.1197	*
Country H – USA	0.2324	0.1314	0.3333	*
Country H – Baylor	0.3169	-0.0055	0.6393	

The variance among the means is significant between Baylor and Country L.

Survey Item 12: Mathematics means learning, remembering, and applying.

The mean of Baylor's data is 4.7865. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.34, 4.35, and 4.36 provide details of the statistical analyses.

Table 4.34

The ANOVA Procedure for Nature of Mathematics Survey Item 12

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	102.7927	34.2642	32.15	3.6355	<0.0001*
Error	2595	2765.6905	1.0658			
Corrected Total	2598	2868.4833				
Note α=0.05						

Because the *F*-value is 32.15, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.35

Goodness of Fit for Nature of Mathematics Survey Item 12

R^2	CV	Root MSE	Mean
0.0358	21.5685	1.0324	4.7865

The R^2 value shows a small effect. The CV shows that 21.5685% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.36

Tukey's HSD Test for Nature of Mathematics Survey Item 12

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5751	0.3406	0.8095	*
Country L – Baylor	0.5991	0.1763	1.0218	*
Country L – Country H	0.8653	0.6207	1.1099	*
USA – Country L	-0.5751	-0.8095	-0.3406	*
USA – Baylor	0.0240	-0.3398	0.3878	
USA – Country H	0.2903	0.1743	0.4063	*
Baylor – Country L	-0.5991	-1.0218	-0.1763	*
Baylor – USA	-0.0240	-0.3878	0.3398	
Baylor – Country H	0.2662	-0.1042	0.6366	
Country H – Country L	-0.8653	-1.1099	-0.6207	*
Country H – USA	-0.2903	-0.4063	-0.1743	*
Country H – Baylor	-0.2662	-0.6366	0.1042	

The variance among the means is significant between Baylor and Country L.

Research Question Two

What beliefs do Baylor PSTMs hold regarding the learning of mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?

The 42 tables that follow are grouped into 14 subgroups, three tables each, corresponding to the survey item indicated by the subheadings. These fourteen survey items are each listed and correspond to the beliefs about the learning of mathematics.

Survey Item 1: The best way to do well in mathematics is to memorize all the formulas.

The mean of Baylor's data is 3.1470. Because there is a statistical significance indicated in the ANOVA, it appears as though at least one of the means of the other countries is significantly different than Baylor's. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is statistical significance between Baylor and Country H. Tables 4.37, 4.38, and 4.39 provide details of the statistical analyses.

Table 4.37

The ANOVA Procedure for Learning of Mathematics Survey Item 1

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	43.0497	14.3499	8.68	3.6355	<0.0001*
Error	2595	4288.8041	1.6527			
Corrected Total	2598	4331.8538				
Note α=0.05						

Because the *F*-value is 8.68, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.38

Goodness of Fit for Learning of Mathematics Survey Item 1

R^2	CV	Root MSE	Mean
0.0099	40.5213	1.2856	3.1470

The R^2 value shows a small effect. The CV shows that 40.5213% of the data are equal to the mean. The mean is between the Likert-scale indication of *slightly disagree* and *slightly agree* with the statement.

Table 4.39

Tukey's HSD Test for Learning of Mathematics Survey Item 1

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	-0.5403	-0.8332	-0.2473	*
Country L – Baylor	-0.4279	-0.9549	0.0991	
Country L – Country H	-0.3941	-0.6995	-0.0886	*
USA – Country L	0.5403	0.2473	0.8332	*
USA – Baylor	0.1123	-0.3407	0.5654	
USA – Country H	0.1462	0.0018	0.2906	*
Baylor – Country L	0.4279	-0.0991	0.9549	
Baylor – USA	-0.1123	-0.5654	0.3407	
Baylor – Country H	0.0339	-0.4274	0.4951	
Country H – Country L	0.3941	0.0886	0.6995	*
Country H – USA	-0.1462	-0.2906	-0.0018	*
Country H – Baylor	-0.0339	-0.4951	0.4274	

The variance among the means is not significant among Baylor and the other three countries. The significance indicated by the ANOVA test was indicative of a group level significance; Tukey's Studentized Range exposed the significance to be among the three countries and did not include any significance with Baylor.

Survey Item 2: *Pupils* need to be taught exact procedures for solving mathematical problems.

The mean of Baylor's data is 3.8044. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.40, 4.41, and 4.42 provide details of the statistical analyses.

Table 4.40

The ANOVA Procedure for Learning of Mathematics Survey Item 2

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	117.9047	39.3136	27.85	3.6355	<0.0001*
Error	2593	3660.6893	1.4118			
Corrected Total	2596	3778.6300				
Note α=0.05						

Because the *F*-value is 27.85, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.41

Goodness of Fit for Learning of Mathematics Survey Item 2

R^2	CV	Root MSE	Mean
0.0312	31.2317	1.1882	3.8044

The R^2 value shows a small effect. The CV shows that 31.2317% of the data are equal to the mean. The mean is between the Likert-scale indication of *slightly disagree* and *slightly agree* with the statement.

Table 4.42

Tukey's HSD Test for Learning of Mathematics Survey Item 2

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.9034	0.6317	1.1750	*
Country L – Baylor	0.9003	0.4128	1.3879	*
Country L – Country H	0.6717	0.3885	0.9549	*
USA – Country L	-0.9034	-1.1750	-0.6317	*
USA – Baylor	-0.0030	-0.4218	0.4157	
USA – Country H	-0.2317	-0.3652	-0.0983	*
Baylor – Country L	-0.9003	-1.3879	-0.4128	*
Baylor – USA	0.0030	-0.4157	0.4218	
Baylor – Country H	-0.2287	-0.6550	0.1976	
Country H – Country L	-0.6717	-0.9549	-0.3885	*
Country H – USA	0.2317	0.0983	0.3652	*
Country H – Baylor	0.2287	-0.1976	0.6550	

The variance among the means is significant between Baylor and Country L.

Survey Item 3: It doesn't really matter if you understand a mathematical problem, if you can get the right answer.

The mean of Baylor's data is 2.1590. Although there is a statistical significance indicated by the *p*-value, Tukey's HSD indicates the means of the other countries are not significantly different from Baylor's, therefore the significance lies within the comparisons of the other countries and no significance is indicated with Baylor. Tables 4.43, 4.44, and 4.45 provide details of the statistical analyses.

Table 4.43

The ANOVA Procedure for Learning of Mathematics Survey Item 3

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	33.9245	11.3082	8.86	3.6355	<0.0001*
Error	2588	3302.5878	1.2761			
Corrected Total	2591	3336.5123				
Note α=0.05						

Because the *F*-value is 8.86, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.44

Goodness of Fit for Learning of Mathematics Survey Item 3

R^2	CV	Root MSE	Mean
0.0102	52.3242	1.1297	2.1590

The R^2 value shows a small effect. The CV shows that 52.3242% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.45

Tukey's HSD Test for Learning of Mathematics Survey Item 3

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5094	0.2493	0.7694	*
Country L – Baylor	0.3421	-0.1225	0.8066	
Country L – Country H	0.4306	0.1597	0.7016	*
USA – Country L	-0.5094	-0.7694	-0.2493	*
USA – Baylor	-0.1673	-0.5654	0.2309	
USA – Country H	-0.0787	-0.2056	0.0482	
Baylor – Country L	-0.3421	-0.8066	0.1225	
Baylor – USA	0.1673	-0.2309	0.5654	
Baylor – Country H	0.0886	-0.3168	0.4939	
Country H – Country L	-0.4306	-0.7016	-0.1597	*
Country H – USA	0.0787	-0.0482	0.2056	
Country H – Baylor	-0.0886	-0.4939	0.3168	

The variance among the means is not significant among Baylor and the other three countries. Tukey's HSD indicates the significance occurs among the other countries.

Survey Item 4: To be good in mathematics you must be able to solve problems quickly.

The mean of Baylor's data is 2.5298. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.46, 4.47, and 4.48 provide details of the statistical analyses.

Table 4.46

The ANOVA Procedure for Learning of Mathematics Survey Item 4

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	440.5804	146.8601	100.31	3.6355	<0.0001*
Error	2593	3796.3568	1.4641			
Corrected Total	2596	4236.9372				
Note α=0.05						

Because the *F*-value is 100.31, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.47

Goodness of Fit for Learning of Mathematics Survey Item 4

R^2	CV	Root MSE	Mean
0.1040	47.8287	1.2100	2.5298

The R^2 value shows a medium effect. The CV shows that 47.8287% of the data are equal to the mean.

Table 4.48

Tukey's HSD Test for Learning of Mathematics Survey Item 4

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.7982	0.5234	1.0730	*
Country L – Baylor	0.8651	0.3696	1.3606	*
Country L – Country H	-0.0741	-0.3608	0.2126	
USA – Country L	-0.7982	-1.0730	-0.5234	*
USA – Baylor	0.0669	-0.3595	0.4934	
USA – Country H	-0.8723	-1.0084	-0.7362	*
Baylor – Country L	-0.8651	-1.3606	-0.3696	*
Baylor – USA	-0.0669	-0.4934	0.3595	
Baylor – Country H	-0.9392	-1.3734	-0.5051	*
Country H – Country L	0.0741	-0.2126	0.3608	
Country H – USA	0.8723	0.7362	1.0084	*
Country H – Baylor	0.9392	0.5051	1.3734	*

The variance among the means is significant between Baylor and Country L and Country H.

Survey Item 5: Pupils learn mathematics best by attending to the teacher's explanations.

The mean of Baylor's data is 3.3042. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.49, 4.50, and 4.51 provide details of the statistical analyses.

