ABSTRACT

Count Regression Models With a Misclassified Binary Covariate:
A Bayesian Approach

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Mismeasurment, and specifically misclassification, are inevitable in a variety of regression applications. Fallible measurement methods are often used when infallible methods are either expensive or not available. Ignoring mismeasurement will result in biased estimates for the associated regression parameters. The models presented in this dissertation are designed to correct this bias and yield variance estimates reflecting the uncertainty that is introduced by flawed measurements. We consider a generalized linear model for a Poisson response. This model accounts for the misclassification associated with the binary exposure covariate. In the first portion of the analysis, diffuse priors are utilized for the regression coefficients and the effective prior sample size technique is implemented to construct informative priors for the misclassification parameters. In the second portion of the analysis we place informative priors on the regression parameters and diffuse priors on the misclassification parameters. We also present results of a simulation study that incorporates prior information for both the regression coefficients and the misclassification parameters.

Next, we extend the Poisson model with a single binary covariate in various ways, including adding a continuous covariate and accounting for clustering through the use of random effects models. We also consider a zero-inflated version of the model. Simulation studies are summarized for each extension.

Finally, we discuss an application in which frequentist and Bayesian logistic regression models are used to predict prevalence of high BMI-for-age among preschool-aged children in Texas.

Count Regression Models With a Misclassified Binary Covariate:
A Bayesian Approach

by

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DEDICATION

To Christopher and Caroline,

Gracias por jugar en mi equipo.

CHAPTER ONE

Introduction

1.1 Overview

The Poisson distribution arises naturally in the study of data taking the form of counts; for instance, a major area of application is epidemiology, where the incidence of diseases is studied. While this dissertation will focus on Bayesian modeling of problems that occur within healthcare-related research topics, the models presented herein can be easily adapted to other scenarios.

In Section 1.2 we introduce the Bayesian Poisson regression model that will serve as the foundation for investigations presented in Chapters 2 and 3. We introduce the notion of covariate mismeasurement in Section 1.3 and briefly discuss the impact experienced when the mismeasurement is not accounted for in a model. In Section 1.4 we outline three specific model extensions that will account for complications regularly encountered in the study of Poisson count outcomes. In Section 1.5 we acquaint the reader with an application in which we use logistic regression to model prevalence of high BMI-for-age in children. We conclude with a dissertation plan in Section 1.6.

The statistical literature is replete with analyses using Poisson rates and counts. This reflects the importance of such data in many fields of study as well as the inherent flexibility of Poisson models. Here are just a few examples: Poisson models have been used for literary analyses, as Mosteller and Wallace (1964) did for the Federalist papers and Efron and Thisted (1976) did with Shakespeare's works. Many medical applications involve Poisson data, including the Rand Health Insurance Experiment work by Keeler and Rolph (1988) that estimated the effects of coinsurance on cost per treatment episode and the number of treatment episodes. A multivariate

Poisson-gamma mixture distribution was used by Nelson (1985) to model correlation and heterogeneity in cross-sectional analyses of felony infractions and of criminal victimization data. Lawless (1987) used a Poisson generalized linear model and a profile log-likelihood to analyze random effects for tumor data, while a mixed effects Poisson regression model was offered by Vonesh (1990) to analyze risk factors associated with continuous ambulatory peritoneal dialysis. Wolfe, Petroni, McLaughlinN, and McMahon (1991) compared extra-Poisson variance models with Poisson error models for estimating diagnosis-specific hospital discharge rates. Disease incidence and mortality rates were analyzed by Bernardinelli and Montomoli (1992) using Gibbs methods and Poisson empirical Bayes methods developed by Clayton and Kaldor (1987). Papageorgiou and Loukas (1988) compared "double-zero" proportion and maximum likelihood estimators for the negative binomial-Poisson distribution using traffic accident data. Christiansen and Morris (1997) fit parametric random-effects models to survival data for transplant patients. Home run data have been analyzed by Albert (1992, 2007) using various Poisson hierarchical methods. More recently, Gibbons et al. (2008) explored a Bayesian mixed effects Poisson model to model rare adverse events in order to study the relationship between anti-depressants and suicide.

Generalized linear models, and Poisson models in particular, are most relevant in epidemiology. In prospective cohort studies, where the researcher is interested in learning about the association between exposure and disease, the Poisson regression model has proven to be an important alternative to the proportional hazards model (Breslow and Day, 1987; Preston, 1998). A grouping of the data into an event-time table classifying cases and person-years by the set of time intervals and covariate categories forms the basis for Poisson regression analysis. Veierod and Laake (2001) presented this as an advantage in large cohort studies, when contrasted

to the proportional hazards model. In clinical trials with discrete data and low probability of events, a Poisson model is the natural and most popular choice (Zaslavsky, 2009).

1.2 The Bayesian Poisson Model

Bayesian Poisson methods are a natural framework for the continual updating inherent in adverse event reporting and for clinical trials in which historical data or expert opinion are readily available. Paramount among advantages associated with the Bayesian paradigm is its consistency with the laws of probability. Inferences in the Bayesian paradigm are easily interpreted and communicated. Furthermore, Bayesian methods can accommodate scenarios that have proven problematic for frequentist methods.

Poisson models are often appropriate when count data do not result from a fixed number of trials. For instance, if y is the number of adverse events due to pharmaceutical drug use in Texas during the coming week, there is no fixed upper limit n for y. The Poisson probability mass function for the j^{th} outcome is

$$f(y_j|\mathbf{x}_j) = \frac{e^{-\lambda_j}\lambda_j^{y_j}}{y_j!}, \qquad y_j = 0, 1, 2, ... \qquad j = 1, ..., J \tag{1.1}$$

In the log linear version of the Poisson regression model, the mean is parameterized as $\lambda_j = \exp(\mathbf{x}_j\boldsymbol{\beta})$ to ensure that $\lambda_j > 0$, where $\mathbf{x}_j = [x_{0j}, ..., x_{k-1j}]$ is a $1 \times k$ vector consisting of an intercept and $k-1$ covariates, and $\boldsymbol{\beta}' = [\beta_0, ..., \beta_{k-1}]$ is a $1 \times k$ vector of regression parameters.

Under the Bayesian framework, prior distributions are required for all unknown parameters in the model. We assume independent priors, $p(\cdot)$, on the k regression parameters introduced above, yielding the joint prior given by

$$p(\beta_0, \beta_1, \ldots, \beta_{k-1}) = p(\beta_0) \times p(\beta_1) \times \ldots \times p(\beta_{k-1}).$$

The resulting joint posterior distribution is

$$p(\beta_0, \beta_1, \ldots, \beta_{k-1}|\mathbf{y}, \mathbf{x}) \propto f(\beta_0, \beta_1, \ldots, \beta_{k-1}|\mathbf{y}, \mathbf{x}) \times p(\beta_0, \beta_1, \ldots, \beta_{k-1}),$$

where $\mathbf{y}$ is a vector of Poisson count responses. For example, we might take each $p(\beta)$ to be a normal distribution with mean μ_β and variance σ^2_β. Such a model is presented in Figure 1.1.

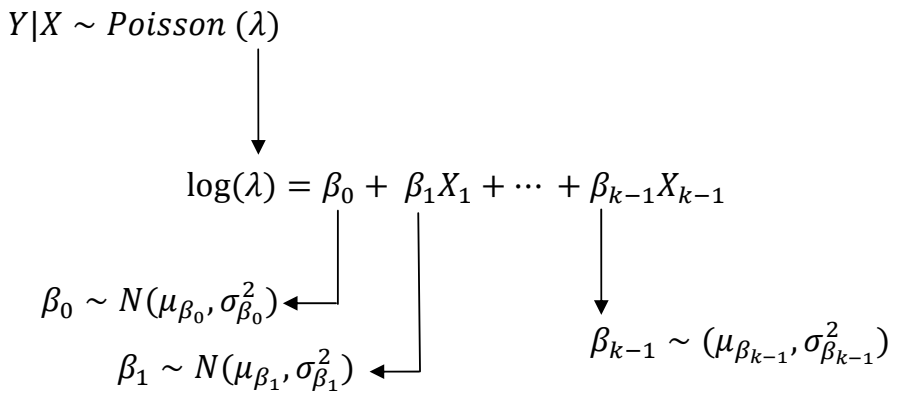


Figure 1.1: Summary of the Bayesian Poisson regression model.

Priors may be chosen to encompass existing knowledge based on results or parameter estimates from historical data or may impart relatively little information in relation to that provided by the data (Congdon, 2005). We consider models for three different prior structures. Initially we assign locally uniform priors to the regression parameters and informative priors to the misclassification parameters. In subsequent simulation studies (see Section 2.5), we consider informative priors for the regression parameters and diffuse priors for the misclassification parameters. We also consider (see Section 2.6) the case where expert opinion or historical data is available for both the regression parameters and the misclassification parameters, and compare the estimates resulting from each of the prior specifications.

1.3 The Impact of Covariate Misclassification

A common problem in medical research is mismeasurement, where the outcome of interest cannot be perfectly observed because of fallible tests or imprecise measurement tools. When a test response is continuous, mismeasurement is called measurement error. When the patient outcome is discrete, mismeasurement is called misclassification. If the Poisson response variable used in a design is estimated using a misclassified covariate, successive decisions may be affected. Safety intervention protocols and/or treatment decisions based on inaccurate responses will compromise the safety and efficiency of the design.

In epidemiologic studies of the relationship between exposure and disease, misclassification of exposure is common and known to introduce bias in the coefficient estimates. The possibility of misclassification should preclude reliance on the maxim that, as long as the misclassification is nondifferential, effect estimates will be conservative underestimates of the true underlying effect estimate. This is because biases both towards and away from null are possible, as illustrated by Veierod and Laake (2001).

Errors in disease exposure classification can yield misleading inferences for covariate effects when the probability of error itself is related to the covariates. More accurate inferences are possible using supplemental data on both true and fallible disease exposure counts at various covariate levels Whittemore and Gong (1991). Using Bayesian modeling techniques, we present a method for incorporating such supplemental data in the form of expert opinion into Poisson regression models via informative priors.

There has been a continuing interest in assessing effects of misclassification on exposure-disease associations, as evidenced by the increase in published misclassification research. Reade-Christopher and Kupper (1991) derived approximate models for assessing the potential bias when misclassification is ignored. More recently,

Veierod and Laake (2001) derived exact expressions for the bias in the Poisson regression coefficients of a categorical exposure variable subject to misclassification. In Chapters 2 and 3, we consider a case in which a binary exposure variable is subject to misclassification. Further assumptions are that the misclassification is nondifferential in variety and that any additional covariates are measured without error.

To examine this problem, we first consider a simple Poisson regression model in which the Poisson count is estimated using a binary exposure covariate subject to misclassification (see Section 2.4). Expanding the work of Liu, Gustafson, Cherry, and Burstyn (2009), we develop a model to incorporate the misclassification in the binary covariate, and results from a simulation experiment are provided to illustrate the effects of ignoring misclassification on conclusions reached by the design.

1.4 Extensions to the Poisson Model w/ Misclassification

A key feature of the Poisson distribution is that its variance equals its mean. Sample counts vary more when their mean is higher. In practice, however, count observations often exhibit variability exceeding that predicted by the Poisson model. This is called *overdispersion*. We may assume that each patient has the same probability of experiencing an adverse event in the next month. More realistically, these probabilities vary, due to factors such as the amount of time in treatment, whether the patient is adherent to a treatment regimen, or even geographical location. Such variation causes adverse event counts to display more variation than predicted by the Poisson model. In Sections 3.3 and 3.4 we present methods that accommodate extra overdispersion through random effects models and zero-inflation models, respectively.

In modeling event counts collected from a clinical trial or after-market safety study, there are usually a relatively large number of zeros (non-events). Commonly

used models such as the Poisson or geometric distributions can underestimate the zero-event probability and hence make it difficult to identify significant covariate effects.

In Section 3.4, we consider model expansions for count responses with excess zeros relative to what standard distributional assumptions, such as the Poisson, can predict. In the literature, "zero-inflated count data" refers to data for which a generalized linear model has lack of fit due to disproportionately many zeros. Such data are common in studies including substantial numbers of subjects prone to zero counts and others yielding non-zero counts. An example of a variable that one might expect to be zero-inflated is the number of times a subject used medical services in the previous year: some subjects may have a zero observation because of chance, whereas others may have a zero observation because they have a fear of doctor's offices or they lack insurance (Deb and Trivedi, 1997).

The zero-inflation model considers the study population a mixture of two subpopulations. One of the subpopulations is regarded as "not at-risk" and must have a response value of zero. This is commonly called a "structural zero". Among subjects in the "at-risk" subpopulation, the responses follow a Poisson distribution, giving a zero-inflated Poisson model (ZIP). Sampling zeros may arise in the at risk population with probability $exp(-\lambda)$, where λ is the Poisson mean. Hence, the membership status is unobservable if the response is zero.

Zero-inflated models for count data have been widely used in statistical science to model a variety of real life count data such as manufacturing defects (Lambert, 1992); sexual behavior (Heilbron, 1994); medical (Bohning, 1998; Cheung, 2002); dental (Bohning et al., 1999; Karlis and Ntzoufras, 2006; Mwalili, Lesaffre, and Declerck, 2008); crime (Famoye and Singh, 2006), and sports data (Karlis and Ntzoufras, 2006). Such models introduce an extra probability parameter to capture an

excess of zero values that cannot be estimated sufficiently by the assumed Poisson model.

In Section 3.2, we extend the Poisson regression model with a misclassified binary covariate to include an additional continuous covariate, and results from subsequent simulation experiments are presented. Finally, in Sections 3.3 and 3.4 random effects and zero-inflation components are incorporated into the model, and simulation studies indicate that prediction is improved compared to models without such adjustments.

1.5 Measurement Error in Logistic Regression - An Application

Age adjusted body mass index (BMI) is the standard method to identify and follow overweight children. Children with values exceeding the 95th percentile as defined by the BMI standards established in 2000 by the CDC are said to have "childhood obesity". Nutritional guidelines, health district budgets, and even medical interventions are often, in part, based on the prevalence of high BMI-for-age that exists in an area. The National Health Examination Survey (NHANES) is the primary tool used by policy makers to estimate the prevalence of high BMI-for age.

In Chapter 4 we present a study in which we investigate age and gender adjusted BMI measurements, taken between Fall 2003 and Spring 2008, for 18,462 children who participated in the Head Start program, which is funded and administered by the US Department of Health and Human Services Administration for Children and Families.

Specifically, data were collected from Head Start centers in several South Texas border counties and one Central Texas county. The data from this study are used in two ways. First, results are compared to the cohort of the NHANES sample consisting of 2-5 year old children, presented by Ogden, Carroll, and Flegal (2008). Second, the prevalence of high BMI-for-age among pre-school children in South Texas

exceeding that of the 2000 CDC growth curves is examined to determine if there are any regional differences between the border counties of South Texas and a central Texas non-border county.

1.6 Plan of the Dissertation

The remainder of the dissertation is organized as follows. In Chapter 2, we extend the work of Liu et al. (2009) to analyze Poisson data, assuming misclassified exposure and no additional covariates. Within this model we examine three different prior structures and compare the merit of each through simulation study. In Chapter 3, three model extensions are considered - the addition of a continuous covariate, incorporation of random effects, and addition of a zero-inflation parameter. In Chapter 4, we focus on extending a logistic regression model by performing a Bayesian analysis of the dataset. Conclusions and remarks on future extensions and research are made in the Discussion sections of Chapters 3 and 4. Each chapter of this dissertation is essentially self-contained with individual literature reviews and conclusions. For details on the code necessary for the implementation of the methods found in this dissertation, please contact the author.

CHAPTER TWO

Poisson Regression When Exposure is Subject to Misclassification

2.1 Overview

Misclassification of a binary exposure variable is a common issue in clinical studies. The exposure misclassification can be especially problematic in retrospective studies, such as case-control studies. Often, "experts" can supply information and estimates regarding other covariates and their relationship to the exposure variable. In order to incorporate the expert beliefs we implement a Bayesian model that accounts for exposure subject to misclassification. In the case of a single covariate reflecting the presence or absence of exposure, there are 3 unknown parameters - the test has unknown sensitivity, specificity, and probability of exposure. However, there is only 1 degree of freedom, since knowing the total sample size and the number of "exposed" patients fixes the number of "unexposed" patients (and vice versa). Having more parameters to estimate than degrees of freedom results in the need for constraints on at least a subset of the model parameters in order to carry out estimation procedures (Joseph et al., 1995). Gustafson (2003) extended the idea of overparameterization to more complex models (specifically linear and logistic regression).

We explore the performance achieved when the constraints, in the form of informative prior distributions, are placed on different subsets of model parameters for a Poisson regression model. First, we study this model's performance when such informative priors are placed on the misclassification parameters with more diffuse priors on the regression parameters. Second, we consider the effect of placing informative priors on the regression parameters. Finally, we consider the case where all of the model parameters are constrained through the use of informative priors.

Our work expands on that by Liu, Gustafson, Cherry, and Burstyn (2009), who examined a Bayesian method to adjust for misclassification in matched case-control studies. They assumed validation data involving gold-standard exposure assessment was unavailable, but that expert prior opinion concerning the nature of the misclassification was available. We present a simple method to assess the bias in Poisson regression coefficients for a binary exposure variable subject to misclassification.

Matched case-control studies are often used to analyze the relationship between a binary exposure variable and the presence or absence of disease. In retrospective studies, misclassification of a variable due to forgetfulness or misreporting is not uncommon. If a non-trivial level of misclassification is anticipated, then a model should be built to account for it. Ignoring misclassification in a study can lead to biased estimates and inaccurate standard errors (Prescott and Garthwaite, 2005).

If a perfect, but expensive "gold standard" measure is available, it can be compared to the ordinary measure for a subset of the study group. For example, in epidemiology, a case-note review provides more reliable but expensive source of information about exposure than the less-expensive postal questionnaire. If studying occupational exposure to chemicals, a patient may not provide accurate recall about the chemicals they handled on a particular day, while employers records may contain all of this information. A full search of this information could be too expensive. Of course, validation data and/or multiple exposure assessments are not always available. In these cases a gold standard does not exist, but expert prior knowledge about exposure-disease association and/or misclassification parameters may be abundant. Constructing a prior distribution based on expert opinion is critical to our approach. (There is a rich literature on the elicitation of prior distributions using expert opinion. See, for example, O'Hagan et al. (2006) and references therein.)

2.2 Impact of Misclassified Binary Variables

Many explanatory variables encountered in statistical practice are categorical rather than continuous in nature. Mismeasurement arises for these variables when the actual and recorded categories for subjects differ. Unlike the case of mismeasurement in a continuous variable, when the surrogate variable can be expressed as a sum of the true variable plus a noise variable, one must characterize the mismeasurement of the categorical variable in terms of classification properties, i.e., given the true classification, how likely is a correct classification? We focus on the impact of misclassification in binary explanatory variables.

Consider the relationship between a Poisson response variable, y, and a binary explanatory variable x. Since x is binary, we can write

$$\log E(y|x) = \beta_0 + \beta_1 x. \tag{2.1}$$

For a study subject, suppose we observe (y, x^*) rather than (y, x), where the binary variable x^* is an imperfect surrogate for x. Under the assumption of nondifferential misclassification, whereby x^* and y are conditionally independent given x, the magnitude of the misclassification can be described by the *sensitivity* and *specificity* of x^* as a surrogate for x. With respect to biomedical applications, we typically refer to $x = 0$ as "unexposed" and $x = 1$ as "exposed". The sensitivity, η, is the probability that a true exposure is correctly identified; that is, $\eta \equiv P(x^* = 1|x = 1)$. The specificity, θ, is the probability that a true nonexposure is correctly identified; that is, $\theta \equiv P(x^* = 0|x = 0)$.

We can show that the extent to which η and θ are less than one is indicative of the severity of the misclassification. Applying the nondifferential property and adapting Gustafson's (2003) linear model to accommodate the the log linear

relationship in the Poisson generalized linear model yields

$$\begin{aligned}\log E(y|x^*) &= E[E(y|x)|x^*] \\ &= \beta_0 + \beta_1 E(x|x^*) \\ &= \beta_0 + \beta_1 x^* P(x=1|x^*=1) + \beta_1(1-x^*)P(x=1|x^*=0) \\ &= \beta_0^* + \beta_1^* x^*,\end{aligned}$$

where

$$\beta_0^* = \beta_0 + \beta_1 P(x=1|x^*=0) \tag{2.2}$$

and

$$\beta_1^* = 1 - P(x=0|x^*=1) - P(x=1|x^*=0). \tag{2.3}$$

Equation (2.3) shows that more attenuation results from larger probabilities of misclassification given the apparent classification. Moving forward, we express the attenuation the other way around, using sensitivity and specificity which are probabilities of misclassification given the true classification. Let $\tau = P(x=1)$ and $\tau^* = P(x^*=1)$, which are the actual and apparent prevalences of exposure, respectively, in the population at hand.

The apparent prevalence of exposure can be written as a function of the actual prevalence of exposure, the sensitivity, and the specificity:

$$\begin{aligned}\tau^* &= P(x=1)P(x^*=1|x=1) + P(x=0)P(x^*=1|x=0) \qquad (2.4)\\ &= \tau\eta + (1-\tau)(1-\theta) \\ &= \tau(\eta+\theta-1) + (1-\theta).\end{aligned}$$

From Bayes' Theorem we have

$$\begin{aligned}P(x=0|x^*=1) &= \frac{P(x=0)P(x^*=1|x=0)}{P(x^*=1)} \qquad (2.5)\\ &= \frac{(1-\tau)(1-\theta)}{\tau^*},\end{aligned}$$

and

$$P(x = 1|x^* = 0) = \frac{\tau(1-\eta)}{1-\tau^*}. \tag{2.6}$$

Incorporating (2.5) and (2.6) into (2.3) yields,

$$\begin{aligned} \alpha \equiv \frac{\beta_1^*}{\beta_1} &= 1 - \frac{(1-\tau)(1-\theta)}{\tau^*} - \frac{\tau(1-\eta)}{1-\tau^*} \\ &= (\eta + \theta - 1)\frac{\tau(1-\tau)}{\tau^*(1-\tau^*)}. \end{aligned} \tag{2.7}$$

Equation (2.7) becomes one when $\eta = 1$ and $\theta = 1$. As terminology, we refer to (2.7) as the *attenuation factor*, while the magnitude of the relative bias is equivalent to $1 - \alpha$ (Gustafson, 2003). The effect of misclassification is to attenuate bias, and the attenuation worsens with the severity of the misclassification. To illustrate this trait, we can examine attenuation as a function of sensitivity and specificity at various levels of prevalence. This is done in the form of contour plots of the attenuation in (2.7), as shown in Figure 2.1. When $\tau = 0.4$, the bias is near-symmetric in η and θ. The contours reveal that substantial attenuation can occur without the misclassification being very severe. For example, $\eta = \theta = 0.9$, which can be interpreted as 10% misclassification, yields $\alpha = 0.79$, interpreted as 21% attenuation. Compared to the analogous case with continuous measurement error, where 10% measurement error produces only 1% attenuation. Gustafson (2003) posits that having 10% of all the measurements "entirely corrupted" in the binary case is much more damaging than having *all* the measurements corrupted by 10% in the continuous case.

The contour plots also reveal that unbalanced misclassification, where θ and η differ, is less damaging than the balanced case of $\theta = \eta$. For example, when $\tau = 0.4$, $\eta = 1$ and $\theta = 0.8$ yields an attenuation factor of 0.77, which is slightly less than the attenuation factor corresponding to the $\eta = \theta = 0.9$ scenario.

The left panel of Figure 2.1 corresponds to $\tau = 0.2$, which can be regarded as a "rare exposure" scenario typifying many epidemiological investigations. Clearly the

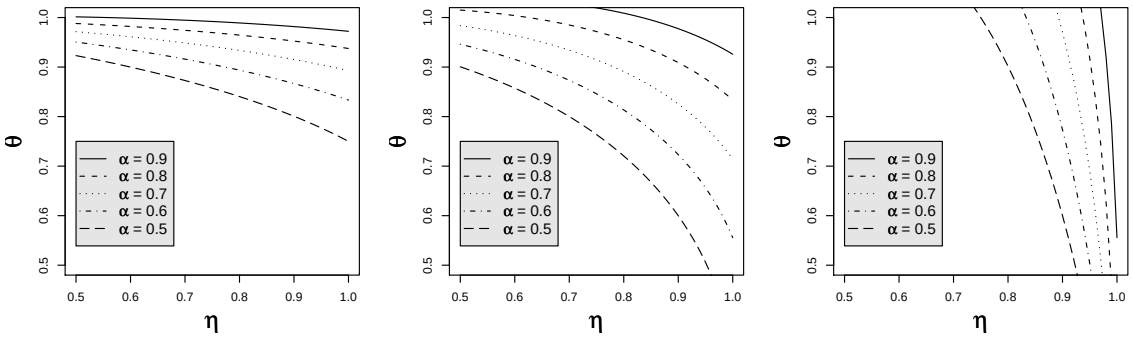


Figure 2.1: Contours of the attenuation factor as a function of sensitivity and specificity. The left panel corresponds to $\tau = P(x = 1) = 0.2$, the center panel corresponds to $\tau = 0.4$, and the right panel corresponds to $\tau = 0.8$.

attenuation worsens more with declining specificity than with declining sensitivity in this scenario. This makes sense, as there are far more true negatives than true positives in the population. Hence the specificity describing the classification of true negatives has a bigger impact than the sensitivity describing the classification of true positives. When η and θ are comparable, the attenuation is now stronger than in the $\tau = 0.4$ case of common exposure. For instance, $\eta = \theta = 0.9$ now yields a stronger attenuation factor of 0.66, compared to 0.79 in the $\tau = 0.4$ case. This is of particular concern in light of the general epidemiological interest in rare exposures. Continuing the consideration of $\eta = \theta = 0.9$, we can see from (2.7) that when $\tau = 0.1$ or $\tau = 0.05$ the attenuation factor drops further to 0.49 and 0.32, an exceedingly substantial attenuation. In the context of rare exposure, even mild nondifferential misclassification can lead to wildly misleading inferences if left unchecked (Gustafson and Greenland, 2006).

The right panel of Figure 2.1 corresponds to $\tau = 0.8$, which can be regarded as a "frequent exposure" scenario. The attenuation worsens more with declining sensitivity than with declining specificity in this instance. Just as the rare exposure attenuation characteristics make intuitive sense, so do the frequent exposure atten-

uation characteristics. There are far more true positives in the population. Hence, the sensitivity that describes the classification of true positives has a larger impact than the specificity describing the classification of true negatives. Just as in the case of $\tau = 0.2$, $\tau = 0.8$ yields an attenuation factor of 0.66 when $\eta = \theta = 0.9$, as the exposure probabilities are symmetric about $\tau = 0.5$.

To illustrate how this pertains to our model, let y be our response which is measured without error. Each subject in a study has a true and an observed exposure status. Misclassification occurs when the true and observed exposure status differ for any patient in the study. A "naive" model attempts to directly model the response using only the observed exposure status, as shown in Figure 2.2, with no regard for the underlying misclassification structure. Failure to incorporate misclassification components results in biased estimates and low coverage in both the frequentist and Bayesian settings.

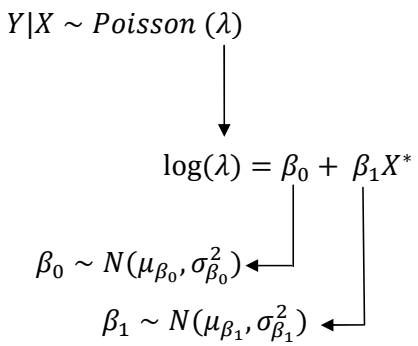


Figure 2.2: Summary of the naive Poisson regression model. This model attempts to model $y|x$ when the data is actually in the form of x^*.

2.3 The Bayesian Model

To build the model that does account for misclassification, we first let y_j be the observed Poisson count, measured without error, for the j^{th} trial/patient, $j = 1, \ldots, J$. Let x_j be the binary exposure status for the j^{th} patient and let $\mathbf{x}_j^*$ be the 1×2 vector of covariates including the intercept and the apparent exposure status

that is susceptible to error, $\mathbf{x}_j^* = (\ 1 \quad x_j^*\ )$. The true Poisson rate is denoted by λ_j. To correct for misclassification of the exposure status, we incorporate the following misclassification parameters and we assume that the sensitivity, η, specificity, θ, and probability of exposure, τ, are independent.

We assume that $y_j|\mathbf{x}_j \sim Poisson(\lambda_j)$, $j = 1, \ldots, J$, where $\lambda_j = g^{-1}(\mathbf{x}_j\boldsymbol{\beta})$ and $g(\cdot)$ is the log link function with regression coefficients vector $\boldsymbol{\beta}' \equiv (\beta_0, \beta_1)$. We relate the covariates to the response using the log link, yielding $\lambda_j = \exp\{\mathbf{x}_j\boldsymbol{\beta}\}$.

The observed data is represented by the density function

$$f(y_j|\mathbf{x}_j) = \frac{\lambda_j^{y_j} e^{-\lambda_j}}{y_j!}, \tag{2.8}$$

and the resulting likelihood is given by

$$f(\boldsymbol{\beta}, \eta, \theta, \tau|\mathbf{y}, \mathbf{x}, \mathbf{x}^*) \propto \prod_{j=1}^{J} [x_j \eta^{x_j^*}(1-\eta)^{1-x_j^*} + (1-x_j)\theta^{1-x_j^*}(1-\theta)^{x_j^*}] \tag{2.9}$$

$$\times\, \tau^{x_j}(1-\tau)^{1-x_j} \lambda_j^{y_j} e^{-\lambda_j},$$

where $\mathbf{y}$ is a $J \times 1$ vector of observed Poisson counts, $\mathbf{X}$ is a $J \times 2$ matrix of covariates including the intercept and the true exposure status, $\mathbf{X}^*$ is a $J \times 2$ matrix of covariates including the intercept and the apparent exposure status, and, as noted above, $\lambda_j = \exp\{\mathbf{x}_j'\boldsymbol{\beta}\}$. We assume that x_{1j}, the exposure status for the j^{th} patient, is subject to misclassification, and we replace x_{1j} with its surrogate, x_{1j}^*, through the relationship,

$$P(x^* = 1|x = 1) = x\eta + (1-x)(1-\theta). \tag{2.10}$$

Assuming independent priors, $p(\cdot)$, for all unknown unknown parameters, the joint prior is given by

$$p(\beta_0, \beta_1, \eta, \theta, \tau) = p(\beta_0) \times p(\beta_1) \times p(\eta) \times p(\theta) \times p(\tau).$$

The resulting joint posterior distribution is

$$p(\beta_0, \beta_1, \eta, \theta, \tau|\mathbf{y}, \mathbf{x}, \mathbf{x}^*) \propto f(\beta_0, \beta_1, \eta, \theta, \tau|\mathbf{y}, \mathbf{x}, \mathbf{x}^*) p(\beta_0, \beta_1, \eta, \theta, \tau). \tag{2.11}$$

This model is shown in Figure 2.3.

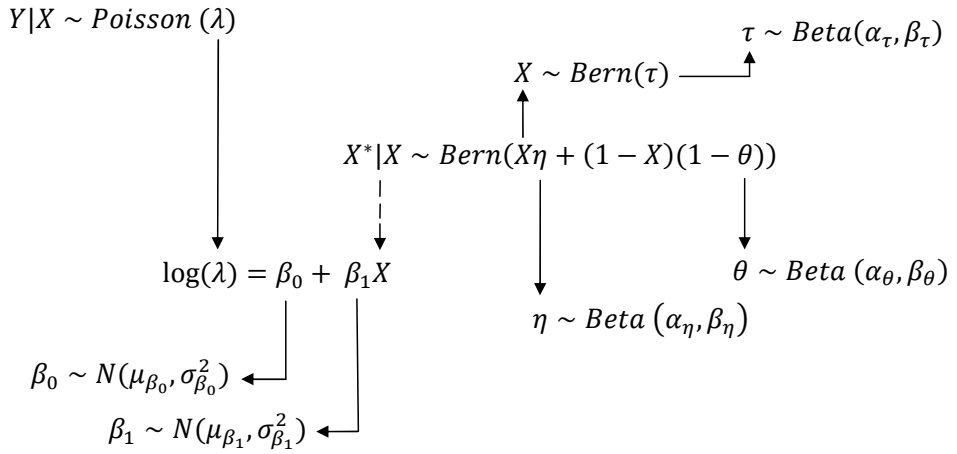


Figure 2.3: Summary of the Poisson regression model with misclassification. The dashed line denotes substitution of a variable with its surrogate.

2.4 Model 1: Informative Priors for the Misclassification Parameters

In the first set of analyses, we place diffuse normal priors on both regression coefficients: $\beta_0 \sim N(\mu_{\beta_0}, \sigma^2_{\beta_0})$ and $\beta_1 \sim N(\mu_{\beta_0}, \sigma^2_{\beta_0})$. We assume independent informative priors on the misclassification components, with sensitivity, specificity, and the probability of exposure each receiving beta priors centered on a value obtained using expert opinion. Because the beta family of distributions offers a wide variety of shapes in the region [0,1], it is considered a natural choice for modeling probabilities. Additionally, the use of informative beta priors ensures useful inferences on model parameters (Joseph, Gyorkos, and Coupal, 1995).

2.4.1 Prior Structures

Following Stamey, Seaman, and Young (2005), we use the notion of equivalent prior sample size (EPSS) to construct beta priors of varying informativeness for the misclassification parameters. For a moderately informative prior, we chose a beta distribution with shape parameters selected so that the EPSS is equivalent to 50 patients by setting the mean of each prior distribution to the prior hypothesized

values of sensitivity, specificity, and probability of exposure. If an equivalent prior sample size, j^*, is considered, and it is believed $\eta = \eta^*$, $\theta = \theta^*$, and $\tau = \tau^*$ are the likely values, we have the equations

$$\begin{array}{lll} a+b=j^*, & c+d=j^*, & e+f=j^*, \\ \frac{a}{a+b}=\eta^*, & \frac{c}{c+d}=\theta^*, & \frac{e}{e+f}=\tau^*. \end{array} \tag{2.12}$$

From this system of equations, if we specify $\eta^* = 0.9$, $\theta^* = 0.7$, and $\tau^* = 0.8$ to be the likely values, and if we utilize an equivalent prior sample size of $j^* = 50$, we have

$$\begin{array}{lll} b=50(1-0.9), & d=50(1-0.7), & f=50(1-0.8), \\ a=50-b, & c=50-d, & e=50-f. \end{array} \tag{2.13}$$

This system of equations results in a $beta(45, 5)$ prior for η, a $beta(35, 15)$ prior for θ, and a $beta(40, 10)$ prior for τ. This specification corresponds to Case 1 of the simulation study outlined in the following section.

Similarly, for a less informative beta prior, we chose shape parameters yielding an equivalent prior sample size of $j^* = 30$. For Case 1 we have

$$\begin{array}{lll} b=30(1-0.9), & d=30(1-0.7), & f=30(1-0.8), \\ a=30-b, & c=30-d, & e=30-f, \end{array} \tag{2.14}$$

which yields three less informative priors for the misclassification parameters: a $beta(27, 3)$ prior for η, a $beta(21, 7)$ prior for θ, and a $beta(24, 6)$ prior for τ.

2.4.2 Simulation Study Design

In this section, we describe an application of the model in (2.11) and methods in Sections 2.3 and 2.4 to simulated data sets, illustrating the impacts of ignoring, and subsequently correcting for, misclassification in the previously introduced Bayesian Poisson regression design. We are interested in investigating the effect of the misclassification on the estimates produced by the naive model. Additionally, we

are interested in learning under which values of sensitivity, specificity, and exposure probability the corrected models produce the most accurate estimates.

Data were generated using the 12 combinations of sensitivity, specificity, and probability of exposure specified in Table 2.1. The scenarios were chosen to reflect what we feel is interesting about the problem of misclassification. We examine situations in which data is generated for varying levels of misclassification to assess the impact on the model's ability to estimate the parameters, with the regression coefficient corresponding to the binary misclassified covariate, β_1, being of particular interest. We then assess the impact when varying levels of exposure are simulated.

Table 2.1: Fixed Values of Sensitivity, Specificity, and Probability of Exposure for the Simulation Study

Case	η	θ	τ
1	0.9	0.7	0.8
2	0.9	0.7	0.4
3	0.9	0.7	0.2
4	0.9	0.5	0.8
5	0.9	0.5	0.4
6	0.9	0.5	0.2
7	0.7	0.9	0.8
8	0.7	0.9	0.4
9	0.7	0.9	0.2
10	0.5	0.9	0.8
11	0.5	0.9	0.4
12	0.5	0.9	0.2

We begin by inflicting minor misclassification on the design with high values of sensitivity and specificity of $\eta = 0.9$ and $\theta = 0.7$ in Cases 1 through 3. In Cases 4 through 6 we maintain the same level of sensitivity, $\eta = 0.9$, and decrease the specificity to $\theta = 0.5$. In our third scenario we lower sensitivity to $\eta = 0.7$ and raise the specificity to $\theta = 0.9$, seen in Cases 9 through 12. Our fourth and final scenario for Cases 10 through 12 finds us decreasing the sensitivity further to $\eta = 0.5$ while

maintaining a specificity of $\theta = 0.9$.

Initally, we generated 100 datasets, each with $J = 300$ Poisson count responses based on a single binary covariate subject to misclassification. A second phase of simulations included datasets made up of $J = 500$ responses. We specify the regression coefficients to be $\beta_0 = 1.9$ and $\beta_1 = 0.8$, and we set the misclassification parameter values according to each of the scenarios in Table 2.1. We call this Data A.

For prior distributions, we used $\beta_0 \sim N(0, 10)$ and $\beta_1 \sim N(0, 10)$, as well as informative beta priors on η, θ, and τ as described in (2.12) such that the equivalent prior sample size is $j^* = 30$. Subsequently, simulations using priors based on an equivalent prior sample size of $j^* = 50$ and $j^* = 100$ were conducted.

To fit this model we used Markov chain Monte Carlo (MCMC) methods implemented in the WinBUGS software package. WinBUGS uses MCMC methods to sample from the desired posterior distribution resulting from the stationary distribution of a Markov chain (Lunn, Thomas, Best, and Spiegelhalter, 2000). This method is particularly useful when the posterior distribution is not in closed form, as the MCMC methods only require that we specify the full conditionals (Gelman, Carlin, Stern, and Rubin, 2003). The simulation study comprised 100 iterations, was implemented on a 3.00 GHz Intel Pentium 4 processor with 4.0 GB RAM, and required 29 hours to reach completion for each of the cases, including 10.5 hours for each corrected model and 8 hours for the naive model.

While we present each of the these as Bayesian models, maximum likelihood estimates and frequentist confidence intervals resulting from the naive and gold standard models are computed for the sake of comparison.

2.4.3 Simulation Study Results

To summarize the simulation results, we report the average of the 100 estimates of the regression coefficients for each of the twelve scenarios. In addition, we recorded the interval estimate lengths and coverage based on 95% confidence intervals and credible sets. These results are displayed in Tables A.1 through A.51 located in Appendix A.

Within the $\eta = 0.9$ and $\theta = 0.7$ cases, the coverage in the corrected models far surpasses the coverage in the naive model. In varying τ, the probability of exposure, we see in Table A.14 that the corrected model with moderately informative priors performs best when $\tau = 0.4$ where the coverage for the regression coefficients β_0 and β_1 is 0.96 and 0.99, respectively. This drops off in the case of $\tau = 0.8$ in Table A.13 and for $\tau = 0.2$ in Table A.15, where the coverage for the regression coefficients β_0 and β_1 is 0.92 and 0.92 when $\tau = 0.8$ and the coverage for the regression coefficients β_0 and β_1 is 0.92 and 0.96 when $\tau = 0.2$. The coverage for η is high, between 0.98 ($\tau = 0.4$) and 1.00 ($\tau = 0.8$ and $\tau = 0.2$), which is to be expected when we consider the sensitivity with which the data was generated.