Table 4.49

The ANOVA Procedure for Learning of Mathematics Survey Item 5

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	86.4054	28.8018	20.24	3.6355	<0.0001*
Error	2593	3689.2788	1.4228			
Corrected Total	2596	3775.6843				
Note α=0.05						

Because the *F*-value is 20.24, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.50

Goodness of Fit for Learning of Mathematics Survey Item 5

R^2	CV	Root MSE	Mean
0.02289	36.0997	1.1928	3.3042

The R^2 value shows a small effect. The CV shows that 36.0997% of the data are equal to the mean. The mean is between the Likert-scale indication of *slightly disagree* and *slightly agree* with the statement.

Table 4.51

Tukey's HSD Test for Learning of Mathematics Survey Item 5

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5433	0.2724	0.8141	*
Country L – Baylor	0.5922	0.1037	1.0806	*
Country L – Country H	0.2051	-0.0775	0.4877	
USA – Country L	-0.5433	-0.8142	-0.2724	*
USA – Baylor	0.0489	-0.3715	0.4692	
USA – Country H	-0.3382	-0.4723	-0.2040	*
Baylor – Country L	-0.5922	-1.0806	-0.1037	*
Baylor – USA	-0.0489	-0.4692	0.3715	
Baylor – Country H	-0.3871	-0.8151	0.0410	
Country H – Country L	-0.2051	-0.4877	0.0776	
Country H – USA	0.3382	0.2040	0.4723	*
Country H – Baylor	0.3871	-0.0410	0.8151	

The variance among the means is significant between Baylor and Country L.

Survey Item 6: When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.

The mean of Baylor's data is 2.1132. Because there is not a statistical significance indicated in the ANOVA, it appears as though none of the means of any of the countries is significantly different than Baylor's, indicating an agreement with the Baylor PSTMs. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is statistical significance between Baylor and Country H and Country L. Tables 4.52, 4.53, and 4.54 provide details of the statistical analyses.

Table 4.52

The ANOVA Procedure for Learning of Mathematics Survey Item 6

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	21.0272	7.0091	5.42	3.6355	0.0010
Error	2594	3353.7026	1.2929			
Corrected Total	2597	3374.7298				
Note α=0.05						

Because the *F*-value is 5.42, the null hypothesis is not rejected: there is not statistical significance in the variation among the means. This indicates that there is not a significant difference in how the respondents answered this question.

Table 4.53

Goodness of Fit for Learning of Mathematics Survey Item 6

R^2	CV	Root MSE	Mean
0.0062	53.8077	1.1370	2.1132

The R^2 value shows an almost negligible effect. The CV shows that 53.8077% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.54

Tukey's HSD Test for Learning of Mathematics Survey Item 6

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	-0.1151	-0.3751	0.1449	
Country L – Baylor	-0.5090	-0.9756	-0.0424	*
Country L – Country H	0.0222	-0.2488	0.2932	
USA – Country L	0.1151	-0.1449	0.3751	
USA – Baylor	-0.3939	-0.7946	0.0068	
USA – Country H	0.1373	0.0096	0.2651	*
Baylor – Country L	0.5090	0.0424	0.9756	*
Baylor – USA	0.3939	-0.0068	0.7946	
Baylor – Country H	0.5090	0.0424	0.9756	*
Country H – Country L	-0.0222	-0.2932	0.2488	
Country H – USA	-0.1373	-0.2651	-0.0096	*
Country H – Baylor	-0.5312	-0.9391	-0.1232	*

Although the ANOVA showed no significance (relationships among all levels), Tukey's *post hoc* showed that there is significance between Baylor and Country H and Country L (relationship on an individual level). Therefore, the variation of means between Baylor and Countries L and H are statistically significant.

Survey Item 7: In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.

The mean of Baylor's data is 5.4732. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating

a disagreement with the Baylor PSTMs. Tables 4.55, 4.56, and 4.57 provide details of the statistical analyses.

Table 4.55

The ANOVA Procedure for Learning of Mathematics Survey Item 7

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	36.7792	12.2597	21.94	3.6355	<0.0001*
Error	2593	1448.6109	0.5587			
Corrected Total	2596	1485.3901				
Note α=0.05						

Because the *F*-value is 21.94, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.56

Goodness of Fit for Learning of Mathematics Survey Item 7

R^2	CV	Root MSE	Mean
0.0248	13.6562	0.7474	5.4732

The R^2 value shows a small effect. The CV shows that 13.6562% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.57

Tukey's HSD Test for Learning of Mathematics Survey Item 7

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	-0.3634	-0.5337	-0.1930	*
Country L – Baylor	-0.3134	-0.6198	-0.0071	*
Country L – Country H	-0.1425	-0.3201	0.0352	
USA – Country L	0.3634	0.1930	0.5337	*
USA – Baylor	0.0499	-0.2134	0.3133	
USA – Country H	0.2209	0.1369	0.3049	*
Baylor – Country L	0.3134	0.0071	0.6198	*
Baylor – USA	-0.0499	-0.3133	0.2135	
Baylor – Country H	0.1710	-0.0972	0.4392	
Country H – Country L	0.1425	-0.0352	0.3201	
Country H – USA	-0.2209	-0.3049	-0.1369	*
Country H – Baylor	-0.1710	-0.4392	0.0972	

The variance among the means is significant between Baylor and Country L.

Survey Item 8: Teachers should allow pupils to figure out their own ways to solve mathematical problems.

The mean of Baylor's data is 4.8796. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.58, 4.59, and 4.60 provide details of the statistical analyses.

Table 4.58

The ANOVA Procedure for Learning of Mathematics Survey Item 8

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	36.6526	12.2175	13.70	3.6355	<0.0001*
Error	2588	2307.7919	0.8917			
Corrected Total	2591	2344.4444				
Note α=0.05						

Because the *F*-value is 13.70, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is not a significant difference in how the respondents answered this question.

Table 4.59

Goodness of Fit for Learning of Mathematics Survey Item 8

R^2	CV	Root MSE	Mean
0.0156	19.3522	0.9443	4.8796

The R^2 value shows a small effect. The CV shows that 19.3522% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.60

Tukey's HSD Test for Learning of Mathematics Survey Item 8

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5085	0.2929	0.7244	*
Country L – Baylor	0.1976	-0.1899	0.5851	
Country L – Country H	0.4694	0.2442	0.6945	*
USA – Country L	-0.5085	-0.7244	-0.2929	*
USA – Baylor	-0.3109	-0.6437	0.2119	
USA – Country H	-0.0391	-0.1454	0.0671	
Baylor – Country L	-0.1976	-0.5851	0.1899	
Baylor – USA	0.3109	-0.0219	0.6437	
Baylor – Country H	0.2718	-0.0671	0.6106	
Country H – Country L	-0.4694	-0.6945	-0.2442	*
Country H – USA	0.0391	-0.0671	0.1454	
Country H – Baylor	-0.2718	-0.6106	0.0671	

The variance among the means is not significant among Baylor and the other three countries. Tukey's Studentized Range supports the ANOVA in that there is statistical significance; it is not among Baylor and the three countries.

Survey Item 9: Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.

The mean of Baylor's data is 2.6136. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.61, 4.62, and 4.63 provide details of the statistical analyses.

Table 4.61

The ANOVA Procedure for Learning of Mathematics Survey Item 9

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	153.6567	51.2189	35.46	3.6355	<0.0001*
Error	2589	3739.1455	1.1442			
Corrected Total	2592	3892.8022				
Note α=0.05						

Because the *F*-value is 35.46, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.62

Goodness of Fit for Learning of Mathematics Survey Item 9

R^2	CV	Root MSE	Mean
0.0395	45.9817	1.2018	2.6136

The R^2 value shows a small effect. The CV shows that 45.9817% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.63

Tukey's HSD Test for Learning of Mathematics Survey Item 9

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.9178	0.6393	1.1964	*
Country L – Baylor	0.9578	0.4625	1.4530	*
Country L – Country H	0.5500	0.2599	0.8401	*
USA – Country L	-0.9178	-1.1964	-0.6393	*
USA – Baylor	0.0399	-0.3836	0.4634	
USA – Country H	-0.3679	-0.5029	-0.2328	*
Baylor – Country L	-0.9578	-1.4530	-0.4625	*
Baylor – USA	-0.0399	-0.4634	0.3836	
Baylor – Country H	-0.4078	-0.8390	0.0234	
Country H – Country L	-0.5500	-0.8401	-0.2599	*
Country H – USA	0.3679	0.2328	0.5029	*
Country H – Baylor	0.4078	-0.0234	0.8390	

The variance among the means is significant between Baylor and Country L.

Survey Item 10: Hands-on mathematics experiences aren't worth the time and expense.

The mean of Baylor's data is 1.7226. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.64, 4.65, and 4.66 provide details of the statistical analyses.

Table 4.64

The ANOVA Procedure for Learning of Mathematics Survey Item 10

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	416.3819	138.7940	144.24	3.6355	<0.0001*
Error	2584	2486.4203	0.9622			
Corrected Total	2587	2902.8022				
Note α=0.05						

Because the *F*-value is 144.24, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.65

Goodness of Fit for Learning of Mathematics Survey Item 10

R^2	CV	Root MSE	Mean
0.1434	59.9463	0.9809	1.7226

The R^2 value shows a medium effect. The CV shows that 59.9463% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.66

Tukey's HSD Test for Learning of Mathematics Survey Item 10

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	1.3410	1.1128	1.5692	*
Country L – Baylor	1.1076	0.7029	1.5129	*
Country L – Country H	0.6347	0.3971	0.8722	*
USA – Country L	-1.3410	-1.5692	-1.1128	*
USA – Baylor	-0.2334	-0.5791	0.1123	
USA – Country H	-0.7063	-0.8166	-0.5960	*
Baylor – Country L	-1.1076	-1.5123	-0.7029	*
Baylor – USA	0.2334	-0.1123	0.5791	
Baylor – Country H	-0.4729	-0.8249	-0.1209	*
Country H – Country L	-0.6347	-0.8722	-0.3971	*
Country H – USA	0.7063	0.5960	0.8166	*
Country H – Baylor	0.4729	0.1209	0.8249	*

The variance among the means is significant among Baylor and Country L and Country H.

Survey Item 11: Time used to investigate why a solution to a mathematical problem works is time well spent.

The mean of Baylor's data is 5.1519. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.67, 4.68, and 4.69 provide details of the statistical analyses.

Table 4.67

The ANOVA Procedure for Learning of Mathematics Survey Item 11

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	135.8516	45.2839	59.72	3.6355	<0.0001*
Error	2584	1959.4695	0.7583			
Corrected Total	2587	2095.3211				
Note α=0.05						

Because the *F*-value is 59.72, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.68

Goodness of Fit for Learning of Mathematics Survey Item 11

R^2	CV	Root MSE	Mean
0.0648	16.9028	0.8708	5.1519

The R^2 value shows an effect between small and medium. The CV shows that 16.9028% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.69

Tukey's HSD Test for Learning of Mathematics Survey Item 11

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	-0.8772	-1.0804	-0.6739	*
Country L – Baylor	-0.9074	-1.2671	-0.5478	*
Country L – Country H	-0.5359	-0.7475	-0.3244	*
USA – Country L	0.8772	0.6739	1.0804	*
USA – Baylor	-0.0303	-0.3372	0.2766	
USA – Country H	0.3412	0.2433	0.4391	*
Baylor – Country L	0.9074	0.5478	1.2671	*
Baylor – USA	0.0303	-0.2766	0.3372	
Baylor – Country H	0.3715	0.0590	0.6839	*
Country H – Country L	0.5359	0.3244	0.7475	*
Country H – USA	-0.3412	-0.4391	-0.2433	*
Country H – Baylor	-0.3715	-0.6839	-0.0590	*

The variance among the means is significant among Baylor and Country L and Country H.

Survey Item 12: Pupils can figure out a way to solve mathematical problems without a teacher's help.

The mean of Baylor's data is 4.4531. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.70, 4.71, and 4.72 provide details of the statistical analyses.

Table 4.70

The ANOVA Procedure for Learning of Mathematics Survey Item 12

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	86.6035	28.8678	25.95	3.6355	<0.0001*
Error	2589	2879.9534	1.1124			
Corrected Total	2592	2966.5569				
Note α=0.05						

Because the *F*-value is 25.95, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.71

Goodness of Fit for Learning of Mathematics Survey Item 12

R^2	CV	Root MSE	Mean
0.0292	23.6843	1.0547	4.4531

The R^2 value shows a small effect. The CV shows that 23.6843% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.72

Tukey's HSD Test for Learning of Mathematics Survey Item 12

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.4571	0.2160	0.6983	*
Country L – Baylor	0.0471	-0.3857	0.4799	
Country L – Country H	0.7224	0.4710	0.9738	*
USA – Country L	-0.4571	-0.6983	-0.2160	*
USA – Baylor	-0.4100	-0.7817	-0.0383	*
USA – Country H	0.2653	0.1468	0.3839	*
Baylor – Country L	-0.0471	-0.4799	0.3857	
Baylor – USA	0.4100	0.0383	0.7817	*
Baylor – Country H	0.6753	0.2969	1.0538	*
Country H – Country L	-0.7224	-0.9738	-0.4710	*
Country H – USA	-0.2653	-0.3839	-0.1468	*
Country H – Baylor	-0.6753	-1.0538	-0.2969	*

The variance among the means is significant among Baylor and USA and Country H.

Survey Item 13: Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.

The mean of Baylor's data is 4.1996. Because there is not a statistical significance indicated in the ANOVA, it appears as though none of the means of any of the countries is significantly different than Baylor's, indicating an agreement with the Baylor PSTMs. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is no statistical significance between Baylor and the three countries. Tables 4.73, 4.74, and 4.75 provide details of the statistical analyses.

Table 4.73

The ANOVA Procedure for Learning of Mathematics Survey Item 13

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	19.0100	6.3367	4.42	3.6355	0.0042
Error	2591	3717.5896	1.4348			
Corrected Total	2594	3736.5996				
Note α=0.05						

Because the *F*-value is 4.42, the null hypothesis is not rejected: there is not statistical significance in the variation among the means. This indicates that there is not a significant difference in how the respondents answered this question.

Table 4.74

Goodness of Fit for Learning of Mathematics Survey Item 13

R^2	CV	Root MSE	Mean
0.0051	28.5225	1.1978	4.1996

The R^2 value shows a small effect. The CV shows that 28.5225% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.75

Tukey's HSD Test for Learning of Mathematics Survey Item 13

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.1847	-0.0883	0.4576	
Country L – Baylor	-0.0448	-0.5358	0.4462	
Country L – Country H	0.0137	-0.2710	0.2983	
USA – Country L	-0.1847	-0.4576	0.0883	
USA – Baylor	-0.2295	-0.6516	0.1927	
USA – Country H	-0.1710	-0.3056	-0.0364	*
Baylor – Country L	0.0448	-0.4462	0.5358	
Baylor – USA	0.2295	-0.1927	0.6516	
Baylor – Country H	0.0584	-0.3713	0.4882	
Country H – Country L	-0.0137	-0.2983	0.2710	
Country H – USA	0.1710	0.0364	0.3056	*
Country H – Baylor	-0.0584	-0.4882	0.3713	

Although the ANOVA showed no significance (relationships among all levels), Tukey's *post hoc* showed that there is significance between some countries (relationship on an individual level). However, the variation of means among Baylor and the three countries are not statistically significant.

Survey Item 14: It is helpful for pupils to discuss different ways to solve particular problems.

The mean of Baylor's data is 5.4163. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating

a disagreement with the Baylor PSTMs. Tables 4.76, 4.77, and 4.78 provide details of the statistical analyses.

Table 4.76

The ANOVA Procedure for Learning of Mathematics Survey Item 14

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	55.5525	18.5175	38.47	3.6355	<0.0001*
Error	2590	1246.7945	0.4814			
Corrected Total	2593	1302.3470				
Note α=0.05						

Because the *F*-value is 38.47, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.77

Goodness of Fit for Learning of Mathematics Survey Item 14

R^2	CV	Root MSE	Mean
0.0427	12.8098	0.6938	5.4163

The R^2 value shows an effect between small and medium. The CV shows that 12.8098% of the data are equal to the mean. The mean is within the Likert-scale indication of *agreement* with the statement.

Table 4.78

Tukey's HSD Test for Learning of Mathematics Survey Item 14

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.0180	-0.1402	0.1761	
Country L – Baylor	0.0563	-0.2282	0.3407	
Country L – Country H	0.3378	0.1729	0.5027	*
USA – Country L	-0.0180	-0.1761	0.1402	
USA – Baylor	0.0383	-0.2062	0.2828	
USA – Country H	0.3199	0.2419	0.3979	*
Baylor – Country L	-0.0563	-0.3407	0.2282	
Baylor – USA	-0.0383	-0.2828	0.2062	
Baylor – Country H	0.2816	0.0326	0.5305	*
Country H – Country L	-0.3378	-0.5027	-0.1729	*
Country H – USA	-0.3199	-0.3979	-0.2419	*
Country H – Baylor	-0.2816	-0.5305	-0.0326	*

The variance among the means is significant between Baylor and Country H.

Research Question Three

What beliefs do Baylor PSTMs hold regarding student achievement in mathematics and in what manner do their beliefs compare to the PSTMs in the USA and the other identified countries participating in TEDS-M?

The 24 tables that follow are grouped into 8 subgroups, three tables each, corresponding to the survey item indicated by the subheadings. These eight survey items are each listed and correspond to the beliefs about achievement in mathematics.

Survey Item 1: Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.

The mean of Baylor's data is 2.6388. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.79, 4.80, and 4.81 provide details of the statistical analyses.

Table 4.79

The ANOVA Procedure for Achievement in Mathematics Survey Item 1

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	315.3785	105.1262	64.72	3.6355	<0.0001*
Error	2590	4207.1601	1.6244			
Corrected Total	2593	4522.5386				
Note α=0.05						

Because the F-value is 64.72, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.80

Goodness of Fit for Achievement in Mathematics Survey Item 1

R^2	CV	Root MSE	Mean
0.0697	48.2993	1.2745	2.6388

The R^2 value shows an effect between small and medium. The CV shows that 48.2993% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.81

Tukey's HSD Test for Achievement in Mathematics Survey Item 1

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.7395	0.4481	1.0309	*
Country L – Baylor	0.9678	0.4448	1.4908	*
Country L – Country H	0.0222	-0.2817	0.3260	
USA – Country L	-0.7395	-1.0309	-0.4481	*
USA – Baylor	0.2283	-0.2209	0.6774	
USA – Country H	-0.7173	-0.8606	-0.5740	*
Baylor – Country L	-0.9678	-1.4908	-0.4448	*
Baylor – USA	-0.2283	-0.6774	0.2209	
Baylor – Country H	-0.9456	-1.4029	-0.4883	*
Country H – Country L	-0.0222	-0.3260	0.2817	
Country H – USA	0.7173	0.5740	0.8606	*
Country H – Baylor	0.9456	0.4883	1.4029	*

The variance among the means is significant among Baylor and Country L and Country H.

Survey Item 2: To be good at mathematics you need to have a kind of "mathematical mind."

The mean of Baylor's data is 3.0583. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating

a disagreement with the Baylor PSTMs. Tables 4.82, 4.83, and 4.84 provide details of the statistical analyses.