A primary interest is the model's ability to correctly estimate β_1, the impact of true positive exposure on the response variable. In examining the resulting output for Case 1 through Case 3, we find from Table A.14 that the average credible set interval width for the posterior mean to be smallest at 0.1589 when $\tau = 0.4$ and moderately informative priors are assigned to the misclassification parameters. When mildly informative priors are assigned, Table A.8 indicates the interval width remains at 0.1589. The associated frequentist gold standard confidence interval widths for β_1 when $\tau = 0.4$ and moderately informative and mildly informative priors are assigned are 0.1464 and 0.1465, respectively. When the probability of exposure is high, $\tau = 0.8$, the interval width for the posterior mean of β_1 is wider at 0.2783 (Table A.13) with moderately informative priors and 0.2801 (Table A.7) with

mildly informative priors. When the probability of exposure is low/rare, $\tau = 0.2$, the interval width for the posterior mean of β_1 is also wider at 0.2649 (Table A.14) with moderately informative priors and 0.3285 (Table A.9) with mildly informative priors. The coverages produced by the models that attempt to account for misclassification are greater than 0.90, while the coverages associated with the naive model estimates range from 0.00 to 0.08.

The naive model produces very poor estimates for β_1, with the Bayesian estimates ranging from 0.1439 when $\tau = 0.4$ to 0.2559 when $\tau = 0.8$. The frequentist estimates are also poor, ranging from 0.2557 when $\tau = 0.8$ to 0.5011 when $\tau = 0.4$. As we would expect, the estimates produced from a model that does not account for misclassification are heavily attenuated.

Keeping the sensitivity high at $\eta = 0.9$, we lower the specificity to $\theta = 0.5$ and again examine the performance of our three different models. Within the $\eta = 0.9$ and $\theta = 0.5$ cases (Cases 4 through 6 of Table 2.1), the coverage in the corrected models again surpasses the coverage in the naive model. In varying τ, the probability of exposure, we see in Tables A.31 and A.28 that the corrected models achieve the best coverage when $\tau = 0.8$, where the coverage for the regression coefficients β_0 and β_1 are 0.95 and 0.96 when moderately informative priors are used and 0.97 and 0.98 when mildly informative priors are used, respectively. For average exposure $\tau = 0.4$, Tables A.32 and A.29 present slightly lower coverages, with the moderately informative priors achieving 0.95 and 0.93 and the mildly informative priors achieving 0.95 and 0.95 coverage. We will see, however, that the interval widths for the posterior estimates are considerably narrower. For low exposure, $\tau = 0.2$ in Tables A.33 and A.30, the coverage for the regression coefficients β_0 and β_1 is 0.97 and 0.94 when moderately informative priors are assigned and 0.94 and 0.96 when mildly informative priors are used.

Again, a primary interest is our models' ability to correctly estimate the impact of true positive exposure on the response variable, quantified in β_1. In examining the resulting output for Case 4 through Case 6, we find from Table A.32 that the average credible set interval width for the posterior mean is narrowest at 0.1595 when $\tau = 0.4$ and moderately informative priors are assigned to the misclassification parameters. When mildly informative priors are assigned, Table A.29 indicates the interval width grows slightly to 0.1605. The associated frequentist gold standard confidence interval widths for β_1 when $\tau = 0.4$ and moderately informative and mildly informative priors are assigned are 0.1463 and 0.1464, respectively. When the probability of exposure is high, $\tau = 0.8$, the interval width for the posterior mean of β_1 is wider at 0.2954 (Table A.31) with moderately informative priors and 0.2953 (Table A.28) with mildly informative priors. When the probability of exposure is low/rare, $\tau = 0.2$, the interval widths for the posterior means are the widest, β_1 is 0.4418 (Table A.33) with moderately informative priors and 0.4819 (Table A.30) with mildly informative priors. The coverages produced by the models that attempt to account for misclassification are greater than 0.90, while the coverages associated with the naive model estimates range from 0.00 (Tables A.25 and A.26) to 0.01 (Table A.27).

The naive model produces very poor estimates for β_1, with the Bayesian estimates ranging from 0.1641 when $\tau = 0.8$ to 0.3768 when $\tau = 0.2$. The naive frequentist estimates are also poor, ranging from 0.1641 when $\tau = 0.8$ to 0.3774 when $\tau = 0.2$. As in Cases 1 through 3 of Table 2.1, the estimates produced from a model that does not account for misclassification are heavily attenuated.

In the third scenario we lower the sensitivity to $\eta = 0.7$, and we raise the specificity to $\theta = 0.9$ and again examine the performance of our three different models. Case 7 through Case 9 (Tables A.34 through A.42) correspond to data generated using $\eta = 0.7$ and $\theta = 0.9$. The coverage in the corrected models again

surpass the coverage in the naive model. In varying τ, the probability of exposure, we see in Tables A.42 and A.39 that the corrected models achieve the best coverage when $\tau = 0.2$, where the coverage for the regression coefficients β_0 and β_1 are 0.97 and 0.94 when moderately informative priors are used and 0.95 and 0.97 when mildly informative priors are used, respectively. For average exposure $\tau = 0.4$, Tables A.41 and A.38 present slightly lower coverages, with the moderately informative priors achieving 0.91 and 0.91 and the mildly informative priors achieving 0.95 and 0.96 coverage. We will see, however, that some of the interval widths for the posterior estimates are considerably narrower when the probability of exposure is $\tau = 0.4$. For common exposure, $\tau = 0.8$, in Tables A.33 and A.30, the coverage for the regression coefficients β_0 and β_1 is 0.97 and 0.94 when moderately informative priors are assigned and 0.94 and 0.96 when mildly informative priors are used.

With respect to β_1, in the resulting output for Case 7 through Case 9 we find from Table A.41 that the average credible set interval width for the posterior mean is narrowest at 0.1587 when $\tau = 0.4$ and moderately informative priors are assigned to the misclassification parameters. When mildly informative priors are assigned, Table A.38 indicates the interval width grows slightly to 0.1589. The associated frequentist gold standard confidence interval widths for β_1 when $\tau = 0.4$ and moderately informative and mildly informative priors are assigned are 0.1464 and 0.1467, respectively. When the probability of exposure is high, $\tau = 0.8$, the interval width for the posterior mean of β_1 is wider at 0.2737 (Table A.40) with moderately informative priors and 0.2787 (Table A.37) with mildly informative priors. When the probability of exposure is low/rare, $\tau = 0.2$, the interval widths for the posterior means are the widest, β_1 is 0.3879 (Table A.42) with moderately informative priors and 0.4685 (Table A.39) with mildly informative priors. The coverages produced by the models that attempt to account for misclassification are greater than or equal to 0.89, while the coverages associated with the naive model estimates range from

0.00 (Tables A.34) when the exposure probability is high to 0.66 (Table A.36) when the exposure probability is low.

The naive model produces better estimates for β_1 that in previous scenarios, with the Bayesian estimates ranging from 0.3725 when $\tau = 0.2$ to 0.4988 when $\tau = 0.4$. The naive frequentist estimates are also poor, ranging from 0.3724 when $\tau = 0.2$ to 0.4986 when $\tau = 0.4$. As in Cases 1 through 6, the estimates produced from the models that do not account for misclassification are heavily attenuated.

In our final scenario we further lower the sensitivity to $\eta = 0.5$, and we raise the specificity to $\theta = 0.9$ and again examine the performance of our three different models. We have decreased our models' ability to correctly detect a positive exposure to 0.5. Case 10 through Case 12 (Tables A.43 through A.51) correspond to data generated using $\eta = 0.5$ and $\theta = 0.9$. The coverage in the corrected models again surpass the coverage in the naive model. As we examine the results obtained by varying τ, the probability of exposure, we see in Tables A.50 and A.47 that the corrected models achieve the best coverage when $\tau = 0.4$, where the coverage for the regression coefficients β_0 and β_1 are 0.98 and 0.97 when moderately informative priors are used and 0.97 and 0.95 when mildly informative priors are used, respectively. For common exposure $\tau = 0.8$, Tables A.51 and A.48 present slightly lower coverages, with the moderately informative priors achieving 0.98 and 0.94 and the mildly informative priors achieving 0.97 and 0.96 coverage. While these coverages are very close to those attained when $\tau = 0.4$, the interval estimates are much wider when $\tau = 0.8$. For rare exposure, $\tau = 0.2$ in Tables A.49 and A.46, the coverage for the regression coefficients β_0 and β_1 is 0.94 and 0.93 when moderately informative priors are assigned and 0.99 and 0.99 when mildly informative priors are used.

Once again, a primary interest is our model's ability to correctly estimate β_1, the impact of true positive exposure on the response variable. In examining the resulting output for Case 10 through Case 12, we find from Table A.50 that

the average credible set interval width for the posterior mean is narrowest at 0.1750 when $\tau = 0.4$ and moderately informative priors are assigned to the misclassification parameters. When mildly informative priors are assigned, Table A.47 indicates the interval width narrows slightly to 0.1519. The associated frequentist gold standard confidence interval widths for β_1 when $\tau = 0.4$ and moderately informative and mildly informative priors are assigned are 0.1464 and 0.1464, respectively. When the probability of exposure is high, $\tau = 0.8$, the interval width for the posterior mean of β_1 is wider at 0.2990 (Table A.49) with moderately informative priors and 0.3034 (Table A.46) with mildly informative priors. When the probability of exposure is low/rare, $\tau = 0.2$, the interval widths for the posterior means are the widest, β_1 is 0.3713 (Table A.51) with moderately informative priors and 0.5957 (Table A.51) with mildly informative priors. The coverages produced by the models that attempt to account for misclassification are greater than or equal to 0.90, while the coverages associated with the naive model estimates range from 0.00 (Tables A.43) when the exposure probability is high to 0.65 (Table A.45) when the exposure probability is low.

The naive model produces estimates for β_1 that are more biased than in the previous scenario, with the Bayesian estimates ranging from 0.2689 when $\tau = 0.2$ to 0.3936 when $\tau = 0.4$. The naive frequentist estimates are also poor, ranging from 0.2686 when $\tau = 0.2$ to 0.3931 when $\tau = 0.4$. As in Cases 1 through 9, the estimates produced from the models that do not account for misclassification are heavily attenuated.

Throughout Chapter 2, we exhibit our simulation results graphically as well as with tables. These graphs follow a common template, which is illustrated in Figure 2.4. This particular figure represents Model 1 under Cases 1 through 3 in Table 2.1, where informative beta priors are placed on the misclassification parameters and diffuse priors are placed on the regression coefficients, as discussed in Section 2.4.

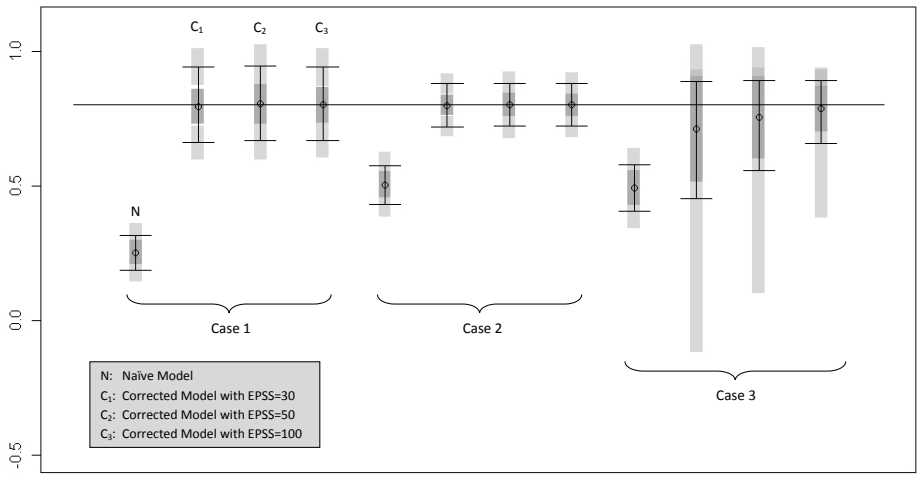


Figure 2.4: Variation within the estimates for β_1 for Cases 1, 2, and 3. Posterior means and credible set intervals are plotted for the naive model and the corrected models with EPSS=30, 50, and 100.

The left four vertical bars represent simulations performed with data generated according to Case 1. The first bar corresponds to the naive model, and the remaining bars correspond to the corrected model with varying degrees of informativeness incorporated into the priors on the misclassification parameters. The four vertical bars in the center represent the simulations performed with data generated according to Case 2, and the four bars to the right represent the simulations performed with data generated according to Case 3. Within each set of four, we have a naive model and three corrected models with increasing equivalent prior sample size. The horizontal line across the entire figure is the true parameter value ($\beta_1 = 0.8$). The horizontal line atop each vertical bar is the simulation average upper bound on the 95% credible interval. The lower horizontal line on each vertical bar is the simulation average lower bound on the interval. The central dot is the simulation average posterior mean. The dark gray box is plus or minus one simulation standard deviation on the posterior mean. The light gray boxes are plus or minus one simulation standard deviation on the upper 97.5% and lower 2.5% bounds.

We look at the first scenario, when $\eta = 0.9$ and $\theta = 0.7$. There is considerable variation in the posterior estimates for β_1 when the probability for exposure is low ($\tau = 0.2$). The variation decreases as the degree of informativeness in the prior distributions increases. The naive models produce estimates with less variation, but the estimates are far from the true value.

2.5 Model 2: Informative Priors for the Regression Parameters

Here we introduce Model 2, in which we examine the effects of placing the information accrued from experts on the regression coefficients and place minimally informative distributions on the misclassification parameters.

2.5.1 Prior Structures

In Model 1 (see Section 2.4), we illustrated Bayesian fitting of a Poisson regression model using a diffuse prior on the regression coefficients. Suppose instead that we have subjective beliefs about the regression vector. A convenient way of representing these beliefs is by use of informative priors on the regression coefficients, β_0 and β_1.

There exists a variety of methods to incorporate expert opinion and/or historical data into prior specification for regression parameters. For Model 2 we choose a straightforward method in which we assign independent informative normal priors and we specify the means to be equal to the true value and the standard deviations to be small.

2.5.2 Simulation Study Design

As with Model 1, we generated 100 datasets, each with $J = 300$ Poisson count responses based on a single binary covariate subject to misclassification. A second phase of simulations performed on a subset of the cases from Table 2.1 included datasets made up of $J = 500$ responses. The analysis is performed on the same

data, Data A, as we set the regression coefficients as $\beta_0 = 1.9$ and $\beta_1 = 0.8$, and we set the misclassification parameter values according to each of the scenarios in Table 2.1.

For prior distributions, we initially specify $\beta_0 \sim N(1.8, 0.09)$ and $\beta_1 \sim N(0.8, 0.09)$, as well as diffuse $beta(1, 1)$ priors on η, θ, and τ. A second phase of simulations are carried out on Model 2, with the standard deviation for the normal priors on the regression parameters decreased from $\sigma = 0.3$ to $\sigma = 0.2$.

2.5.3 Simulations Study Results

To summarize the simulation results, we report the average of the 100 estimates of the regression coefficients for each of the twelve scenarios. In addition, we recorded the interval estimate lengths, and coverage based on 95% confidence intervals and credible sets. These results are displayed in Tables A.52 through A.69 located in Appendix A.

Upon inspection of the results in Tables A.52 through A.54 we find the interval widths for the Bayesian estimates of each model parameter to be larger than anticipated, particularly with the cases that model low exposure ($\tau = 0.2$). For the first scenario, where $\eta = 0.9$ and $\theta = 0.7$, the average credible set width is 0.5339 (Table A.54). The average credible set widths for β_1 when the probability for exposure is low for the other scenarios are 0.3684, 0.5620, and 0.4919 as found in Tables A.63, A.66, and A.69, respectively.

As a diagnostic measure, we plot the estimates for β_1 produced during each MCMC iteration of our simulation study. In Figure 2.5 we see the variation between estimates for β_1 is much greater when the exposure probability is low ($\tau = 0.2$), than when the exposure probability is higher ($\tau = 0.8$).

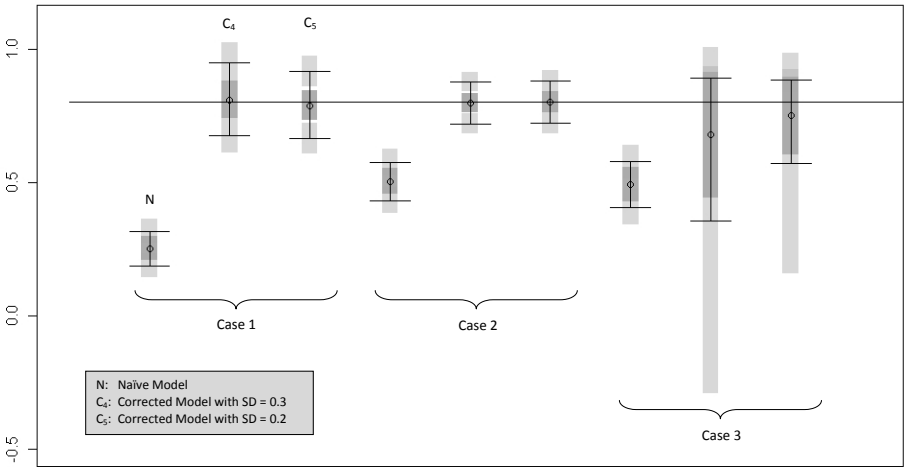


Figure 2.5: Variation within the estimates for β_1 for Cases 1, 2, and 3. Posterior means and credible set intervals are plotted for the naive model and the corrected models with $\sigma = 0.3$ and $\sigma = 0.2$.

Further investigation of the posterior distributions for β_1 when the exposure probability is low reveals problems with convergence. In Figure 2.6 we present the prior and posterior distributions for β_1 when Case 3 is considered.

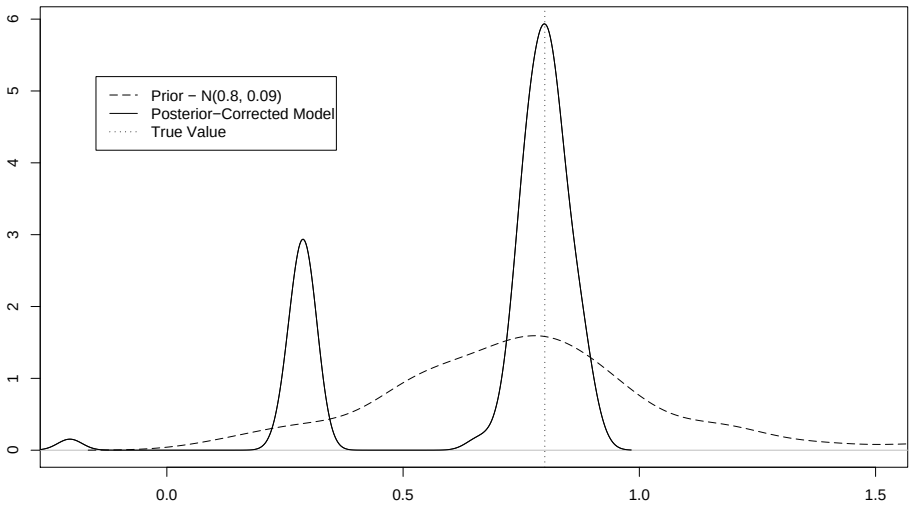


Figure 2.6: Prior and posterior distributions for β_1 when Case 3 is considered.

Figure 2.6 illustrates the difficulty the model is having with convergence. The posterior distributions for β_1 under Model 2 are not unimodal and exhibit a large

amount of variability. Investigation of the variation between estimates for the other model parameters produced results consistent with those presented above.

We consider a subset of the cases presented in Table 2.1 for further study. For Case 1 through Case 3, we make the priors on β_0 and β_1 more informative by decreasing the standard deviations from $\sigma = 0.3$ to $\sigma = 0.2$. We find the more informative priors do lead to a decrease in credible set widths. For Case 3 we find the interval width for β_0 decreases by 40%, from 0.2942 when $\sigma = 0.3$ (Table A.54) to 0.1781 when $\sigma = 0.2$ (Table A.57). The interval width for β_1 decreases 42%, from 0.5339 when $\sigma = 0.3$ to 0.3102 when $\sigma = 0.2$. Similar interval width decreases were observed for each of the misclassification parameters. We do see, however, that the posterior distribution continues to exhibit multi-modality, as presented in Figure 2.7.

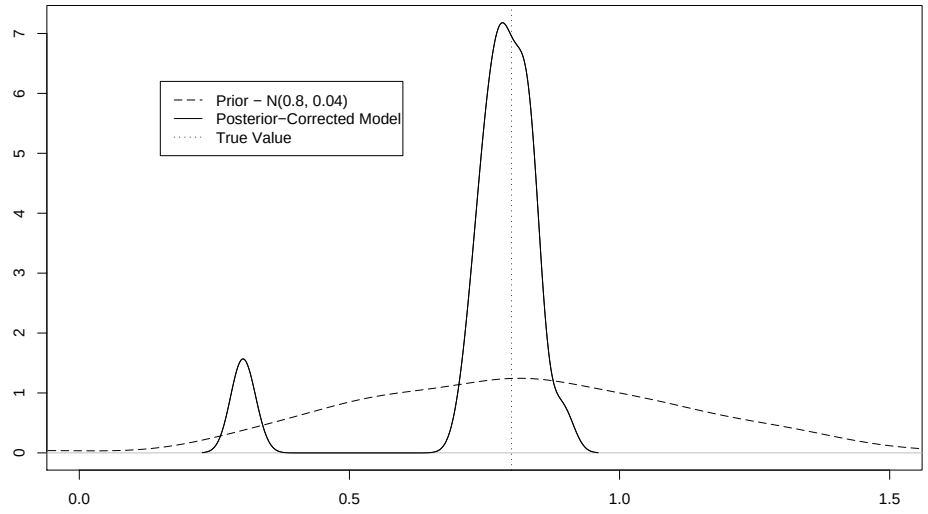


Figure 2.7: Prior and posterior distributions for β_1 when the standard deviation on the prior is reduced to $\sigma = 0.2$.

The simulation is repeated once more, with the study sample size increased from $J = 300$ to $J = 500$ patients. The interval widths continue to decrease, as evidenced in Tables A.58 through A.60 where the interval width for β_0 decreases

by 30 percent, from 0.1781 when $J = 300$ to 0.1276 when $J = 500$ (Table A.60). The interval width for β_1 also decreases from 0.3102 when $J = 300$ to 0.2179 when $J = 500$.

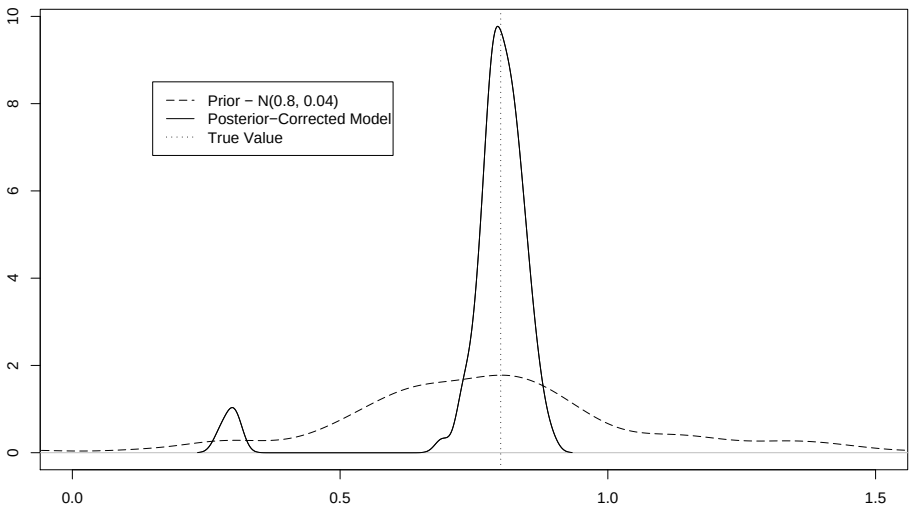


Figure 2.8: Prior and posterior distributions for β_1 when the standard deviation on the prior is reduced to $\sigma = 0.2$ and the study sample size is increased to $J = 500$.

While the posterior distribution only exhibits a small amount of variation toward the left tail (Figure 2.8), we are not entirely satisfied with the performance of Model 2. With regard to the usefulness of expert opinion and/or historical data in overparameterized models, prior information appears to be most helpful when applied to the misclassification parameters as opposed to the regression parameters. In the next section, we investigate model performance when expert opinion and/or historical data are available for all model parameters.

2.6 Model 3: Informative Priors on All Model Parameters

We conclude with Model 3, an analysis that places informative distributions on both the regression parameters and the misclassification parameters.

2.6.1 Prior Structures

Occasionally a researcher is presented with expert opinion and historical data related to both the misclassification parameters and the regression parameters. Model 3 incorporates this wealth of information via informative normal priors on the regression parameters and informative beta priors on the misclassification parameters.

The normal priors are centered at the true value and small standard deviations are assigned, yielding $\beta_0 \sim N(1.9, 0.04)$ and $\beta_1 \sim N(0.8, 0.04)$. The notion of equivalent prior sample size is again employed to construct informative priors for the misclassification parameters.

2.6.2 Simulation Study Design

For Model 3 we generated 100 datasets, each with $J = 300$ Poisson count responses based on a single binary covariate subject to misclassification. A second phase of simulations performed on a subset of the cases from Table 2.1 included datasets made up of $J = 500$ responses. We continue to use Data A, in which set the regression coefficients as $\beta_0 = 1.9$ and $\beta_1 = 0.8$, and we set the misclassification parameter values according to each of the scenarios in Table 2.1.

For prior distributions, we specify $\beta_0 \sim N(1.8, 0.04)$ and $\beta_1 \sim N(0.8, 0.04)$, as well as informative beta priors for η, θ, and τ representing three degrees of informativeness. Initially we consider an equivalent prior sample size of $j^* = 30$ for all twelve cases. We increase the informative nature of these priors and repeat the simulations on a subset of cases from Table (2.1) using an equivalent prior sample size of $j^* = 50$ and $j^* = 100$.

2.6.3 Simulation Study Results

To summarize the simulation results, we report the average of the 100 estimates of the regression coefficients for each of the twelve scenarios. In addition, we recorded

the interval estimate lengths and coverage based on 95% confidence intervals and credible sets. These results are displayed in Tables A.70 through A.96 located in Appendix A.

We perform a number of adjustments throughout the simulation study in our attempts to learn which simulation characteristics are most predictive of successful estimation of model parameters. Our primary interest lies in the Model 3's ability to correctly estimate β_1, the impact of true positive exposure on the response variable. While we perform each simulation adjustment on all cases, we present the detailed results corresponding to the first scenario only in Tables A.70 through A.87. Recall that in our first scenario, we begin by inflicting minor misclassification on the design with high values of sensitivity and specificity of $\eta = 0.90$ and $\theta = 0.70$ in Cases 1 through 3 of Table 2.1.

Tables A.70 through A.72 contain the estimates produced when Model 3 is fit to data simulated according to Cases 1, 2, and 3 using a study sample size of $J = 300$, and the beta priors on the misclassification parameters are constructed using an equivalent prior sample size of $j^* = 30$. For frequent exposure, $\tau = 0.8$, Model 3 produces posterior means of 1.9033 and 0.7972 for β_0 and β_1, respectively. The associated credible set widths are narrower and the coverages, 0.99 and 0.96, are higher than those produced by Model 1 (Table A.7) and Model 2 (A.52).

Tables A.73 through A.75 contain the results obtained when the study sample size is increased to $J = 500$. We also note that when the probability of exposure is low ($\tau = 0.2$), the posterior means for β_0 and β_1 of 1.9005 and 0.8014, respectively, are accompanied by credible set widths of 0.0885 and 0.1417 and coverages of 0.98 and 0.99. Model 3 clearly outperforms Model 1 and Model 2, where the cases associated with low exposure probabilities tended to produce the most biased estimates with the widest credible set widths. Table A.75 indicates that Model 3 performs as

well as, and perhaps better than, the "gold standard" model - in which a frequentist Poisson regression model is fit with the true data.

Next, we increase the degree of information incorporated into the prior distributions for the misclassification parameters and consider moderately informative priors with equivalent prior sample size of $j^* = 50$. In Tables A.76 through A.81 we see that, as expected, Model 3 produces more accurate posterior means and narrower interval widths than the analogous analyses performed with Model 1 (Tables A.13 through A.18)and Model 2 (Tables A.55 through A.57). When the probability of exposure is frequent $\tau = 0.8$, Model 3 produces coverages for β_0 and β_1 that are higher than those produced by the gold standard analysis, though with slightly wider intervals (Table A.76). When common exposure probabilities are considered, Model 3 performs well - producing the exact same coverages of 0.96 for β_0 and 0.97 for β_1 as the gold standard. When the probability of exposure is low (Table A.77), Model 3 achieves greater coverage than the gold standard. Increasing the study sample size to $J = 500$ results narrower credible set widths and slightly lower coverages, as evidenced in Tables A.79 through A.81.

Finally, we examine the case where we have a great amount of historical data, and we construct the prior distributions for the misclassification parameters to be representative of the information gained from 100 patients in a previous study ($j^* = 100$). We first consider a study sample size specified to be $J = 300$. Increasing the level of information in the misclassification priors results in a marked improvement of the coverage of our parameter of interest, β_1. When $j^* = 30$, the posterior mean produced by Model 3 is 0.8044 with a credible set width of 0.1829 and a coverage of 0.90 (Table A.72). Increasing the prior information to $j^* = 100$ results in a posterior mean of 0.7999 with a credible set width of 0.1802 and coverage of 0.97 (Table A.87). This increase in accuracy is comparable to that obtained when the equivalent prior sample size is $j^* = 30$ with the study sample size increased to $J = 500$. However,

obtaining the information from the additional 70 patients in the historical data is likely to be much less expensive than enrolling an additional 200 patients in the study.

To assess the variation among the estimates, we plot the posterior means and 95% credible set bounds for β_1 that are produced during each MCMC iteration of our simulation study when the study sample size is $J = 300$. We include the estimates produced by the naive model as well. In Figure 2.9 we see that the variation between estimates for β_1 is consistent across the corrected models. Increasing the equivalent prior sample size from $j^* = 30$ to $j^* = 50$, and then from $j^* = 50$ to $j^* = 100$, results in slightly less variation. Case 2 ($\tau = 0.4$) produces estimates with the least amount of variability.

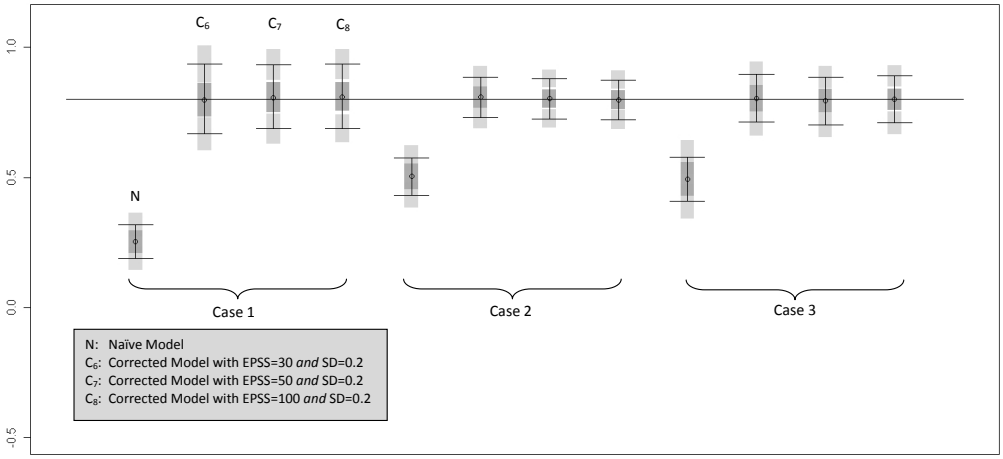


Figure 2.9: Variation within the estimates for β_1 produced by Model 3.

Tables A.88 through A.96 contain the estimates produced by three additional scenarios. In Cases 4 through 6 of Table 2.1, we maintain the same level of sensitivity, $\eta = 0.9$, and decrease the specificity to $\theta = 0.5$. In our third scenario we lower sensitivity to $\eta = 0.7$ and raise the specificity to $\theta = 0.9$, seen in Cases 9 through 12. Our fourth and final scenario for Cases 10 through 12 from Table 2.1 finds

us decreasing the sensitivity further to $\eta = 0.5$ while maintaining a specificity of $\theta = 0.9$. Each scenario produced results consistent with those described in detail above.

While viewing the average posterior means and average credible set widths is an important element of the statistical analysis, we also present graphical representations of the accuracy achieved by each of the models presented here. We see from Figure 2.10 that the simulation variation for estimates of β_1 is greater under Model 2 than Models 1 or 3. Model 3 leads to the smallest amount of within simulation variation. Increasing the equivalent prior sample size leads to more accurate estimates and higher coverages, while increasing the study sample size results in decreased credible set width.

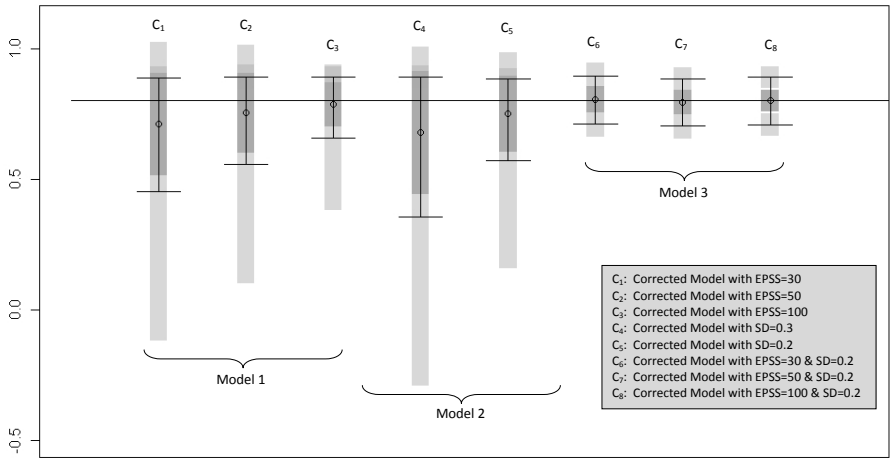


Figure 2.10: Variation within the estimates for β_1 produced by Models 1, 2, and 3 when Data A is simulated according to Case 3.

Model 3 leads to estimates and coverages that achieve, and in some cases exceed, that of the gold standard analysis in which a frequentist Poisson regression model is fit to the true data.

2.7 Discussion

In our Bayesian model for Poisson outcomes using surrogate data, we find that expert opinion and/or historical data that provides information about the misclassification parameters is more useful in fitting overparameterized models than similar information about the regression parameters. When little is known about the misclassification parameters, one must be able to put very informative priors on the regression parameters and the study sample size must be quite large in order to obtain model convergence and reliable estimates. Not surprisingly, the estimates with the least bias, smallest interval width, and greatest coverage are achieved when expert opinion and/or historical data are available for both the misclassification parameters and the regression parameters.

Nondifferential misclassification of a binary explanatory variable yields attenuated estimates of associated effects. While binary misclassification seems to be more damaging than continuous covariate measurement error in general, they share some key features. In both cases a primary determinant of how bad the bias will be is the strength of correlation between the mismeasured explanatory variable and other precisely measured explanatory variables. While we have focused on the case of a single misclassified exposure covariate in this chapter, we extend the model to consider an additional precisely measured explanatory variable in the next chapter. Also, the mismeasurement bias does not depend on the actual distribution of the response variable. Generally speaking, the bias due to mismeasurement worsens as the proportion of subjects exposed gets close to zero (or close to one). In epidemiological contexts it is often natural to conduct studies with low exposure prevalences, so there is a clear need for methods which adjust inferences to account for misclassification. Some such methods are introduced and considered by Gustafson (2003).

CHAPTER THREE

Poisson Regression With Misclassified Binary Covariate - Extensions

3.1 Overview

In this chapter, we consider three complications common to Poisson regression in biomedical settings and propose modeling strategies for each. These include the following:

(1) Poisson regression when there is a continuous covariate in addition to the binary covariate subject to misclassification,

(2) Incorporating random effects to account for residual variability that exceeds that expected under the Poisson mean-variance property, and

(3) Accounting for excess zeros in the Poisson count via zero-inflated Poisson models.

Each of these results in an extension of the model introduced in Chapter II.

This chapter is organized as follows. In Section 3.2 we expand the model from Chapter 2 and present the Bayesian Poisson regression model with an additional continuous covariate. We also present the results of a simulation study designed to examine model performance. In Section 3.3 we introduce the concept of random effects as a means of accounting for excess variability. We also summarize simulation results for the model with random effects. In Section 3.4 we discuss an alternative method for accounting for excess variability when the data is subject to zero-inflation. We present a small simulation study and then illustrate the methods using an example. In Section 3.5 we make concluding remarks.

3.2 Continuous Covariate

It has been suggested that acknowledging and accounting for misclassification in a model is less pivotal when additional covariates are present in addition to the covariate subject to misclassification (Veierod and Laake, 2001). We show that failure to account for covariate effects when attempting to distinguish between exposed and non-exposed patients can result in poor accuracy and considerable bias. If the misclassification rates are independent of any covariates in the model (nondifferential misclassification), the direction of the bias is always towards zero (Armstrong, 1998).

To adjust for the bias induced by misclassification, we can use patient information in addition to exposure status, such as blood glucose levels, blood pressure, and age. In what follows, we examine the extent to which such information improves prediction of exposure status.

3.2.1 The Bayesian Model

Let y_j be the observed Poisson count, measured without error, for the j^{th} patient, $j = 1, \ldots, J$. Let x_{1j} be the binary exposure status for the j^{th} patient and let x_{2j} be the value associated with a continuous covariate. Now let $\mathbf{x}_j^*$ be a 1×3 vector of covariates including the intercept, the apparent exposure status that is susceptible to error, and the value of the continuous covariate measured without error, that is,

$$\mathbf{x}_j^{*'} = (\ 1 \quad x_{1j}^* \quad x_{2j}\ ).$$

The true Poisson rate is denoted by λ_j, and is once again related to the covariates through its link function, $\log(\lambda_j) = \beta_{0j} + \beta_{1j}x_{1j} + \beta_{2j}x_{2j}$. To correct for misclassification of the exposure status, we incorporate the following misclassification parameters and assume that the sensitivity, η, specificity, θ, and probability of exposure, τ are

independent. For an individual with true exposure status x_j we have

$$\begin{aligned} P(x^*_{1j} = 1) &= P(x^*_{1j} = 1|x_{1j} = 1) \times P(x_{1j} = 1) \\ &\quad + P(x^*_{1j} = 1|x_{1j} = 0) \times P(x_{1j} = 0) \\ &= x_{1j}\eta + (1 - x_{1j})(1 - \theta). \end{aligned}$$

We assume that $y_j|\mathbf{x}_j \sim Poisson(\lambda_j)$, $j = 1, \ldots, J$, where $\mathbf{x}_j$ is the 1×3 vector that contains the true covariate values for the j^{th} patient, $\boldsymbol{\beta}' \equiv (\beta_0, \beta_1, \beta_2)$ are the regression coefficients, and we relate the covariates to the response using the log link yielding $\lambda_j = \exp\{\mathbf{x}_j\boldsymbol{\beta}\}$.