Table 4.82

The ANOVA Procedure for Achievement in Mathematics Survey Item 2

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	581.5977	193.8659	132.78	3.6355	<0.0001*
Error	2588	3778.6057	1.4600			
Corrected Total	2591	4360.2033				
Note α=0.05						

Because the *F*-value is 132.78, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.83

Goodness of Fit for Achievement in Mathematics Survey Item 2

R^2	CV	Root MSE	Mean
0.1334	39.5103	1.2083	3.0583

The R^2 value shows a medium effect. The CV shows that 39.5103% of the data are equal to the mean. The mean is between the Likert-scale indication of *slightly disagree* and *slightly agree* with the statement.

Table 4.84

Tukey's HSD Test for Achievement in Mathematics Survey Item 2

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	1.4102	1.1348	1.6856	*
Country L – Baylor	1.4032	0.9078	1.8984	*
Country L – Country H	0.5245	0.2373	0.8116	*
USA – Country L	-1.4102	-1.6856	-1.1348	*
USA – Baylor	-0.0070	-0.4329	0.4188	
USA – Country H	-0.8857	-1.0216	-0.7499	*
Baylor – Country L	-1.4032	-1.8985	-0.9078	*
Baylor – USA	0.0070	-0.4188	0.4329	
Baylor – Country H	-0.8787	-1.3123	-0.4452	*
Country H – Country L	-0.5245	-0.8116	-0.2373	*
Country H – USA	0.8857	0.7499	1.0216	*
Country H – Baylor	0.8787	0.4452	1.3123	*

The variance among the means is significant among Baylor and Country L and Country H.

Survey Item 3: Mathematics is a subject in which natural ability matters a lot more than effort.

The mean of Baylor's data is 2.4954. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.85, 4.86, and 4.87 provide details of the statistical analyses.

Table 4.85

The ANOVA Procedure for Achievement in Mathematics Survey Item 3

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	153.5004	51.1668	40.07	3.6355	<0.0001*
Error	2588	3304.4441	1.2768			
Corrected Total	2591	3457.9444				
Note α=0.05						

Because the *F*-value is 40.07, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.86

Goodness of Fit for Achievement in Mathematics Survey Item 3

R^2	CV	Root MSE	Mean
0.0444	45.2827	1.1300	2.4954

The R^2 value shows an effect between small and medium. The CV shows that 45.2827% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.87

Tukey's HSD Test for Achievement in Mathematics Survey Item 3

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.7456	0.4873	1.0040	*
Country L – Baylor	0.6475	0.1838	1.1112	*
Country L – Country H	0.2946	0.0252	0.5639	*
USA – Country L	-0.7456	-1.0040	-0.4873	*
USA – Baylor	-0.981	-0.4964	0.3001	
USA – Country H	-0.4512	-0.5782	-0.3240	*
Baylor – Country L	-0.6475	-1.1112	-0.1838	*
Baylor – USA	0.0981	-0.3001	0.4964	
Baylor – Country H	-0.3530	-0.7584	0.0525	
Country H – Country L	-0.2946	-0.5639	-0.0252	*
Country H – USA	0.4511	0.3240	0.5782	*
Country H – Baylor	0.3530	-0.0525	0.7584	

The variance among the means is significant between Baylor and Country L.

Survey Item 4: Only the more able pupils can participate in multi-step problem solving activities.

The mean of Baylor's data is 2.2711. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.88, 4.89, and 4.90 provide details of the statistical analyses.

Table 4.88

The ANOVA Procedure for Achievement in Mathematics Survey Item 4

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	632.7642	210.9214	187.55	3.6355	<0.0001*
Error	2589	2911.6423	1.1246			
Corrected Total	2592	3544.4065				
Note α=0.05						

Because the *F*-value is 187.55, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.89

Goodness of Fit for Achievement in Mathematics Survey Item 4

R^2	CV	Root MSE	Mean
0.1785	46.6943	1.0605	2.2711

The R^2 value shows an effect between medium and large. The CV shows that 46.6943% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.90

Tukey's HSD Test for Achievement in Mathematics Survey Item 4

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	1.2910	1.0477	1.5343	*
Country L – Baylor	1.1258	0.6902	1.5614	*
Country L – Country H	0.3052	0.0516	0.5588	*
USA – Country L	-1.2910	-1.5343	-1.0477	*
USA – Baylor	-0.1652	-0.5389	0.2085	
USA – Country H	-0.9858	-1.1051	-0.8667	*
Baylor – Country L	-1.1258	-1.5614	-0.6902	*
Baylor – USA	0.1652	-0.2085	0.5389	
Baylor – Country H	-0.8206	-1.2012	-0.4401	*
Country H – Country L	-0.3052	-0.5588	-0.0516	*
Country H – USA	0.9858	0.8666	1.1051	*
Country H – Baylor	0.8206	0.4401	1.2012	*

The variance among the means is significant among Baylor and Country L and Country H.

Survey Item 5: In general, boys tend to be naturally better at mathematics than girls.

The mean of Baylor's data is 2.3413. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.91, 4.92, and 4.93 provide details of the statistical analyses.

Table 4.91

The ANOVA Procedure for Achievement in Mathematics Survey Item 5

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	130.3450	43.4483	24.55	3.6355	<0.0001*
Error	2589	4582.6014	1.7700			
Corrected Total	2592	4712.9464				
Note α=0.05						

Because the *F*-value is 24.55, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.92

Goodness of Fit for Achievement in Mathematics Survey Item 5

R^2	CV	Root MSE	Mean
0.0277	56.8241	1.3304	2.3413

The R^2 value shows small effect. The CV shows that 56.8241% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.93

Tukey's HSD Test for Achievement in Mathematics Survey Item 5

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.8758	0.5716	1.1800	*
Country L – Baylor	0.6912	0.1452	1.2371	*
Country L – Country H	0.5587	0.2415	0.8759	*
USA – Country L	-0.8758	-1.1800	-0.5716	*
USA – Baylor	-0.1846	-0.6535	0.2842	
USA – Country H	-0.3171	-0.4668	-0.1675	*
Baylor – Country L	-0.6912	-1.2371	-0.1452	*
Baylor – USA	0.1846	-0.2842	0.6535	
Baylor – Country H	-0.1325	-0.6099	0.3449	
Country H – Country L	-0.5587	-0.8759	-0.2415	*
Country H – USA	0.3171	0.1675	0.4668	*
Country H – Baylor	0.1325	-0.3449	0.6099	

The variance among the means is significant between Baylor and Country L.

Survey Item 6: Mathematical ability is something that remains relatively fixed throughout a person's life.

The mean of Baylor's data is 2.8837. Because there is a statistical significance indicated in the ANOVA, it appears as though at least one of the means of the other countries is significantly different than Baylor's. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is no statistical significance between Baylor and any of the other countries. Tables 4.94, 4.95, and 4.96 provide details of the statistical analyses.

Table 4.94

The ANOVA Procedure for Achievement in Mathematics Survey Item 6

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	155.3098	51.7699	27.22	3.6355	<0.0001*
Error	2584	4914.6821	1.9020			
Corrected Total	2587	5069.9919				
Note α=0.05						

Because the *F*-value is 27.22, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.95

Goodness of Fit for Achievement in Mathematics Survey Item 6

R^2	CV	Root MSE	Mean
0.0306	47.8247	1.3791	2.8837

The R^2 value shows small effect. The CV shows that 47.8247% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.96

Tukey's HSD Test for Achievement in Mathematics Survey Item 6

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.9979	0.6793	1.3165	*
Country L – Baylor	0.5495	-0.0182	1.1173	
Country L – Country H	0.6974	0.3655	1.0294	*
USA – Country L	-0.9979	-1.3165	-0.6793	*
USA – Baylor	-0.4484	-0.9344	0.0377	
USA – Country H	-0.3004	-0.4557	-0.1452	*
Baylor – Country L	-0.5495	-1.1173	0.0182	
Baylor – USA	0.4484	-0.0377	0.9344	
Baylor – Country H	0.1479	-0.3470	0.6428	
Country H – Country L	-0.6974	-1.0294	-0.3655	*
Country H – USA	0.3004	0.1452	0.4557	*
Country H – Baylor	-0.1479	-0.6428	0.3470	

The variance among the means is not significant among Baylor and the three countries. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is statistical significance among the other three countries.

Survey Item 8: Some people are good at mathematics and some aren't.

The mean of Baylor's data is 3.7040. Because there is a statistical significance indicated, the mean of at least one of the countries is not the same as Baylor's, indicating a disagreement with the Baylor PSTMs. Tables 4.97, 4.98, and 4.99 provide details of the statistical analyses.

Table 4.97

The ANOVA Procedure for Achievement in Mathematics Survey Item 7

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	281.9042	93.9681	48.33	3.6355	<0.0001*
Error	2587	5030.0449	1.9444			
Corrected Total	2590	5311.9491				
Note α=0.05						

Because the *F*-value is 48.33, the null hypothesis is rejected: there is statistical significance in the variation among the means. This indicates that there is a difference in how the respondents answered this question.

Table 4.98

Goodness of Fit for Achievement in Mathematics Survey Item 7

R^2	CV	Root MSE	Mean
0.0531	37.6461	1.3944	3.7040

The R^2 value shows an effect between small and medium. The CV shows that 37.6461% of the data are equal to the mean. The mean is between the Likert-scale indication of *slightly disagree* and *slightly agree* with the statement.

Table 4.99

Tukey's HSD Test for Achievement in Mathematics Survey Item 7

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	1.2810	0.9622	1.5999	*
Country L – Baylor	1.3265	0.7543	1.8987	*
Country L – Country H	0.8175	0.4851	1.1500	*
USA – Country L	-1.2810	-1.6000	-0.9622	*
USA – Baylor	0.0455	-0.4460	0.5369	
USA – Country H	-0.4635	-0.6204	-0.3066	*
Baylor – Country L	-1.3265	-1.8987	-0.7543	*
Baylor – USA	-0.0455	-0.5369	0.4460	
Baylor – Country H	-0.5090	-1.0093	-0.0086	*
Country H – Country L	-0.8175	-1.1500	-0.4851	*
Country H – USA	0.4635	0.3066	0.6204	*
Country H – Baylor	0.5090	0.0086	1.0093	*

The variance among the means is significant among Baylor and Country L and Country H.