The observed counts are distributed Poisson with probability mass function

$$f(y_j|\mathbf{x}_j) = \frac{\lambda_j^{y_j} e^{-\lambda_j}}{y_j!}, \tag{3.1}$$

so that the resulting likelihood is

$$\begin{aligned} f(\boldsymbol{\beta}, \eta, \theta, \tau|\mathbf{y}, \mathbf{x}, \mathbf{x}^*) \propto \prod_{j=1}^{J} &[x_j \eta^{x^*_j}(1 - \eta)^{1-x^*_j} + (1 - x_j)\theta^{1-x^*_j}(1 - \theta)^{x^*_j}] \\ &\times \tau^{x_j}(1 - \tau)^{1-x_j}\lambda_j^{y_j} e^{-\lambda}, \end{aligned} \tag{3.2}$$

where $\mathbf{y}$ is a $j \times 1$ vector of observed Poisson count responses. We assume that x_{1j}, the exposure status for the j^{th} patient, is subject to misclassification, and we replace x_{1j} with its surrogate, x^*_{1j}.

3.2.2 Prior Distributions

We assume independent informative beta priors on the values of sensitivity, specificity, and probability of exposure. The beta family of distributions offers a wide variety of shapes with support [0,1], and is therefore considered a natural choice for modeling probabilities. We place normal priors on all three regression coefficients, $\beta_0 \sim N(\mu_{\beta_0}, \sigma^2_{\beta_0})$, $\beta_1 \sim N(\mu_{\beta_1}, \sigma^2_{\beta_1})$, and $\beta_2 \sim N(\mu_{\beta_2}, \sigma^2_{\beta_2})$. The beta and normal priors may be fit using prior data and expert opinion. See, for example, Johnson,

Gastwirth, and Pearson (2001) or Joseph, Gyorkos, and Coupal (1995). Assuming prior independence of all unknown parameters, the joint prior is given by

$$p(\beta_0, \beta_1, \beta_2, \eta, \theta, \tau) = p(\beta_0) \times p(\beta_1) \times p(\beta_2) \times p(\eta) \times p(\theta) \times p(\tau), \tag{3.3}$$

where $p(\cdot)$ is the generic notation for a prior density. The resulting joint posterior distribution is

$$p(\beta_0, \beta_1, \beta_2, \eta, \theta, \tau | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) \propto f(\boldsymbol{\beta}, \eta, \theta, \tau | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) p(\beta_0, \beta_1, \beta_2, \eta, \theta, \tau). \tag{3.4}$$

This model is shown in Figure 3.1.

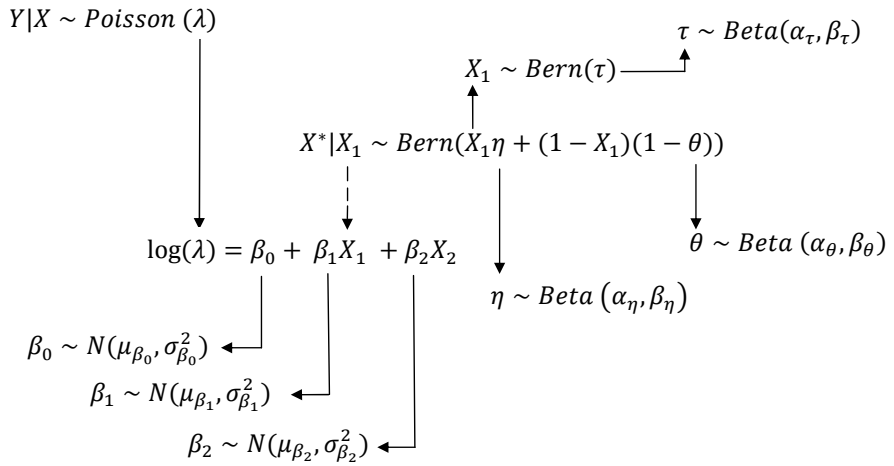


Figure 3.1: Summary of the Poisson regression model with additional continuous covariate. The dashed line denotes substitution of a variable with its surrogate. All hyperprior beta parameters are assumed known, along with the normal parameters.

3.2.3 Simulation Study Design

To investigate the performance of our expanded model, the following simulation study was conducted. First $J = 300$ outcomes were generated from a Poisson distribution where the rate of event occurrence, λ_j, is related to the covariates via the log link. Given the data $(\mathbf{X}^*, \mathbf{y})$, where $\mathbf{X}^*$ is the $J \times 3$ matrix of covariate information for all J patients and $\mathbf{y}$ is the $J \times 1$ vector of observed event counts, we fit the model described in Section 3.2.1. For this simulation, we fix the regression

coefficients in the generalized linear model arbitrarily to be $\boldsymbol{\beta}' = (1.9, 0.8, -0.2)$ while we allow η, θ, and τ to vary across the cases. We call this Data B.

The regression coefficients receive diffuse normal priors, $\beta_0 \sim N(0, 10)$, $\beta_1 \sim N(0, 10)$, and $\beta_2 \sim N(0, 10)$. Given that the variance used here, 10, is at least five times the magnitude of regression coefficient values, we felt this was a relatively non-informative choice. We investigated two beta priors within each corrected model. For a moderately informative prior, we chose a beta distribution with shape parameters selected so that the equivalent prior sample size (EPSS) is equivalent to 50 patients by setting the mean of each prior distribution to the prior hypothesized values of sensitivity, specificity, and probability of exposure. From Chapter 2, if an equivalent prior sample size, j^*, is considered, and it is believed $\eta = \eta^*$, $\theta = \theta^*$, and $\tau = \tau^*$ are the likely values, we have the equations

$$\begin{array}{lll} a + b = j^*, & c + d = j^*, & e + f = j^*, \\ \frac{a}{a+b} = \eta^*, & \frac{c}{c+d} = \theta^*, & \frac{e}{e+f} = \tau^*. \end{array} \tag{3.5}$$

From this system of equations, for Case 1 we specify $\eta^* = 0.9$, $\theta^* = 0.7$, $\tau^* = 0.8$, and $j^* = 50$, we have

$$\begin{array}{lll} b = 50(1 - 0.9), & d = 50(1 - 0.7), & f = 50(1 - 0.8), \\ a = 50 - b, & c = 50 - d, & e = 50 - f. \end{array} \tag{3.6}$$

This system of equations results in a $beta(45, 5)$ prior for η, a $beta(35, 15)$ prior for θ, and a $beta(40, 10)$ prior for τ for Case 1 of our simulation study. Similarly, for a mildly informative beta prior, we chose shape parameters yielding an equivalent prior sample size of $j^* = 30$. For Case 1 we have

$$\begin{array}{lll} b = 30(1 - 0.9), & d = 30(1 - 0.7), & f = 30(1 - 0.8), \\ a = 30 - b, & c = 30 - d, & e = 30 - f, \end{array} \tag{3.7}$$

which yields three less informative priors for the misclassification parameters: a $beta(27, 3)$ prior for η, a $beta(21, 7)$ prior for θ, and a $beta(24, 6)$ prior for τ.

Thus, we use somewhat diffuse priors for the regression coefficients and informative priors for the misclassification parameters whose true values change for each of the twelve configurations of the simulation, provided in Table 3.1.

Table 3.1: Fixed Values of Sensitivity, Specificity, and Probability of Exposure for the Continuous Covariate Simulation Study.

Case	η	θ	τ
1	0.9	0.7	0.8
2	0.9	0.7	0.4
3	0.9	0.7	0.2
4	0.9	0.5	0.8
5	0.9	0.5	0.4
6	0.9	0.5	0.2
7	0.7	0.9	0.8
8	0.7	0.9	0.4
9	0.7	0.9	0.2
10	0.5	0.9	0.8
11	0.5	0.9	0.4
12	0.5	0.9	0.2

We performed the analysis using the naive model and the model accounting for exposure misclassification specified in (3.7). For each of the 100 samples in our simulation, we record the coverage, posterior mean, and 95% credible set for each parameter in both models. The results are summarized in Tables B.1 through B.36, located in Appendix B. In each table we provide the average posterior mean (across the 100 replications), the interval width, and the coverage for the naive and corrected

models (with differing prior structure) corresponding to each configuration listed in Table 3.1. (See Section 2.4.3 for a discussion of within simulation variability.)

To fit this model we used Markov chain Monte Carlo (MCMC) methods implemented in the WinBUGS software package. WinBUGS uses MCMC methods to sample from the desired posterior distribution resulting from the stationary distribution of a Markov chain. This method is particularly useful when the posterior distribution is not in closed form, as the MCMC methods only require that we specify the full conditionals as described in Gelman et al. (2003). We used three independent chains with dispersed starting values, each of length 25,000 after a burn-in of 5,000. The simulation study comprised 100 iterations, was implemented on a 3.00 GHz Intel Pentium 4 processor with 4.0 GB RAM, and required 34 hours to reach completion for each of the cases, including 12 hours for each corrected model and 10 hours for the naive model.

3.2.4 Simulation Study Results

As in Chapter 2, in our first scenario we investigate the estimates obtained when we specify the sensitivity to be $\eta = 0.9$ and the specificity to be $\theta = 0.7$. Tables B.1 through B.9 contain the results of Cases 1 through 3 where frequent exposure is modeled by $\tau = 0.8$, common exposure is modeled by $\tau = 0.4$, and rare exposure is modeled by $\tau = 0.2$, respectively. When expert opinion is available and moderately informative priors can be assigned to the misclassification parameters, the model produces the best estimates when the true probability of exposure is $\tau = 0.4$. In Table B.5 we see that the coverages for each of the regression coefficients, β_0, β_1, and β_2 are 0.97, 0.97, and 0.93, respectively, exceeding the corresponding coverages obtained when $\tau = 0.8$ (Table B.2) and $\tau = 0.2$ (Table B.8). We also see that widths of the credible sets when $\tau = 0.4$ are slightly narrower than those associated with $\tau = 0.2$ and much narrower than those for $\tau = 0.8$. Finally, the corrected model

results from Table B.5 also indicate that the frequentist gold standard estimates associated with $\tau = 0.4$ produce higher coverages and narrower confidence interval widths than the other values of τ.

Tables B.3, B.6, and B.9 present the simulation results for a corrected model that uses mildly informative priors on the misclassification parameters. Examination of the regression coefficient estimates indicates that the model produces better results when the probability of exposure is $\tau = 0.8$, commonly known as frequent exposure. According to Table B.3 the corrected model with mildly informative priors results in a coverage of 0.99, 0.97, and 0.95 for β_0, β_1, and β_2, respectively. While the credible set widths are wider than those associated with the common exposure ($\tau = 0.4$) and ($\tau = 0.2$), the coverages reported for $\tau = 0.8$ are considerably higher than those reported in Table B.6 and B.9. Table B.6 reports the coverages for β_0, β_1, and β_2 to be 0.85, 0.95, and 0.90, respectively, when $\tau = 0.4$. Table B.9 reports coverages of 0.91, 0.96, and 0.91 for the aforementioned regression coefficients when the probability of exposure is low ($\tau = 0.2$).

In all three cases of this first scenario, the corrected models produce better coverages than the naive models. While corrected models result in coverages at or above 0.85, the naive model produces coverages in the 0.0 to 0.87 range (Tables B.1, B.4, and B.7). In each case the corrected model produces higher coverages, but it is notable that the coverage for the naive models is significantly higher with the addition of the continuous covariate measured without error than in the data with a single binary covariate from Chapter II. The naive model produces biased estimates whose magnitude is inflated for β_0 and attenuated for β_1. The estimates for β_2 have small bias, although the latter fluctuates toward and away from zero in the different cases. The frequentist estimates produced by the corrected models perform well in this scenario, with coverages comparable to those produced by the Bayesian model and slightly narrower confidence intervals.

In our second scenario, we investigate the estimates obtained when we maintain a high sensitivity of $\eta = 0.9$ and lower the specificity to $\theta = 0.5$. Tables B.10 through B.18 contain the results of Cases 4 through 6 of Table 3.1 where frequent exposure is modeled by $\tau = 0.8$, common exposure is modeled by $\tau = 0.4$, and rare exposure is modeled by $\tau = 0.2$, respectively. When expert opinion is available and moderately informative priors can be assigned to the misclassification parameters, this model produces the best estimates when the true probability of exposure is $\tau = 0.8$. This makes intuitive sense when you consider that Case 4 models a high probability of positive exposure using a diagnostic test with a probability of detecting the positive exposure. In Table B.11 we see that the coverages for each of the regression coefficients, β_0, β_1, and β_2 are 0.97, 0.98, and 0.97, respectively, exceeding the corresponding coverages obtained when $\tau = 0.4$ (Table B.14) and $\tau = 0.2$ (Table B.17). We also see that widths of the credible sets when $\tau = 0.8$, are slightly narrower than those associated with $\tau = 0.4$ but somewhat wider than those for $\tau = 0.2$. Finally, the corrected model results from Table B.11 also indicate that the frequentist gold standard estimates associated with $\tau = 0.4$ produce higher coverages and narrower confidence interval widths than the common exposure probability $\tau = 0.4$ and higher coverages but wider interval widths than the rare exposure probability $\tau = 0.2$.

Tables B.12, B.15, and B.18 present the simulation results for a corrected model that uses mildly informative priors on the misclassification parameters. As can be seen, an exposure probability of $\tau = 0.2$ yields the best regression coefficient estimates. According to Table B.18 the corrected model with mildly informative priors results in a coverage of 0.98, 0.99, and 0.95 for β_0, β_1, and β_2, respectively. Regression coefficient credible set widths are wider than for common exposure ($\tau = 0.4$), but are more narrow than those with frequent exposure ($\tau = 0.8$). Coverages for $\tau = 0.2$ are somewhat higher than those reported in Tables B.12 and B.15.

Coverages for β_0, β_1, and β_2, reported for $\tau = 0.4$ in Table B.15, are 0.98, 0.94, and 0.94, respectively. The aforementioned regression coefficients exhibited coverages of 0.96, 0.96, and 0.94 when $\tau = 0.8$, as seen in Table B.12.

In all three cases of this second scenario, the corrected models produce better coverages than the naive models. While corrected models result in coverages at or above 0.92, the naive model produces coverages in the 0.0 to 0.90 range (Tables B.10, B.13, and B.16). In each case the corrected model produced higher coverages, but it is notable that, like the first scenario, the coverage for the naive models is significantly higher with the addition of the continuous covariate measured without error than in the data with a single binary covariate from Chapter II. The naive model produces biased estimates, again with magnitudes inflated for β_0 and attenuated for β_1. The estimates for β_2 have small bias is very small, and it fluctuates toward and away from zero in the different cases. The frequentist gold standard estimates produced by the corrected models perform well in this scenario, with coverages comparable to those produced by the Bayesian model and slightly narrower confidence interval widths.

In our third scenario we investigate the estimates obtained when we decrease the sensitivity to $\eta = 0.7$ and raise the specificity to $\theta = 0.9$. Tables B.10 through B.18 contain the results of Cases 7 through 9 in Table 3.1. As in Cases 4 through 5, when expert opinion is available and moderately informative priors can be assigned to the misclassification parameters, this model produces the best estimates when the true probability of exposure is $\tau = 0.8$. In Table B.20 we see that the coverages for each of the regression coefficients, β_0, β_1, and β_2 are 0.97, 0.97, and 0.96, respectively, exceeding the corresponding coverages obtained when $\tau = 0.4$ (Table B.23) and $\tau = 0.2$ (Table B.26). Note, however, that the credible set widths when modeling $\tau = 0.8$ are wider than those associated with $\tau = 0.4$ and $\tau = 0.2$. Finally, the corrected model results from Tables B.20, B.23 and B.26 present almost

identical coverages associated with the confidence interval around the gold standard frequentist estimates. Similar coverages leads us to look at the width of the associated interval, and we find that the credible set widths produced when $\tau = 0.4$ and $\tau = 0.2$ are significantly smaller than those produced when $\tau = 0.8$.

Tables B.21, B.24, and B.27 present the simulation results for a corrected model that uses mildly informative priors on the misclassification parameters. In studying the resulting estimates of the regression coefficients, we see that the model produces similar results across values of τ, but considering our primary coefficient of interest, β_1, the coverage of 0.97 when $\tau = 0.4$ is the highest and the credible set width associated with it is narrower than that of $\tau = 0.8$ or $\tau = 0.2$. According to Table B.23 the corrected model with mildly informative priors results in a coverage of 0.89, 0.97, and 0.93 for β_0, β_1, and β_2, respectively. Table B.21 reports the coverages for β_0, β_1, and β_2 to be 0.94, 0.95, and 0.98, respectively, when $\tau = 0.8$ and Table B.27 reports coverages of 0.91, 0.93, and 0.95 for the aforementioned regression coefficients when the probability of exposure is low ($\tau = 0.2$).

In all three cases of this scenario, the corrected models produced better coverages than the naive models. While corrected models resulted in coverages at or above 0.89, the naive model produced coverages in the 0.0 to 0.89 range (Tables B.19, B.22, and B.25). In each case the corrected model produced higher coverages, but it is notable that, like the first scenario, the coverage for the naive models is significantly higher with the addition of the continuous covariate measured without error than in the data with a single binary covariate from Chapter II. Again, the naive model produces biased estimates: magnitudes for β_0 are inflated, and those for β_1 are attenuated. The estimates for β_2 exhibited a small bias, and it fluctuated toward and away from zero in the different cases. The frequentist estimates produced by the corrected models perform well in this scenario, with coverages comparable to those produced by the Bayesian model when the probability of exposure was high

$\tau = 0.8$ and higher than those produced by the Bayesian model when the probability of exposure was common $\tau = 0.4$ and low $\tau = 0.2$. The frequentist gold standard estimates were also accompanied by slightly narrower confidence interval widths in this scenario.

In our final scenario we investigate the estimates obtained when we further decrease the sensitivity to $\eta = 0.5$ and maintain a high specificity of $\theta = 0.9$. Tables B.28 through B.36 contain the results of Cases 10 through 12 where frequent exposure is modeled by $\tau = 0.8$, common exposure is modeled by $\tau = 0.4$, and rare exposure is modeled by $\tau = 0.2$, respectively. When expert opinion is available and moderately informative priors can be assigned to the misclassification parameters, this model produces the best estimates when the true probability of exposure is $\tau = 0.4$. In Table B.32 we see that the coverages for each of the regression coefficients, β_0, β_1, and β_2 are 0.96, 0.95, and 0.97, respectively, which are similar to the coverages obtained when $\tau = 0.8$ (Table B.29). We do, however, see that the credible set widths when modeling $\tau = 0.4$ are considerably narrower than those associated with $\tau = 0.8$ and $\tau = 0.2$. The corrected model results from Tables B.29, B.32 and B.35 present high coverages associated with the confidence interval around the frequentist estimates. Similar coverages leads us to look at the width of the associated interval, and we find that the CI interval widths produced when $\tau = 0.4$ and $\tau = 0.2$ are slightly smaller than those produced when $\tau = 0.8$.

Tables B.30, B.33, and B.36 present the simulation results for a corrected model that uses mildly informative priors on the misclassification parameters. In studying the resulting estimates of the regression coefficients, we see that the model produces similar results across values of τ, but considering our primary coefficient of interest, β_1, the coverage of 0.99 when $\tau = 0.8$ is the highest, although the credible set width associated with it is twice as wide as the interval associated with $\tau = 0.4$. According to Table B.30 the corrected model with mildly informative priors results in

a coverage of 0.96, 0.99, and 0.94 for β_0, β_1, and β_2, respectively. Table B.33 reports the coverages for β_0, β_1, and β_2 to be 0.97, 0.96, and 0.96, respectively, when $\tau = 0.4$ and Table B.36 reports coverages of 0.96, 0.95, and 0.97 for the aforementioned regression coefficients when the probability of exposure is low ($\tau = 0.2$).

In this scenario, the corrected model fails to produce better coverages than the naive model for all but one of the regression coefficients, namely β_2. Because our primary interest is the estimation of the covariate subject to misclassification, we will not put significant weight on this finding as the coverages and estimates for β_1 produced by the naive model differ grossly from nominal. While corrected models resulted in coverages at or above 0.89, the naive model produced coverages in the 0.0 to 0.95 range (Tables B.28, B.31, and B.34). In now familiar fashion, the naive model yields biased estimates, with inflated magnitudes for β_0 and attenuated magnitudes for β_1. And, once again, the estimates for β_2 exhibited only a small bias, and it fluctuated toward and away from zero in the different cases. The frequentist estimates produced by the corrected models perform well in this scenario, with coverages comparable to those produced by the Bayesian model. The frequentist gold standard estimates were also accompanied by slightly narrower confidence interval widths in this scenario.

Just as in Chapter 2, we exhibit our simulation results graphically as well as with tables. Figure 3.2 corresponds to the continuous covariate model under Cases 1 through 3 in Table 3.1, where informative beta priors are placed on the misclassification parameters and diffuse priors are placed on the regression coefficients, as discussed in Section 3.2.2.

The left three vertical bars represent simulations performed with data generated according to Case 1. The first bar corresponds to the naive model, and the remaining bars correspond to the corrected model with varying degrees of informativeness incorporated into the priors on the misclassification parameters. The three

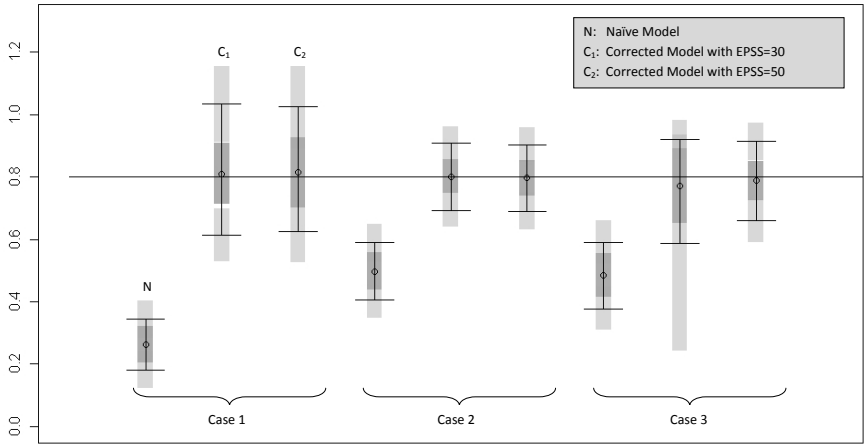


Figure 3.2: Variation within the estimates for β_1 for Cases 1, 2, and 3. Posterior means and credible set intervals are plotted for the naive model and the corrected models with EPSS=30 and EPSS=50.

vertical bars in the center represent the simulations performed with data generated according to Case 2, and the three bars to the right represent the simulations performed with data generated according to Case 3. Within each set of three, we have a naive model and two corrected models with increasing equivalent prior sample size. The horizontal line across the entire figure is the true parameter value ($\beta_1 = 0.8$). The horizontal line atop each vertical bar is the simulation average upper bound on the 95% credible interval. The lower horizontal line on each vertical bar is the simulation average lower bound on the interval. The central dot is the simulation average posterior mean. The dark gray box is plus or minus one simulation standard deviation on the posterior mean. The light gray boxes are plus or minus one simulation standard deviation on the upper 97.5% and lower 2.5% bounds.

Ultimately, we see that nondifferential misclassification of a binary explanatory variable yields attenuated estimates of associated effects. While binary misclassification seems to be more damaging than continuous measurement error in general (see Section 2.2), they share key features (Gustafson, 2003). In each case a primary

predictor of the extent of the bias is the strength of the relationship between the mismeasured explanatory variable and the precisely measured explanatory variable.

In the previous chapter we noted that, generally speaking, the bias due to misclassification worsens as the proportion of subjects exposed nears the extremes of $[0, 1]$. In epidemiological contexts it is not uncommon to conduct studies with low exposure prevalences, and we reiterate the clear need for methods which can adjust inferences to account for misclassification, such as those presented by Gustafson (2003).

3.3 Random Effects

One of the assumptions for the generalized linear model (GLM) is that the responses are independent. However, it is reasonable to assume that observations within a cluster will tend to be more alike than observations from different clusters. For example, repeated observations on a patient in a longitudinal study are unlikely to be independent. In sample surveys responses from members of the same community are likely to be correlated. In another example, Hougaard, Lee, and Whitmore (1997) established that, for counts of the number of epileptic seizures, there is very large individual variation in the seizure rate. Likewise, in genetic epidemiology, observations on members of the same family will most certainly be correlated (Zeger and Karim, 1991).

If prior experience or expert opinion leads us to believe there is clustering among observations, or lack of independence of any nature, analysis that assumes independence is inappropriate (Agresti, 2002). The dependence structure may be accommodated by including random effects, resulting in a mixed model. The generalized linear mixed model (GLMM) is an extension of the GLM that allows both fixed effects and random effects. Breslow and Clayton (1993), Agresti et al. (2000),

and Gibbons et al. (2008) have each explored a mixed model for Poisson counts when the covariates are assumed to be measured without error.

Often, residual variability under under a Poisson distribution exceeds that expected under the postulated variance-mean relationship. One approach to handling such a violation is to use a GLMM. Thus for a Poisson outcome, $y_j \sim Poisson(\lambda_j)$, we might stipulate a model for the mean, λ_j, containing both fixed and random effects:

$$\log(\lambda_j) = \mathbf{x}_j\boldsymbol{\beta} + \varepsilon_j,$$

where $\mathbf{x}_j$ is a vector of covariates with corresponding coefficient vector $\boldsymbol{\beta}$, and where the ε_j are parametric in this study (e.g. normal), but could possibly be semi-parametric (e.g. where we could plausibly use a Dirichlet process prior for the ε_j with a normal baseline model). Congdon (2005) notes that this approach translates into adding a set of parameters which increase in number with the sample size, making the likelihood nonregular and raising the question about how many effective parameters are in the model.

3.3.1 The Bayesian Model

In this section we describe the Bayesian GLMM that accommodates a Poisson count with misclassified binary predictor variable. We let y_j be the observed count that is produced by a Poisson process with rate parameter λ_j, where $\log \lambda_j = \mathbf{x}_j'\boldsymbol{\beta}$ for the j^{th} covariate pattern. Here, $\boldsymbol{\beta}$ is a vector of regression coefficients corresponding to $\mathbf{x}_j$, a $1 \times k$ vector of covariates, one of which is the binary covariate subject to exposure misclassification.

Agresti (2002) describes the GLMM as a two stage model. In the initial stage, the observed responses follow a traditional generalized linear model (GLM), conditioned on the random effects. As a result of this conditioning on random effects, observations within a cluster, such as for a patient group or a testing site, may be

independent. At the second stage, the random effects are assumed independent, with $\varepsilon_j \sim N(0, \sigma^2_\varepsilon)$.

For the mixed model, the likelihood function of the observed data is

$$\begin{aligned} f(\boldsymbol{\beta}, \eta, \theta, \tau, \boldsymbol{\varepsilon}, \sigma^2_\varepsilon | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) &= \prod_{j=1}^{J} f(y_i | \boldsymbol{\beta}, \eta, \theta, \tau, \varepsilon_j) f(\varepsilon_j | \sigma^2_\varepsilon) \\ &= \prod_{j=1}^{J} [x_j \eta^{x^*_j} (1-\eta)^{1-x^*_j} + (1-x_j)\theta^{1-x^*_j}(1-\theta)^{x^*_j}] \\ &\quad \times \tau^{x_j}(1-\tau)^{1-x_j} \lambda^{y_j} e^{-\lambda} \\ &\quad \times \prod_{j=1}^{J} (2\pi\sigma^1)^{-1/2} exp[-\varepsilon_j^2/(2\sigma^2)], \end{aligned}$$

where $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_j)$ and $g(\lambda_j) = \mathbf{x}_j \boldsymbol{\beta} + \varepsilon_j$, conditioned on the random effects.

3.3.2 Prior Distributions

We assume independent informative beta priors on the values of sensitivity, specificity, and probability of exposure. As in Section 3.2.3, we use beta priors for the misclassification parameters. We place diffuse normal priors on both regression coefficients, $\beta_0 \sim N(\mu_{\beta_0}, \sigma^2_{\beta_0})$ and $\beta_1 \sim N(\mu_{\beta_1}, \sigma^2_{\beta_1})$. These diffuse normal priors are relatively noninformative compared to the likelihood because they are "flat" where the likelihood is peaked. We place a normal prior on the random effects component, $\varepsilon \sim N(\mu_\varepsilon, \sigma^2_\varepsilon)$, and a hyperprior on the variance component of the random effects $\sigma_\varepsilon \sim unif(0, B)$, where B is an upper bound. Assuming prior independence of all unknown parameters, the joint prior distribution is given by

$$p(\boldsymbol{\beta}, \eta, \theta, \tau, \boldsymbol{\varepsilon}) = p(\boldsymbol{\beta}) \times p(\eta) \times p(\theta) \times p(\tau) \times p(\boldsymbol{\varepsilon}), \tag{3.8}$$

and the joint posterior distribution is

$$p(\boldsymbol{\beta}, \eta, \theta, \tau, \boldsymbol{\varepsilon} | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) \propto f(\boldsymbol{\beta}, \eta, \theta, \tau, \eta, \boldsymbol{\varepsilon} | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) \times p(\boldsymbol{\beta}, \eta, \theta, \tau, \eta, \boldsymbol{\varepsilon}). \tag{3.9}$$

A graphical summary of this model is displayed in Figure 3.3.

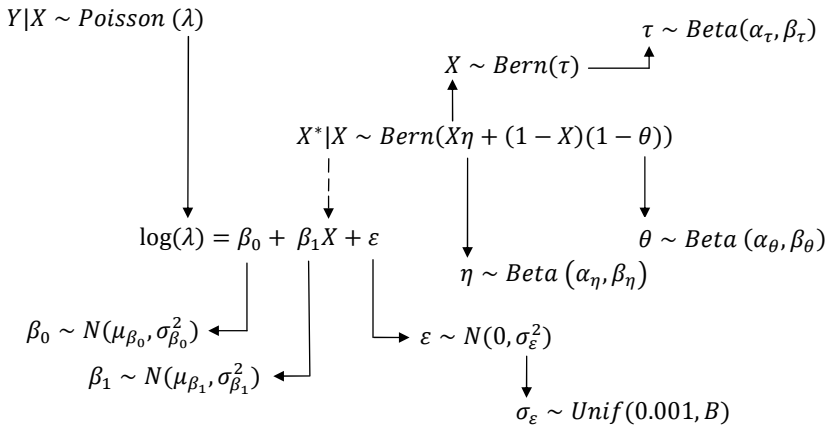


Figure 3.3: Summary of the Poisson regression model with random effects. (The dashed line denotes substitution of a variable with its surrogate.)

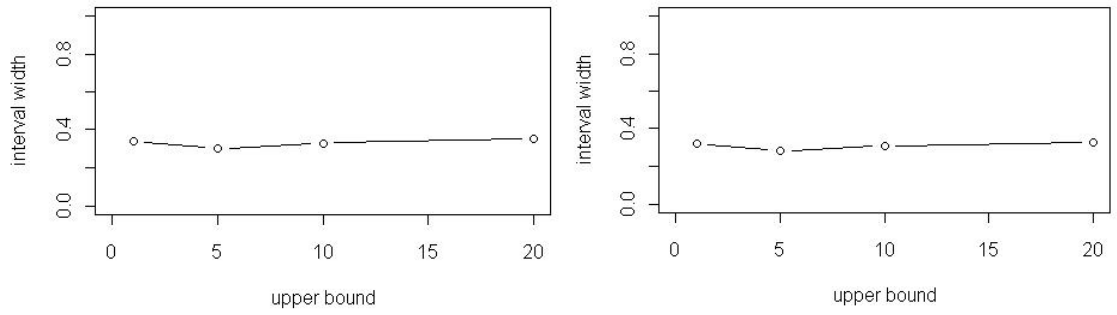
3.3.3 Simulation Study Design

To compare the fixed effect model to the mixed effect model when random effects are indeed present, data are generated according to the model specified in (3.9). We assume independent beta priors for the sensitivity, specificity, and probability of exposure, with "likely" values $\eta = (0.9, 0.7, 0.5)$, $\theta = (0.9, 0.7, 0.5)$ and $\tau = (0.8, 0.4, 0.2)$. The equivalent prior sample size, $j^* = 30$, yields mildly informative priors, and the equivalent prior sample size, $j^* = 50$, yields moderately informative priors.

Using (3.8) we obtain the priors $\eta \sim beta(27, 3)$, $\theta \sim beta(21, 9)$, and $\tau \sim beta(24, 6)$ for Case 1 of Table 3.1 when considering an equivalent prior sample size of $j^* = 30$. The regression coefficients receive diffuse normal distributions, $\beta_0 \sim N(0, 10)$ and $\beta_1 \sim N(0, 10)$. We place a $uniform(0.001, 5)$ prior on the standard deviation of the random effect (Gelman, 2006). We call this Data C.

To fit this model we used Markov chain Monte Carlo (MCMC) methods implemented in the WinBUGS software package, as in Section 3.2.3. We used three independent chains, each with 25,000 iterations after a 5,000 burn-in. The results are presented in Tables B.37 through B.52 of Appendix B.

It is important to check the robustness of the posterior to changes in the upper bound, B, for the uniform prior on σ_ε. To do so, we consider a sequence of values for B, and run the model for each value in the sequence, each time monitoring the posterior summaries for σ_ε. Figure 3.4 indicates that the posterior estimate interval widths stabilize quickly, and we choose $B = 5$ as the upper bound for our simulation.



(a) (b)

Figure 3.4: Investigation of the effect of changing the upper bound on the uniform prior for σ_ε. The 95% posterior interval widths for (a) β_0 and (b) β_1 are plotted against the upper bound used in simulation.

3.3.4 Simulation Study Results

We now compare the operating characteristics and estimates that result from three competing models, as follows:

(1) The model using random effects, as described in Section 3.3;

(2) The model using fixed effects, effectively ignoring additional variability (see Section 2.2);

(3) The gold standard model that fits the Poisson regression to the true data.

We record the coverage, posterior mean, and 95% credible set for each parameter in both models. We consider a simulation size of $N = 100$. The simulation results are provided in Tables B.37 through B.96, which can be found in Appendix B. (Within simulation variability is discussed as well, just as in Section 2.4.3.)

Tables B.37 through B.60 contain the results produced by the fixed effects and random effects models when an equivalent prior sample size of $j^* = 30$ is used to calculate priors for the misclassification parameter combinations listed in Table 3.1. Here, $J = 300$ observations are generated for each simulation iteration.

Tables B.31 through B.84 contain the results produced by the fixed effects and random effects models when an equivalent prior sample size is increased to $j^* = 50$ for the construction of priors for the same misclassification parameter combinations. Again, we have $J = 300$ observations generated for each simulation iteration.

Tables B.85 through B.96 contain the results produced by the fixed effects and random effects models for Case 1 through Case 6 when the equivalent prior sample size remains at $j^* = 50$. This time the study sample size is increased to $J = 500$ observations for each simulation iteration.

In our first scenario we again investigate the estimates obtained when we specify the sensitivity to be $\eta = 0.9$ and the specificity to be $\theta = 0.7$. Tables B.37 through B.42 contain the results of Cases 1 through 3 where frequent exposure is modeled by $\tau = 0.8$, common exposure is modeled by $\tau = 0.4$, and rare exposure is modeled by $\tau = 0.2$. Table B.37 presents the results obtained from the fixed effects model, where we fail to account for excess variability. Comparing the results to Table B.38 where we do account for the variability using the random effects model, we find the resulting estimates and coverages to be similar. The credible set widths are comparable at 0.2786 for the fixed effects model and 0.2841 for the mixed model. The frequentist gold standard analysis produces estimates of 0.8000 (96% coverage) and 0.7960 (97% coverage), respectively.

Examining the common exposure ($\tau = 0.4$) under the same scenario in Tables B.39 and B.40, we find that the fixed and random effects models again perform comparably, with the fixed effects model achieving 96% coverage for β_1 with a credible

set width of 0.1592 and the random effects model achieving 95% coverage for β_1 with a credible set width of 0.1594.

Tables B.41 and B.42 pertain to the low exposure ($\tau = 0.2$), and we see a significant increase in interval widths, with the fixed effect model producing a posterior mean of 0.7320 and interval width of 0.3871 for β_1. The increase is also experienced in the random effects model, which produces a posterior mean of 0.7362 and interval width of 0.3888. We see increases in the interval width of every other model parameter as well. We investigate this apparent model instability by plotting the posterior mean and credible set for β_1 produced for each iteration of the simulation, as presented in Figure 3.5. We find the posteriors to be multi-modal and we conclude that the model is having difficulty converging when the probability of exposure is low.

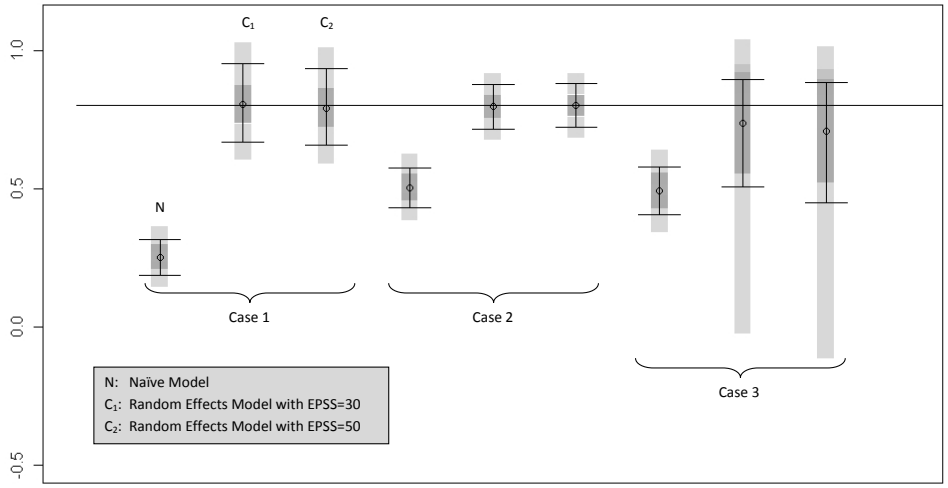


Figure 3.5: Posterior means and credible sets for β_1 in Cases 1, 2, and 3 when EPSS=30 and EPSS=50.

Investigating further, we conduct another simulation in which the same fixed effects and random effects models are fitted with moderately informative priors constructed using an equivalent prior sample size of $j^* = 50$. Tables B.65 and B.66 contain the results of this simulation. We see that coverages for the model parameters

increase, but the interval widths for β_1 produced by the Bayesian models are still much wider than those achieved with higher exposure probabilities.

The simulation for Case 3 is conducted a third time, maintaining the equivalent prior sample size for the informative priors of $j^* = 50$ but increasing the study sample size to $J = 500$. The results are presented in Tables B.89 and B.90. There is a sharp decrease in the interval widths of all model parameters for both the fixed and random effects models. The credible set width for β_1 produced by the random effects model is just 0.1873, a significant decrease from 0.4351, the width produced by the same model when the study size was $J = 300$.

In Figure 3.6, we plot the posterior distributions for β_1 from the simulations described above.

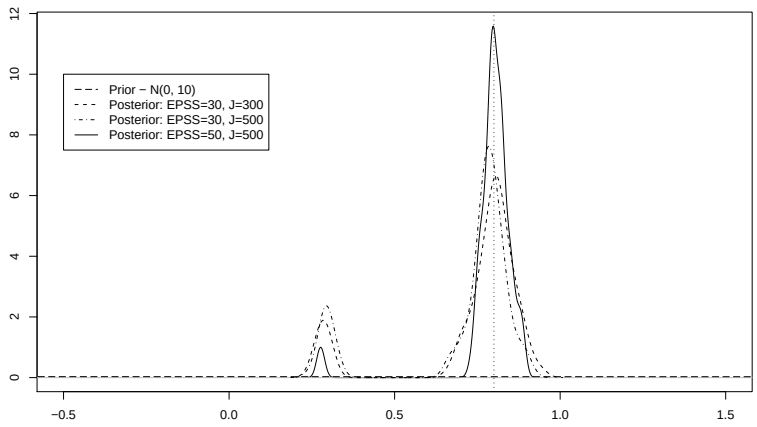


Figure 3.6: Prior and posterior distributions for our parameter of interest, β_1, when data is simulated according to Case 3. An increase in EPSS and study size results in a posterior with most of its mass centered on the true value.

In our second scenario we maintain a sensitivity of $\eta = 0.9$ and lower the specificity to $\theta = 0.5$. As we can see from Tables B.41 through B.43, the random effects model outperforms the fixed effects model when frequent exposure (Table B.42) or common exposure (Table B.44) are assumed. When $\tau = 0.8$, the random effects model achieves 93% coverage for β_1 compared to 91% achieved by the fixed

effects model. When $\tau = 0.4$ the random effects model model is again superior, achieving 98% coverage for β_1 compared to 95% coverage that results from the use of the fixed effects model.

When considering Case 6, we see that the low exposure probability again results in model instability. From Tables B.47 and B.48 we see the credible set widths produced for β_1 by the fixed and random effects models are 0.4347 and .5710, respectively. In Tables B.71 and B.72 we find that increasing the equivalent prior sample size to $j^* = 50$ does not resolve the problem, and credible set widths of 0.3380 and 0.5379 are reported for β_1. Maintaining an equivalent prior sample size of $j^* = 50$ and increasing the study size to $J = 500$ as in the previous scenario does produce more accurate posterior means and narrower credible set widths. For example, from Table B.96 we have that the credible set width for β_1 resulting from use of the random effects model is 0.2072, a reduction of greater than 50% from the credible set obtained when the study sample was of size $J = 300$.

In our third scenario we assume a lower sensitivity of $\eta = 0.7$ and a high specificity of $\theta = 0.9$. In this scenario, found in Tables B.45 through B.48, the random effects model does not perform as well as the fixed effects model. Continuing with the consideration of our regression coefficients, β_0 and β_1, the random effects model results in coverage for β_0 and β_1 of 0.94 and 0.93, respectively, while the fixed effects model results in coverages of 0.96 and 0.96. The disparity grows when we assume a common exposure $\tau = 0.4$ as the random effects model results in coverage for β_0 and β_1 of 0.89 and 0.91, respectively, while the fixed effects model results in coverages of 0.96 and 0.95.

In our final scenario we investigate the estimates obtained when we lower the sensitivity to $\eta = 0.5$ but we maintain high specificity $\theta = 0.9$, with the results presented in Tables B.49 through B.52. We continue to see the increased credible set widths for cases corresponding to low exposure probabilities. We anticipate that

this can be resolved by increasing the study sample size, just as we outlined for the first two scenarios.

3.4 Zero-inflation

In biomedical or healthcare research, outcomes of interest often consist of count variables. For such counts, the Poisson regression model is commonly used to explain the relationship between outcome variable and a set of explanatory variables. However, it is often the case that there is a higher proportion of zero counts than would be predicted by the Poisson distribution, possibly due to a distinct subpopulation of subjects whose only response is zero counts. Suppose, for example, that the outcome is the number of postoperative complications related to a surgical procedure. Patients who are at small risk have zero complications. Patients who are at a higher risk of postoperative complications will exhibit Poisson distributed numbers of complications. When extra-zero counts are observed, it has been suggested that applying the basic Poisson regression model is problematic. The relative errors incurred by ignoring the presence of extra zeros were studied by Gupta, Gupta, and Tripathi (1996), who showed that more error is observed for small values of the count if a basic Poisson model is used instead of a modified Poisson model which adjusts for extra-zeros.

In order to adjust for these extra zero counts, various modifications of the Poisson regression model have been proposed. Lambert (1992) described a zero-inflated Poisson (ZIP) model as a two-component mixture model where one component has a degenerate distribution at zero and the other is a Poisson count model. An observed zero count would arise from the degenerate component with probability π_0. Hur et al. (2002) extended that model for the case of clustered data (e.g., patients observed within hospitals) and present random-effects ZIP models. Mwalili, Lesaffre, and Declerck (2008) attempted to correct for misclassification of dental caries

data in a zero-inflated negative binomial regression model. Bayesian analysis of zero-inflated models has been described in detail by Ghosh, Mukhopadhyay, and Lu (2006). We investigate a Bayesian treatment of the zero-inflated Poisson model, where we extend the previously studied models to include a binary covariate subject to misclassification.

3.4.1 The Bayesian Model

Suppose $f_D(y|\phi)$ is a probability density for y, with parameter vector ϕ. The zero-inflated version of this density, denoted by $Y \sim ZID(\pi_0, \phi)$, has a density of the form

$$f_{ZID}(y) = \pi_0 I(y = 0) + (1 - \pi_0) f_D(y|\phi)$$

where $f_D(y|\phi)$ is a probability density or mass function with parameter vector ϕ and $0 < \pi_0 < 1$. From the equation above, the probability density at zero is equal to $\pi_0 + (1 - \pi_0) f_D(0|\phi)$, while the density at $y > 0$ is given by $(1 - \pi_0) f_D(y|\phi)$. Moreover, the mean and variance of this distribution are equal to

$$E(Y) = (1 - \pi_0) E(Y_D),$$

and

$$V(Y) = (1 - \pi_0) \left(V(Y_D) + \pi_0 E(Y_D) \right),$$

respectively.

The zero-inflated Poisson (ZIP) is the simplest ZID. The full ZIP model has the following representation:

$$Y_i \sim ZIP(\pi_0, \lambda_i) \qquad \text{and} \qquad \log(\lambda_i) = \mathbf{X}_i \boldsymbol{\beta},$$

where $\mathbf{X}_i$ is a design matrix and $\boldsymbol{\beta}$ is a corresponding vector of coefficients.

As noted by Ntzoufras (2008), the excessive proportion of zeros, π_0, is usually assumed constant across all observations, but covariates can also be incorporated

here with little difficulty. For example, we may model this dependence as

$$\log\left(\frac{\pi_{0i}}{1-\pi_{0i}}\right) = \mathbf{X}_i^Z \boldsymbol{\beta}^Z,$$

where $\mathbf{X}_i^Z$ and $\boldsymbol{\beta}^Z$ are a design matrix and a vector of coefficients, respectively.

In Figure 3.7, we see the difference that even a small value of π_0 makes in the distribution of the data. For this example 100 responses were generated from a Poisson distribution with event rate $\lambda = 3$. We also generated 100 responses from a zero-inflated Poisson distribution with the same event rate but a probability of zero-state of $\pi_0 = 0.2$. The regression estimates produced by a basic Poisson model would be grossly inaccurate if applied to zero-inflated data.

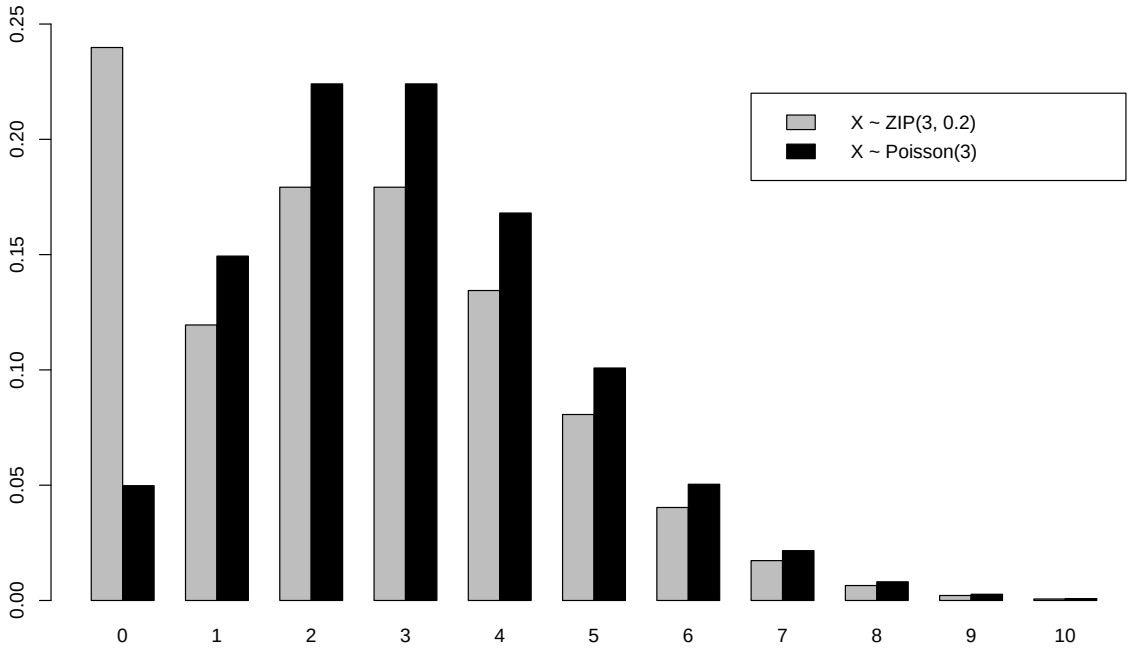


Figure 3.7: Comparison of data generated from a zero-inflated Poisson distribution constant rate λ and relatively low probability of zero-state with data generated from a basic Poisson distribution with the same rate.

3.4.2 Prior Distributions

We place normal priors on both regression coefficients, $\beta_0 \sim N(\mu_{\beta_0}, \sigma^2_{\beta_0})$ and $\beta_1 \sim N(\mu_{\beta_1}, \sigma^2_{\beta_1})$, where the variances will be large, creating diffuse distributions.

We assume independent informative beta priors on the values of sensitivity, specificity, and probability of exposure. We also choose a beta distribution for π_0, the probability of zero-inflation. When available, expert opinion will be incorporated in the construction of an informative beta prior for π_0. Assuming prior independence of all unknown parameters, the joint prior distribution is given by

$$p(\boldsymbol{\beta}, \eta, \theta, \tau, \pi_0) = p(\boldsymbol{\beta}) \times p(\eta) \times p(\theta) \times p(\tau) \times p(\pi_0), \tag{3.10}$$

and the joint posterior distribution is

$$p(\boldsymbol{\beta}, \eta, \theta, \tau, \pi_0 | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) \propto f(\boldsymbol{\beta}, \eta, \theta, \tau, \pi_0 | \mathbf{y}, \mathbf{x}, \mathbf{x}^*) \times p(\boldsymbol{\beta}, \eta, \theta, \tau, \pi_0), \tag{3.11}$$

where η, θ, and τ are defined as in Section 2.2. A graphical summary of this model is displayed in Figure 3.8.

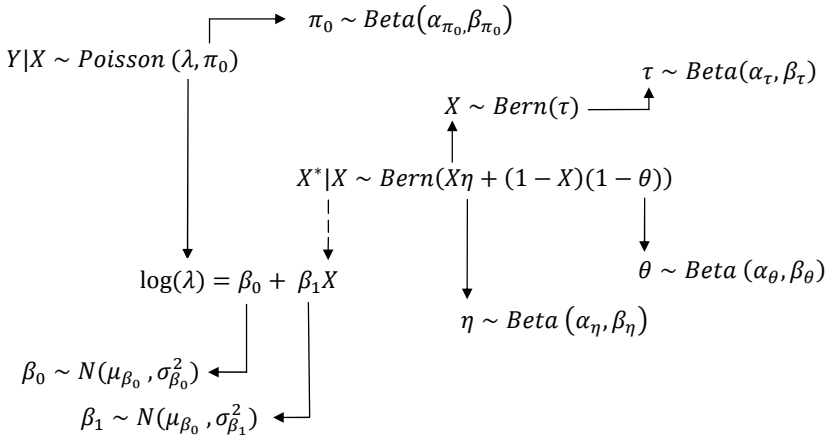


Figure 3.8: Summary of the zero-inflated Poisson regression model. The dashed line denotes substitution of a variable with its surrogate.

3.4.3 Simulation Study Design

To compare the zero-inflated Poisson model to the tradition Poisson model when zero-inflation is indeed present, data are generated according to the model specified in (3.14). We assume independent beta priors for the sensitivity, specificity,

and probability of exposure, with "likely" values $\eta = (0.9, 0.7)$, $\theta = (0.9, 0.7)$ and $\tau = (0.8, 0.2)$, and the equivalent prior sample size $j^* = 50$ yields moderately informative priors.

Using (3.8) we obtain the priors $\eta \sim beta(45, 5)$, $\theta \sim beta(35, 15)$, $\tau \sim beta(40, 10)$, and $\pi_0 \sim beta(5, 45)$ for Case 1 of Table 3.2 when considering an equivalent prior sample size of $j^* = 50$. The regression coefficients receive the diffuse normal distributions, $\beta_0 \sim N(0, 10)$ and $\beta_1 \sim N(0, 10)$. Just as in Section 3.2.2, we feel this is a relatively non-informative choice because the variance is of considerably greater magnitude than the coefficients leading to priors that are "flat" where the likelihood is peaked . We examine the cases where the the probabilities of exposure are "frequent" and "low", $\tau = 0.8$ and $\tau = 0.2$, as stated in Table 3.2. We call this Data D.

Table 3.2: Fixed Values of Sensitivity, Specificity, Probability of Exposure, and Probability of Zero-Inflation for the ZIP Simulation Study

Case	η	θ	τ	π_0
1	0.9	0.7	0.8	0.1
2	0.9	0.7	0.8	0.2
3	0.9	0.7	0.2	0.1
4	0.9	0.7	0.2	0.2
5	0.7	0.9	0.8	0.1
6	0.7	0.9	0.2	0.1

We performed the analysis using the naive model and the model accounting for zero inflation specified in (3.11). We record the coverage, posterior mean, and 95% credible set for each parameter in both models. We consider a simulation size of $N = 100$, and sample sizes of $J = 300$. We summarize the simulation results in

Tables B.97 through B.108, located in Appendix B. In each table we provide the average posterior mean (across the 100 replications), the interval width, and the coverage for the naive and corrected models corresponding to each configuration listed in Table 3.2.

3.4.4 Simulation Study Results

In our first scenario we investigate the estimates obtained when we specify the sensitivity to be $\eta = 0.9$ and the specificity to be $\theta = 0.7$. Tables B.97 through B.104 contain the results of Cases 1 through 4 where frequent exposure is modeled by $\tau = 0.8$ and rare exposure is modeled by $\tau = 0.2$. When expert opinion is available and moderately informative priors can be assigned to the misclassification and zero-inflation parameters, the model produces the best estimates when the true probability of exposure is high, $\tau = 0.8$, and probability of zero-inflation is very low ($\pi_0 = 0.1$).

In the results presented in Appendix B, two tables are presented for each case listed in Table 3.2. The first contains the results when a basic Poisson model is used to analyze the zero-inflated data. Here, "naive" refers to a model that does account for misclassification, but to account for zero-inflation. The second contains the results obtained when a zero-inflated Poisson model is used to analyze the same zero-inflated data. For both the basic and zero-inflated models, Bayesian and frequentist estimates, interval widths, and coverages are provided.

In Table B.97 we see that the posterior means are highly inaccurate and the credible set widths are large. With no expert opinion available for the regression coefficients, the basic Poisson model fails to converge for Case 1. In Table B.98, the posterior means are close to the true values and the credible set widths are within the expected range. The zero-inflated model achieves coverages between 0.93 and 1.00 for each parameter in the model.

To confirm the suspicions of nonconvergence in the basic Poisson model, we consider Case 1 from Table 3.2. We compare the posterior distribution for β_1 produced by the basic Poisson model to the posterior distribution for β_1 produced by the zero-inflated Poisson model in Figure 3.9 below.

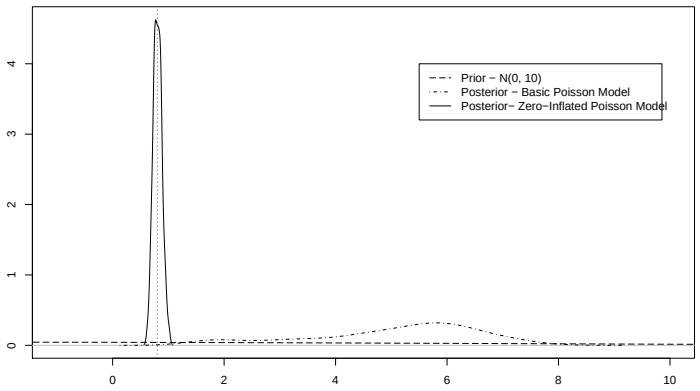


Figure 3.9: Prior and posterior distributions for β_1 when Case 1 is considered.

The frequentist parameter estimates presented in Tables B.97 and B.98 are stable, but the very low coverages for β_0 and β_1 that result from the use of a basic Poisson regression model on the true data are much lower than those that result from the use of a zero-inflated Poisson regression model on the true data, the "gold standard" in this case. The basic Poisson model results in coverages of 0.56 and 0.90 for β_0 and β_1, respectively. The zero-inflated Poisson model fit to the gold standard data results in coverages of 0.96 and 0.95.

The results for Case 2, in which we raise the probability of zero-inflation to $\pi_0 = 0.2$, are indicative of convergence problems similar to that of Case 1 for the basic Poisson model. In comparing Case 1 and Case 2, we can see from Table B.98 and B.100 that when the probability of zero-inflation grows, the posterior means for β_0 and β_1 are less accurate and produce wider credible set intervals. The Bayesian zero-inflated model does continue to outperform the frequentist gold

standard zero-inflated model in its estimation and coverage of β_0 and performs similarly to the frequentist model in the estimation and coverage of β_1.

Tables B.101 through B.104 contain the results obtained when the true rate of exposure is lowered to $\tau = 0.2$. The basic Poisson model continues to struggle with convergence issues. For Case 3, when the probability of zero-inflation is very low ($\pi_0 = 0.1$), the zero-inflated Poisson model achieves higher coverage than the frequentist zero-inflated model. Here the Bayesian model produces coverages of 0.95 for β_0 and 0.97 for β_1, compared to 0.92 and 0.94 for the frequentist model.

In our second scenario we investigate the estimates obtained when we lower the sensitivity to $\eta = 0.7$ and raise the specificity to be $\theta = 0.9$. Tables B.105 through B.108 contain the results of Case 5 and Case 6 where frequent exposure is modeled by $\tau = 0.8$ and rare exposure is modeled by $\tau = 0.2$. We can see in Tables B.106 and B.108 that the posterior means for the regression parameters and their associated credible set widths are reminiscent of those produced by the basic Poisson model. In changing the parameterization of the misclassification parameters by lowering the sensitivity and raising the specificity, we find that both the basic and zero-inflated Poisson models fail to converge.

This simulation study, while small, does give us a glimpse at the vulnerabilities and potential of the Bayesian zero-inflated Poisson models. Without expert opinion for the regression parameters and/or without a larger sample size, the Bayesian models are not robust with respect to zero-inflated data. When the information and sample size that we have available are sufficient for model convergence, the zero-inflated Bayesian model outperforms each of the other models. In the next section we present an example in which our data set is very large. In this case we will see that model convergence can be achieved even when we have no expert opinion or prior information regarding many of the model parameters, as long as we have expert opinion concerning the probability of zero-inflation.

3.4.5 Example

Cost and access continue to be the fundamental issues in the debate over the future of the American health-care system. An important element of this debate concerns the health-care needs of the elderly. During the past two decades, the population aged 65 and over increased more than twice as fast as the younger population and they account for a disproportionate share of medical care expenditures (Deb and Trivedi, 1997). We wish to model instances of medical care utilization by the elderly in the United States using data from the National Medical Expenditure Survey, 1987. A feature of these data is that they include a high proportion of zero counts, as seen in Figure 3.10.

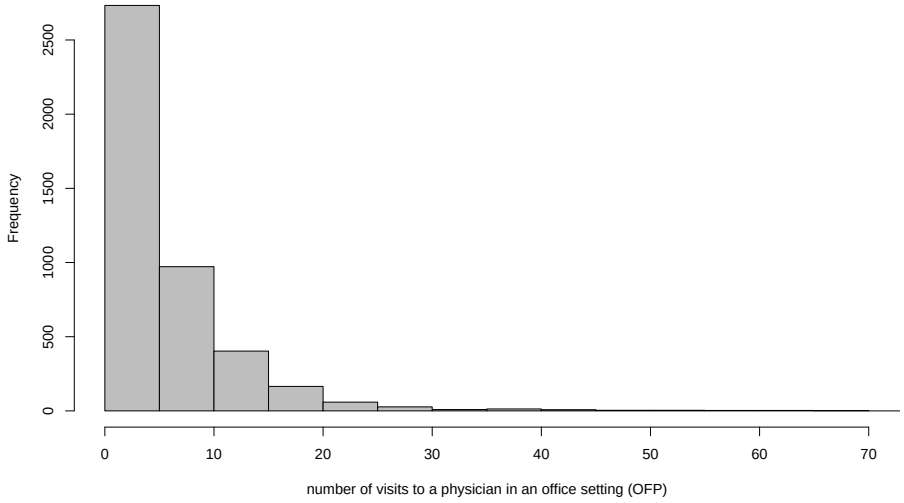


Figure 3.10: Number of visits to a physician in an office setting - as reported by the National Medical Expenditure Survey.

The data are obtained from the National Medical Expenditure Survey (NMES) which was conducted in 1987 and 1988 to provide a comprehensive picture of how Americans use and pay for health services. The NMES is based upon a representative, national probability sample of the civilian, non-institutionalized population and individuals admitted to long-term care facilities during 1987. Under the house-

hold survey of the NMES, more than 38,000 individuals in 15,000 households across the United States were interviewed quarterly about their health insurance coverage, the services they used, and the cost and source of payments of those services. These data were verified by cross-checking information provided by survey respondents with providers of health-care services. In addition to health-care data, NMES provides information on health status, employment, sociodemographic characteristics, and economic status.

For our example we consider a subsample of individuals ages 66 and over, a total of 4,406 observations. Deb and Trivedi (1997) considered six mutually exclusive measures of utilization, but for the sake of illustration we will consider a single outcome - visits to a physician in an office setting (OFP). Additionally, we consider a single economic predictor variable - employment status (EMP). The number of visits to a physician takes on the role of a Poisson outcome measured without error and employment status plays the role of a binary covariate subject to misclassification, thus replicating the model presented in Figure 2.3, with the added complexity of apparent zero-inflation.

We perform two analyses, the first using a "naive" model in which zero-inflation is not accounted for in the traditional Poisson regression model, and the second using a model that incorporates an additional parameter, π_0, to account for zero-inflation. Because we have little historical data regarding the sensitivity and specificity for employment responses, we place diffuse $beta(1, 1)$ priors on η, θ, and τ. We place an informative $beta(5, 45)$ prior on the probability of zero-inflation π_0.

To fit each model we used Markov chain Monte Carlo (MCMC) methods implemented in the WinBUGS software package. We used three independent chains, each with 25,000 iterations after a 5,000 burn-in. The results of the first analysis in which we fail to account for zero-inflation are presented in Table 3.3.

Table 3.3: Posterior Summaries for the Naive Model

Parameter	Mean	SD	MCerror	2.5%	50%	97.5%
β_0	1.5340	0.7504	0.0345	0.9744	1.0150	2.6130
β_1	0.5306	1.5010	0.0691	-1.612	1.5820	1.6160
η	0.8978	0.0157	6.596E-4	0.8773	0.8916	0.9296
θ	0.0920	0.0161	6.611E-4	0.0677	0.0872	0.1207
τ	0.4284	0.2025	0.0093	0.2699	0.2912	0.7271

We see that the standard errors for our parameters of interest, β_0 and β_1, are quite large as well as the 95% credible set widths. In viewing the posterior densities and trace plots produced by WinBUGS and presented as Figure 3.11, we see that the model is not converging.

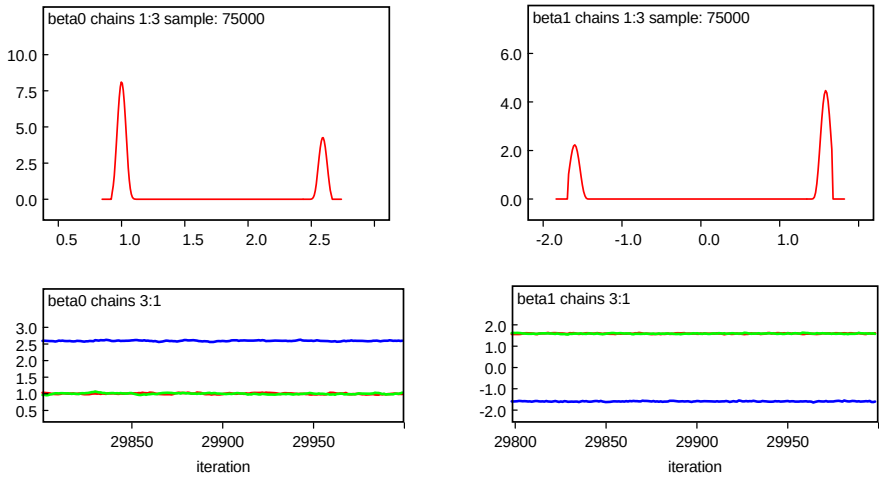


Figure 3.11: Posterior densities and trace plots for β_0 and β_1.

We implement MCMC methods again within WinBUGS to fit the second model in which we account for zero-inflation through an additional parameter, π_0. The results of this analysis are presented in Table 3.4.

Table 3.4: Posterior Summaries For The Zero-Inflated Model

Parameter	Mean	SD	MCerror	2.5%	50%	97.5%
β_0	1.419	0.01379	1.439E-4	1.391	1.419	1.445
β_1	1.361	0.01298	8.507E-5	1.335	1.361	1.386
η	0.8914	0.005483	2.117E-5	0.8805	0.8915	0.9019
θ	0.08604	0.01038	4.937E-5	0.06669	0.08572	0.1074
τ	0.2185	0.008847	7.075E-5	0.2014	0.2184	0.236
π_0	0.1437	0.005508	2.33E-5	0.133	0.1436	0.1546

Compared to the results from the naive model, the standard deviations and Monte Carlo error have decreased significantly. Again, we examine the posterior densities and trace plots for β_0 and β_1 in Figure 3.12 and see that convergence is achieved and the posterior densities are smooth and unimodal. Similarly, we present the posterior densities for each model parameter in Figure B.1 through Figure B.6, located in Appendix B.

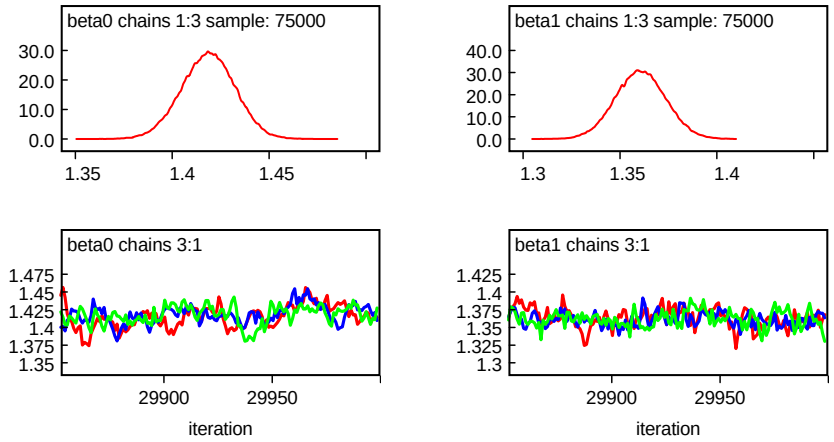


Figure 3.12: Posterior densities and trace plots for β_0 and β_1.

3.5 Discussion

In this chapter, we have extended our Bayesian model to analyze data in which the response follows a Poisson distribution and is predicted by a covariate subject to misclassification as well as 1) an additional continuous covariate measured without error, 2) a random effects component to account for excess variability that arises when responses are correlated or not independent of one another, and 3) a zero-inflation component to account for the excess variability that arises when there are two different states within a single population.

The proposed models performed well for a simulated data set with known characteristics. To further verify the robustness of our model, we should generate data with different regression coefficient design points. It would also be of interest to build upon the work of Hur et al. (2002) and examine the operating characteristics achieved when combining the random effects and zero-inflation models in the Bayesian setting. Additional variability would surely be at a minimum if zero-inflation and clustering effects were accounted for.

CHAPTER FOUR

Using Logistic Regression to Analyze the Prevalence of Obesity: An Application

4.1 Overview

According to a recent report from the Centers for Disease Control and Prevention (CDC) (Sharma et al., 2009), obesity prevalence among low-income, preschool-aged children in the United States has stabilized around 14.6% since 2003. The prevalence of overweight children may have stabilized, but it has done so at such a high level that it remains a concern, particularly in the Mexican American population where the prevalence of high body mass index (BMI) is greater. Among the risks associates with increased body weight is diabetes (Weiss et al., 2004).

Age adjusted body mass index (BMI) is the standard method to identify and follow overweight children. Children with values exceeding the 95th percentile as defined by the BMI standards established in 2000 by the CDC are said to have "childhood obesity" (Barlow, 2007). The population as a whole has grown more obese, and obesity in children has increased since that time as well (Weiss et al., 2004), though, as noted above, the prevalence has appeared stable from about 2003 through 2008.

This chapter focuses on a study investigating age and gender adjusted BMI for 18,462 children from Fall 2003 through Spring 2008. These children were participants in the Head Start program, which is funded and administered by the US Department of Health and Human Services Administration for Children and Families. Specifically, data were collected from Head Start centers in several South Texas border counties and one Central Texas county. In our analysis, we use the data from this study in two ways. First, results are compared to the cohort of the NHANES sample consisting of 2-5 year old children, presented by Ogden, Carroll, and Flegal

(2008). Second, those children who are found to have a BMI-for-age that exceeds that of the 2000 CDC growth curve are examined to determine if there are any regional differences between the border counties of South Texas and a non-border county in central Texas. Our results suggest that prevalence estimates for high BMI children in the predominantly latino population exceeds those obtained by Ogden using the Mexican American subset of the 2-5 year cohort within the NHANES sample. Furthermore, our analyses suggest that there are some regional differences among the prevalence of high BMI between border and non-border counties in Texas.

This chapter is organized as follows. In Section 4.2 we describe the data collection process and provide descriptive statistics. In Section 4.3 we perform two analyses. First, we compute the BMI-for-age for each child and compare the the results to those reported by Ogden et al. (2008) in order gauge how representative the Mexican American cohort of the national survey sample is of the Mexican American population in Texas. Second, we perform logistic regression on our sample in an attempt to determine which covariates are most likely to predict whether a child's BMI-for-age exceeds that of the 2000 CDC growth curve. Section 4.4 contains a Bayesian treatment of the logistic regression problem. Conclusions and comments on future research are contained in Section 4.5.

4.2 Data Collection

BMI is an inexpensive and easy-to-perform method of screening for weight-related health problems (Pietrobelli et al., 1998). It is calculated from a person's weight and height as

$$BMI = \frac{weight(kg)}{(height(m))^2} \quad \text{or} \quad BMI = \frac{703 \cdot weight(lbs)}{(height(in))^2}.$$

BMI is considered a reliable indicator of body fatness for most people, including children (Freedman and Sherry, 2009). For children and teens, the CDC considers age and gender specific BMI, referred to as BMI-for-age, with each gender having

its own set of comparative values. The BMI standards for children younger than 6 years of age were established in 2000 by Kuczmarski et al. (2002) on behalf of the CDC. The CDC has established age and gender specific BMI growth charts to allow establishment of standards for overweight in children.

In the study yielding the data we analyze here, BMI values were computed for 18,462 children who participated in the Head Start program from Fall 2003 through Spring 2008 at centers in three south Texas border counties and one Central Texas county. The gender and date of birth were recorded for each participant at the time of enrollment in the program and their height and weight were measured within the first 30 days of enrollment at the Head Start centers.

The results of this study are compared to the cohort of the NHANES sample consisting of 1,770 2-5 year old children that participated in the NHANES study between 2003 and 2006, studied by Ogden et al. (2008). The data in our study consists primarily of Mexican American children, with more than 90% of the legal guardians identifying the children as Mexican American at the time of program enrollment. A secondary consideration was to determine whether or not there are differences in childhood obesity rates when comparing the Rio Grande region of Texas with a more northernly county. Figure 4.1 presents a map of the Texas counties from which samples were obtained. The southern-most counties, Dimmit, Cameron, and Hidaldo, are located on the Texas/Mexico border. The northern most county, Bastrop, is located in central Texas.

The Texas data are summarized in Table 4.1 and Table 4.2. Table 4.1 contains a cross tabulation of age and year. Age-in-months are grouped by age-year, for children 2 years old (24-35 months) through 5 years old (60-71 months). Observation dates are grouped by calendar year (January-December) for 2003-2008. In the fall of 2003, 495 observations were obtained, and another 436 in 2004. Participation increased in 2005, when data were received for 1,280 children, and continued through

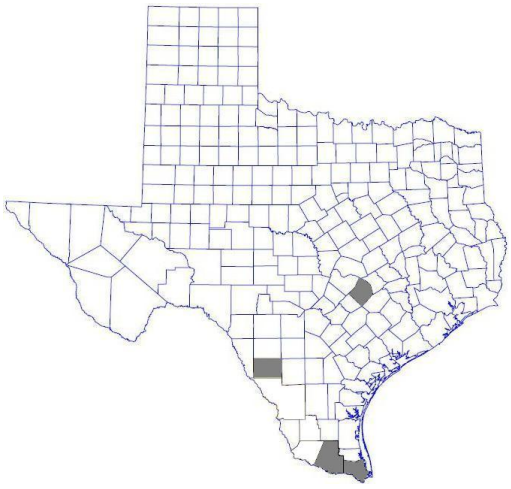


Figure 4.1: Texas Counties. The shaded counties yielded data for our study.

2006 (3,875 observations) and 2007 (9,021 observations). During the spring of 2008, data for 3,355 children were submitted.

Table 4.2 contains a cross tabulation of county and year. One county reported in 2003 and 2004. Three counties reported in 2006 and 2008, and four counties reported in 2005 and 2007. Throughout the study, Bastrop County reported BMI measurements for 947 program participants, Cameron County reported for 4092, Dimmit County reported for 1467, and Hidalgo County reported for 11,956.

Table 4.1: Cross Tabulation of Age by Year

age (in years)	year						
	2003	2004	2005	2006	2007	2008	Total
2	70	54	162	508	355	22	1171
3	416	297	674	2877	4465	737	9466
4	9	84	422	450	4045	1776	6786
5	0	1	12	10	154	820	997
Total	495	436	1280	3875	9021	3355	18462

Table 4.2: Cross Tabulation of County by Year

age (in years)	year						
	2003	2004	2005	2006	2007	2008	Total
Bastrop	0	0	276	0	324	347	947
Cameron	0	0	11	535	2925	621	4092
Dimmit	0	0	24	378	1065	0	1467
Hidalgo	495	436	969	2962	4707	2387	11956
Total	495	436	1280	3875	9021	3355	18462

4.3 Data Analysis

In this section we set the stage for later modeling efforts by considering, largely via descriptive statistics, questions about the similarities that may or may not exist between the Mexican American cohort of the NHANES study and our cohort of Head Start children. Our data analysis consists of two parts. First, observed results from this study are compared with the findings given in Ogden. In the second part statistical models are used to estimate the prevalence of obesity using the CDC BMI growth charts for specified cut points. In addition, these models allow allow detection of trends in obesity rates among pre-school aged children in south and central Texas. Furthermore, we investigate the possible prevalence differences in these rates in the border regions of south Texas when compared to a non-border county in central Texas.

4.3.1 Comparison with Ogden et al. Findings

Ogden et al. (2008) used the NHANES data for 2003-2006 to determine the prevalence of children age 2-5 exceeding the CDC BMI. They reported that 24.4% exceeded the 85th percentile, 12.5% exceeded the 95th percentile and 8.5% exceeded

the 97th percentile, with males having slightly but not significantly higher BMI than females. Non-Hispanic black and Mexican American children were more likely to have a high BMI than non-Hispanic white children. The highest prevalence was found in the Mexican American children: in the 2-5 year age group 29.9% were at or above the 85th percentile, 16.7% at or above the 95th percentile, and 13.4% at or above the 97th percentile. Again, the males were more likely than females to have a higher BMI when compared to their non-Hispanic white counterparts.

In comparison, the prevalence among Mexican American children in the national sample was higher, with 32.4% of the 2-5 year old Mexican American males exceeding the 85th percentile, 18.8% exceeding the 95th percentile, and 16% exceeding the 97th percentile. Only 27.3% of the 2-5 year old Mexican American females exceeded the 85th percentile, 14.5% exceeded the 95th percentile, and 10.8% exceeded the 97th percentile.

The observed prevalence of high BMI in the Texas study is higher than the results reported by Ogden et al. (2008) We find that among the Texas pre-school aged males, 40.79% exceeded the 85th percentile, 20.01% exceeded the 95th percentile, and 15.60% exceeded the 97th percentile. Comparatively, Ogden et al. report that only 32.4% of the 2-5 year old Mexican American males in the NHANES study exceeded the 85th percentile, 18.8% exceeded the 95th percentile, and 16% exceeded the 97th percentile. The results for Texas pre-school aged females indicate that 36.73% exceed the 85th percentile, 19.04% exceed the 95th percentile, and 14.81% exceed the 97th percentile. Using the NHANES sample, Ogden et al. report that only 27.3% of the 2-5 year old Mexican American females exceeded the 85th percentile, 14.5% exceeded the 95th percentile, and 10.8% exceeded the 97th percentile. The results for the four Texas counties are summarized in Table 4.3.

Table 4.3: Percent Overweight and Obese by County

	BMI > 85%		BMI > 95%	
County	Male	Female	Male	Female
Bastrop	41.51	37.01	19.29	16.02
Cameron	43.80	40.20	23.92	19.28
Dimmit	38.94	35.81	17.47	19.52
Hidalgo	39.82	35.62	18.94	19.21

To this point we have compared the obesity prevalence in this study with the Ogden results. In order to illustrate the extent of obesity found in this study, we compare the distribution of pre-school aged BMI with the results found in the CDC tables. In the next section, we will model the prevalence of different levels of obesity as defined by the CDC BMI tables using logistic regression. Before making these comparisons, we want to illustrate how the distribution of the Texas pre-school aged BMI compares with the CDC tables in the tails of the distribution (those with the highest incidence of childhood obesity). That is, we compare the distribution of the Texas data with the CDC tables by computing the percent change from the CDC BMI baseline values at different cut points.

Figure 4.2 illustrates the percent change in the sample percentiles of BMI as compared to those given in the CDC growth curve at the 50th, 85th, 95th, and 97th percentile for the ages 2-5. For example, when considering the 95th percentile for both the Texas data and the CDC distribution of 3 year old males and females we observe that the percent change is 18.4%. This means that the 95th percentile in the Texas data for three year old children when ignoring gender differences is 18.4% higher than would be expected from the CDC BMI tables. Figure 4.3 presents the percent change from CDC baseline results by gender.