Survey Item 8: Some ethnic groups are better at mathematics than others.

The mean of Baylor's data is 2.4478. Because there is not a statistical significance indicated in the ANOVA, it appears as though none of the means of any of the countries is significantly different than Baylor's, indicating an agreement with the Baylor PSTMs. However, Tukey's Studentized Range indicates, on an individual level rather than a grouped level, there is no statistical significance between Baylor and the other three countries. Tables 4.100, 4.101, and 4.102 provide details of the statistical analyses.

Table 4.100

The ANOVA Procedure for Achievement in Mathematics Survey Item 8

Source of Variation	*df*	SS	MS	*F*	*F*-Crit	*p*
Model	3	34.8303	11.6101	5.62	3.6359	0.0008
Error	2198	4539.6638	2.0654			
Corrected Total	2201	4574.4941				
Note α=0.05						

Because the *F*-value is 5.65, the null hypothesis is not rejected: there is not statistical significance in the variation among the means. This indicates that there is not a significant difference in how the respondents answered this question.

Table 4.101

Goodness of Fit for Achievement in Mathematics Survey Item 8

R^2	CV	Root MSE	Mean
0.0076	58.7120	1.4371	2.4478

The R^2 value shows a small effect. The CV shows that 58.7120% of the data are equal to the mean. The mean is within the Likert-scale indication of *disagreement* with the statement.

Table 4.102

Tukey's HSD Test for Achievement in Mathematics Survey Item 8

Country Comparison	Difference Between Means	Simultaneous 95% Confidence Intervals		
Country L – USA	0.5158	0.1849	0.8466	*
Country L – Baylor	0.3657	-0.2254	0.9567	
Country L – Country H	0.4164	0.0459	0.7869	*
USA – Country L	-0.5158	-0.8466	-0.1849	*
USA – Baylor	-0.1501	-0.6566	0.3564	
USA – Country H	-0.0994	-0.3102	0.1115	
Baylor – Country L	-0.3657	-0.9567	0.2254	
Baylor – USA	0.1501	-0.3564	0.6566	
Baylor – Country H	0.0508	-0.4825	0.5840	
Country H – Country L	-0.4164	-0.7869	-0.0459	*
Country H – USA	0.0994	-0.1115	0.3102	
Country H – Baylor	-0.0508	-0.5840	0.4825	

Although the ANOVA showed no significance (relationships among all levels), Tukey's *post hoc* showed that there is significance between some countries (relationship on an individual level). However, the variation of means among Baylor and the three countries are not statistically significant.

Summary Tables

To summarize the preceding statistical analyses, five additional tables are provided. The beliefs regarding the nature of mathematics and the learning of mathematics are divided into two tables: one addressing the calculational viewpoint and the other addressing the conceptual viewpoint. The beliefs regarding achievement in mathematics are not broken down into cluster tables since those questions were not designed from a conceptual or calculational viewpoint. Throughout the remaining tables, significance is indicated by an asterisk.

The first two tables, Table 4.103 and Table 4.104, address the beliefs regarding the nature of mathematics and are organized into calculational and conceptual clusters.

Table 4.103

Beliefs Regarding the Nature of Mathematics – Calculational

Survey Item	Baylor's Mean	Country L	USA	Country H
1. Mathematics is a collection of rules and procedures that prescribe how to solve a problem.	4.5874			
2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.	4.6636	*		
5. When solving mathematical tasks you need to know the correct procedures else you would be lost.	4.0231	*		
7. Fundamental to mathematics is its logical rigor and preciseness.	4.5706			*
11. To do mathematics requires much practice, correct application of routines, and problem solving strategies.	5.0000	*		
12. Mathematics means learning, remembering, and applying.	4.7865	*		

Table 4.104

Beliefs Regarding the Nature of Mathematics – Conceptual

Survey Item	Baylor's Mean	Country L	USA	Country H
3. Mathematics involves creativity and new ideas.	4.7524			
4. In mathematics, many things can be discovered and tried out for oneself.	4.9441			*
6. If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).	5.0551			*
8. Mathematical problems can be solved correctly in many ways.	5.3638			
9. Many aspects of mathematics have practical relevance.	5.2126			*
10. Mathematics helps solve everyday problems and tasks.	5.3394			*

The next two tables, Table 4.105 and Table 4.106, address the beliefs regarding the learning of mathematics and are organized into calculational and conceptual clusters.

Table 4.105

Beliefs Regarding the Learning of Mathematics – Calculational

Survey Item	Baylor's Mean	Country L	USA	Country H
1. The best way to do well in mathematics is to memorize all the formulas.	3.1470			
2. Pupils need to be taught exact procedures for solving mathematical problems.	3.8044	*		
3. It doesn't really matter if you understand a mathematical problem, if you can get the right answer.	2.1590			
4. To be good in mathematics you must be able to solve problems quickly.	2.5298	*		*
5. Pupils learn mathematics best by attending to the teacher's explanations.	3.3042	*		
6. When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process being followed.	2.1132	*		*
9. Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.	2.6136	*		
10. Hands-on mathematics experiences aren't worth the time and expense.	1.7226	*		*

Table 4.106

Beliefs Regarding the Learning of Mathematics – Conceptual

Survey Item Number	Baylor's Mean	Country L	USA	Country H
7. In additional to getting a right answer in mathematics, it is important to understand why the answer is correct.	5.4732	*		
8. Teachers should allow pupils to figure out their own ways to solve mathematical problems,	4.8796			
11. Time used to investigate why a solution to a mathematical problem works is time well spent.	5.1519	*		*
12. Pupils can figure out a way to solve mathematical problems without a teacher's help.	4.4531		*	*
13. Teachers should encourage students to find their own solutions to mathematical problems even they are inefficient.	4.1996			
14. It is helpful for pupils to discuss different ways to solve particular problems.	5.4163			*

The fifth and final table, Table 4.107, is an overview of the statistical significance of the survey items relating to the beliefs regarding achievement in mathematics. These questions were not affiliated with either calculational or conceptual viewpoints; therefore, there is only one summary table for this research question.

Table 4.107

Beliefs Regarding Achievement in Mathematics

Survey Item	Baylor's Mean	Country L	USA	Country H
1. Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.	2.6388	*		*
2. To be good at mathematics you need to have a kind of "mathematical mind."	3.0583	*		*
3. Mathematics is a subject in which natural ability matters a lot more than effort.	2.4954	*		
4. Only the more able pupils can participate in multi-step problem solving activities.	2.2711	*		*
5. In general, boys tend to be naturally better at mathematics than girls.	2.3413	*		
6. Mathematical ability is something that remains relatively fixed throughout a person's life.	2.8837			
7. Some people are good at mathematics and some aren't.	3.7040	*		*
8. Some ethnic groups are better at mathematics than others.	2.4478			

Conclusion

The findings of the statistical analyses performed on the datasets were presented in this chapter. Statistical significance was noted between Baylor and Country L and Country H but only once between Baylor and USA. Summaries of the calculational viewpoints and the conceptual viewpoints were also provided. In the following chapter, discussion about the meanings and implications of these findings will be presented.

CHAPTER FIVE

Discussion

Historically, different types of interventions to affect change in student achievement have been implemented: methods teachers utilize in their classrooms, professional development to assist current in-service teachers, and researching how students learn. The TEDS-M research study provided data for researchers to study mathematics teacher education programs as well as PSTMs enrolled in those programs. This research study was designed to address one component of that exhaustive project: what beliefs do PSTMs hold toward the end of their program of study and does it reflect the current conceptual and inclusive viewpoints that research shows to be beneficial?

Rather than restrict the research study to only Baylor PSTMs, an effort was made to provide context. This was accomplished by including data from countries that participated in TEDS-M. The choices were culled further by identifying countries whose students had also participated in the most recent TIMSS and PISA (2015) studies. Country L was chosen from countries whose students ranked on the lower end of the scale; the USA was chosen because it is the country where Baylor University is located; and Country H was chosen from countries whose students ranked on the higher end of the scale.

Connection to the Research Questions

Research Question One

What beliefs do Baylor PSTMs hold regarding the nature of mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in the TEDS-M?

Nature of Mathematics – Calculational: Baylor's mean on the six questions in this cluster had a range of 4.0231 to 5.0000. These values would be comparable to ranking between *slightly agree* to *agree* on the Likert scale. There was no statistical significance at all between Baylor and the USA; only one question was statistically significant between Baylor and Country H. Because research has shown that learning is deeper and more meaningful when emphasis is not completely on a calculational viewpoint, these results show a modicum of agreement yet a striking disconnect with the responses from the PSTMs from Country L. These results could lend even more credence to the literature from Chapter Two that supports a more progressive, conceptual viewpoint of the nature of mathematics.

The following bullet points expand on Table 4.103, addressing the relationship between Baylor's mean and whether there was significance or not.

- *Mathematics is a collection of rules and procedures that prescribe how to solve a problem.*

  Baylor's mean was 4.5874, aligning with the *agreement* side of the Likert scale; there was no significance between Baylor and the countries noted in the analysis. This is a closed definition, not allowing for discovery, creativity, or the exploration of a variety of methods that could solve a problem. Agreement with this statement could stem from simply acknowledging the existence of definitions, rules, and procedures. The PSTMs from all three countries believed similarly to Baylor's PSTMs.

- *Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.*

  Baylor's mean was 4.6636, aligning with the *agreement* side of the Likert scale; there was significance noted between Baylor and Country L; Baylor's mean was lower than the mean of Country L indicating the PSTMs of Country L more strongly agree with the statement. The phrasing of the question allows for the involvement of the remembering and applying but does not exclude creativity and discovery on the part of the students. A conceptual viewpoint could be supported in this statement; for example, if students discover patterns and rules on their own, they may apply those findings in solving problems later.

- *When solving mathematical tasks, you need to know the correct procedures else you would be lost.*

  Baylor's mean was 4.0231, barely aligning with the *slightly agree* portion of the Likert scale. There was significance noted between Baylor and Country L; Baylor's mean was lower than the mean of Country L indicating the PSTMs of Country L more strongly agree with the statement. The question seems to indicate there are only certain ways to solve problems and, without knowing those methods, students would be confused.

- *Fundamental to mathematics is its logical rigor and preciseness.*

  Baylor’s mean was 4.5706, aligning with the *agreement* side of the Likert scale. There was significance noted between Baylor and Country H; Baylor’s mean was lower than the mean of Country H indicating the PSTMs of Country H more strongly agree with the statement. Although supporting a calculational framework in intent, this statement upholds the logic inherent in mathematics as well as the detail necessary in carrying out mathematics. Agreement with the statement does not negate the conceptual framework.

- *To do mathematics requires much practice, correct application of routines, and problem solving strategies.*

  Baylor’s mean was 5.0000, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country L. The mean of Baylor is lower than that of Country L, indicating the PSTMs of Country L have a stronger agreement with the statement. Much practice and diligence is required when exploring mathematics; applying the wrong methodologies to mathematics will not result in understanding.

- *Mathematics means learning, remembering, and applying.*

  Baylor’s mean was 4.7865, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country L. The mean of Baylor is lower than that of Country L, indicating the PSTMs of Country L have a stronger agreement with the statement. The tone of the statement leaves little room for the conceptual framework; however, learning, remembering, and applying within discovery would support the conceptual framework.

The significance among the statements in the preceding bullet points in this section, without fail, indicated that Baylor had a lower mean in comparison to the countries that showed significance. That significance can be generalized, showing that Baylor does not agree as strongly with calculational statements; when there was significance indicated, the significance indicated a stronger agreement with the calculational statements than Baylor.

Nature of Mathematics – Conceptual: Baylor's mean on the six questions in this cluster had a range of 4.7524 to 5.3638. These values would be comparable to *agree* on the Likert scale. There was no significance whatsoever between Baylor and the USA or Baylor and Country L. However, Baylor and Country H had significant results for four of the six questions. Once again, Baylor and the USA have similar results; the significance between Baylor and Country H does not necessarily support the premise that conceptual, progressive views will increase understanding. Since the students in Country H are among the top scoring on PISA and TIMSS, it would appear on the surface that those students have a very solid understanding of the nature of mathematics. Another point of interest is the general consensus among Baylor PSTMs and Country L PSTMs. Because these TEDS-M questions address a conceptual view, a higher rate of significance between Baylor and Country L was anticipated because of the lower performance of Country L on the PISA and TIMSS tests (2015).

The following bullet points expand on Table 4.104, addressing the relationship between Baylor's mean and whether there was significance or not.

- *Mathematics involves creativity and new ideas.*

  Baylor's mean was 4.7524, aligning with the *agreement* side of the Likert scale; there was no significance between Baylor and the countries noted in the analysis indicating the PSTMs from the other three countries believed essentially the same as the PSTMs from the other countries. There is agreement that mathematics is also conducive to letting students create and develop their own ideas while exploring.

- *In mathematics, many things can be discovered and tried out for oneself.*

  Baylor's mean was 4.944, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country H. The mean of Baylor is higher than that of Country H, indicating the PSTMs of Baylor have a stronger agreement with the statement. This statement strongly supports a conceptual framework, allowing for students' discovery rather than passively taking in information.

- *If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).*

  Baylor's mean was 5.0551, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country H. The mean of Baylor is higher than that of Country H, indicating the PSTMs of Baylor have a stronger agreement with the statement. This statement could also align with the statement in the previous section – *To do mathematics requires much practice, correct application of routines, and problem solving strategies* – explaining why Baylor had a higher mean on that particular question.

- *Mathematical problems can be solved correctly in many ways.*

  Baylor's mean was 5.3638, aligning with *agree* on the Likert scale. There is no significance between the means of Baylor and the other countries indicating similar beliefs among the PSTMs.

- *Many aspects of mathematics have practical relevance.*

  Baylor's mean was 5.2126, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country H. The mean of Baylor is higher than that of Country H, indicating the PSTMs of Baylor have a stronger agreement with the statement. Rather than seeing mathematics as abstract, agreement with this statement would indicate that mathematics is applicable in everyday life.

- *Mathematics helps solve everyday problems and tasks.*

  Baylor's mean was 5.3394, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country H. The mean of Baylor is higher than that of Country H, indicating the PSTMs of Baylor have a stronger agreement with the statement. Just as the preceding statement indicated, agreement with this statement removes the stigma of mathematics being simply rules, algorithms, and procedures and gives importance to application in a person's daily life.

The significance among the statements in the preceding bullet points in this section, without fail, indicated that Baylor had a higher mean in comparison to the countries that showed significance. That significance can be generalized, showing that

Baylor PSTMs hold a stronger agreement with conceptual statements compared to PSTMs of the countries on the statements indicating significance.

Summary of the Nature of Mathematics: Overall, Baylor's mean never entered the area indicating disagreement in the Likert scale in either the calculational or conceptual statements in this section. However, in the calculational statements, any significance found in the data showed that Baylor had a lower mean when compared to the countries having significance. This shows that Baylor PSTMs do not agree as strongly with the calculational statements as the PSTMs in the other countries. Likewise, in the conceptual statements, any significance noted in the analyses indicated the means of Baylor were higher than those of the countries with significance.

The authors of TEDS-M did not force PSTMs to identify solely with calculational beliefs or conceptual beliefs; however, they did hypothesize that PSTMs would identify more strongly with one set of beliefs over the other. The means of Baylor were higher in the conceptual statements and slightly lower in the calculational statements (Tatto, et al, 2012).

The research of Ernest (1989) and Carney (2014) indicates that teacher beliefs regarding the nature of mathematics have significant impacts in classroom instruction and student achievement. Philipp (2007) and Staub and Stern (2002) classify beliefs of a conceptual or progressive nature where mathematics is discoverable, different methodologies are encouraged, and there is practical relevance in mathematics. The beliefs of Baylor PSTMs aligned more strongly with these findings and did support a conceptual belief system. Where statistical significance was found, the Baylor PSTMs had a higher agreement with conceptual statements than did the countries showing significance.

Research Question Two

What beliefs do Baylor PSTMs hold regarding the learning of mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in the TEDS-M?

Learning of Mathematics – Calculational: Baylor's mean on the seven questions in this cluster range from 1.7226 to 3.8044, which all fall from *disagree* to almost *slightly agree* on the Likert scale. Just as in the nature of mathematics calculational cluster, five of these seven questions showed significance between Baylor and Country L. Only two questions were significant between Baylor and Country H. Not surprisingly, there is no significant difference present between Baylor and the USA.

The following bullet points expand on Table 4.105, addressing the relationship between Baylor's mean and whether there was significance or not.

- *The best way to do well in mathematics is to memorize all the formulas.*
  Baylor's mean was 3.1470, aligning with the *disagreement* side or even a neutral area (between the 1-3 of *disagreement* and 4-6 of *agreement*) of the Likert scale; there was no significance between Baylor and the countries noted in the analysis indicating the PSTMs from the other three countries believed essentially the same as the PSTMs from the other countries. In general, PSTMs do not agree that rote memorization is the best route to master understanding of mathematics.

- *Pupils need to be taught exact procedures for solving mathematical problems.*

  Baylor's mean was 3.8044, aligning with the *disagreement* side or even a neutral area (between the 1-3 of *disagreement* and 4-6 of *agreement*) of the Likert scale. There is significance between the means of Baylor and Country L. The mean of Baylor is lower than that of Country L, indicating the PSTMs of Country L have a stronger agreement with the statement. The tone of the statement leaves little room for the conceptual framework; there is no allowance for discovery or creativity.

- *It doesn't really matter if you understand a mathematical problem, if you can get the right answer.*

  Baylor's mean was 2.1590, aligning with *disagree* on the Likert scale. There is no significance among the means of Baylor and the other countries. This statement is purely calculational with no regard for fostering student understanding or ownership of the learning experience. The lack of significance indicates a common disagreement with the statement.

- *To be good in mathematics you must be able to solve problems quickly.*

  Baylor's mean was 2.5298, aligning with *disagree* on the Likert scale. There is significance between the means of Baylor and both Country L and Country H. The mean of Baylor is lower than both means of Country L and Country H, indicating the PSTMs of Baylor more strongly disagree with the statement. Again, the strictly calculational tone of this statement allows no room for student discovery or development of understanding while processing problems.

- *Pupils learn mathematics best by attending to the teacher's explanations.*

  Baylor's mean was 3.3042, aligning with the *disagreement* side or even a neutral area (between the 1-3 of *disagreement* and 4-6 of *agreement*) of the Likert scale. There is significance between the means of Baylor and Country L. The mean of Baylor is lower than that of Country L, indicating the PSTMs of Country L have a stronger agreement with the statement. Again, the statement strongly supports a calculational viewpoint where student learning is more passive than active.

- *When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.*

  Baylor's mean is 2.1132, aligning with *disagreement* on the Likert scale. There was significance between Baylor and Country L as well as Country H. Baylor's mean was higher than both of the means of the other two countries indicating the PSTMs of the other two countries had a stronger disagreement than did the PSTMs of Baylor. This single statement does not follow the pattern of every other occurrence of significance in the calculational statements: the PSTMs of the other two countries more strongly disagree with the statement than do Baylor PSTMs. It should be noted that the Baylor PSTMs' mean still lands in the *disagreement* range.