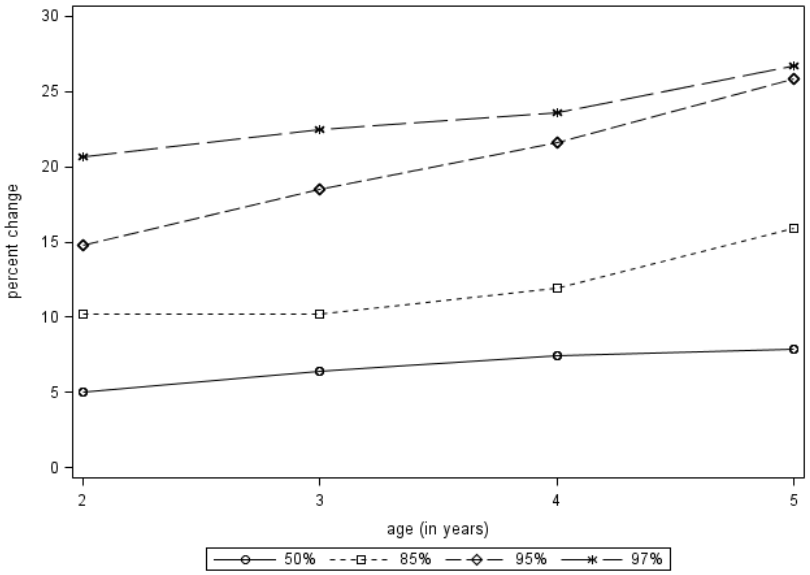
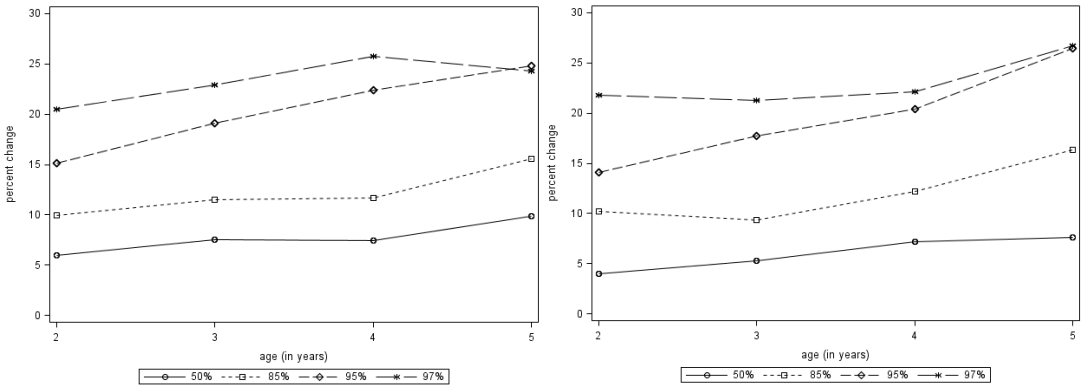


Figure 4.2: Percent change from CDC by age (combined genders)



(a) (b)

Figure 4.3: Percent change from CDC by age for (a) females and (b) males

Figure 4.4 illustrates the percent change in the percentiles of the BMI for the Texas pre-school aged children as compared to the expected percentiles using the CDC BMI tables at four different percentiles, the 50th, 85th, 95th, and 97th for the ages 2-5. In Bastrop County, the BMI corresponding to the 50th percentile for a preschool child is 16.4, which is 7.5% higher than the BMI corresponding to the 50th

percentile in the 2000 CDC charts, 15.2. The 85th, 95th, and 97th percentiles for Bastrop County are 10%, 19%, and 22% higher than that of the 2000 CDC growth charts, respectively.

In Cameron County, the BMI corresponding to the 50th percentile for a preschool child is 16.6, which is 8.5% higher than the BMI corresponding to the 50th percentile in the 2000 CDC charts, 15.3. The 85th, 95th, and 97th percentiles for Cameron County are 13%, 21%, and 25% higher than that of the 2000 CDC growth charts, respectively.

In Dimmit County, the BMI corresponding to the 50th percentile for a preschool child is 16.4, which is 6.6% higher than the BMI corresponding to the 50th percentile in the 2000 CDC charts, 15.4. The 85th, 95th, and 97th percentiles in Dimmit County are 9%, 16%, and 19% higher than that of the 2000 CDC growth charts, respectively.

In Hidalgo County, the BMI corresponding to the 50th percentile for a preschool child is 16.3, which is 6.5% higher than the BMI corresponding to the 50th percentile in the 2000 CDC charts, 15.3. The BMI corresponding to the 85th, 95th, and 97th percentiles in Hidalgo county are 11%, 20%, and 24% higher than that of the 2000 CDC growth charts, respectively.

Figure 4.5 presents the percent change from CDC baseline results by county and gender.

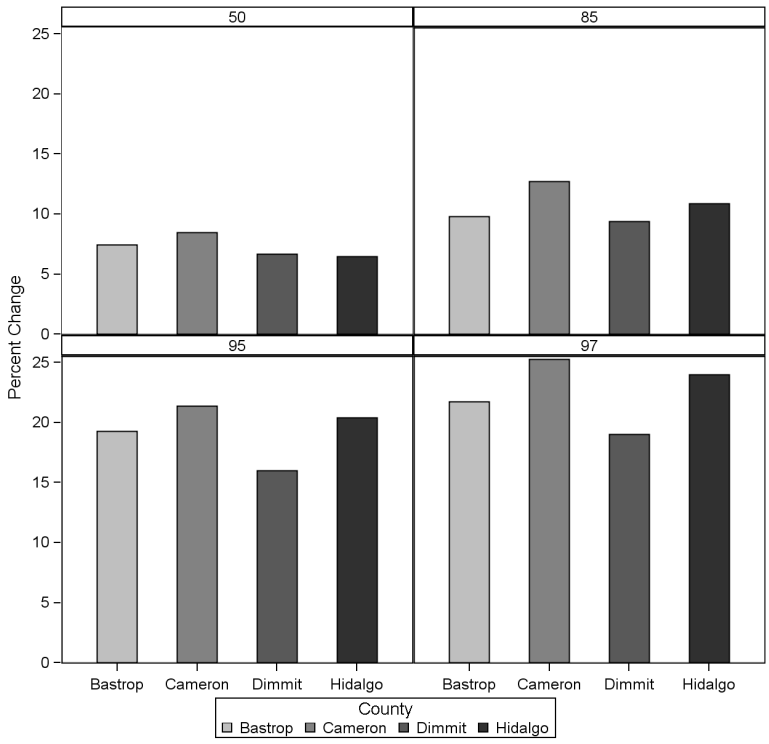
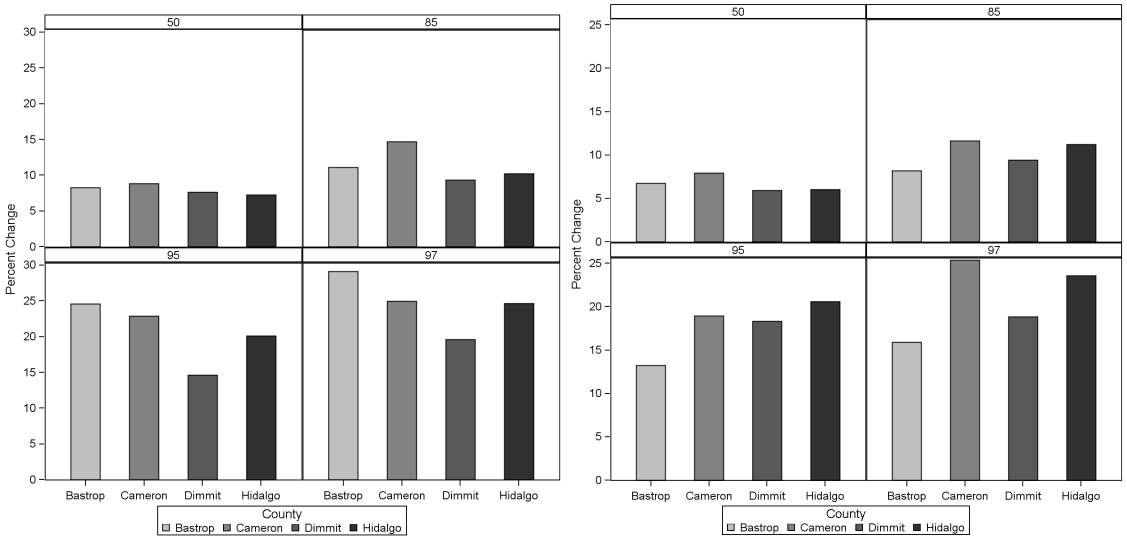


Figure 4.4: Percent change from CDC by county (combined genders)



(a) (b)

Figure 4.5: Percent change from CDC by county for (a) females and (b) males

4.3.2 Estimation of Obesity Prevalence

We now turn to the question of whether or not obesity prevalence is uniform across Texas counties. Here, descriptive statistics will not suffice. As we shall see, variability in BMI results are clearly a function of covariates, and therefore more sophisticated statistical modeling is required. In this part of the analysis we are interested in in determining if there are there are differences in childhood obesity prevalence between the Texas border counties and non-border counties. The procedure is to estimate the probability of exceeding the CDC BMI threshold at specified cut points for the three South Texas counties and one Central Texas county for the 2003 - 2008 time period while controlling for gender and year as covariates. The four cut points are at the 50th, 85th, 95th, and 97th percentiles.

The probability of exceeding the CDC BMI threshold at a specified cut point is estimated using the logistic regression model where y_i is a binary response indicating whether the observed BMI of the i^{th} child exceeds the BMI associated with a specified CDC cut point. The logistic regression model is

$$Y_i \sim Bernoulli(\pi_i) \quad \text{and} \quad \pi_i = \frac{e^{\mathbf{x}_i'\boldsymbol{\beta}}}{1 + e^{\mathbf{x}_i\boldsymbol{\beta}}},$$

where $\pi_i = g^{-1}(\mathbf{x}_i\boldsymbol{\beta})$ and $g(\cdot)$ is the logit link function. Here $\mathbf{X}$ is an $N \times 9$ design model matrix made up of an intercept and 8 dummy variables used to model the covariates *year* $(X_1, \ldots, X_4)$, *county* $(X_5, \ldots, X_7)$, and *gender* (X_8), and $\mathbf{x}_i$ is the 1×9 vector of covariate values for the i^{th} child. Let $\boldsymbol{\beta}$ be a 9×1 vector of regression coefficients associated with the covariates in $\mathbf{X}$.

The density function for the collected data is

$$f(y_i|x_i) = \pi_i^{y_i} \times (1 - \pi_i)^{1-y_i}.$$

The likelihood function is

$$f(\boldsymbol{\beta}|\mathbf{y}, \mathbf{X}) \propto \prod_{i=1}^{N} \pi_i^{y_i}(1 - \pi_i)^{1-y_i}, \tag{4.1}$$

where $\mathbf{y}' = (y_1, y_2, \ldots, y_N)$ is a $1 \times N$ vector of binary responses indicating the presence or absence of a BMI-for-age exceeding that of the CDC growth curve.

The reference subgroups for the logistic regression are chosen to facilitate comparisons between levels of covariates that are thought to be significant predictors of prevalence:

- Bastrop County; it is located in Central Texas thus allowing for a direct comparison to the border counties.
- Females; based on results from the first portion of our analysis, we suspect gender to be a significant covariate.
- 2003; a primary interest to the medical researcher involved the investigation of a possible time trend, with the physician believing the prevalence may increase incrementally between 2003 and 2008.

Thus, the i^{th} child with covariate profile $\mathbf{x}_i = (\ 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ )$ would represent a female child from Bastrop County whose BMI was recorded in 2003.

Table 4.4 contains the results obtained when using frequentist logistic regression to model the probability that a child's BMI-for-age exceeds that of the CDC growth curve using county, gender, and year as predictor variables. A question of primary interest concerns whether or not there is a temporal trend in the prevalence of high BMI. The results indicate that there is not a temporal effect in these data when using the 85th, 95th, and 97th percentile cut points. The "year" effect is excluded as a covariate in subsequent models.

Table 4.4: Logistic Regression Results Using Year, Gender, and County

		≥ 50%		≥ 85%		≥ 95%		≥ 97%	
Effect	DF	Chi-square	p-value	Chi-square	p-value	Chi-square	p-value	Chi-square	p-value
county	3	7.2892	0.0632	19.9178	0.0002	14.6291	0.0022	20.7074	0.0001
gender	1	93.7589	< .0001	28.5076	< .0001	2.3637	0.1242	1.9378	0.1639
year	5	24.4677	0.0002	2.2023	0.8205	2.2909	0.8076	4.5446	0.4739

Table 4.5 summarizes the results for the second model of predicting the prevalence using "county" and "gender" as covariates. The results indicate there is a significant county effect at each cut point and a significant gender effect at the 50th and 85th percentile cut points. The gender effect was insignificant at the highest level of obesity. These differences were then investigated using the parameter estimates and odds ratios.

Table 4.5: Logistic Regression Results Using Gender and County

		≥ 50%		≥ 85%		≥ 95%		≥ 97%	
Effect	DF	Chi-square	p-value	Chi-square	p-value	Chi-square	p-value	Chi-square	p-value
county	3	11.7872	0.0081	23.1155	< .0001	14.6016	0.0022	20.7232	0.0001
gender	1	93.3843	< .0001	28.4611	< .0001	2.3018	0.1292	1.8505	0.1737

Table 4.6 contains the maximum likelihood estimates (MLE) for the parameters in the reduced model. With Bastrop County as the reference county, only Cameron County (the southernmost border country) is significant using the 85th, 95th and 97th percentile cut points. Dimmit County (the northernmost border county) does not appear to be significantly different from Bastrop County at any of the cut points. The prevalence of high BMI in Hidalgo County is significantly different than that of Bastrop county at the 50th and 85th percentile cut points. The MLEs and p-values for the gender comparison – with females serving as the ref-

erence group – indicate that there is a significant difference between the prevalence of high BMI among males and females at the 50th and 85th percentile cut points, with females having lower prevalence than males.

Table 4.6: Maximum Likelihood Estimates at Each Cut Point

			≥ 50%		≥ 85%		≥ 95%		≥ 97%	
Parameter		DF	MLE	*p*-value	MLE	*p*-value	MLE	*p*-value	MLE	*p*-value
intercept		1	1.2120	< .0001	-0.4414	< .0001	-1.4416	< .0001	-1.7728	< .0001
county	Cameron	1	0.0174	0.6572	0.1179	0.0004	0.1495	0.0003	0.2036	< .0001
county	Dimmit	1	-0.0761	0.1641	-0.0653	0.1732	-0.0423	0.4804	-0.0439	0.5139
county	Hidalgo	1	-0.0868	0.0073	-0.0615	0.0271	-0.00245	0.9440	0.0276	0.4828
gender	Female	1	-0.1716	< .0001	-0.0805	< .0001	-0.0280	0.1456	-0.0282	0.1847

The odds ratios are given in Table 4.7 using *Bastrop County* and *Female* as the reference groups. Figure 4.6 illustrates the odds ratios with 95% Wald confidence intervals for prevalence of high BMI as compared to the CDC growth curves for the 50th, 85th, 95th, and 97th percentile. Each of the pairwise comparisons are given in this figure. For example, the odds ratio for males versus females at the 85th percentile cut point is 1.182 with 95% Wald confidence limits of [1.112, 1.257]. Because the confidence interval for this ratio is greater than one there is a significant difference in the estimated proportions of males that exceed the CDC BMI 85th percentile as compared to the estimated proportion of females that exceed the CDC BMI 85th percentile.

Table 4.7: Odds Ratios with 95% Wald Intervals

	≥ 50%	≥ 85%	≥ 95%	≥ 97%
county				
Cameron vs. Bastrop	0.879 (0.738, 1.048)	1.115 (0.964, 1.290)	1.289 (1.072, 1.551)	1.478 (1.196, 1,862)
Dimmit vs. Bastrop	0.801 (0.652, 0.983)	0.928 (0.780, 1.105)	1.064 (0.854, 1.327)	1.154 (0.897, 1.485)
Hidalgo vs. Bastrop	0.793 (0.673, 0.935)	0.932 (0.813, 1.069)	1.108 (0.930, 1.319)	1.240 (1.014, 1.516)
gender				
Male vs. Female	1.416 (1.319, 1.519)	1.182 (1.112, 1.257)	1.060 (0.983, 1.143)	1.059 (0.975, 1.151)

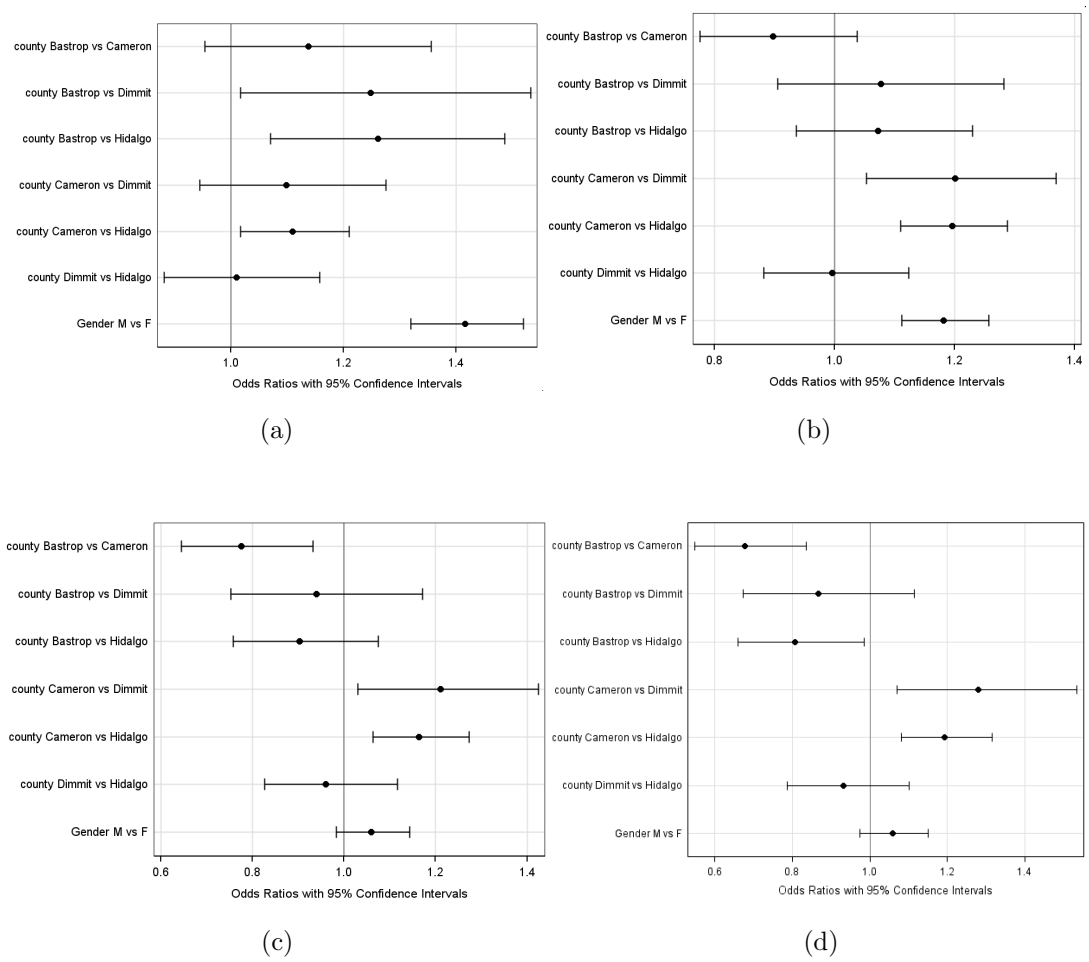


Figure 4.6: Odds Ratios with 95% Wald Intervals at the (a) 50th, (b) 85th, (c) 95th, and (d) 97th percentile cut points.

Figure 4.7 contains the corresponding prediction estimates and 95% confidence intervals. For example a randomly selected female student from Bastrop County has an estimated probability of 39% [36.3%, 42.5%] of exceeding the CDC 85th percentile cut point. The largest disparity is found in Cameron County, where the estimated probability of a male student of exceeding the 85th percentile cut point is 42%.

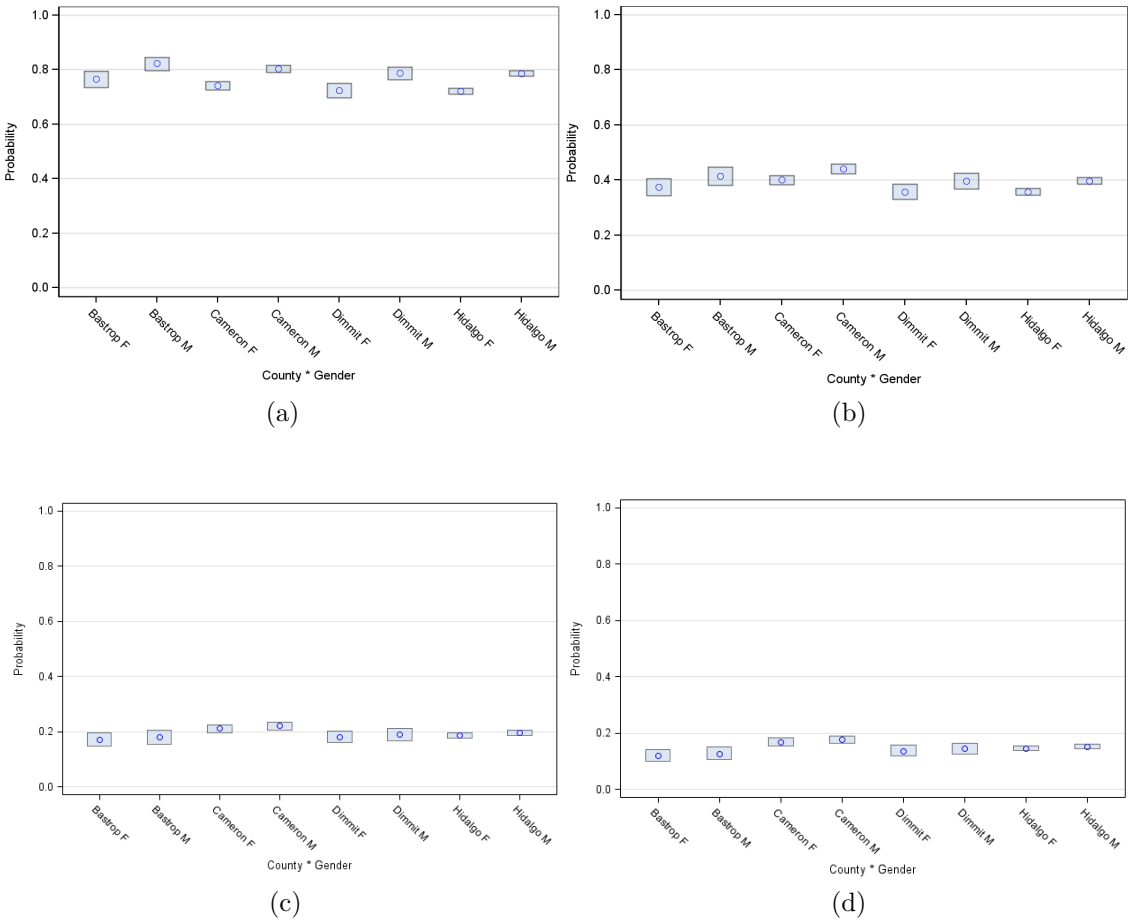


Figure 4.7: Prediction Estimates with 95% Confidence Bands for the (a) 50th, (b) 85th, (c) 95th, and (d) 97th percentile cut points.

4.4 Bayesian Analysis

4.4.1 Model Development

We now develop the Bayesian model for modeling the binary outcome as a function of "county" and "gender". We let $y = 1$ if a child's BMI-for-age exceeds a predetermined percentile for his or her age and gender, and $y = 0$ otherwise. All covariates in the model are assumed to be measured without error, as we are interested in examining the relationship between the response y and the predictor variables.

As in Section 4.4, let $\pi_i = P(y_i = 1|\mathbf{x}_i)$ be the probability that the i^{th} child, with covariate profile $\mathbf{x}_i$, has a BMI-for-age exceeding that of the CDC growth curve. The logistic regression model we wish to fit is the same as that described in Section 4.4. We assume that $(y_i|\mathbf{x}_i) \sim$ Bernoulli(π_i), where $\pi_i = g^{-1}(\mathbf{x}_i\boldsymbol{\beta})$ and $g(\cdot)$ is the logit link function. Again, the density function for the collected data is

$$f(y_i|\mathbf{x}_i) = \pi_i^{y_i} \times (1 - \pi_i)^{1-y_i},$$

and the likelihood function is

$$f(\boldsymbol{\beta}|\mathbf{y}, \mathbf{X}) \propto \prod_{i=1}^{N} \pi_i^{y_i}(1 - \pi_i)^{1-y_i}, \tag{4.2}$$

where $\mathbf{y}' = (y_1, y_2, \ldots, y_N)$ is a $1 \times N$ vector of binary responses indicating the presence or absence of a BMI-for-age exceeding that of the CDC growth curve, and $\mathbf{x}_i$ is a $1 \times k$ vector containing the covariate profile for the i^{th} child. The design matrix is $\mathbf{X} \equiv (1 : X_1 : \cdots : X_{k-1})$, where $\mathbf{1}$ is an $N \times 1$ vector of ones, $X_1, \ldots, X_{k-1}$ are $N \times 1$ vectors of covariate values, and ":" denotes horizontal concatenation.

Under the Bayesian framework, a prior distribution, $p(\cdot)$, is required for each unknown parameter in the model. For each of the $k = 5$ regression coefficients, we assume independent diffuse Normal priors, $\beta_k \sim N(0, 10^6)$. Assuming prior independence of all unknown parameters, the joint prior distribution for model is given by

$$p(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4) = p(\beta_0) \times p(\beta_1) \times p(\beta_2) \times p(\beta_3) \times p(\beta_4).$$

The resulting joint posterior distribution is

$$p(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4|\mathbf{y}, \mathbf{X}) \propto f(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4|\mathbf{y}, \mathbf{X})p(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4). \tag{4.3}$$

For ease of illustration, we write the product of the independent normal priors as a k-dimensional multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and $k \times k$

covariance matrix $\mathbf{\Sigma}$. We take the diagonal elements of $\mathbf{\Sigma}$ to be 10^6 with zero off diagonal elements. This model is depicted in Figure 4.8.

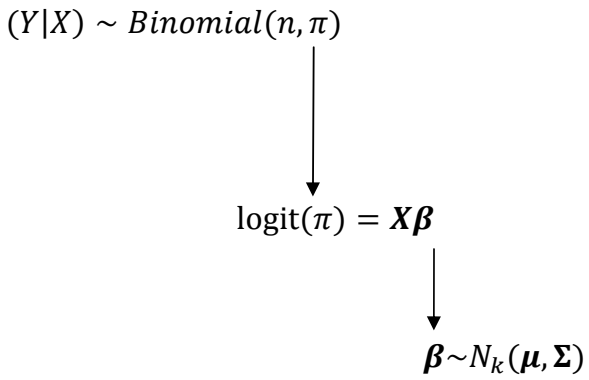


Figure 4.8: Summary of Bayesian Logistic Regression Model

4.4.2 Chains / Convergence

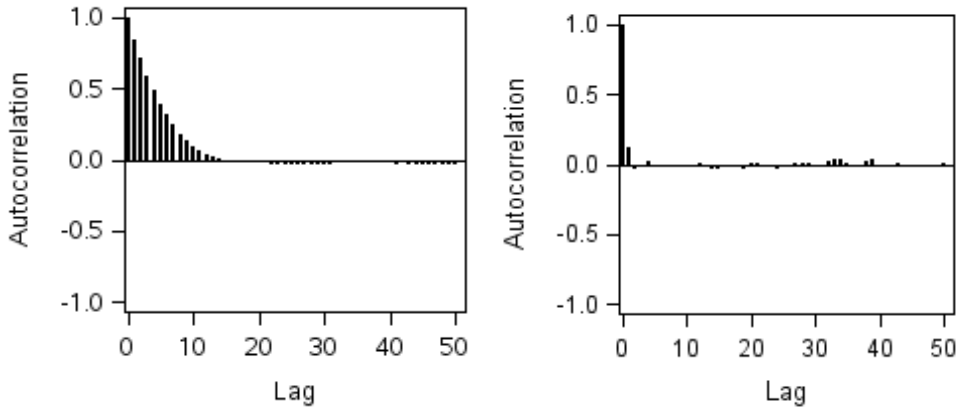
The data analysis for this paper was performed using *SAS* software, Version 9.2 of the *SAS* System for Windows, taking advantage of the newly implemented `bayes` statement within the GENMOD procedure.

One long chain is mathematically justifiable (see Gilks et al., 1996, p. 13, and references therein), but our means for assessing convergence rely on multiple chains. Specifically, the Gelman and Rubin diagnostics (Gelman and Rubin, 1992; Brooks and Gelman, 1997) are based on analyzing multiple (m) simulated Markov Chain Monte Carlo chains by comparing the variances within each chain and the variance between chains. A large deviation between these two variances indicates nonconvergence. In *SAS* PROC GENMOD, the first chain is used for posterior inference, such as mean and standard deviation; the other $m-1$ chains are used for computing the diagnostics and are discarded afterward. This test can be computationally costly, because it prolongs the simulation m-fold. It is best to choose different initial values for all chains. The initial values should be as dispersed from each other as possible

so that the Markov chains can fully explore different parts of the parameter space before they converge to the target.

Therefore, in the first stage we run three independent chains of 10,000 iterations each with a burn-in of 2,000. We specify dispersed starting values and rely on the results to establish convergence. In the second stage, we repeat the simulation with one long chain of 30,000 iterations following a burn-in of 5,000 to obtain posterior estimates and Monte Carlo error from *SAS*.

We illustrate the merits of thinning through a comparison of the autocorrelation plots produced when we run the full and reduced models for our data. Using analysis for the 85th percentile, we present autocorrelation plots for the regression coefficient pertaining to the dummy covariate "year2003", defined as X_0 as in Section 4.3.2, which demonstrated the most dramatic difference. Figure 4.9(a) shows the autocorrelation plot for 2003 with no thinning. Figure 4.9(b) shows the autocorrelation plot for 2003 using thinning, where we retained every 10th update. This results in 3,000 usable iterations. Thinning the chain yielded the desired improvement in the correlation, and we proceed to perform the Bayesian analysis using thinning.



(a) (b)

Figure 4.9: Autocorrelations for 2003 at the 85th Percentile with (a) no thinning, and (b) thin = 10.

4.4.3 Model Selection

There are several methods that provide a measure of "evidence in favor of a model". We examine two such methods that make use of the penalized likelihood ratio form. The *Akaike information criterion* (AIC) is a measure of goodness of model fit that balances model fit against model simplicity (Akaike, 1974). AIC has the form

$$AIC \equiv -2 \log L + 2k,$$

where k is the number of parameters estimated in the model, and $\log L$ is the natural log of the likelihood evaluated at the value of the estimated parameters. For the full model, where we consider multiple levels of "year", "county", and "gender", $k = 9$; for the reduced model, where we remove "year" as a covariate, $k = 5$. The AIC is useful in comparing and selecting nonnested model specifications (Gill, 2008), but Carlin and Louis (2009) note that the AIC has a strong bias toward models that overfit with extra parameters because the penalty component is linear with respect to increases in the number of explanatory variables, and the log likelihood often increases more rapidly.

The *Bayesian information criterion* (BIC), occasionally referred to as the Schwarz criterion, is a similar measure. It penalizes the complexity of the model where complexity refers to the number of parameters in model (Schwarz, 1978). BIC is defined by

$$BIC \equiv -2 \log L + k \log N,$$

where k is the number of parameters estimated in the model, $\log L$ is the natural log of the likelihood, and N is the total sample size. A penalty for overparameterization enters the model through the second term in the BIC. However, the penalty for additional parameters is stronger than that of the AIC. As noted by Leonard and Hsu (2001), the BIC is not recommended for use if $\log(N/2\pi) < 2$, that is, if

$N \leq 46$. Our sample size, $N = 18462$, is very large and, thus, not subject to this constraint. PROC GENMOD uses the full log likelihoods, with all terms included, for computing both the BIC and the AIC.

In our analysis, we compare the BIC and AIC for the full and reduced models. Table 4.8 contains the Bayesian information criterion and Akaike information criterion for both the full and reduced models at each of the four cut points.

Table 4.8: Information Criterion for Full and Reduced Model

	$\geq 50\%$		$\geq 85\%$		$\geq 95\%$		$\geq 97\%$	
	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC
Full Model	18992.33	18914.78	23068.23	22990.67	17105.81	17028.26	14768.85	14691.30
Reduced Model	18968.07	18929.29	23021.66	22982.88	17059.32	17020.54	14724.47	14685.70

Clearly, the smaller BIC and AIC values correspond to the reduced model at each cut point.

4.4.4 Bayesian Results

The 85th percentile has been established by the CDC as the cutoff for "overweight" status and is widely accepted among the medical community as such. It is a focal point of pediatric obesity prevention, and as such we present the results of the Bayesian logistic regression analysis at this cutpoint, and then move on to the results at the 50th, 95th, and 97th percentiles. Table 4.9 presents the posterior mean, standard deviation, and several percentiles for each variable.

Table 4.9: Posterior Summaries For The 85th Percentile

Parameter	N	Mean	SD	Percentiles		
				25%	50%	75%
Intercept	3000	−0.4412	0.0237	−0.4576	−0.4408	−0.4251
Cameron	3000	0.1177	0.0334	0.0941	0.1179	0.1404
Dimmit	3000	−0.0665	0.0478	−0.0985	−0.0664	−0.0345
Hidalgo	3000	−0.0614	0.0273	−0.0797	−0.0616	−0.0428
Male	3000	0.0837	0.0157	0.0730	0.0838	0.0946

Note that none of the posterior credible sets contain zero. Figure 4.10 through Figure 4.14 contain plots of the posterior distributions for $\beta_0, ..., \beta_4$.

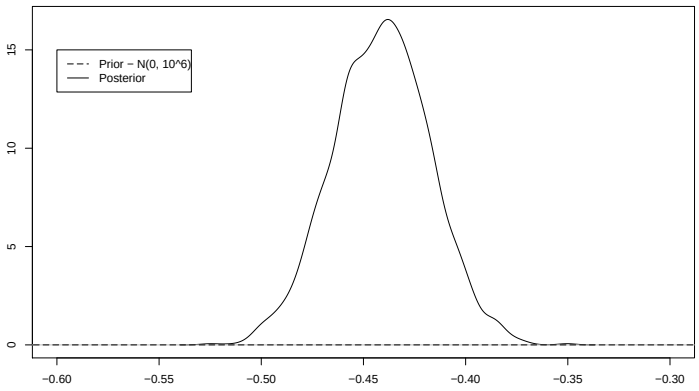


Figure 4.10: Prior and posterior distributions for β_0. The diffuse normal prior appears to be constant at zero in this plot but, of course, is everywhere positive.

Much information was gained about β_0, which left alone in the model corresponds to a female subject from Bastrop County, from the observed data through the Bayesian analysis. This is indicated by the increase in precision compared to the prior. Figure 4.11 through Figure 4.14 shows the same for the parameters β_1 through β_4.

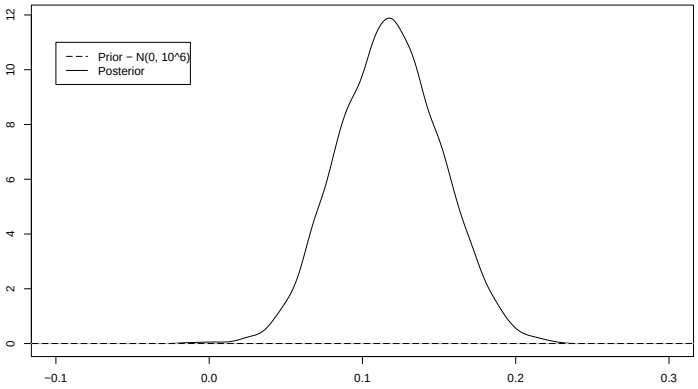


Figure 4.11: Prior and posterior distributions for β_1 corresponding to Cameron County.

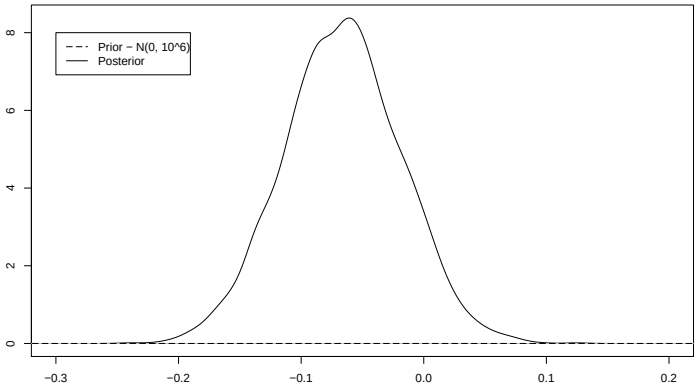


Figure 4.12: Prior and posterior distributions for β_2 corresponding to Dimmit County.

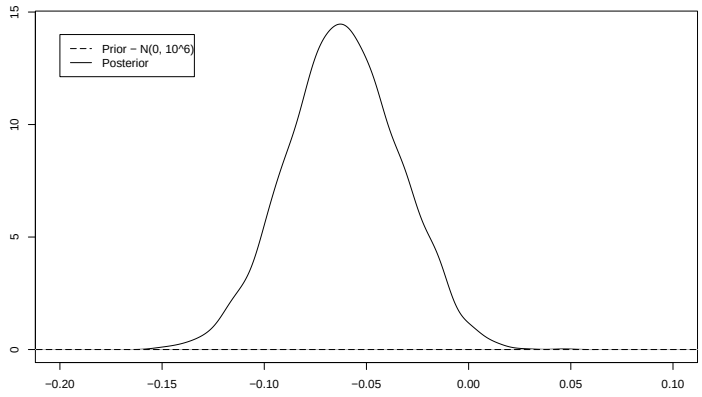


Figure 4.13: Prior and posterior distributions for β_3 corresponding to Hidalgo County.

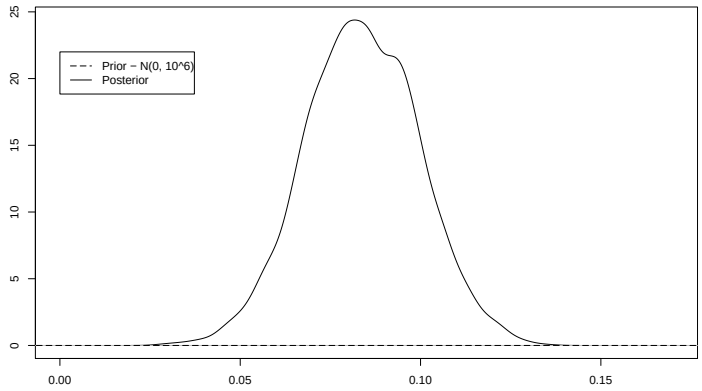


Figure 4.14: Prior and posterior distributions for β_4 corresponding to males.

Tables C.1, C.2, and C3 present the posterior means, standard deviations, and quartiles for each predictor variable at the 50th, 95th, and 97th percentiles, respectively.

The Bayesian approach also enables us to produce posterior distributions for the odds of a child having a BMI-for-age that exceeds that of the 2000 CDC growth curve given that the child is a member one level of a covariate versus another level. In Table 4.7 (Section 4.3.2) we presented the odds ratios, and their respective Wald

95% confidence intervals resulting from the frequentist analysis, for the county and gender comparisons of primary interest. In Figure 4.15 we present the posterior densities for the odds that, given a child has a BMI-for-age that exceeds that of the 50th percentile of the CDC growth curve, the child is a male. The posterior densities corresponding to the 85th, 95th, and 97th percentiles are given as well. Note that, while all four posterior modes are greater than one, there is a significant shift from the 85th to the 95th percentile.