- *Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.*

  Baylor's mean was 2.6136, aligning with *disagreement* on the Likert scale. There was significance between Baylor and Country L. The mean of Baylor is lower than that of Country L, meaning PSTMs of Baylor disagrees more strongly with the statement than do the PSTMs of Country L. The term “standard” procedure implies that the discovery and creativity of students would not be encouraged since only preferred methods would be advocated.

- *Hands-on mathematics experiences aren’t worth the time and expense.* Baylor’s mean was 1.7226, aligning with *disagreement* on the Likert scale. There was significance between Baylor and Country L as well as Country H. Baylor’s mean was lower than both of the means of the other two countries indicating Baylor’s PSTMs had a stronger disagreement than did the PSTMs of the other two countries. This statement once again is purely calculational, not providing any ownership to the students in their learning experiences.

The significance among the statements in the preceding bullet points in this section indicated that Baylor had a lower mean in comparison to the countries that showed significance with the one exception noted. The significance can be generalized, showing that Baylor PSTMs hold a stronger disagreement with calculational statements compared to PSTMs of the countries on those statements indicating significance.

Learning of Mathematics – Conceptual: Of the seven questions in this cluster, the range of Baylor’s mean is from 4.1996 to 5.4732. In this cluster, the only result that found a significance between Baylor and the USA is identified: this one statement addressed the ability of students to find an answer without the intervention of a teacher. Three questions were significant between Baylor and Country L while four were significant between Baylor and Country H.

The following bullet points expand on Table 4.106, addressing the relationship between Baylor’s mean and whether there was significance or not.

- *In addition to getting the right answer in mathematics, it is important to understand why the answer is correct.*

  Baylor's mean was 5.4732, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country L. The mean of Baylor is higher than that of Country L, indicating the PSTMs of Baylor have a stronger agreement with the statement. Conceptually, students need to make sense of their answers and determine if the method they used is valid.

- *Teachers should allow pupils to figure out their own ways to solve mathematical problems.*

  Baylor's mean was 4.8796, aligning with *agree* on the Likert scale. There is no significance between Baylor's mean and the means of the other participating countries. The statement allows for students to take ownership of their learning and develop methodologies that facilitate their own understanding.

- *Time used to investigate why a solution to a mathematical problem works is time well spent.*

  Baylor's mean was 5.1519, aligning with *agree* on the Likert scale. There is significance between the means of Baylor and Country L and Country H. The mean of Baylor is higher than that of both Country L and Country H, indicating the PSTMs of Baylor have a stronger agreement with the statement. The understanding of the mechanics behind processes developed by students will further enhance their understanding of the mathematical concepts themselves.

- *Pupils can figure out a way to solve mathematical problems without a teacher's help.*

  Baylor's mean on this statement was 4.4531, indicating *agreement* on the Likert scale. There is significance between the means of Baylor PSTMs and Country H PSTMs and this statement is the only statement of the 34 survey questions where Baylor and the USA results are also significant. Baylor's mean is higher than those of the USA and Country H, indicating a stronger agreement with the statement. A conceptual framework allows for students to develop their own methods of solving problems; however, the significance in the analyses could possibly be due to the teacher's role as facilitator rather than passive observer.
- *Teachers should encourage students to find their own solutions to mathematical problems even if they are inefficient.*

  Baylor's mean on this statement was 4.1996, indicating *slightly agree* on the Likert scale. There was no significance found between Baylor and the three countries; the beliefs of the PSTMs were mostly consistent. The slightly lower mean on this question could be due to the fact that part of the appeal of mathematics is the efficiency with which problems can be solved; this trait could be learned over time.
- *It is helpful for pupils to discuss different ways to solve particular problems.*

  Baylor's mean on this statement was 5.4163, indicating *agree* on the Likert scale. There was significance between Baylor's mean and the mean of Country H. Baylor's mean was higher than that of Country H, indicating a higher incidence of agreement with the statement. Part of the conceptual framework is students being able to discuss their findings. Discussion would open doors for other students to explore methodologies that perhaps had not occurred to them, thereby increasing their understanding.

The significance among the statements in the preceding bullet points in this section indicated that Baylor had a higher mean in comparison to the countries that showed significance. The significance can be generalized, showing that Baylor PSTMs hold a stronger agreement with conceptual statements compared to PSTMs of the countries on those statements indicating significance. As noted, this section contains the one instance of Baylor and USA having significance between their means.

Summary of the Learning of Mathematics: Overall, Baylor's mean never entered the area indicating agreement in the calculational statements; some of the means did fall between a Likert scale value of 3 and 4 which is between slightly disagree and slightly agree – a neutral zone of sorts. Baylor's mean also never entered the area indicating disagreement in the Likert scale in the conceptual statements in this section. However, in the calculational statements, any significance found in the data showed that Baylor had a lower mean when compared to the countries having significance with the one exception noted in bulleted list. Generally, this shows that Baylor PSTMs do not disagree more strongly with the calculational statements as the PSTMs in the other countries. Likewise, in the conceptual statements, any significance noted in the analyses indicated the means of Baylor were higher than those of the countries with significance, indicating Baylor PSTMs agreed more strongly with the conceptual statements.

As Weldeana and Abraham (2014) perpetuated in their research, teacher beliefs, consciously or subconsciously, regarding the learning of mathematics can negatively or positively affect students. A more calculational belief system can undermine students' self-efficacy, cause them to feel helpless when learning mathematics, and overall affect their self-esteem and self-confidence. However, teachers holding a conceptual belief system regarding the learning of mathematics have seen a higher self-efficacy in their students and more confidence in their learning and themselves. Weldeana and Abraham incorporated research from other studies that also found a conceptual belief system regarding the learning of mathematics benefits students: Boaler (1997), Madden (2008), McLeod (1992), Renga and Dalla (1993), and Stevens (2005).

The beliefs of Baylor PSTMs aligned more strongly with these findings and did support a conceptual belief system. Where statistical significance was found, the Baylor PSTMs had a higher agreement with the conceptual statements than did the countries showing significance.

Research Question Three

What beliefs do Baylor PSTMs hold regarding student achievement in mathematics and in what manner do their beliefs compare to the PSTMs at other institutions in the USA and the other identified countries participating in the TEDS-M?

Achievement in Mathematics: There is a striking amount of significant differences between Baylor and County L and Country H in this set. Of the eight questions, six of those showed significance between Baylor and Country L; none between Baylor and the USA; and four between Baylor and Country H. There are several incidents of significance within these questions. This final cluster of survey statements are not classified according to calculational or conceptual beliefs; therefore, all eight statements are grouped together and will be discussed as a complete group.

The following bullet points expand on Table 4.107, addressing the relationship between Baylor's mean and whether there was significance or not.

- *Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.*

  Baylor's mean on this statement is 2.6388 aligning with *disagreement* on the Likert scale. There is significance between Baylor's mean and the means of both Country L and Country H. Baylor's mean is lower than the other two, indicating a stronger disagreement with the statement than the PSTMs of Country L and Country H.
- *To be good at mathematics, you need to have a kind of "mathematical mind."*

  Baylor's mean on this statement is 3.0583, indicating *slightly disagree* on the Likert scale. Again, there is significance between the PSTMs of Baylor and those of Country L and Country H. Baylor's mean is lower than those of Countries L and H indicating a stronger disagreement with the statement.
- *Mathematics is a subject in which natural ability matters a lot more than effort.*

  Baylor's mean was 2.4954 on this statement, indicating *disagree* on the Likert scale. One instance of significance was found in the analysis of this question; Baylor's mean was significantly lower than that of Country L, indicating a stronger disagreement with the statement.

- *Only the more able pupils can participate in multi-step problem solving activities.* Baylor's mean on this statement was 2.2711, indicating *disagree* on the Likert scale. There was significance between the means of Baylor and Country L as well as between Baylor and Country H. Baylor's mean was significantly lower than those of the other countries, indicating a stronger disagreement with the statement.
- *In general, boys tend to be naturally better at mathematics than girls.* Baylor's mean on this statement was 2.3413, indicating *disagree* on the Likert scale. There was significance between Baylor and Country L, with Baylor having a lower mean than that of Country L. This indicates a stronger disagreement with the statement.
- *Mathematical ability is something that remains relatively fixed throughout a person's life.* The mean of Baylor PSTMs on this statement was 2.8837, indicating *disagreement* on the Likert scale. There was no significance between Baylor and the other three countries indicating a similar view on this statement among the PSTMs participating in the survey.
- *Some people are good at mathematics and some aren't.* Baylor's mean on this question fell at 3.7040, which is in a natural zone or headed towards *slightly agree*. Both Countries L and H had significant results compared to Baylor; both of their means were higher than Baylor's indicating they agreed at least somewhat with the statement.

- *Some ethnic groups are better at mathematics than others.*

  Baylor's mean was 2.4478 on this survey item, indicating *slightly disagree* on the Likert scale. There was no significance noted between Baylor and any of the other three countries.

Summary of Achievement in Mathematics: Since all eight of these survey questions take on a more antiquated lens of the learners of mathematics, this summary will address the eight questions as a whole. Overall, Baylor's mean fell in the range of 1-3 on the Likert scale, indicating some level of disagreement, with the exception of two of the statements. Both of those statements fell in the range between 3 and 4, falling between a slightly disagree to slightly agree interpretation.

Country L was significantly different on every statement except for two. Their means were higher than the means of Baylor, indicating they did not disagree with those statements as strongly as the Baylor PSTMs did.