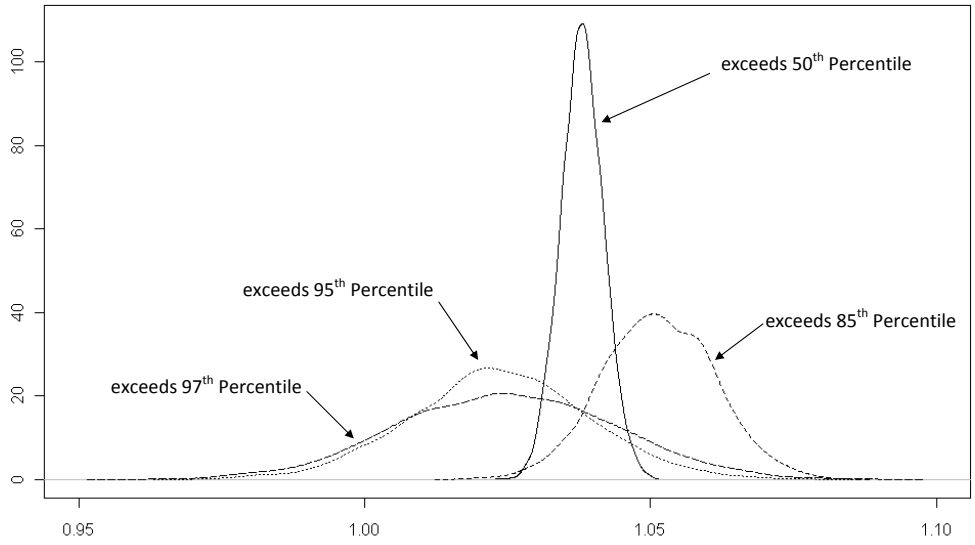


Figure 4.15: Posterior distribution of the odds that a child with a BMI-for-age that exceeds the CDC growth curve is a male (versus female).

4.4.5 Induced Prior

Although we have used diffuse priors throughout this analysis, the induced prior on the i^{th} probability, i.e., the distribution of

$$\pi_i = \frac{\exp \mathbf{x}_i \boldsymbol{\beta}}{1 + \exp \mathbf{x}_i \boldsymbol{\beta}} \tag{4.4}$$

is in fact informative. This is easy to see by simulating values of π_i, computing (4.4), and plotting the resulting relative frequency distribution. We have done this for the case of π_{26}, where the 26^{th} child corresponds to a female from Bastrop county with

covariate profile $\mathbf{x}_{26} = (\ 1 \quad 0 \quad 0 \quad 0 \quad 0\ )$. Figure 4.16 displays the relative frequency of 10,000 simulated values of π_{26}. Clearly, the transformation induces an informative prior. We must verify that, despite this informativeness, we have not biased the resulting posteriors. In fact, our sample size is sufficiently large that this is not the case, as can be seen in Figure 4.17.

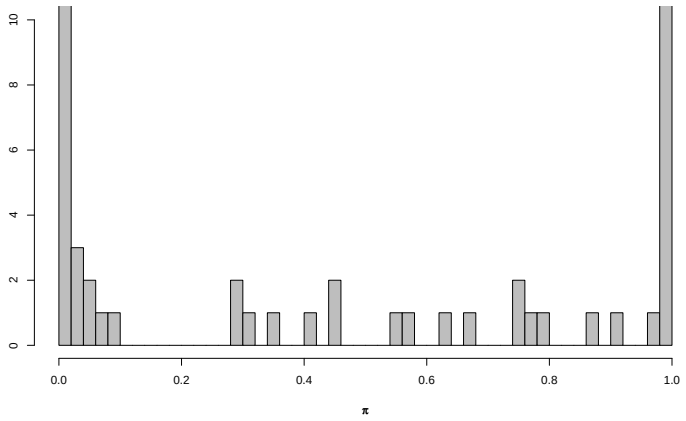


Figure 4.16: Induced prior on π_i for a female student from Bastrop County ($x_{0i} = 1$).

In our example, the data set is comprised of more than 18,000 observations and the data is likely to overwhelm this issue with the induced prior. In Figure 4.17 we see that while the induced prior places most of the density at the end points of the [0,1] interval, the posterior distribution for π_{26} is narrowly centered around 0.39. In fact, the posterior distribution is so heavily centered at 0.39 that our general plotting methods cannot capture the entire density.

Figure 4.18 captures the posterior distribution for π_{26}, the probability that a female student from Bastrop County has a BMI-for-age that exceeds that of the CDC growth curve.

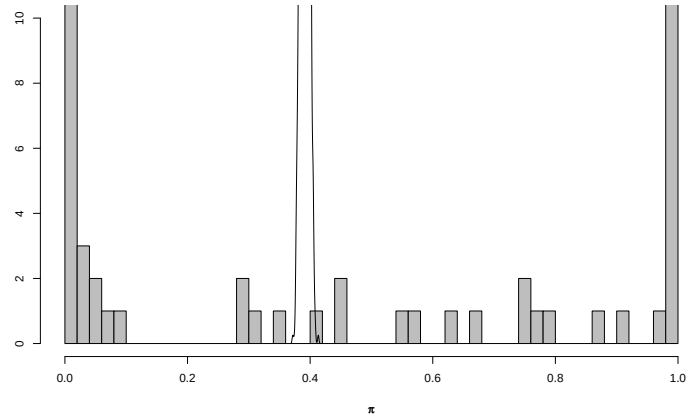


Figure 4.17: Induced prior on π_{26} and the resulting posterior distribution for π_{26} .

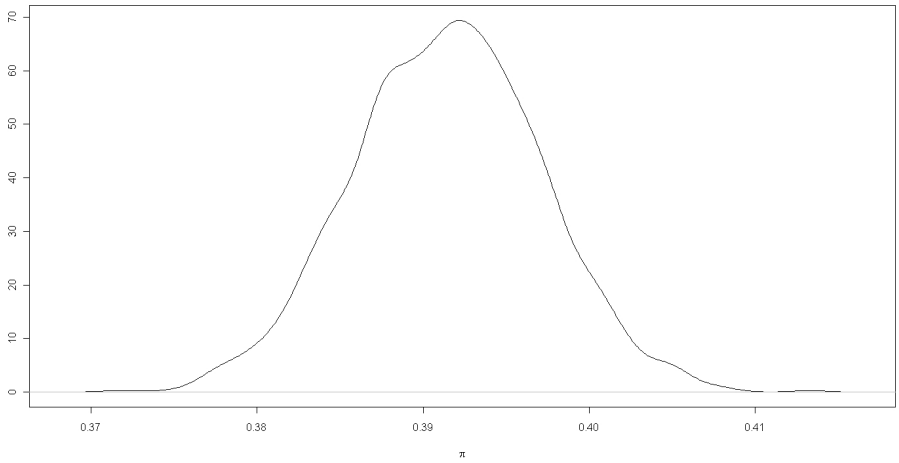


Figure 4.18: Posterior distribution for π_{26} .

4.5 Conclusion

Our study revealed a significantly higher prevalence of children with high BMI for age in our populations in both central Texas and the United States/Mexico border counties from both the CDC 2000 BMI charts, the analysis of NHANES data by Ogden et al. (2008) even when compared to the Hispanic subpopulation. In addition, based on the Pediatric Nutrition Surveillance System (PedNSS), the CDC estimated the prevalence of obese preschool children in Texas as 14.5% in 2003 and 16.2% in 2008 (Sharma et al., 2009). The prevalence in our population exceeds

that as well. The only estimate of prevalence similar to our population is that of Anderson and Whitaker (2009) in a study of prevalence of obesity in 4 year old children from different ethnic groups in 2005 where the overall prevalence at the 95th percentile was 22% in the Hispanic group.

Although there was no significant change in the prevalence of high BMI during the years 2003-2008, the five-year-old children were consistently the highest BMI-for-age group over all suggesting that the same increasing prevalence of high BMI within the 6-11 year will be seen as in the analysis of Ogden et al. (2008). An additional concern is that the percent deviation from the CDC growth curve increases in our population between age 2-5 (Figure 4.2). The growth curve depicts a decrease of BMI for age during this age span since the height generally changes faster than the weight suggesting a very significant weight increase in our population since the height values are increasing appropriately. The average BMI of children who exceeded the 85th percentile was 19.13, and the average BMIs of the children who exceeded the 95th and 97th percentiles were 20.76 and 21.42, respectively. These levels of BMI increase morbidity in children. Endothelial dysfunction, dyslipidemia, type 2 diabetes, and hepatic steatosis have all been recently reported (Pena et al., 2006; Haines et al., 2007; Alisi et al., 2009). In the predominantly Mexican American population in our area, the risk of these disorders is even higher than in the average population of obese children due to the combination of Hispanic and Native American heritage. Additionally, a survey of the Head Start population in a southern border county of Texas did reveal that although the caloric intake was high, the protein and fiber content was lower in the diet provided by the parents than that provided by the Head Start facilities. In particular, there was a high consumption of foods with high-sugar and high-fat contents (Mier et al., 2007). Each of the four counties that contributed data to the survey analyzed in this study was provided a nutrition intervention activity to improve awareness of the importance of avoiding

sugar containing beverages and consuming more low fat protein sources (Mier et al., 2005).

4.6 Discussion

The observed prevalence of high BMI in Mexican American children age 2-5 in the Texas study is higher than the results for Health and Nutrition Examination Survey (NHANES) for 2003-2006. There was no change in the overall prevalence of high BMI from 2003-2008 in the population. Furthermore, the analysis suggests that there are some regional differences among the prevalence of high BMI between border and non-border counties in Texas. Although the prevalence of high BMI was stable during the 2003-2008 time period for the 2-5 year old Head Start population in our sample, it far exceeds the national level and the Texas Mexico border counties had the highest prevalence demonstrating a critical need for dietary and exercise education and interventions in this underserved area.

The data reporting was not uniform across the period of analysis. One border county (Hidalgo) reported in 2003 and 2004, with Bastrop, Cameron, and Dimmit counties reporting in 2005 through 2008. In the future, the study will be expanded to include two additional South Texas counties and one additional Central Texas county. More data from 2008 and new data for 2009 will be available soon, and will be incorporated into the analysis. Occasionally, measurements such as height and weight suffer from measurement error, usually as a result of faulty equipment or inconsistent data collecting practices. Failing to control for such measurement error can result in attenuation of the standard errors, and inaccurate confidence intervals. Future analyses will include components that control for such measurement error.

APPENDICES

APPENDIX A

Chapter II Tables

Table A.1: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.4243	0.1042	0.00	2.4247	0.1043	0.00
β_1	0.8	N(0, 10)	0.2559	0.1300	0.00	0.2557	0.1301	0.00
η	0.9							
θ	0.7							
τ	0.8							

Table A.2: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0982	0.0977	0.00	2.0985	0.0977	0.00
β_1	0.8	N(0, 10)	0.5010	0.1439	0.00	0.5011	0.1439	0.00
η	0.9							
θ	0.7							
τ	0.4							

Table A.3: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9903	0.0948	0.08	1.9907	0.0949	0.08
β_1	0.8	N(0, 10)	0.4923	0.1681	0.00	0.4927	0.1684	0.00
η	0.9							
θ	0.7							
τ	0.2							

Table A.4: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.4266	0.0802	0.00	2.4269	0.0802	0.00
β_1	0.8	N(0, 10)	0.2556	0.1003	0.00	0.2555	0.1004	0.00
η	0.9							
θ	0.7							
τ	0.8							

Table A.5: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.1059	0.0751	0.00	2.1062	0.0752	0.00
β_1	0.8	N(0, 10)	0.4875	0.1115	0.00	0.4876	0.1117	0.00
η	0.9							
θ	0.7							
τ	0.4							

Table A.6: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9907	0.0732	0.03	1.9909	0.0733	0.03
β_1	0.8	N(0, 10)	0.4850	0.1312	0.00	0.4853	0.1313	0.00
η	0.9							
θ	0.7							
τ	0.2							

Table A.7: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8828	0.2889	0.95	1.8909	0.1968	0.94
β_1	0.8	N(0, 10)	0.8167	0.2801	0.95	0.8079	0.2076	0.93
η	0.9	beta(27, 3)	0.9013	0.1053	1.00			
θ	0.7	beta(21, 9)	0.7008	0.1192	0.92			
τ	0.8	beta(24, 6)	0.7970	0.1134	0.93			

Table A.8: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9023	0.1416	0.96	1.9013	0.1131	0.95
β_1	0.8	N(0, 10)	0.7955	0.1589	0.97	0.7957	0.1465	0.94
η	0.9	beta(27, 3)	0.8996	0.0975	0.98			
θ	0.7	beta(21, 9)	0.7036	0.1668	0.97			
τ	0.4	beta(12, 18)	0.3995	0.1324	0.98			

Table A.9: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9253	0.1842	0.94	1.8999	0.0980	0.92
β_1	0.8	N(0, 10)	0.7409	0.3285	0.93	0.7962	0.1640	0.92
η	0.9	beta(27, 3)	0.8849	0.1141	0.95			
θ	0.7	beta(21, 9)	0.6854	0.2572	1.00			
τ	0.2	beta(4, 24)	0.2186	0.1551	0.99			

Table A.10: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8965	0.2205	0.94	1.8930	0.1516	0.96
β_1	0.8	N(0, 10)	0.8054	0.2129	0.94	0.8076	0.1599	0.96
η	0.9	beta(27, 3)	0.9020	0.1322	0.98			
θ	0.7	beta(21, 9)	0.7008	0.0956	0.96			
τ	0.8	beta(24, 6)	0.7937	0.0922	0.93			

Table A.11: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8997	0.1106	0.96	1.9028	0.0872	0.97
β_1	0.8	N(0, 10)	0.7972	0.1230	0.92	0.7974	0.1132	0.93
η	0.9	beta(27, 3)	0.9021	0.0782	0.99			
θ	0.7	beta(21, 9)	0.7028	0.1376	0.96			
τ	0.4	beta(12, 18)	0.4003	0.1058	0.96			

Table A.12: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9027	0.1056	0.95	1.9003	0.0758	0.93
β_1	0.8	N(0, 10)	0.7895	0.1745	0.94	0.8034	0.1269	0.94
η	0.9	beta(27, 3)	0.8984	0.0745	0.94			
θ	0.7	beta(21, 9)	0.6948	0.1963	1.00			
τ	0.2	beta(4, 24)	0.2075	0.0997	0.97			

Table A.13: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8919	0.2866	0.92	1.8951	0.1962	0.93
β_1	0.8	N(0, 10)	0.8090	0.2783	0.92	0.8063	0.2070	0.92
η	0.9	beta(45, 5)	0.9026	0.1297	1.00			
θ	0.7	beta(35, 15)	0.6995	0.1147	0.94			
τ	0.8	beta(40, 10)	0.7977	0.1081	0.99			

Table A.14: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8994	0.1402	0.96	1.9016	0.1125	0.96
β_1	0.8	N(0, 10)	0.7976	0.1589	0.99	0.7979	0.1464	0.99
η	0.9	beta(45, 5)	0.9040	0.0887	0.98			
θ	0.7	beta(35, 15)	0.6989	0.1547	1.00			
τ	0.4	beta(20, 30)	0.3982	0.1251	1.00			

Table A.15: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9113	0.1500	0.92	1.9000	0.0977	0.94
β_1	0.8	N(0, 10)	0.7751	0.2649	0.96	0.8050	0.1645	0.96
η	0.9	beta(45, 5)	0.8958	0.0896	1.00			
θ	0.7	beta(35, 15)	0.6935	0.2109	1.00			
τ	0.2	beta(10, 40)	0.2067	0.1256	0.98			

Table A.16: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8897	0.2245	0.96	1.8937	0.1520	0.97
β_1	0.8	N(0, 10)	0.8101	0.2170	0.95	0.8061	0.1603	0.98
η	0.9	beta(45, 5)	0.8985	0.1201	0.99			
θ	0.7	beta(35, 15)	0.7025	0.0924	0.92			
τ	0.8	beta(40, 10)	0.7984	0.0884	0.98			

Table A.17: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9008	0.1094	0.97	1.8987	0.0875	0.92
β_1	0.8	N(0, 10)	0.7983	0.1230	1.00	0.8002	0.1134	1.00
η	0.9	beta(45, 5)	0.9014	0.0745	0.97			
θ	0.7	beta(35, 15)	0.7004	0.1300	0.97			
τ	0.4	beta(20, 30)	0.3982	0.1019	0.95			

Table A.18: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9097	0.1193	0.91	1.9008	0.0758	0.93
β_1	0.8	N(0, 10)	0.7794	0.2050	0.89	0.8015	0.1267	0.92
η	0.9	beta(45, 5)	0.8958	0.0784	0.98			
θ	0.7	beta(35, 15)	0.6949	0.1880	1.00			
τ	0.2	beta(10, 40)	0.2091	0.1049	0.98			

Table A.19: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9000	0.2788	0.97	1.8939	0.1982	0.96
β_1	0.8	N(0, 10)	0.7997	0.2728	0.97	0.8043	0.2088	0.97
η	0.9	beta(90, 10)	0.8984	0.1043	1.00			
θ	0.7	beta(70, 30)	0.7019	0.1038	0.98			
τ	0.8	beta(80, 20)	0.7976	0.0970	1.00			

Table A.20: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8983	0.1393	0.97	1.9005	0.1134	0.93
β_1	0.8	N(0, 10)	0.8012	0.1587	0.94	0.8009	0.1464	0.92
η	0.9	beta(90, 10)	0.9052	0.0776	1.00			
θ	0.7	beta(70, 30)	0.7031	0.1309	1.00			
τ	0.4	beta(40, 60)	0.4038	0.1126	0.97			

Table A.21: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9072	0.1238	0.98	1.9020	0.0975	0.92
β_1	0.8	N(0, 10)	0.7853	0.2346	0.95	0.8003	0.1650	0.93
η	0.9	beta(90, 10)	0.8991	0.0718	1.00			
θ	0.7	beta(70, 30)	0.6966	0.1589	1.00			
τ	0.2	beta(20, 80)	0.1974	0.0964	0.99			

Table A.22: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9005	0.2212	0.95	1.8985	0.1525	0.97
β_1	0.8	N(0, 10)	0.7989	0.2151	0.96	0.8011	0.1607	0.95
η	0.9	beta(90, 10)	0.9016	0.0967	1.00			
θ	0.7	beta(70, 30)	0.7003	0.0869	0.98			
τ	0.8	beta(80, 20)	0.7997	0.0817	0.98			

Table A.23: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9044	0.1077	0.94	1.9036	0.0873	0.63
β_1	0.8	N(0, 10)	0.7950	0.1224	0.97	0.7960	0.1132	0.95
η	0.9	beta(90, 10)	0.9011	0.0678	0.97			
θ	0.7	beta(70, 30)	0.7007	0.1156	1.00			
τ	0.4	beta(40, 60)	0.3994	0.0947	1.00			

Table A.24: Model 1: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9036	0.0879	0.96	1.9036	0.0756	0.96
β_1	0.8	N(0, 10)	0.7953	0.1434	0.94	0.7975	0.1272	0.94
η	0.9	beta(90, 10)	0.9000	0.0588	0.98			
θ	0.7	beta(70, 30)	0.6992	0.1410	1.00			
τ	0.2	beta(20, 80)	0.1994	0.0792	0.97			

Table A.25: Naive Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.5106	0.0844	0.00	2.5109	0.0844	0.00
β_1	0.8	N(0, 10)	0.1641	0.1251	0.00	0.1641	0.1252	0.00
η	0.9							
θ	0.5							
τ	0.8							

Table A.26: Naive Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.1889	0.0871	0.00	2.1892	0.0878	0.00
β_1	0.8	N(0, 10)	0.3706	0.1523	0.00	0.3709	0.1526	0.00
η	0.9							
θ	0.5							
τ	0.4							

Table A.27: Naive Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0367	0.0902	0.01	2.0371	0.0903	0.01
β_1	0.8	N(0, 10)	0.3768	0.1830	0.00	0.3774	0.1839	0.00
η	0.9							
θ	0.5							
τ	0.2							

Table A.28: Model 1: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9009	0.3113	0.97	1.9047	0.1964	0.94
β_1	0.8	N(0, 10)	0.8014	0.2953	0.98	0.7979	0.2071	0.93
η	0.9	beta(27, 3)	0.9002	0.1053	1.00			
θ	0.5	beta(15,15)	0.5087	0.1268	0.98			
τ	0.8	beta(24, 6)	0.7991	0.1239	0.98			

Table A.29: Model 1: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8968	0.1524	0.95	1.8997	0.1133	0.97
β_1	0.8	N(0, 10)	0.8017	0.1605	0.95	0.8014	0.1464	0.96
η	0.9	beta(27, 3)	0.8989	0.0944	0.95			
θ	0.5	beta(15, 15)	0.4997	0.1753	0.99			
τ	0.4	beta(12, 18)	0.4046	0.1414	0.98			

Table A.30: Model 1: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9404	0.2591	0.94	1.8972	0.0980	0.94
β_1	0.8	N(0, 10)	0.6980	0.4819	0.96	0.8035	0.1644	0.94
η	0.9	beta(27, 3)	0.8882	0.1234	1.00			
θ	0.5	beta(15, 15)	0.4693	0.2766	0.98			
τ	0.2	beta(4, 24)	0.2346	0.2043	0.99			

Table A.31: Model 1: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8946	0.3098	0.95	1.9073	0.1947	0.94
β_1	0.8	N(0, 10)	0.8046	0.2954	0.96	0.7931	0.2055	0.96
η	0.9	beta(45, 5)	0.9028	0.1281	1.00			
θ	0.5	beta(25, 25)	0.5013	0.1281	0.96			
τ	0.8	beta(40, 10)	0.7989	0.1215	0.98			

Table A.32: Model 1: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8993	0.1468	0.95	1.8994	0.1127	0.95
β_1	0.8	N(0, 10)	0.8036	0.1595	0.93	0.8046	0.1463	0.94
η	0.9	beta(45, 5)	0.9039	0.0860	0.94			
θ	0.5	beta(25, 25)	0.5090	0.1635	0.99			
τ	0.4	beta(20, 30)	0.3971	0.1314	0.96			

Table A.33: Model 1: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9337	0.2296	0.97	1.9002	0.0979	0.94
β_1	0.8	N(0, 10)	0.7098	0.4418	0.94	0.7999	0.1641	0.94
η	0.9	beta(45, 5)	0.8953	0.1030	0.97			
θ	0.5	beta(25, 25)	0.4843	0.2435	0.99			
τ	0.2	beta(10, 40)	0.2252	0.1770	0.98			

Table A.34: Naive Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.2614	0.1547	0.00	2.2650	0.1548	0.00
β_1	0.8	N(0, 10)	0.3934	0.1690	0.00	0.3927	0.1691	0.00
η	0.7							
θ	0.9							
τ	0.8							

Table A.35: Naive Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0010	0.1227	0.17	2.0016	0.1227	0.17
β_1	0.8	N(0, 10)	0.4988	0.1511	0.00	0.4986	0.1512	0.00
η	0.7							
θ	0.9							
τ	0.4							

Table A.36: Naive Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9428	0.1125	0.65	1.9433	0.1125	0.66
β_1	0.8	N(0, 10)	0.3725	0.1573	0.00	0.3724	0.1574	0.00
η	0.7							
θ	0.9							
τ	0.2							

Table A.37: Model 1: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8935	0.2866	0.92	1.8970	0.1980	0.94
β_1	0.8	N(0, 10)	0.8037	0.2787	0.92	0.8001	0.2087	0.95
η	0.7	beta(21, 9)	0.7025	0.2229	1.00			
θ	0.9	beta(27, 3)	0.8977	0.0819	0.98			
τ	0.8	beta(24, 6)	0.8004	0.1132	0.96			

Table A.38: Model 1: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8948	0.1423	0.95	1.8945	0.1136	0.98
β_1	0.8	N(0, 10)	0.8025	0.1589	0.96	0.8028	0.1467	0.96
η	0.7	beta(21, 9)	0.8967	0.1406	0.99			
θ	0.9	beta(27, 3)	0.8967	0.1151	0.98			
τ	0.4	beta(12, 18)	0.4025	0.1331	0.99			

Table A.39: Model 1: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9413	0.2463	0.95	1.8993	0.0978	0.96
β_1	0.8	N(0, 10)	0.7035	0.4685	0.97	0.7977	0.1648	0.94
η	0.7	beta(21, 9)	0.6793	0.1782	0.96			
θ	0.9	beta(27, 3)	0.8691	0.2355	1.00			
τ	0.2	beta(4, 24)	0.2275	0.1959	0.99			

Table A.40: Model 1: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9034	0.2802	0.91	1.9077	0.1958	0.89
β_1	0.8	N(0, 10)	0.7971	0.2737	0.93	0.7937	0.2065	0.93
η	0.7	beta(35, 15)	0.7027	0.1943	0.99			
θ	0.9	beta(45, 5)	0.8988	0.0778	0.94			
τ	0.8	beta(40, 10)	0.8002	0.1072	0.98			

Table A.41: Model 1: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8967	0.1398	0.91	1.9005	0.1130	0.91
β_1	0.8	N(0, 10)	0.8001	0.1587	0.91	0.7984	0.1464	0.93
η	0.7	beta(35, 15)	0.7031	0.1315	0.97			
θ	0.9	beta(45, 5)	0.9001	0.1044	0.96			
τ	0.4	beta(20, 30)	0.4021	0.1257	0.99			

Table A.42: Model 1: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9299	0.1975	0.97	1.9002	0.0979	0.94
β_1	0.8	N(0, 10)	0.7359	0.3879	0.94	0.7999	0.1641	0.94
η	0.7	beta(35, 15)	0.6917	0.1407	0.97			
θ	0.9	beta(45, 5)	0.8804	0.1843	0.99			
τ	0.2	beta(10, 40)	0.2128	0.1568	0.98			

Table A.43: Naive Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.3188	0.1704	0.00	2.3199	0.1706	0.00
β_1	0.8	N(0, 10)	0.3098	0.1831	0.00	0.3089	0.1833	0.00
η	0.5							
θ	0.9							
τ	0.2							

Table A.44: Naive Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0240	0.1405	0.11	2.0248	0.1405	0.11
β_1	0.8	N(0, 10)	0.3936	0.1634	0.00	0.3931	0.1635	0.00
η	0.5							
θ	0.9							
τ	0.4							

Table A.45: Naive Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9540	0.1316	0.64	1.9547	0.1315	0.65
β_1	0.8	N(0, 10)	0.2689	0.1640	0.00	0.2686	0.1641	0.00
η	0.5							
θ	0.9							
τ	0.2							

Table A.46: Model 1: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8809	0.3197	0.99	1.8939	0.1977	0.90
β_1	0.8	N(0, 10)	0.8169	0.3034	0.99	0.8047	0.2084	0.92
η	0.5	beta(15, 15)	0.5026	0.2345	0.99			
θ	0.9	beta(27, 3)	0.8985	0.0794	0.96			
τ	0.2	beta(4, 24)	0.8013	0.1233	0.93			

Table A.47: Model 1: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8958	0.1513	0.97	1.8983	0.1136	0.95
β_1	0.8	N(0, 10)	0.8026	0.1519	0.95	0.8021	0.1464	0.96
η	0.5	beta(15, 15)	0.5012	0.1471	0.97			
θ	0.9	beta(27, 3)	0.8986	0.1109	0.97			
τ	0.4	beta(12, 18)	0.4069	0.1409	0.98			

Table A.48: Model 1: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9595	0.3078	0.97	1.8984	0.0978	0.98
β_1	0.8	N(0, 10)	0.6672	0.5957	0.96	0.8039	0.1646	0.96
η	0.5	beta(15, 15)	0.4867	0.1783	0.98			
θ	0.9	beta(27, 3)	0.8740	0.2208	0.98			
τ	0.2	beta(4, 24)	0.2391	0.2380	0.99			

Table A.49: Model 1: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8939	0.3131	0.94	1.9077	0.1980	0.96
β_1	0.8	N(0, 10)	0.8037	0.2990	0.93	0.7915	0.2086	0.94
η	0.5	beta(25, 25)	0.4949	0.2072	1.00			
θ	0.9	beta(45, 5)	0.8991	0.0758	0.97			
τ	0.2	beta(10, 40)	0.8041	0.1158	0.97			

Table A.50: Model 1: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9033	0.1554	0.98	1.9009	0.1130	0.98
β_1	0.8	N(0, 10)	0.7947	0.1750	0.97	0.7991	0.1464	0.97
η	0.5	beta(25, 25)	0.5012	0.1404	0.98			
θ	0.9	beta(45, 5)	0.8971	0.1063	0.99			
τ	0.4	beta(20, 30)	0.3994	0.1341	0.97			

Table A.51: Model 1: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9213	0.1969	0.98	1.8953	0.0979	0.96
β_1	0.8	N(0, 10)	0.7397	0.3713	0.94	0.8044	0.1651	0.94
η	0.5	beta(25, 25)	0.4956	0.1360	0.97			
θ	0.9	beta(45, 5)	0.8888	0.1577	1.00			
τ	0.2	beta(10, 40)	0.2143	0.1566	0.99			

Table A.52: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.8914	0.2852	0.94	1.8905	0.1965	0.95
β_1	0.8	N(0.8, 0.09)	0.8091	0.2718	0.94	0.8093	0.2073	0.95
η	0.9	beta(1, 1)	0.8784	0.2183	0.99			
θ	0.7	beta(1, 1)	0.6969	0.1289	0.97			
τ	0.8	beta(1, 1)	0.7923	0.1290	0.98			

Table A.53: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9040	0.1430	0.96	1.9023	0.1128	0.96
β_1	0.8	N(0.8, 0.09)	0.7984	0.1572	0.96	0.7969	0.1463	0.96
η	0.9	beta(1, 1)	0.8892	0.1104	0.91			
θ	0.7	beta(1, 1)	0.6983	0.1945	0.98			
τ	0.4	beta(1, 1)	0.3964	0.1435	0.96			

Table A.54: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9536	0.2942	0.99	1.8970	0.0982	0.98
β_1	0.8	N(0.8, 0.09)	0.6781	0.5339	0.96	0.8054	0.1636	0.97
η	0.9	beta(1, 1)	0.8500	0.2379	0.96			
θ	0.7	beta(1, 1)	0.6417	0.3946	0.94			
τ	0.2	beta(1, 1)	0.2553	0.2468	0.92			

Table A.55: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $\sigma = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.04)	1.9153	0.2650	0.97	1.9059	0.1968	0.97
β_1	0.8	N(0.8, 0.04)	0.7884	0.2524	0.96	0.7952	0.2075	0.95
η	0.9	beta(1, 1)	0.8690	0.2282	0.95			
θ	0.7	beta(1, 1)	0.7010	0.1280	0.94			
τ	0.8	beta(1, 1)	0.7924	0.1289	0.96			

Table A.56: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $\sigma = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.04)	1.9015	0.1415	0.91	1.8991	0.1133	0.95
β_1	0.8	N(0.8, 0.04)	0.8016	0.1549	0.96	0.8042	0.1465	0.92
η	0.9	beta(1, 1)	0.8966	0.1086	0.91			
θ	0.7	beta(1, 1)	0.7018	0.1928	0.93			
τ	0.4	beta(1, 1)	0.4014	0.1433	0.94			

Table A.57: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $\sigma = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.04)	1.9199	0.1781	0.96	1.9009	0.0097	0.94
β_1	0.8	N(0.8, 0.04)	0.7495	0.3102	0.98	0.7946	0.1649	0.95
η	0.9	beta(1, 1)	0.8803	0.1482	0.93			
θ	0.7	beta(1, 1)	0.6691	0.3346	0.95			
τ	0.2	beta(1, 1)	0.2198	0.1743	1.00			

Table A.58: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $\sigma = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.04)	1.9019	0.2145	1.00	1.9008	0.1517	0.99
β_1	0.8	N(0.8, 0.04)	0.7986	0.2048	0.98	0.7986	0.1600	0.97
η	0.9	beta(1, 1)	0.8917	0.1705	0.96			
θ	0.7	beta(1, 1)	0.7009	0.0997	0.95			
τ	0.8	beta(1, 1)	0.7952	0.0987	0.97			

Table A.59: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $\sigma = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.04)	1.8957	0.1106	0.98	1.8978	0.0876	0.97
β_1	0.8	N(0.8, 0.04)	0.8045	0.1210	0.98	0.8035	0.1134	0.97
η	0.9	beta(1, 1)	0.8991	0.0841	0.95			
θ	0.7	beta(1, 1)	0.6989	0.1945	0.98			
τ	0.4	beta(1, 1)	0.4021	0.1109	0.92			

Table A.60: Model 2: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $\sigma = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.04)	1.9128	0.1276	0.96	1.8992	0.0757	0.96
β_1	0.8	N(0.8, 0.04)	0.7775	0.2179	0.94	0.8038	0.1273	0.93
η	0.9	beta(1, 1)	0.8882	0.0997	0.94			
θ	0.7	beta(1, 1)	0.6852	0.2530	0.99			
τ	0.2	beta(1, 1)	0.2082	0.1229	0.96			

Table A.61: Model 2: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.8989	0.3017	0.97	1.8927	0.1978	0.95
β_1	0.8	N(0.8, 0.09)	0.8037	0.2824	0.94	0.8079	0.2084	0.93
η	0.9	beta(1, 1)	0.8789	0.2091	0.96			
θ	0.5	beta(1, 1)	0.4987	0.1355	0.95			
τ	0.8	beta(1, 1)	0.7925	0.1395	1.00			

Table A.62: Model 2: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.8972	0.1585	0.98	1.8963	0.1134	0.96
β_1	0.8	N(0.8, 0.09)	0.8029	0.1722	0.96	0.8035	0.1466	0.97
η	0.9	beta(1, 1)	0.8966	0.1086	0.96			
θ	0.5	beta(1, 1)	0.4994	0.2035	0.97			
τ	0.4	beta(1, 1)	0.4017	0.1537	0.96			

Table A.63: Model 2: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9320	0.2141	0.97	1.9042	0.0976	0.94
β_1	0.8	N(0.8, 0.09)	0.7274	0.3684	0.92	0.7960	0.1644	0.96
η	0.9	beta(1, 1)	0.8834	0.1399	0.97			
θ	0.5	beta(1, 1)	0.4831	0.3277	0.95			
τ	0.2	beta(1, 1)	0.2317	0.1991	0.95			

Table A.64: Model 2: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.8897	0.2776	0.96	1.9021	0.1967	0.94
β_1	0.8	N(0.8, 0.09)	0.8101	0.2669	0.96	0.7995	0.2074	0.97
η	0.7	beta(1, 1)	0.7049	0.3027	0.99			
θ	0.9	beta(1, 1)	0.8931	0.0878	0.96			
τ	0.8	beta(1, 1)	0.8014	0.1237	0.99			

Table A.65: Model 2: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9093	0.1622	0.92	1.9012	0.1123	0.92
β_1	0.8	N(0.8, 0.09)	0.7810	0.2024	0.92	0.7963	0.1465	0.92
η	0.7	beta(1, 1)	0.6964	0.1690	0.99			
θ	0.9	beta(1, 1)	0.8901	0.1539	0.96			
τ	0.4	beta(1, 1)	0.3936	0.1470	0.93			

Table A.66: Model 2: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9606	0.3057	0.95	1.9019	0.0980	0.96
β_1	0.8	N(0.8, 0.09)	0.6778	0.5620	0.95	0.8058	0.1632	0.95
η	0.7	beta(1, 1)	0.6417	0.2743	0.97			
θ	0.9	beta(1, 1)	0.8249	0.3379	0.93			
τ	0.2	beta(1, 1)	0.2553	0.2552	0.89			

Table A.67: Model 2: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9060	0.3026	0.98	1.9036	0.1963	0.99
β_1	0.8	N(0.8, 0.09)	0.7929	0.2832	0.98	0.7953	0.2071	0.99
η	0.5	beta(1, 1)	0.5005	0.3069	0.91			
θ	0.9	beta(1, 1)	0.8954	0.0870	0.95			
τ	0.8	beta(1, 1)	0.7946	0.1407	0.98			

Table A.68: Model 2: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.8999	0.1517	0.96	1.9010	0.1131	0.97
β_1	0.8	N(0.8, 0.09)	0.8014	0.1578	0.94	0.7999	0.1463	0.97
η	0.5	beta(1, 1)	0.5008	0.1161	1.00			
θ	0.9	beta(1, 1)	0.8948	0.1297	0.96			
τ	0.4	beta(1, 1)	0.4018	0.1552	0.96			

Table A.69: Model 2: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $\sigma = 0.3$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(1.9, 0.09)	1.9499	0.2749	0.93	1.9007	0.0978	0.91
β_1	0.8	N(0.8, 0.09)	0.6859	0.4919	0.97	0.7961	0.1644	0.98
η	0.5	beta(1, 1)	0.4688	0.2121	0.95			
θ	0.9	beta(1, 1)	0.8586	0.2609	0.99			
τ	0.2	beta(1, 1)	0.2475	0.2481	0.97			

Table A.70: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9033	0.2777	0.99	1.9006	0.1975	0.95
β_1	0.8	N(0, 10)	0.7972	0.2683	0.96	0.7991	0.2081	0.95
η	0.9	beta(27, 3)	0.9016	0.1519	1.00			
θ	0.7	beta(21, 9)	0.7066	0.1194	0.99			
τ	0.8	beta(24, 6)	0.7984	0.1343	0.93			

Table A.71: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8909	0.1424	0.90	1.8959	0.1135	0.98
β_1	0.8	N(0, 10)	0.8078	0.1578	0.96	0.8070	0.1465	0.94
η	0.9	beta(27, 3)	0.9002	0.0970	0.93			
θ	0.7	beta(21, 9)	0.6925	0.1662	0.99			
τ	0.4	beta(12,18)	0.4063	0.1321	0.92			

Table A.72: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8977	0.1147	0.93	189943	0.0979	0.91
β_1	0.8	N(0, 10)	0.8044	0.1829	0.90	0.8030	0.1641	0.92
η	0.9	beta(27, 3)	0.8997	0.0811	0.99			
θ	0.7	beta(21, 9)	0.6963	0.2174	0.99			
τ	0.2	beta(6, 24)	0.2015	0.1104	0.95			

Table A.73: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8974	0.2108	0.98	1.9015	0.1510	0.94
β_1	0.8	N(0, 10)	0.7999	0.2031	0.99	0.7968	0.1594	0.96
η	0.9	beta(27, 3)	0.9009	0.1339	0.99			
θ	0.7	beta(21, 9)	0.6986	0.0952	0.95			
τ	0.8	beta(24, 6)	0.7995	0.0913	0.97			

Table A.74: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8997	0.1095	0.95	1.8977	0.0877	0.96
β_1	0.8	N(0, 10)	0.8027	0.1208	0.96	0.8028	0.1134	0.96
η	0.9	beta(27, 3)	0.8985	0.0787	0.94			
θ	0.7	beta(21, 9)	0.6972	0.1393	0.96			
τ	0.4	beta(12,18)	0.4016	0.1056	0.98			

Table A.75: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9005	0.0885	0.98	1.8998	0.0757	0.98
β_1	0.8	N(0, 10)	0.8014	0.1417	0.99	0.8018	0.1274	0.95
η	0.9	beta(27, 3)	0.9048	0.0633	0.94			
θ	0.7	beta(21, 9)	0.6995	0.1874	0.98			
τ	0.2	beta(6, 24)	0.1976	0.0877	0.96			

Table A.76: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8920	0.2539	0.97	1.8914	0.1976	0.95
β_1	0.8	N(0, 10)	0.8073	0.2473	0.97	0.8077	0.2083	0.95
η	0.9	beta(45, 5)	0.9018	0.1302	1.00			
θ	0.7	beta(35, 15)	0.7039	0.1132	0.98			
τ	0.8	beta(40, 10)	0.7985	0.1055	0.98			

Table A.77: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9073	0.1353	0.96	1.8989	0.1135	0.96
β_1	0.8	N(0, 10)	0.8020	0.1537	0.97	0.8030	0.1464	0.97
η	0.9	beta(45, 5)	0.8975	0.0910	1.00			
θ	0.7	beta(35, 15)	0.7084	0.1527	0.99			
τ	0.4	beta(20, 30)	0.3750	0.1252	0.90			

Table A.78: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8958	0.1129	0.99	1.8983	0.0977	0.98
β_1	0.8	N(0, 10)	0.7943	0.1814	0.97	0.7997	0.1653	0.95
η	0.9	beta(45, 5)	0.9020	0.0770	0.97			
θ	0.7	beta(35, 15)	0.7005	0.1910	1.00			
τ	0.2	beta(10, 40)	0.2010	0.1052	0.99			

Table A.79: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8985	0.2092	0.94	1.8974	0.1526	0.92
β_1	0.8	N(0, 10)	0.8027	0.2022	0.95	0.8038	0.1609	0.92
η	0.9	beta(45, 5)	0.8994	0.1211	1.00			
θ	0.7	beta(35, 15)	0.6987	0.0923	0.97			
τ	0.8	beta(40, 10)	0.7996	0.0878	0.94			

Table A.80: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9047	0.1076	0.93	1.8994	0.0875	0.93
β_1	0.8	N(0, 10)	0.8020	0.1537	0.97	0.8005	0.1134	0.92
η	0.9	beta(45, 5)	0.8990	0.0752	0.98			
θ	0.7	beta(35, 15)	0.7068	0.1299	0.97			
τ	0.4	beta(20, 30)	0.3815	0.1027	0.88			

Table A.81: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8968	0.0879	0.96	1.8965	0.0757	0.96
β_1	0.8	N(0, 10)	0.8005	0.1420	0.94	0.8026	0.1278	0.96
η	0.9	beta(45, 5)	0.9003	0.0626	0.97			
θ	0.7	beta(35, 15)	0.7030	0.1691	0.96			
τ	0.2	beta(10, 40)	0.1974	0.0847	0.97			

Table A.82: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8877	0.2533	0.99	1.8937	0.1964	0.92
β_1	0.8	N(0, 10)	0.8093	0.2474	0.99	0.8051	0.2072	0.98
η	0.9	beta(90, 10)	0.9001	0.1037	1.00			
θ	0.7	beta(70, 10)	0.6995	0.1033	0.98			
τ	0.8	beta(80, 20)	0.8044	0.0950	0.98			

Table A.83: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9047	0.1342	0.94	1.9037	0.1135	0.96
β_1	0.8	N(0, 10)	0.7979	0.1536	0.96	0.7988	0.1462	0.96
η	0.9	beta(90, 10)	0.9002	0.0789	0.97			
θ	0.7	beta(70, 10)	0.7034	0.1311	0.99			
τ	0.4	beta(20, 30)	0.3983	0.1127	1.00			

Table A.84: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9000	0.1105	0.97	1.8991	0.0978	0.97
β_1	0.8	N(0, 10)	0.7999	0.1802	0.97	0.7989	0.1646	0.97
η	0.9	beta(90, 10)	0.8986	0.0702	0.98			
θ	0.7	beta(70, 10)	0.7012	0.1522	1.00			
τ	0.2	beta(10, 40)	0.1982	0.0935	1.00			

Table A.85: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8980	0.2049	0.98	1.8992	0.1523	0.97
β_1	0.8	N(0, 10)	0.8026	0.1992	0.96	0.8017	0.1606	0.99
η	0.9	beta(90, 10)	0.9013	0.0968	1.00			
θ	0.7	beta(70, 10)	0.6993	0.0864	0.97			
τ	0.8	beta(80, 20)	0.8000	0.0805	0.95			

Table A.86: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8990	0.1064	0.94	1.8993	0.0873	0.91
β_1	0.8	N(0, 10)	0.8008	0.1206	0.94	0.8012	0.1133	0.95
η	0.9	beta(90, 10)	0.9008	0.0677	1.00			
θ	0.7	beta(70, 10)	0.6991	0.1157	1.00			
τ	0.4	beta(20, 30)	0.3973	0.0945	0.99			

Table A.87: Model 3: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 100$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8945	0.0874	0.95	1.8969	0.0758	0.96
β_1	0.8	N(0, 10)	0.8033	0.1404	0.95	0.8022	0.1275	0.93
η	0.9	beta(90, 10)	0.9007	0.0584	0.99			
θ	0.7	beta(70, 10)	0.6983	0.1404	0.99			
τ	0.2	beta(10, 40)	0.2003	0.0788	0.99			

Table A.88: Model 3: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8918	0.2964	0.98	1.8962	0.1987	0.93
β_1	0.8	N(0, 10)	0.8065	0.2818	0.96	0.8029	0.2093	0.95
η	0.9	beta(27, 3)	0.9058	0.1461	1.00			
θ	0.5	beta(15, 15)	0.5080	0.1260	0.97			
τ	0.8	beta(24, 6)	0.8024	0.1204	0.96			

Table A.89: Model 3: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8957	0.1494	0.96	1.9004	0.1124	0.94
β_1	0.8	N(0, 10)	0.8030	0.1587	0.97	0.8009	0.1464	0.95
η	0.9	beta(27, 3)	0.9010	0.0923	0.99			
θ	0.5	beta(15, 15)	0.4996	0.1770	0.98			
τ	0.4	beta(12, 18)	0.3979	0.1401	0.94			

Table A.90: Model 3: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9075	0.1399	0.94	1.9021	0.0977	0.96
β_1	0.8	N(0, 10)	0.7803	0.2306	0.94	0.7969	0.1644	0.95
η	0.9	beta(27, 3)	0.8996	0.0855	0.98			
θ	0.5	beta(15, 15)	0.4953	0.2374	1.00			
τ	0.2	beta(6, 24)	0.2055	0.1304	0.97			

Table A.91: Model 3: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9033	0.2737	0.96	1.9129	0.1957	0.95
β_1	0.8	N(0, 10)	0.7934	0.2661	0.95	0.7858	0.2065	0.96
η	0.7	beta(21, 9)	0.7071	0.2236	1.00			
θ	0.9	beta(27, 3)	0.8970	0.0820	0.96			
τ	0.8	beta(24, 6)	0.8028	0.1126	0.94			

Table A.92: Model 3: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9026	0.1461	0.98	1.8972	0.1134	0.96
β_1	0.8	N(0, 10)	0.7960	0.1726	0.97	0.8034	0.1465	0.97
η	0.7	beta(21, 9)	0.7012	0.1431	0.97			
θ	0.9	beta(27, 3)	0.9034	0.1163	0.98			
τ	0.4	beta(12, 18)	0.4004	0.1325	0.92			

Table A.93: Model 3: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9052	0.1480	0.95	1.8931	0.0983	0.96
β_1	0.8	N(0, 10)	0.7847	0.2595	0.96	0.8105	0.1639	0.94
η	0.7	beta(21, 9)	0.6986	0.1340	0.95			
θ	0.9	beta(27, 3)	0.8912	0.1691	1.00			
τ	0.2	beta(6, 24)	0.2083	0.1320	0.98			

Table A.94: Model 3: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8963	0.2877	0.95	1.8985	0.1964	0.97
β_1	0.8	N(0, 10)	0.8064	0.2737	0.95	0.8040	0.2071	0.95
η	0.5	beta(15, 15)	0.5111	0.2323	0.99			
θ	0.9	beta(27, 3)	0.8985	0.0795	0.92			
τ	0.8	beta(24, 6)	0.7982	0.1213	1.00			

Table A.95: Model 3: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9046	0.1485	0.96	1.9039	0.1132	0.96
β_1	0.8	N(0, 10)	0.7265	0.1580	0.92	0.7934	0.1463	0.93
η	0.5	beta(15, 15)	0.5005	0.1470	0.95			
θ	0.9	beta(27, 3)	0.9043	0.1097	0.97			
τ	0.4	beta(12, 18)	0.4034	0.1416	0.94			

Table A.96: Model 3: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9221	0.1983	1.00	1.8991	0.0979	0.97
β_1	0.8	N(0, 10)	0.7358	0.3633	0.95	0.7946	0.1647	0.93
η	0.5	beta(15, 15)	0.4873	0.1495	0.96			
θ	0.9	beta(27, 3)	0.8837	0.1843	0.98			
τ	0.2	beta(6, 24)	0.2189	0.1683	0.97			

APPENDIX B

Chapter III Tables

Table B.1: Naive Model: $\eta = 0.9\ \theta = 0.7\ \tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.4245	0.1745	0.00	2.4256	0.1751	0.00
β_1	0.8	N(0, 10)	0.2574	0.1631	0.00	0.2570	0.1633	0.00
β_2	-0.2	N(0, 10)	-0.1989	0.0558	0.88	-0.1990	0.0559	0.89
η	0.9							
θ	0.7							
τ	0.8							

Table B.2: Corrected Model: $\eta = 0.9\ \theta = 0.7\ \tau = 0.8\ EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8984	0.4349	0.96	1.9071	0.2738	0.98
β_1	0.8	N(0, 10)	0.7968	0.4027	0.96	0.7909	0.2608	0.98
β_2	-0.2	N(0, 10)	-0.1992	0.0618	0.93	-0.2001	0.0557	0.93
η	0.9	beta(45, 5)	0.9044	0.1391	1.00			
θ	0.7	beta(35, 15)	0.6971	0.1249	0.96			
τ	0.8	beta(40, 10)	0.7984	0.1284	0.98			

Table B.3: Corrected Model: $\eta = 0.9\ \theta = 0.7\ \tau = 0.8\ EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8898	0.4406	0.99	1.9142	0.2712	0.95
β_1	0.8	N(0, 10)	0.8078	0.4086	0.97	0.7832	0.2584	0.95
β_2	-0.2	N(0, 10)	-0.1992	0.0621	0.95	-0.1989	0.0559	0.99
η	0.9	beta(27, 3)	0.8999	0.1691	1.00			
θ	0.7	beta(21, 9)	0.7002	0.1309	0.95			
τ	0.8	beta(24, 6)	0.8001	0.1383	0.98			

Table B.4: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.1117	0.1808	0.06	2.1124	0.1822	0.05
β_1	0.8	N(0, 10)	0.4897	0.1802	0.00	0.4896	0.1819	0.00
β_2	-0.2	N(0, 10)	-0.2029	0.0636	0.87	-0.2029	0.0642	0.85
η	0.9							
θ	0.7							
τ	0.4							

Table B.5: Corrected Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8974	0.2632	0.97	1.8967	0.1956	0.97
β_1	0.8	N(0, 10)	0.7931	0.2469	0.97	0.8029	0.1846	0.94
β_2	-0.2	N(0, 10)	-0.1993	0.0771	0.93	-0.1996	0.0643	0.93
η	0.9	beta(45, 5)	0.9030	0.1169	0.99			
θ	0.7	beta(35, 15)	0.7023	0.1938	0.98			
τ	0.4	beta(20, 30)	0.4009	0.1591	0.97			

Table B.6: Corrected Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8975	0.2372	0.85	1.8966	0.1945	0.90
β_1	0.8	N(0, 10)	0.7951	0.2106	0.95	0.8008	0.1696	0.90
β_2	-0.2	N(0, 10)	-0.1936	0.0772	0.90	-0.1957	0.0642	0.85
η	0.9	beta(27, 3)	0.8967	0.0868	1.00			
θ	0.7	beta(21, 9)	0.7121	0.1383	1.00			
τ	0.4	beta(12, 18)	0.3983	0.1221	1.00			

Table B.7: Naive Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9879	0.1908	0.56	1.9890	0.1909	0.56
β_1	0.8	N(0, 10)	0.4856	0.2146	0.00	0.4861	0.2146	0.00
β_2	-0.2	N(0, 10)	-0.2004	0.7078	0.86	-0.2005	0.0708	0.87
η	0.9							
θ	0.7							
τ	0.2							

Table B.8: Corrected Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8878	0.2443	0.93	1.8895	0.1917	0.94
β_1	0.8	N(0, 10)	0.7842	0.3025	0.92	0.8058	0.2071	0.94
β_2	-0.2	N(0, 10)	-0.1976	0.0829	0.98	-0.1993	0.0706	0.95
η	0.9	beta(45, 5)	0.9005	0.0894	0.96			
θ	0.7	beta(35, 15)	0.6939	0.2130	1.00			
τ	0.2	beta(10, 40)	0.2069	0.1284	0.98			

Table B.9: Corrected Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9069	0.2558	0.91	1.9004	0.1917	0.90
β_1	0.8	N(0, 10)	0.7704	0.3299	0.96	0.8001	0.2063	0.96
β_2	-0.2	N(0, 10)	-0.1969	0.0832	0.91	-0.1973	0.0703	0.94
η	0.9	beta(27, 3)	0.9003	0.1006	0.94			
θ	0.7	beta(21, 9)	0.7001	0.2552	1.00			
τ	0.2	beta(6, 24)	0.2083	0.1375	0.98			

Table B.10: Naive Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.5089	0.1575	0.00	2.5098	0.1577	0.00
β_1	0.8	N(0, 10)	0.1621	0.1577	0.00	0.1620	0.1578	0.00
β_2	-0.2	N(0, 10)	-0.2002	0.0558	0.90	-0.2002	0.0558	0.90
η	0.9							
θ	0.5							
τ	0.8							

Table B.11: Corrected Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8931	0.4641	0.97	1.8985	0.2759	0.96
β_1	0.8	N(0, 10)	0.8005	0.4253	0.98	0.7977	0.2627	0.95
β_2	-0.2	N(0, 10)	-0.1997	0.0635	0.97	-0.1991	0.0558	0.97
η	0.9	beta(45, 5)	0.8979	0.1421	1.00			
θ	0.5	beta(25, 25)	0.5031	0.1283	0.99			
τ	0.8	beta(40, 10)	0.7982	0.1437	1.00			

Table B.12: Corrected Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8881	0.4844	0.96	1.8919	0.2761	0.95
β_1	0.8	N(0, 10)	0.8128	0.4433	0.96	0.8108	0.2636	0.95
β_2	-0.2	N(0, 10)	-0.1989	0.0633	0.94	-0.1993	0.0556	0.94
η	0.9	beta(27, 3)	0.9019	0.1631	1.00			
θ	0.5	beta(15, 15)	0.5052	0.1362	0.99			
τ	0.8	beta(24, 6)	0.7994	0.1581	1.00			

Table B.13: Naive Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.1914	0.1737	0.01	2.1923	0.1738	0.01
$\beta_1 1$	0.8	N(0, 10)	0.3785	0.1916	0.00	0.3788	0.1915	0.00
$\beta_2 2$	-0.2	N(0, 10)	-.2024	0.0643	0.85	-0.2024	0.0644	0.85
η	0.9							
θ	0.5							
τ	0.4							

Table B.14: Corrected Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8881	0.4844	0.96	1.8919	0.2761	0.95
β_1	0.8	N(0, 10)	0.8128	0.4433	0.96	0.8108	0.2636	0.95
β_2	-0.2	N(0, 10)	-0.1989	0.0633	0.94	-0.1993	0.0556	0.94
η	0.9	beta(27, 3)	0.9019	0.1631	1.00			
θ	0.5	beta(15, 15)	0.5052	0.1362	0.99			
τ	0.8	beta(24, 6)	0.7994	0.1581	1.00			

Table B.15: Corrected Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8857	0.2646	0.98	1.8894	0.1962	0.97
β_1	0.8	N(0, 10)	0.8154	0.2168	0.94	0.8103	0.1847	0.95
β_2	-0.2	N(0, 10)	-0.1970	0.0819	0.94	-0.1977	0.0644	0.93
η	0.9	beta(27, 3)	0.8997	0.1044	0.94			
θ	0.5	beta(15, 15)	0.4923	0.1908	0.94			
τ	0.4	beta(12, 18)	0.3992	0.1733	0.97			

Table B.16: Naive Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0421	0.1853	0.21	2.0431	0.1855	0.21
β_1	0.8	N(0, 10)	0.3627	0.2341	0.00	0.3637	0.2342	0.00
β_2	-0.2	N(0, 10)	-0.2003	0.0706	0.90	-0.2004	0.0707	0.90
η	0.9							
θ	0.5							
τ	0.2							

Table B.17: Corrected Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.1912	0.2614	0.96	1.8992	0.1919	0.94
β_1	0.8	N(0, 10)	0.7602	0.3377	0.93	0.7905	0.2070	0.96
β_2	-0.2	N(0, 10)	-0.2014	0.0859	0.95	-0.1987	0.0704	0.96
η	0.9	beta(45, 5)	0.8999	0.0867	0.97			
θ	0.5	beta(25, 25)	0.4902	0.2244	1.00			
τ	0.2	beta(10, 40)	0.2055	0.1458	0.99			

Table B.18: Corrected Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9156	1.8970	0.98	0.3175	0.1929	0.94
β_1	0.8	N(0, 10)	0.7338	0.7995	0.99	0.4521	0.2077	0.98
β_2	-0.2	N(0, 10)	-0.2003	-0.2007	0.95	0.0874	0.0709	0.92
η	0.9	beta(27, 3)	0.8961		0.95	0.1053		
θ	0.5	beta(15, 15)	0.4875		1.00	0.2751		
τ	0.2	beta(6, 24)	0.2175		0.98	0.1904		

Table B.19: Naive Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.2829	0.2262	0.00	2.2849	0.2263	0.00
β_1	0.8	N(0, 10)	0.3803	0.2124	0.00	0.3789	0.2124	0.00
β_2	-0.2	N(0, 10)	-0.2037	0.0556	0.88	-0.2038	0.0556	0.89
η	0.7							
θ	0.9							
τ	0.8							

Table B.20: Corrected Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8914	0.4179	0.97	1.8973	0.2747	0.96
β_1	0.8	N(0, 10)	0.8071	0.3891	0.97	0.8003	0.2615	0.94
β_2	-0.2	N(0, 10)	-0.1991	0.0616	0.96	-0.1978	0.0556	0.95
η	0.7	beta(35, 15)	0.6996	0.2091	1.00			
θ	0.9	beta(45, 5)	0.8998	0.0854	0.97			
τ	0.8	beta(40, 10)	0.8016	0.1232	1.00			

Table B.21: Corrected Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9014	0.4131	0.94	1.9056	0.2738	0.95
β_1	0.8	N(0, 10)	0.8011	0.3841	0.95	0.7965	0.2607	0.99
β_2	-0.2	N(0, 10)	-0.2028	0.6168	0.98	-0.2022	0.0557	0.94
η	0.7	beta(21, 9)	-0.7066	0.2448	1.00			
θ	0.9	beta(27, 3)	0.9001	0.0916	0.97			
τ	0.8	beta(24, 6)	0.7967	0.1357	0.98			

Table B.22: Naive Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0130	0.2038	0.41	2.0144	0.2038	0.41
β_1	0.8	N(0, 10)	0.4816	0.1903	0.00	0.4810	0.1903	0.00
β_2	-0.2	N(0, 10)	-0.2002	0.0639	0.88	-2.003	0.0639	0.88
η	0.7							
θ	0.9							
τ	0.4							

Table B.23: Corrected Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8989	0.2527	0.94	1.8986	0.1963	0.96
β_1	0.8	N(0, 10)	0.7941	0.2282	0.94	0.7986	0.1847	0.92
β_2	-0.2	N(0, 10)	-0.1990	0.0778	0.92	-0.1993	0.0642	0.95
η	0.7	beta(35, 15)	0.7003	0.1456	0.99			
θ	0.9	beta(45, 5)	0.9018	0.1172	1.00			
η	0.4	beta(20, 30)	0.4002	0.1463	0.99			

Table B.24: Corrected Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8535	0.2584	0.89	1.8876	0.1960	0.91
β_1	0.8	N(0, 10)	0.8120	0.2189	0.97	0.8071	0.1849	0.95
β_2	-0.2	N(0, 10)	-0.1994	0.0785	0.93	-0.1977	0.0639	0.93
η	0.7	beta(21, 9)	0.7463	0.1648	0.85			
θ	0.9	beta(27, 3)	0.8381	0.1411	0.59			
τ	0.4	beta(12, 18)	0.4426	0.1669	0.89			

Table B.25: Naive Model: $\eta = 0.7\ \theta = 0.9\ \tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9438	0.2023	0.84	1.9451	0.2025	0.83
β_1	0.8	N(0, 10)	0.3819	0.1979	0.00	0.3818	0.1979	0.00
β_2	-0.2	N(0, 10)	-0.2013	0.0703	0.88	-0.2013	0.0703	0.88
η	0.7							
θ	0.9							
τ	0.2							

Table B.26: Corrected Model: $\eta = 0.7\ \theta = 0.9\ \tau = 0.2\ EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9018	0.2337	0.93	1.9056	0.1914	0.94
β_1	0.8	N(0, 10)	0.7875	0.2675	0.92	0.8002	0.2061	0.96
β_2	-0.2	N(0, 10)	-0.2014	0.0833	0.92	-0.2022	0.0704	0.95
η	0.7	beta(35, 15)	0.7051	0.1252	0.97			
θ	0.9	beta(45, 5)	0.9029	0.1377	1.00			
τ	0.2	beta(10, 40)	0.2091	0.1271	0.99			

Table B.27: Corrected Model: $\eta = 0.7\ \theta = 0.9\ \tau = 0.2\ EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9132	0.2664	0.91	1.9057	0.1903	0.96
β_1	0.8	N(0, 10)	0.7651	0.3502	0.93	0.8024	0.2072	0.96
β_2	-0.2	N(0, 10)	-0.2013	0.0837	0.95	-0.2013	0.0702	0.98
η	0.7	beta(21, 9)	0.6977	0.1451	0.98			
θ	0.9	beta(27, 3)	0.8857	0.1924	1.00			
τ	0.2	beta(6, 24)	0.2104	0.1585	0.98			

Table B.28: Naive Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.3188	0.2414	0.02	2.3212	0.2411	0.02
β_1	0.8	N(0, 10)	0.3112	0.2274	0.00	0.3095	0.2272	0.00
β_2	-0.2	N(0, 10)	-0.2004	0.0558	0.95	-0.2005	0.0558	0.95
η	0.5							
θ	0.9							
τ	0.8							

Table B.29: Corrected Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8628	0.4688	0.94	1.8968	0.2743	0.93
β_1	0.8	N(0, 10)	0.8276	0.4309	0.96	0.7978	0.2606	0.94
β_2	-0.2	N(0, 10)	-0.1994	0.0633	0.95	-0.1996	0.0559	0.90
η	0.5	beta(25, 25)	0.5089	0.2192	1.00			
θ	0.9	beta(45, 5)	0.8946	0.0841	0.99			
τ	0.8	beta(40, 10)	0.8018	0.1385	0.99			

Table B.30: Corrected Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8981	0.4649	0.96	1.8997	0.2736	0.94
β_1	0.8	N(0, 10)	0.7981	0.4232	0.99	0.7989	0.2613	0.97
β_2	-0.2	N(0, 10)	-0.1993	0.0635	0.94	-0.2004	0.0558	0.93
η	0.5	beta(15, 15)	0.5094	0.2556	1.00			
θ	0.9	beta(27, 3)	0.9029	0.0876	0.99			
τ	0.8	beta(24, 6)	0.7954	0.1577	0.99			

Table B.31: Naive Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.0357	0.2216	0.34	2.0375	0.2219	0.34
β_1	0.8	N(0, 10)	0.3745	0.2054	0.00	0.3736	0.2057	0.00
β_2	-0.2	N(0, 10)	-0.2001	0.0645	0.85	-0.2002	0.0645	0.85
η	0.5							
θ	0.9							
τ	0.4							

Table B.32: Corrected Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8993	0.2629	0.96	1.8965	0.1965	0.96
β_1	0.8	N(0, 10)	0.7972	0.2166	0.95	0.7972	0.1846	0.97
β_2	-0.2	N(0, 10)	-0.1982	0.0818	0.97	-0.1973	0.0642	0.98
η	0.5	beta(25, 25)	0.5047	0.1475	0.96			
θ	0.9	beta(45, 5)	0.8972	0.1152	0.98			
τ	0.4	beta(20, 30)	0.3958	0.1631	1.00			

Table B.33: Corrected Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8978	0.2637	0.97	1.8950	0.1963	0.93
β_1	0.8	N(0, 10)	0.8081	0.2163	0.96	0.8031	0.1844	0.94
β_2	-0.2	N(0, 10)	-0.2012	0.0821	0.96	-0.1988	0.0644	0.96
η	0.5	beta(15, 15)	0.4988	0.1579	0.96			
θ	0.9	beta(27, 3)	0.9024	0.1232	1.00			
τ	0.4	beta(12, 18)	0.3987	0.1755	0.98			

Table B.34: Naive Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9447	0.2210	0.87	1.9463	0.2218	0.88
β_1	0.8	N(0, 10)	0.2689	0.2069	0.00	0.2683	0.2070	0.00
β_2	-0.2	N(0, 10)	-0.1974	0.0702	0.89	-0.1974	0.0703	0.89
η	0.5							
θ	0.9							
τ	0.2							

Table B.35: Corrected Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9115	0.3079	0.94	1.8985	0.1921	0.95
β_1	0.8	N(0, 10)	0.7384	0.4679	0.96	0.8022	0.2084	0.92
β_2	-0.2	N(0, 10)	-0.2017	0.0879	0.89	-0.2036	0.0707	0.91
η	0.5	beta(25, 25)	0.5007	0.1382	0.94			
θ	0.9	beta(45, 5)	0.8891	0.1651	1.00			
τ	0.2	beta(10, 40)	0.2103	0.1653	1.00			

Table B.36: Corrected Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9209	0.3219	0.96	1.8951	0.1922	0.95
β_1	0.8	N(0, 10)	0.7287	0.4738	0.95	0.8038	0.2072	0.99
β_2	-0.2	N(0, 10)	-0.2002	0.0872	0.97	-0.1992	0.0705	0.96
η	0.5	beta(15, 15)	0.4968	0.1549	0.97			
θ	0.9	beta(27, 3)	0.8857	0.1995	0.99			
τ	0.2	beta(6, 24)	0.2183	0.1966	0.98			

Table B.37: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8968	0.2881	0.98	1.9005	0.1949	0.95
β_1	0.8	N(0, 10)	0.8032	0.2786	1.00	0.8000	0.2058	0.96
η	0.9	beta(27, 3)	0.9054	0.1509	1.00			
θ	0.7	beta(21, 9)	0.7031	0.1195	0.95			
τ	0.8	beta(24, 6)	0.7963	0.1151	0.97			
σ	0.1							

Table B.38: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8928	0.2942	0.94	1.9021	0.1959	0.94
β_1	0.8	N(0, 10)	0.8049	0.2841	0.95	0.7960	0.2067	0.97
η	0.9	beta(27, 3)	0.9075	0.1483	1.00			
θ	0.7	beta(21, 9)	0.7018	0.1200	0.99			
τ	0.8	beta(24, 6)	0.7991	0.1148	0.97			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00	1.5946		

Table B.39: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8986	0.1431	0.95	1.9024	0.1131	0.97
β_1	0.8	N(0, 10)	0.8007	0.1592	0.96	0.7987	0.1462	0.96
η	0.9	beta(27, 3)	0.9012	0.0968	0.98			
θ	0.7	beta(21, 9)	0.7010	0.1661	0.98			
τ	0.4	beta(12, 18)	0.4046	0.1322	0.99			
σ	0.1							

Table B.40: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8980	0.1402	0.94	1.8982	0.1132	0.96
β_1	0.8	N(0, 10)	0.7960	0.1594	0.95	0.7980	0.1469	0.96
η	0.9	beta(27, 3)	0.9032	0.0953	0.95			
θ	0.7	beta(21, 9)	0.7049	0.1670	0.98			
τ	0.4	beta(12, 18)	0.4001	0.1318	0.95			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00	1.8562		

Table B.41: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9312	0.2093	0.97	1.8996	0.0979	0.98
β_1	0.8	N(0, 10)	0.7320	0.3872	0.98	0.8006	0.1639	0.94
η	0.9	beta(27, 3)	0.8896	0.1237	0.98			
θ	0.7	beta(21, 9)	0.6796	0.2713	1.00			
τ	0.2	beta(6, 24)	0.2212	0.1697	0.99			
σ	0.1							

Table B.42: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9287	0.2120	0.96	1.8982	0.0982	0.93
β_1	0.8	N(0, 10)	0.7362	0.3888	0.91	0.8051	0.1642	0.96
η	0.9	beta(27, 3)	0.8888	0.1248	0.99			
θ	0.7	beta(21, 9)	0.6754	0.2721	1.00			
τ	0.2	beta(6, 24)	0.2245	0.1730	0.97			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00	2.0218		

Table B.43: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8989	0.3140	0.91	1.8988	0.1960	0.92
β_1	0.8	N(0, 10)	0.8024	0.2972	0.91	0.8027	0.2068	0.96
η	0.9	beta(27, 3)	0.9010	0.1487	1.00			
θ	0.5	beta(15, 15)	0.4992	0.1272	0.99			
τ	0.8	beta(24, 6)	0.7964	0.1260	1.00			
σ	0.1							

Table B.44: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8846	0.3138	0.94	1.8902	0.1983	0.97
β_1	0.8	N(0, 10)	0.8132	0.2982	0.93	0.8075	0.2090	0.96
η	0.9	beta(27, 3)	0.9043	0.1476	0.99			
θ	0.5	beta(15, 15)	0.5018	0.1262	0.96			
τ	0.8	beta(24, 6)	0.7998	0.1226	1.00			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00	1.6062		

Table B.45: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8948	0.1510	0.96	1.8967	0.1133	0.97
β_1	0.8	N(0, 10)	0.8047	0.1603	0.95	0.8029	0.1465	0.95
η	0.9	beta(27, 3)	0.9052	0.0918	0.96			
θ	0.5	beta(15, 15)	0.4951	0.1754	0.96			
τ	0.4	beta(12, 18)	0.4016	0.1402	0.99			
σ	0.1							

Table B.46: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9004	0.1522	0.98	1.9015	0.1132	0.97
β_1	0.8	N(0, 10)	0.7953	0.1607	0.98	0.7954	0.1466	0.99
η	0.9	beta(27, 3)	0.9026	0.0935	1.00			
θ	0.5	beta(15, 15)	0.5017	0.1763	0.96			
τ	0.4	beta(12, 18)	0.4029	0.1418	0.93			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00		1.8511	

Table B.47: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9327	0.2358	0.94	1.8961	0.0981	0.96
β_1	0.8	N(0, 10)	0.7231	0.4347	0.96	0.8042	0.1643	0.98
η	0.9	beta(27, 3)	0.8909	0.1156	0.99			
θ	0.5	beta(15, 15)	0.4774	0.2700	0.98			
τ	0.2	beta(6, 24)	0.2266	0.1919	0.96			
σ	0.1							

Table B.48: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9561	0.2996	0.98	1.8959	0.0981	0.97
β_1	0.8	N(0, 10)	0.6630	0.5710	0.97	0.7998	0.1654	0.98
η	0.9	beta(27, 3)	0.8854	0.1366	0.99			
θ	0.5	beta(15, 15)	0.4673	0.2945	0.98			
τ	0.2	beta(6, 24)	0.2414	0.2354	1.00			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00		2.0191	

Table B.49: Fixed Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8832	0.2865	0.96	1.8970	0.1956	0.93
β_1	0.8	N(0, 10)	0.8130	0.2783	0.96	0.8015	0.2064	0.90
η	0.7	beta(21, 9)	0.7065	0.2224	1.00			
θ	0.9	beta(27, 3)	0.8987	0.0813	0.95			
τ	0.8	beta(24, 6)	0.8012	0.1125	0.98			
σ	0.1							

Table B.50: Random Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9034	0.2846	0.94	1.9080	0.1956	0.96
β_1	0.8	N(0, 10)	0.7944	0.2768	0.93	0.7914	0.2064	0.98
η	0.7	beta(21, 9)	0.7068	0.2232	0.99			
θ	0.9	beta(27, 3)	0.8992	0.0815	0.95			
τ	0.8	beta(24, 6)	0.8002	0.1137	0.95			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00	1.5953		

Table B.51: Fixed Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8970	0.1425	0.96	1.9005	0.1131	0.96
β_1	0.8	N(0, 10)	0.7956	0.4595	0.95	0.7957	0.1465	0.93
η	0.7	beta(21, 9)	0.6997	0.1411	0.99			
θ	0.9	beta(27, 3)	0.9017	0.1130	0.97			
τ	0.4	beta(12, 18)	0.4045	0.1334	0.96			
σ	0.1							

Table B.52: Random Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8918	0.1416	0.89	1.8939	0.1138	0.95
β_1	0.8	N(0, 10)	0.8081	0.1587	0.91	0.8085	0.1468	0.96
η	0.7	beta(21, 9)	0.7023	0.1398	0.99			
θ	0.9	beta(27, 3)	0.9031	0.1113	0.99			
τ	0.4	beta(12, 18)	0.4052	0.1315	0.97			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00	1.8480		

Table B.53: Fixed Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9479	0.2571	0.94	1.8994	0.0980	0.95
β_1	0.8	N(0, 10)	0.6918	0.4884	0.98	0.7987	0.1641	0.95
η	0.7	beta(21, 9)	0.6758	0.1796	0.97			
θ	0.9	beta(27, 3)	0.8643	0.2381	1.00			
τ	0.2	beta(6, 24)	0.2328	0.1992	0.96			
σ	0.1							

Table B.54: Random Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9301	0.2089	0.96	1.8978	0.0981	0.94
β_1	0.8	N(0, 10)	0.7214	0.3897	0.97	0.7995	0.1651	0.96
η	0.7	beta(21, 9)	0.6924	0.1597	0.96			
θ	0.9	beta(27, 3)	0.8812	0.2043	1.00			
τ	0.2	beta(6, 24)	0.2231	0.1703	0.98			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00	2.0142		

Table B.55: Fixed Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8820	0.3135	0.95	1.8965	0.1969	0.95
β_1	0.8	N(0, 10)	0.8174	0.2981	0.98	0.8048	0.2076	0.96
η	0.5	beta(15, 15)	0.4972	0.2332	0.99			
θ	0.9	beta(27, 3)	0.8982	0.1147	0.94			
τ	0.8	beta(24, 6)	0.8018	0.1222	0.97			
σ	0.1							

Table B.56: Random Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9030	0.3112	0.95	1.9080	0.1956	0.96
β_1	0.8	N(0, 10)	0.7950	0.2959	0.96	079614	0.2064	0.98
η	0.5	beta(15, 15)	0.5035	0.2344	1.00			
θ	0.9	beta(27, 3)	0.8988	0.0799	0.95			
τ	0.8	beta(24, 6)	0.7995	0.1245	0.96			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00	1.5953		

Table B.57: Fixed Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9030	0.1483	0.97	1.9004	0.1125	0.97
β_1	0.8	N(0, 10)	0.8003	0.1593	0.98	0.8002	0.1464	0.96
η	0.5	beta(15, 15)	0.4945	0.1452	0.96			
θ	0.9	beta(27, 3)	0.9029	0.1112	0.97			
τ	0.4	beta(12, 18)	0.3921	0.1409	0.9			
σ	0.1							

Table B.58: Random Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8913	0.1504	0.95	1.8939	0.1138	0.95
β_1	0.8	N(0, 10)	0.8087	0.7596	0.92	0.8085	0.1468	0.96
η	0.5	beta(15, 15)	0.5034	0.1469	0.97			
θ	0.9	beta(27, 3)	0.9040	0.1083	0.97			
τ	0.4	beta(12, 18)	0.4054	0.1398	0.96			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00	1.8480		

Table B.59: Fixed Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9663	0.3161	0.95	1.9007	0.0980	0.90
β_1	0.8	N(0, 10)	0.6429	0.6132	0.96	0.7930	0.1639	0.93
η	0.5	beta(15, 15)	0.4748	0.1859	0.98			
θ	0.9	beta(27, 3)	0.8714	0.2296	1.00			
τ	0.2	beta(6, 24)	0.2480	0.2454	0.98			
σ	0.1							

Table B.60: Random Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 30$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9722	0.3316	0.95	1.9039	0.0978	0.96
β_1	0.8	N(0, 10)	0.6240	0.6612	0.95	0.7909	0.1647	0.96
η	0.5	beta(15, 15)	0.4813	0.1876	0.97			
θ	0.9	beta(27, 3)	0.8631	0.2414	0.98			
τ	0.2	beta(6, 24)	0.2510	0.2568	0.94			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00	2.0106		

Table B.61: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8935	0.2900	0.97	1.9031	0.1954	0.93
β_1	0.8	N(0, 10)	0.8062	0.2811	0.95	0.7974	0.2062	0.91
η	0.9	beta(45, 5)	0.8978	0.1340	1.00			
θ	0.7	beta(35, 15)	0.6944	0.1150	0.98			
τ	0.8	beta(40, 10)	0.7990	0.1091	0.97			
σ	0.1							

Table B.62: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9092	0.2831	0.94	1.9077	0.1950	0.94
β_1	0.8	N(0, 10)	0.7916	0.2746	0.92	0.7921	0.2059	0.96
η	0.9	beta(45, 5)	0.9010	0.1323	1.00			
θ	0.7	beta(35, 15)	0.7052	0.1148	0.98			
τ	0.8	beta(40, 10)	0.7950	0.1100	1.00			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.63: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8960	0.1496	0.92	1.8974	0.1131	0.93
β_1	0.8	N(0, 10)	0.7967	0.1746	0.97	0.8009	0.1466	0.97
η	0.9	beta(45, 5)	0.9009	0.0935	1.00			
θ	0.7	beta(35, 15)	0.6914	0.1586	0.98			
τ	0.4	beta(20, 30)	0.4034	0.1274	1.00			
σ	0.1							

Table B.64: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9007	0.1409	0.95	1.9016	0.1133	0.96
β_1	0.8	N(0, 10)	0.8008	0.1585	0.95	0.8013	0.1465	0.96
η	0.9	beta(45, 5)	0.9042	0.0892	0.99			
θ	0.7	beta(35, 15)	0.7015	0.1536	0.99			
τ	0.4	beta(20, 30)	0.4027	0.1251	0.96			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.65: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9126	0.1663	0.99	1.8996	0.0979	0.96
β_1	0.8	N(0, 10)	0.7595	0.2986	0.96	0.8004	0.1645	0.95
η	0.9	beta(45, 5)	0.8953	0.1677	0.97			
θ	0.7	beta(35, 15)	0.6887	0.2187	1.00			
τ	0.2	beta(10, 40)	0.2116	0.1329	1.00			
σ	0.1							

Table B.66: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9403	0.2250	0.95	1.9026	0.0978	0.97
β_1	0.8	N(0, 10)	0.7078	0.4351	0.91	0.7979	0.1648	0.92
η	0.9	beta(45, 5)	0.8905	0.1203	0.96			
θ	0.7	beta(35, 15)	0.6761	0.2601	1.00			
τ	0.2	beta(10, 40)	0.2217	0.1725	1.00			
σ	0.1	unif(0.001, 5)	0.0101	0.0016	0.00			

Table B.67: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8918	0.3084	0.97	1.8958	0.1970	0.95
β_1	0.8	N(0, 10)	0.8095	0.2941	0.97	0.8056	0.2077	0.98
η	0.9	beta(45, 5)	0.9027	0.1278	1.00			
θ	0.5	beta(15, 15)	0.4997	0.1214	0.97			
τ	0.8	beta(40, 10)	0.7984	0.1161	0.99			
σ	0.1							

Table B.68: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8875	0.3169	0.96	1.8931	0.1986	0.95
β_1	0.8	N(0, 10)	0.8103	0.3019	0.94	0.8057	0.2092	0.93
η	0.9	beta(45, 5)	0.9020	0.1292	1.00			
θ	0.5	beta(15, 15)	0.5002	0.1212	0.98			
τ	0.8	beta(40, 10)	0.8022	0.1162	0.98			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.69: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9026	0.1497	0.96	1.9029	0.1133	0.94
β_1	0.8	N(0, 10)	0.7979	0.1605	0.95	0.7978	0.1463	0.97
η	0.9	beta(45, 5)	0.9005	0.0873	0.98			
θ	0.5	beta(15, 15)	0.4949	0.1632	0.94			
τ	0.4	beta(20, 30)	0.4004	0.1333	0.97			
σ	0.1							

Table B.70: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8925	0.1512	0.93	1.8954	0.1139	0.95
β_1	0.8	N(0, 10)	0.8073	0.1605	0.92	0.8027	0.1469	0.95
η	0.9	beta(45, 5)	0.8982	0.0886	0.96			
θ	0.5	beta(15, 15)	0.4969	0.1627	1.00			
τ	0.4	beta(20, 30)	0.4040	0.1336	0.95			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.71: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9112	0.1835	0.96	1.8956	0.0979	0.96
β_1	0.8	N(0, 10)	0.5171	0.3380	0.96	0.8066	0.1652	0.97
η	0.9	beta(45, 5)	0.8946	0.0933	0.98			
θ	0.5	beta(15, 15)	0.4929	0.2273	1.00			
τ	0.2	beta(10, 40)	0.2146	0.1468	0.97			
σ	0.1							