The beliefs of Baylor's PSTMs align more strongly with what research indicates is more beneficial. Tang and Hsieh (2014), Upadyaya and Eccles (2014), Holder and Kessels (2017), and Parks and Kennedy (2007) have all found through their research studies that beliefs regarding student achievement that are biased toward gender, age, or ethnicity does have an effect on students. Students can be obscurely or overtly affected by beliefs of teachers; consequently, student achievement can be affected. Having beliefs of equity and tolerance between genders, among ethnicities, and acknowledging that all ages can learn mathematics is supported by research to benefit students. Baylor's PSTMs align strongly with this belief system.

Future Research

Based on the findings, indications, and limitations of this study, the following items are noted for further research.

- The TEDS-M study focused on PSTMs in the final year of their program. A qualitative, longitudinal study examining beliefs of PSTMs as they progress through their program would provide further insight into the effect the teacher program has upon PSTM beliefs.
- Studies across grade bands is also recommended. Methods courses for primary PSTMs are different than those of secondary PSTMs. The belief systems may be different for PSTMs in preparing to teach in higher secondary grades.
- Research involving the reflections of PSTMs on their own belief systems is warranted. Do their beliefs affect their teaching in the classroom? Is what they say they believe reflected in what they do in the classroom?
- Do Baylor PSTMs enter the program already believing what this study purports or are their beliefs affected by Baylor's teacher education program?

Conclusions

In 2017, AMTE published Standards specifically designed to address the characteristics to be cultivated in PSTMs before their official entrance into classrooms as practicing teachers. Affecting change in the design of teacher education programs is a necessary step towards fulfilling these standards.

In Indicator C.1.3 of the AMTE Standards, the call for PSTMs to show productive mathematical dispositions is made. Mathematics should be seen as worthwhile to the PSTMs as well as to the students with which they interact. The Standards recognize that all people have the capability of thinking mathematically. PSTMs should exhibit “a commitment to sense making in mathematical thinking, teaching, and learning and to develop[ing] *habits of mind*, including curiosity, imagination, inventiveness, risk-taking, and persistence.” By developing these characteristics, PSTMs will have the courage to be able to utilize their own critical thinking skills in order to investigate alternative methods of learning and instruction.

In Chapter Nine of the AMTE Standards, special notice is given to equity in mathematics. There is a need to “confront an education system that marginalizes and disenfranchises a significant part of our citizenry.” This statement is based on previous research conducted by AMTE (2015) and NCTM (2000, 2014a, 2014b). The authors of the Standards offer assumptions that should be made about the content of every teacher education program. Among those assumptions is that a deep, integrated focus on equity within each program that prepares PSTMs must be present in order to assure that every individual learner can experience success. Educators of PSTMs must ensure that equity, social justice, and diversity are present consistently. The Standards recognize that changing superficialities will not be enough; beliefs must be addressed and hard questions must be asked and answered by educators and PSTMs in order to advocate equity in the achievement of mathematics. As stated in the Preliminaries 3.3 of the Standards, “Such analysis is essential because the current mathematics education system is unjust and grounded in a legacy of segregation, systems of power and privilege, and deficit thinking based on race, ethnicity, class, language, and gender.”

Utilizing the most recent Standards published by AMTE (2017), the continuing need for early, intentional monitoring of beliefs of PSTMs in teacher education programs has been addressed. These interventions would serve to strengthen the conceptual belief systems of students in teacher preparation programs as well as continuing to strive towards eradicating pre-existing bias among gender and ethnic groups. The beliefs of Baylor's PSTMs that participated in the TEDS-M study appear to already align with what research indicates are healthy beliefs that promote student achievement and understanding which suggests the teacher education program at Baylor cultivates a conceptual belief system regarding the nature and learning of mathematics and a non-biased belief system regarding how students achieve in mathematics.

APPENDICES

APPENDIX A

PSTM Beliefs Questions from TEDS-M

Beliefs Regarding the Nature of Mathematics

1. Mathematics is a collection of rules and procedures that prescribe how to solve a problem.
2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.
3. Mathematics involves creativity and new ideas.
4. In mathematics many things can be discovered and tried out by oneself.
5. When solving mathematical tasks you need to know the correct procedures else you would be lost.
6. If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).
7. Fundamental to mathematics is its logical rigor and preciseness.
8. Mathematical problems can be solved correctly in many ways.
9. Many aspects of mathematics have practical relevance.
10. Mathematics helps solve everyday problems and tasks.
11. To do mathematics requires much practice, correct application of routines, and problem solving strategies.
12. Mathematics means learning, remembering, and applying.

Beliefs Regarding the Learning of Mathematics

1. The best way to do well in mathematics is to memorize all the formulas.
2. Pupils need to be taught exact procedures for solving mathematical problems.
3. It doesn’t really matter if you understand a mathematical problem, if you can get the right answer.
4. To be good in mathematics you must be able to solve problems quickly.
5. Pupils learn mathematics best by attending to the teacher’s explanations.
6. When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.
7. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
8. Teachers should allow pupils to figure out their own ways to solve mathematical problems.
9. Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.
10. Hands-on mathematics experiences aren’t worth the time and expense.
11. Time used to investigate why a solution to a mathematical problem works is time well spent.
12. Pupils can figure out a way to solve mathematical problems without a teacher’s help.
13. Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.
14. It is helpful for pupils to discuss different ways to solve particular problems.

Beliefs Regarding Mathematics Achievement

1. Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.
2. To be good at mathematics you need to have a kind of "mathematical mind."
3. Mathematics is a subject in which natural ability matters a lot more than effort.
4. Only the more able pupils can participate in multi-step problem solving activities.
5. In general, boys tend to be naturally better at mathematics than girls.
6. Mathematical ability is something that remains relatively fixed throughout a person's life.
7. Some people are good at mathematics and some aren't.
8. Some ethnic groups are better at mathematics than others.

APPENDIX B

Affidavit of Non-Disclosure

International Association for the
Evaluation of Educational Achievement

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50 years of international educational research

Affidavit of Nondisclosure and User Agreement for Using the TEDS-M 2008 Databases

To be completed by anyone seeking to use the TEDS-M 2008 International Database

By signing this form, I agree that when using TEDS-M data collected by the IEA, or its designees, I will not:

1. use or reveal any individually identifiable information furnished, acquired, retrieved or assembled by me or others, for any purpose other than statistical reporting purposes;
2. make any disclosure or publication whereby any sampled unit or respondent (including, but not limited to, students, teachers, faculty, schools, or institutions) could be identified, or the data furnished by or related to a sampled unit or respondent could be identified;
3. match the records on the subject data to any other data files so as to re-identify the survey units on the files, even if the matching has a valid statistical purpose;
4. distribute the TEDS-M database or make it accessible in any electronic network to anyone;
5. disclose subject data for any administrative purposes nor use it in any manner to change the status, condition, or public perception of any sampled unit or respondent regarding whom subject data is maintained;
6. make disclosures of individual subject data to any individual;
7. make any publication or other release of subject data listing information regarding individual sampled units or respondents even if the individual respondent identifiers have been removed.

I also understand and agree to the following:

1. I may publish the results, analysis, or other information developed as a result of any research based on subject data covered by the affidavit only in summary or aggregated statistical form so that the identity of individual sampled units or respondents contained in the subject data is not revealed;
2. I will acknowledge the source of the data in all results, publications, documents and communications by providing a source citation next to each table and graph in the following form: "Source: IEA TEDS-M 2008";
3. When publication or other release of research results could raise reasonable questions regarding disclosure of individually identifiable information contained in subject data, I will provide copies of the proposed publication or release to the IEA Secretariat, using the contact information listed below, before that disclosure is made so that IEA may advise whether the disclosure is authorized;

TEDS-M 2008 Affidavit of non-disclosure and user agreement page 1 of 3

4. I will not publish or otherwise release research results if the IEA advises that such disclosure is not authorized;
5. When using the TEDS-M 2008 databases, I will not reveal information identifying the German Bundesländer;
6. I shall notify IEA immediately upon receipt of any legal, investigatory, or other demand for disclosure of individually identifiable data;
7. I will notify IEA immediately upon discovering any disclosure of subject data to unauthorized parties or agencies;
8. I will return to the IEA all individual subject data as well as all notes, reports, files, any kind of documents and copies of the same, or destroy those data under IEA supervision or by approved IEA procedures, when any of the clauses above are breached, or whenever I have finished working with the data, whichever occurs first;
9. Upon request, I will provide the IEA with written confirmation that I have complied with numeral 8 above, and that I have not given access to these items to any third person.
10. I have no ownership or any other intellectual property of the subject data and any related documentation or accompanying software which at all times shall be and remain the sole and exclusive property of the IEA.
11. I shall at all times indemnify and save harmless the IEA and its officers, servants and agents from and against all claims, losses, damages, costs, expenses, actions and other proceedings made, sustained, brought, prosecuted, threatened to be brought or prosecuted, in any manner based upon, caused by, or in any way attributable to the use of the subject data and related documentation provided pursuant hereto.
12. I will notify the IEA within 15 days of any changes happening to the contact information provided below.

Signed and agreed by:

Name: Sandra J. Bentley

Organization: Baylor University, Waco, Texas

Title: Doctoral Candidate - Curriculum and Teaching

Signature and Date: Sandra J. Bentley 3/21/13

For Students, please ask a representative of your education institution to sign this agreement:

Name of Representative: Trena L. Wilkerson Trena L. Wilkerson

Title: Associate Professor Baylor University

Signature and Date: Trena L. Wilkerson 3/21/13

Please print and complete this form manually, and send it (scanned copy) by email or fax to the IEA Secretariat:

Fax: +31 20 420 7136
Email: department@iea.nl (please use subject: TEDS-M 2008 Database Request)

Or by post mail to:

IEA Secretariat
TEDS-M Databases
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