Table B.72: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9437	0.2687	0.96	1.8986	0.0981	0.99
β_1	0.8	N(0, 10)	0.6839	0.5379	0.96	0.8032	0.1654	0.96
η	0.9	beta(45, 5)	0.8934	0.1106	0.98			
θ	0.5	beta(15, 15)	0.4972	0.2555	1.00			
τ	0.2	beta(10, 40)	0.2317	0.1978	0.97			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00			

Table B.73: Fixed Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8936	0.2818	0.91	1.8970	0.2008	0.95
β_1	0.8	N(0, 10)	0.8087	0.2753	0.93	0.8063	0.2112	0.94
η	0.7	beta(35, 15)	0.7029	0.1954	1.00			
θ	0.9	beta(45, 5)	0.9017	0.7613	0.98			
τ	0.8	beta(40, 10)	0.8038	0.1055	0.96			
σ	0.1							

Table B.74: Random Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8936	0.2735	0.96	1.8879	0.1975	0.93
β_1	0.8	N(0, 10)	0.8069	0.2671	0.96	0.8121	0.2082	0.94
η	0.7	beta(35, 15)	0.6963	0.1942	1.00			
θ	0.9	beta(45, 5)	0.8989	0.0773	0.95			
τ	0.8	beta(40, 10)	0.7964	0.1070	0.98			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.75: Fixed Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8970	0.1425	0.96	1.9005	0.1131	0.96
β_1	0.8	N(0, 10)	0.7956	0.1595	0.95	0.7957	0.1465	0.93
η	0.7	beta(35, 15)	0.6997	0.1411	0.99			
θ	0.9	beta(45, 5)	0.9017	0.1130	0.97			
τ	0.4	beta(20, 30)	0.4045	0.1334	0.96			
σ	0.1							

Table B.76: Random Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8918	0.1416	0.89	1.8939	0.1138	0.95
β_1	0.8	N(0, 10)	0.8081	0.1587	0.91	0.8085	0.1468	0.96
η	0.7	beta(35, 15)	0.7023	0.1398	0.99			
θ	0.9	beta(45, 5)	0.9031	0.1113	0.99			
τ	0.4	beta(20, 30)	0.4052	0.1315	0.97			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00	1.8480		

Table B.77: Fixed Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9252	0.1966	0.89	1.8987	0.0979	0.92
β_1	0.8	N(0, 10)	0.7326	0.3842	0.98	0.7978	0.1641	0.95
η	0.7	beta(35, 15)	0.6890	0.1450	0.98			
θ	0.9	beta(45, 5)	0.8808	0.1847	1.00			
τ	0.2	beta(10, 40)	0.2166	0.1539	0.98			
σ	0.1							

Table B.78: Random Effects Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9296	0.2033	0.97	1.8997	0.0979	0.96
β_1	0.8	N(0, 10)	0.7305	0.4014	0.94	0.8052	0.1648	0.97
η	0.7	beta(35, 15)	0.6873	0.1429	0.95			
θ	0.9	beta(45, 5)	0.8782	0.1882	1.00			
τ	0.2	beta(10, 40)	0.2153	0.1582	0.99			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.79: Fixed Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8954	0.3033	0.92	1.8943	0.1970	0.91
β_1	0.8	N(0, 10)	0.8062	0.2890	0.91	0.8079	0.2077	0.92
η	0.5	beta(15, 15)	0.5058	0.2048	1.00			
θ	0.9	beta(45, 5)	0.8974	0.0769	0.96			
τ	0.8	beta(40, 10)	0.7976	0.1226	0.99			
σ	0.1							

Table B.80: Random Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.8$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8936	0.3016	0.99	1.8969	0.1950	0.97
β_1	0.8	N(0, 10)	0.8069	0.2878	0.99	0.8043	0.2058	0.97
η	0.5	beta(15, 15)	0.5003	0.2044	1.00			
θ	0.9	beta(45, 5)	0.9008	0.0755	0.99			
τ	0.8	beta(40, 10)	0.7961	0.1160	0.97			
σ	0.1	unif(0.001, 5)	0.0101	0.0016	0.00			

Table B.81: Fixed Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8869	0.1500	0.95	1.8895	0.1136	0.94
β_1	0.8	N(0, 10)	0.8094	0.1602	0.96	0.8077	0.1469	0.97
η	0.5	beta(15, 15)	0.5048	0.1384	0.98			
θ	0.9	beta(45, 5)	0.9018	0.1016	0.99			
τ	0.4	beta(20, 30)	0.4010	0.1395	1.00			
σ	0.1							

Table B.82: Random Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.4$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9052	0.1498	0.98	1.9041	0.1137	0.96
β_1	0.8	N(0, 10)	0.7862	0.1605	0.93	0.7869	0.1466	0.92
η	0.5	beta(15, 15)	0.4997	0.1396	0.99			
θ	0.9	beta(45, 5)	0.9030	0.1019	1.00			
τ	0.4	beta(20, 30)	0.4055	0.1355	0.99			
σ	0.1	unif(0.001, 5)	0.0099	0.0016	0.00			

Table B.83: Fixed Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9389	0.2436	0.93	1.9013	0.0978	0.98
β_1	0.8	N(0, 10)	0.6981	0.4785	0.92	0.8020	0.1638	0.95
η	0.5	beta(15, 15)	0.4900	0.1502	1.00			
θ	0.9	beta(45, 5)	0.8841	0.1799	1.00			
τ	0.2	beta(10, 40)	0.2288	0.1682	0.97			
σ	0.1							

Table B.84: Random Effects Model: $\eta = 0.5$ $\theta = 0.9$ $\tau = 0.2$ $J = 300$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9611	0.3294	0.94	1.8937	0.0984	0.94
β_1	0.8	N(0, 10)	0.6439	0.6612	0.91	0.8070	0.1643	0.91
η	0.5	beta(15, 15)	0.4838	0.1699	0.98			
θ	0.9	beta(45, 5)	0.8653	0.2207	1.00			
τ	0.2	beta(10, 40)	0.2399	0.2346	0.97			
σ	0.1	unif(0.001, 5)	0.0100	0.0016	0.00			

Table B.85: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8930	0.2725	0.97	1.9025	0.1948	0.93
β_1	0.8	N(0, 10)	0.8057	0.2798	0.95	0.7986	0.2057	0.91
η	0.9	beta(45, 5)	0.8992	0.1140	0.99			
θ	0.7	beta(35, 15)	0.6952	0.1098	0.98			
τ	0.8	beta(40, 10)	0.7998	0.1013	0.97			
σ	0.1							

Table B.86: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9054	0.2826	0.94	1.9070	0.1941	0.95
β_1	0.8	N(0, 10)	0.7932	0.2730	0.93	0.7918	0.2045	0.97
η	0.9	beta(45, 5)	0.9001	0.1314	1.00			
θ	0.7	beta(35, 15)	0.7057	0.1140	0.98			
τ	0.8	beta(40, 10)	0.7958	0.1100	0.99			
σ	0.1	unif(0.001, 5)	0.0100	0.0012	0.00			

Table B.87: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8961	0.1102	0.96	1.8998	0.0875	0.96
β_1	0.8	N(0, 10)	0.8037	0.1230	0.94	0.8009	0.1133	0.95
η	0.9	beta(45, 5)	0.8998	0.0752	0.97			
θ	0.7	beta(35, 15)	0.7009	0.1293	0.98			
τ	0.4	beta(20, 30)	0.4016	0.1018	0.95			
σ	0.1							

Table B.88: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.4$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8946	0.1094	1.00	1.8972	0.0877	1.00
β_1	0.8	N(0, 10)	0.8089	0.1219	1.00	0.8057	0.1134	1.00
η	0.9	beta(45, 5)	0.9016	0.0744	1.00			
θ	0.7	beta(35, 15)	0.7044	0.1288	1.00			
τ	0.4	beta(20, 30)	0.4003	0.1011	1.00			
σ	0.1	unif(0.001, 5)	0.0099	0.0012	0.00			

Table B.89: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9068	0.1037	0.98	1.9013	0.0757	0.98
β_1	0.8	N(0, 10)	0.7867	0.1756	0.98	0.7953	0.1270	0.97
η	0.9	beta(45, 5)	0.8980	0.0692	0.98			
θ	0.7	beta(35, 15)	0.6960	0.1784	0.99			
τ	0.2	beta(10, 40)	0.2027	0.0952	0.99			
σ	0.1							

Table B.90: Random Effects Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9059	0.1087	0.94	1.8995	0.0758	0.90
β_1	0.8	N(0, 10)	0.7909	0.1873	0.94	0.8051	0.1271	0.94
η	0.9	beta(45, 5)	0.8981	0.0706	0.95			
θ	0.7	beta(35, 15)	0.6972	0.1805	0.98			
τ	0.2	beta(10, 40)	0.2044	0.0987	0.96			
σ	0.1	unif(0.001, 5)	0.0099	0.0012	0.00			

Table B.91: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8900	0.3072	0.97	1.8962	0.1965	0.95
β_1	0.8	N(0, 10)	0.8084	0.2935	0.98	0.8050	0.2069	0.99
η	0.9	beta(45, 5)	0.9018	0.1262	1.00			
θ	0.5	beta(15, 15)	0.4999	0.1201	0.97			
τ	0.8	beta(40, 10)	0.7997	0.1150	1.00			
σ	0.1							

Table B.92: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.8$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8929	0.2429	0.98	1.8940	0.1520	0.96
β_1	0.8	N(0, 10)	0.8084	0.2295	0.94	0.8063	0.1603	1.00
η	0.9	beta(45, 5)	0.9026	0.1138	1.00			
θ	0.5	beta(15, 15)	0.4993	0.0985	0.96			
τ	0.8	beta(40, 10)	0.7962	0.0968	0.92			
σ	0.1	unif(0.001, 5)	0.0099	0.0012	0.00			

Table B.93: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9038	0.1156	0.95	1.9018	0.0872	0.97
β_1	0.8	N(0, 10)	0.7946	0.1234	0.91	0.7973	0.1132	0.90
η	0.9	beta(45, 5)	0.8995	0.0729	0.95			
θ	0.5	beta(15, 15)	0.5026	0.1373	0.99			
τ	0.4	beta(20, 30)	0.3971	0.1092	0.97			
σ	0.1							

Table B.94: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.4$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8989	0.1173	0.92	1.9006	0.0878	0.92
β_1	0.8	N(0, 10)	0.8043	0.1233	1.00	0.8033	0.1133	1.00
η	0.9	beta(45, 5)	0.9031	0.0721	1.00			
θ	0.5	beta(15, 15)	0.5056	0.1359	1.00			
τ	0.4	beta(20, 30)	0.4044	0.1089	1.00			
σ	0.1	unif(0.001, 5)	0.0100	0.0012	0.00			

Table B.95: Fixed Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9169	0.1438	0.99	1.8996	0.0759	0.98
β_1	0.8	N(0, 10)	0.7618	0.2534	0.99	0.7977	0.1268	0.93
η	0.9	beta(45, 5)	0.8954	0.0765	0.98			
θ	0.5	beta(15, 15)	0.4887	0.1970	1.00			
τ	0.2	beta(10, 40)	0.2124	0.1242	0.98			
σ	0.1							

Table B.96: Random Effects Model: $\eta = 0.9$ $\theta = 0.5$ $\tau = 0.2$ $J = 500$ $EPSS = 50$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9083	0.1213	0.95	1.9005	0.0757	0.97
β_1	0.8	N(0, 10)	0.7804	0.2072	0.99	0.8016	0.1275	0.98
η	0.9	beta(45, 5)	0.8967	0.0710	0.91			
θ	0.5	beta(15, 15)	0.5011	0.1898	0.98			
τ	0.2	beta(10, 40)	0.2060	0.1089	0.99			
σ	0.1	unif(0.001, 5)	0.0100	0.0012	0.00			

Table B.97: Basic Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	-1.5188	2.8307	0.00	1.7960	0.2082	0.56
β_1	0.8	N(0, 10)	4.1298	2.8076	0.00	0.8012	0.2194	0.90
η	0.9	beta(45, 5)	0.7345	0.1851	0.02			
θ	0.7	beta(35, 15)	0.6159	0.1113	0.19			
τ	0.8	beta(40, 10)	0.8608	0.0830	0.24			
π_0	0.1							

Table B.98: Zero-Inflated Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8940	0.3042	0.99	1.8926	0.2089	0.96
β_1	0.8	N(0, 10)	0.8051	0.2952	0.96	0.8068	0.2203	0.95
η	0.9	beta(45, 5)	0.8999	0.1339	1.00			
θ	0.7	beta(35, 15)	0.6961	0.1183	0.99			
τ	0.8	beta(40, 10)	0.7964	0.1125	0.99			
π_0	0.1	beta(5, 45)	0.1046	0.0636	0.93			

Table B.99: Basic Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $\pi_0 = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	-1.5188	2.8030	0.00	1.7967	0.1949	0.95
β_1	0.8	N(0, 10)	4.1298	2.8076	0.00	0.8012	0.2194	0.90
η	0.9	beta(45, 5)	0.7345	0.1851	0.02			
θ	0.7	beta(35, 15)	0.6159	0.1113	0.19			
τ	0.8	beta(40, 10)	0.8608	0.0830	0.24			
π_0	0.2							

Table B.100: Zero-Inflated Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.8$ $\pi_0 = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.8904	0.3406	0.92	1.8993	0.2022	0.59
β_1	0.8	N(0, 10)	0.8078	0.3315	0.96	0.8014	0.2378	0.98
η	0.9	beta(45, 5)	0.9056	0.1334	1.00			
θ	0.7	beta(35, 15)	0.6961	0.1183	0.97			
τ	0.8	beta(40, 10)	0.8077	0.1141	0.99			
π_0	0.2	beta(10, 40)	0.2001	0.0834	0.94			

Table B.101: Basic Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.6962	0.2173	0.13	1.7927	0.1035	0.01
β_1	0.8	N(0, 10)	0.6782	1.2365	0.29	0.7982	0.1726	0.87
η	0.9	beta(45, 5)	0.8895	0.0885	0.99			
θ	0.7	beta(35, 15)	0.6328	0.2100	0.77			
τ	0.2	beta(10, 40)	0.2330	0.1296	0.83			
π_0	0.1							

Table B.102: Zero-Inflated Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9142	0.1750	0.95	1.8944	0.1039	0.92
β_1	0.8	N(0, 10)	0.7547	0.3208	0.97	0.8074	0.1728	0.94
η	0.9	beta(45, 5)	0.8963	0.0975	0.97			
θ	0.7	beta(35, 15)	0.6846	0.2237	1.00			
τ	0.2	beta(10, 40)	0.2137	0.1355	0.97			
π_0	0.1	beta(5, 45)	0.0979	0.0621	0.96			

Table B.103: Basic Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $\pi_0 = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	-2.3693	3.2587	0.14	1.6731	0.1098	0.00
β_1	0.8	N(0, 10)	4.2545	4.3664	0.14	0.7918	0.1839	0.74
η	0.9	beta(45, 5)	0.8896	0.1237	0.98			
θ	0.7	beta(35, 15)	0.8414	0.1297	0.55			
τ	0.2	beta(10, 40)	0.3584	0.1541	0.09			
π_0	0.2							

Table B.104: Zero-Inflated Poisson Model: $\eta = 0.9$ $\theta = 0.7$ $\tau = 0.2$ $\pi_0 = 0.2$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9138	0.1720	0.95	1.8997	0.1096	0.95
β_1	0.8	N(0, 10)	0.7723	0.3181	0.95	0.8061	0.1843	0.95
η	0.9	beta(45, 5)	0.8956	0.0952	0.97			
θ	0.7	beta(35, 15)	0.6926	0.2242	1.00			
τ	0.2	beta(10, 40)	0.2058	0.1353	0.97			
π_0	0.2	beta(10, 40)	0.1987	0.0833	0.93			

Table B.105: Basic Poisson Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	-1.3699	2.4647	0.00	1.7854	0.2083	0.44
β_1	0.8	N(0, 10)	3.9811	2.4450	0.00	0.8061	0.2196	0.89
η	0.7	beta(35, 15)	0.5448	0.2060	0.12			
θ	0.9	beta(45, 5)	0.8197	0.0893	0.09			
τ	0.8	beta(40, 10)	0.8529	0.0823	0.32			
π_0	0.1							

Table B.106: Zero-Inflated Poisson Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.8$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.9936	0.5091	0.96	1.9000	0.2085	0.95
β_1	0.8	N(0, 10)	0.6610	0.6420	0.97	0.7999	0.2198	0.95
η	0.7	beta(35, 15)	0.6695	0.2894	1.00			
θ	0.9	beta(45, 5)	0.8839	0.1348	0.96			
τ	0.8	beta(40, 10)	0.7723	0.1842	0.99			
π_0	0.1	beta(5, 45)	0.1024	0.0631	1.00			

Table B.107: Basic Poisson Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	1.7024	0.3413	0.22	1.7929	0.1036	0.05
β_1	0.8	N(0, 10)	0.5450	1.8678	0.37	0.7993	0.1723	0.72
η	0.7	beta(35, 15)	0.6916	0.1365	0.98			
θ	0.9	beta(45, 5)	0.8468	0.1929	0.86			
τ	0.2	beta(10, 40)	0.2300	0.1461	0.91			
π_0	0.1							

Table B.108: Zero-Inflated Poisson Model: $\eta = 0.7$ $\theta = 0.9$ $\tau = 0.2$ $\pi_0 = 0.1$

	TRUE Value	Prior	Posterior Mean	Cred Set Width	Coverage	Par Estimate	Conf Int Width	Coverage
β_0	1.9	N(0, 10)	2.1762	0.6271	0.86	1.8966	0.1040	0.96
β_1	0.8	N(0, 10)	0.0989	1.3630	0.88	0.8059	0.1729	0.99
η	0.7	beta(35, 15)	0.6115	0.2867	0.90			
θ	0.9	beta(45, 5)	0.7010	0.4662	0.90			
τ	0.2	beta(10, 40)	0.3534	0.3996	0.88			
π_0	0.1	beta(5, 45)	0.1001	0.0627	0.93			

Figure B.1: Posterior densities and trace plots for β_0.

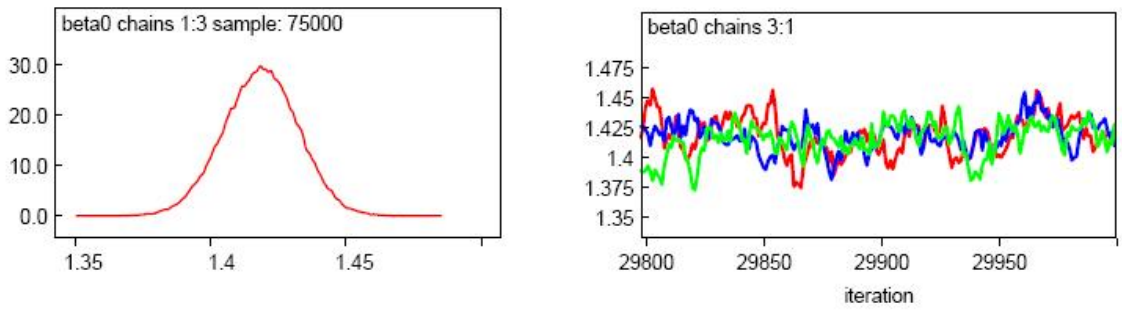


Figure B.2: Posterior densities and trace plots for β_1.

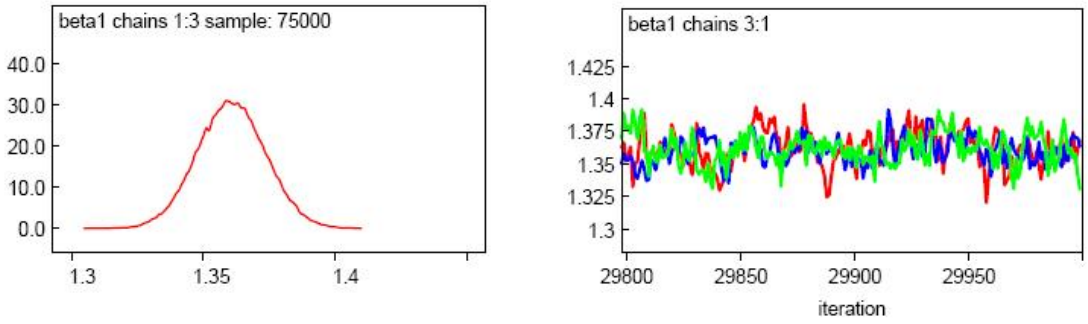


Figure B.3: Posterior densities and trace plots for η.

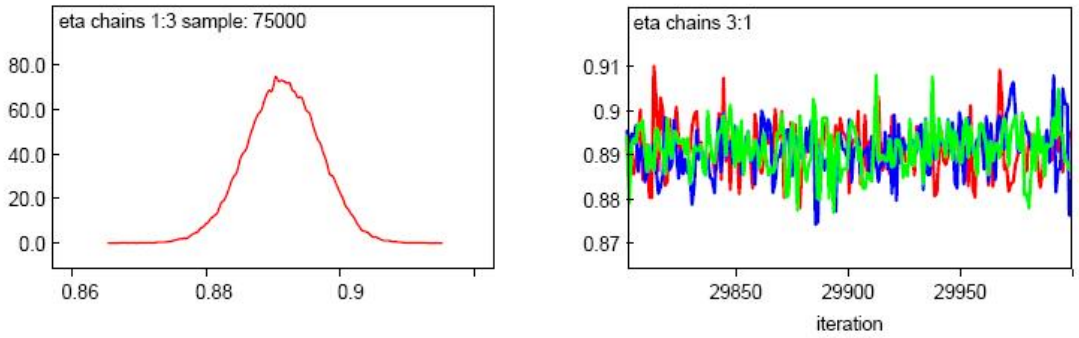


Figure B.4: Posterior densities and trace plots for θ.

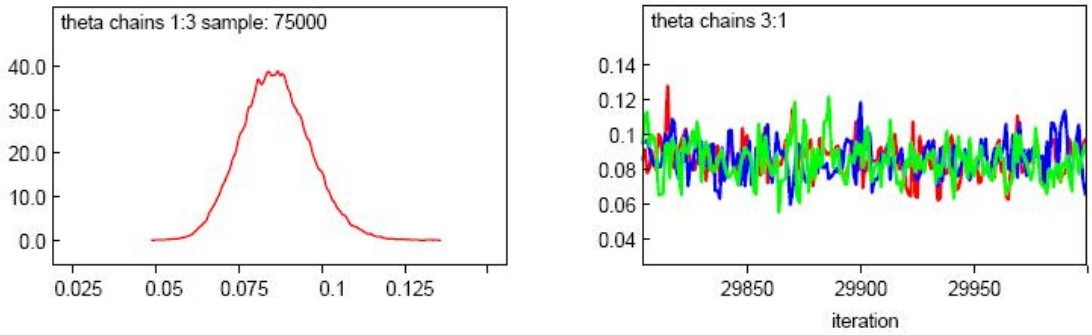


Figure B.5: Posterior densities and trace plots for τ.

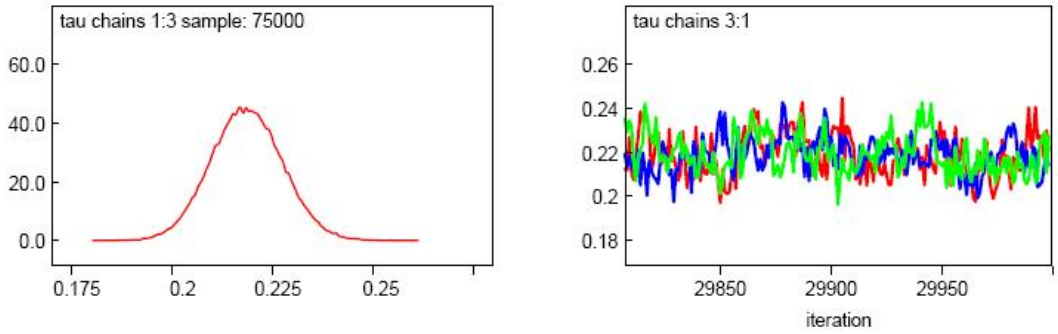
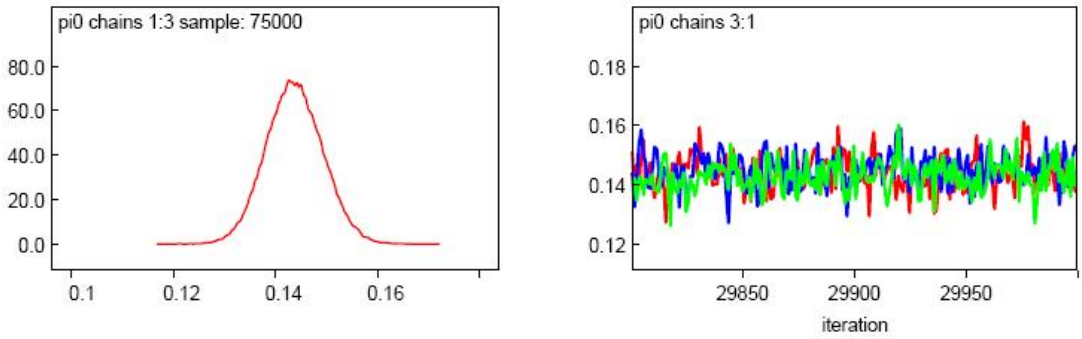


Figure B.6: Posterior densities and trace plots for π_0.



APPENDIX C

Chapter IV Tables

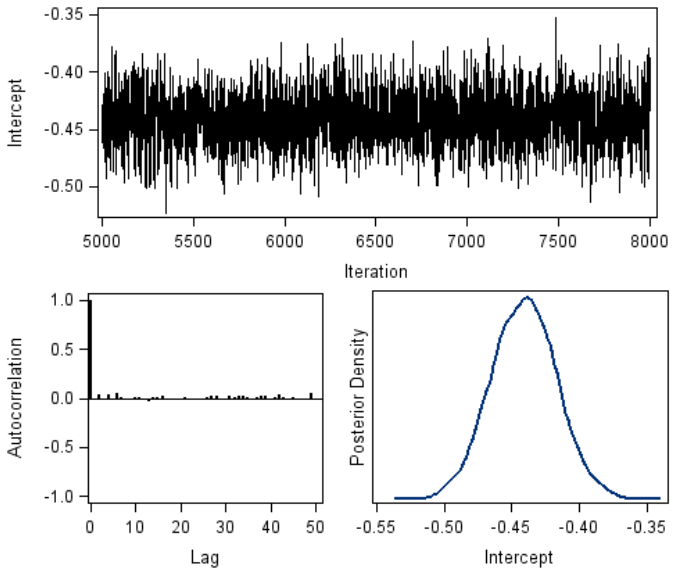


Figure C.1: Trace, Autocorrelation, and Posterior Density Plots for β_0 (Intercept)

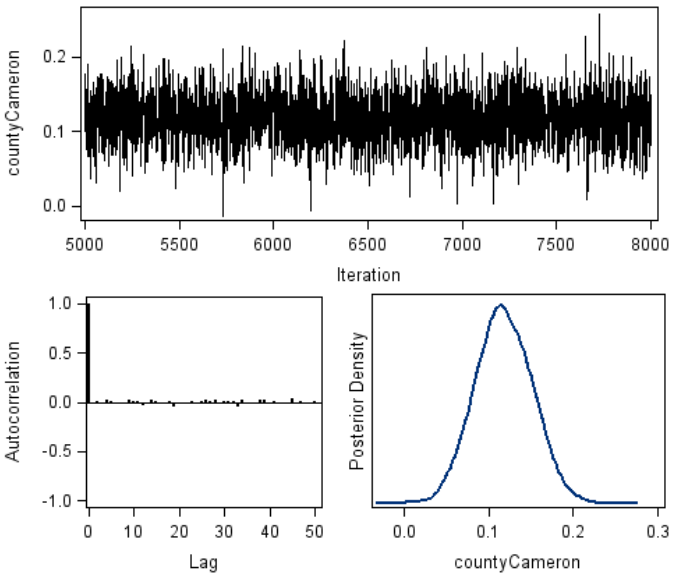


Figure C.2: Trace, Autocorrelation, and Posterior Density Plots for β_1 (Cameron)

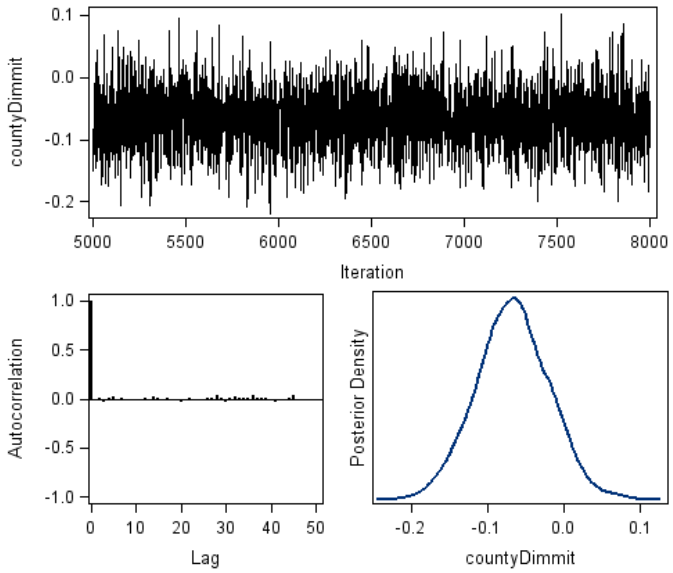


Figure C.3: Trace, Autocorrelation, and Posterior Density Plots for β_2 (Dimmit)

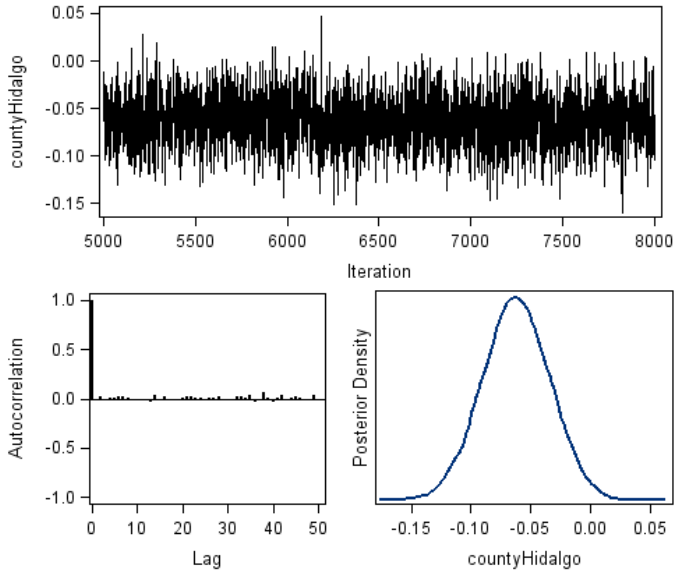


Figure C.4: Trace, Autocorrelation, and Posterior Density Plots for β_3 (Hidalgo)

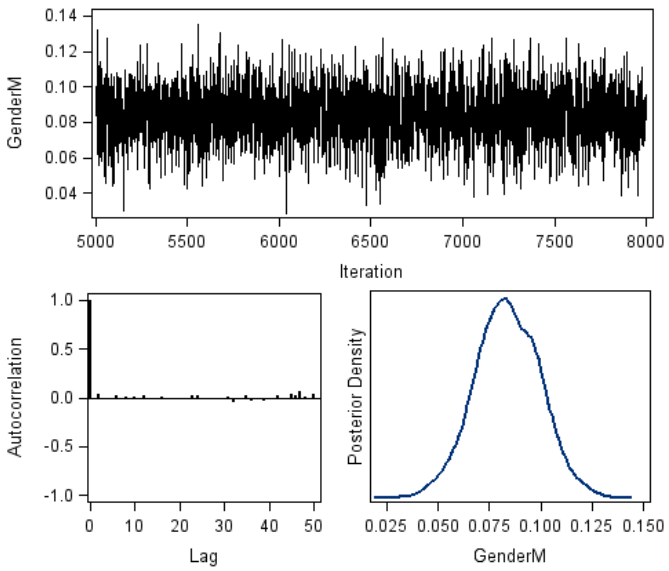


Figure C.5: Trace, Autocorrelation, and Posterior Density Plots for β_4 (Males)

Table C.1: Posterior Summaries For The 50th Percentile

Parameter	N	Mean	SD	Percentiles		
				25%	50%	75%
Intercept	3000	1.2136	0.0282	1.1942	1.2128	1.2322
Cameron	3000	0.0171	0.0395	−0.0101	0.0172	0.0439
Dimmit	3000	−0.0772	0.0545	−0.1137	−0.0773	−0.0402
Hidalgo	3000	−0.0874	0.0318	−0.1097	−0.0866	−0.0652
Male	3000	0.1746	0.0177	0.1629	0.1745	0.1862

Table C.2: Posterior Summaries For The 95th Percentile

Parameter	N	Mean	SD	Percentiles		
				25%	50%	75%
Intercept	3000	−1.4434	0.0300	−1.4636	−1.4429	−1.4225
Cameron	3000	0.1506	0.0408	0.1231	0.1514	0.1776
Dimmit	3000	−0.0418	0.0593	−0.0809	−0.0403	−0.00273
Hidalgo	3000	−0.00119	0.0351	−0.0247	−0.00134	0.0232
Male	3000	0.0288	0.0189	0.0167	0.0287	0.0412

Table C.3: Posterior Summaries For The 97th Percentile

Parameter	N	Mean	SD	Percentiles		
				25%	50%	75%
Intercept	3000	−1.7749	0.0350	−1.7989	−1.7746	−1.7512
Cameron	3000	0.2037	0.0456	0.1727	0.2032	0.2342
Dimmit	3000	−0.0455	0.0684	−0.0919	−0.0463	0.000747
Hidalgo	3000	0.0297	0.0402	0.00357	0.0300	0.0560
Male	3000	0.0285	0.0217	0.0136	0.0285	0.0434

BIBLIOGRAPHY

Agresti, A. (2002), *Catagorical Data Analysis*, Chichester: Wiley, 2nd ed.

Agresti, A., Booth, J. G., Hobert, J. P., and Caffo, B. (2000), "Random-Effects Modeling of Categorical Response Data," *Sociological Methodology 2000, Vol 30*, 30, 27–80.

Akaike, H. (1974), "New Look at Statistical-Model Identification," *Ieee Transactions On Automatic Control*, AC19, 716–723.

Albert, J. (1992), "A Bayesian-Analysis of a Poisson Random Effects Model for Home Run Hitters," *American Statistician*, 46, 246–253.

— (2007), *Bayesian Computation with R*, Springer.

Alisi, A., Manco, M., Vania, A., and Nobili, V. (2009), "Pediatric Nonalcoholic Fatty Liver Disease in 2009," *Journal Of Pediatrics*, 155, 469–474.

Anderson, S. E. and Whitaker, R. C. (2009), "Prevalence of Obesity Among US Preschool Children in Different Racial and Ethnic Groups," *Archives Of Pediatrics & Adolescent Medicine*, 163, 344–348.

Armstrong, B. G. (1998), "Effect of Measurement Error on Epidemiological Studies of Environmental and Occupational Exposures," *Occupational And Environmental Medicine*, 55, 651–656.

Barlow, S. E. (2007), "Expert Committee Recommendations Regarding the Prevention, Assessment, and Treatment of Child and Adolescent Overweight and Obesity: Summary Report," *Pediatrics*, 120, S164–S192.

Bernardinelli, L. and Montomoli, C. (1992), "Empirical Bayes Versus Fully Bayesian-Analysis of Geographical Variation in Disease Risk," *Statistics In Medicine*, 11, 983–1007.

Bohning, D. (1998), "Zero-Inflated Poisson Models and C.A.MAN: A Tutorial Collection of Evidence," *Biometrical Journal*, 40, 833–843.

Bohning, D., Dietz, E., Schlattmann, P., Mendonca, L., and Kirchner, U. (1999), "The Zero-Inflated Poisson Model and the Decayed, Missing and Filled Teeth Index in Dental Epidemiology," *Journal Of The Royal Statistical Society Series A-Statistics In Society*, 162, 195–209.

Breslow, N. and Day, N. (1987), *Statistical Methods in Cancer Research II: The Design and Analysis of Cohort Studies*, Lyon: IARC.

Breslow, N. E. and Clayton, D. G. (1993), "Approximate Inference in Generalized Linear Mixed Models," *Journal Of The American Statistical Association*, 88, 9–25.

Brooks, S. and Gelman, A. (1997), "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, 7, 434–455.

Carlin, B. and Louis, T. (2009), *Bayesian Methods for Data Analysis*, Boca Raton: Chapman & Hall / CRC.

Cheung, Y. (2002), "Zero-Inflated Models for Regression Analysis of Count Data: A Study of Growth and Development," *Statistics In Medicine*, 21, 1461–1469.

Christiansen, C. L. and Morris, C. N. (1997), "Hierarchical Poisson Regression Modeling," *Journal Of The American Statistical Association*, 92, 618–632.

Clayton, D. and Kaldor, J. (1987), "Empirical Bayes Estimates of Age-Standardized Relative Risks for Use in Disease Mapping," *Biometrics*, 43, 671–681.

Congdon, P. (2005), *Bayesian Models for Categorical Data*, Chichester: Wiley.

Deb, P. and Trivedi, P. K. (1997), "Demand for Medical Care by the Elderly: A Finite Mixture Approach," *Journal Of Applied Econometrics*, 12, 313–336.

Efron, B. and Thisted, R. (1976), "Estimating Number of Unseen Species - How Many Words Did Shakespeare Know," *Biometrika*, 63, 435–447.

Famoye, F. and Singh, K. (2006), "Zero-Inflated Generalized Poisson Regression Model with an Application to Domestic Violence Data," *Journal of Data Science*, 4, 117–130.

Freedman, D. S. and Sherry, B. (2009), "The Validity of BMI as an Indicator of Body Fatness and Risk Among Children," *Pediatrics*, 124, S23–S34.

Gelman, A. (2006), "Prior Distributions for Variance Parameters in Hierarchical Models(Comment on an Article by Browne and Draper)," *Bayesian Analysis*, 1, 515–533.

Gelman, A., Carlin, J., Stern, H., and Rubin, D. (2003), *Bayesian Data Analysis*, Boca Raton: Chapman & Hall/CRC, 2nd ed.

Gelman, A. and Rubin, D. (1992), "Inference From Iterative Simulation Using Multiple Sequences," *Statistical Science*, 7, 457–511.

Ghosh, S. K., Mukhopadhyay, P., and Lu, J. C. (2006), "Bayesian Analysis of Zero-Inflated Regression Models," *Journal Of Statistical Planning And Inference*, 136, 1360–1375.

Gibbons, R. D., Segawa, E., Karabatsos, G., Amatya, A. K., Bhaumik, D. K., Brown, C. H., Kapur, K., Marcus, S. M., Hur, K., and Mann, J. J. (2008), "Mixed-Effects Poisson Regression Analysis of Adverse Event Reports: The Relationship Between Antidepressants and Suicide," *Statistics In Medicine*, 27, 1814–1833.

Gilks, W., Richardson, S., and Spiegelhalter, D. (1996), *Introducing Markov Chain Monte Carlo*, London: Chapman & Hall.

Gill, J. (2008), *Bayesian Methods: A Social and Behavioral Sciences Approach*, Boca Raton: Chapman & Hall/CRC.

Gupta, P. L., Gupta, R. C., and Tripathi, R. C. (1996), "Analysis of Zero-Adjusted Count Data," *Computational Statistics & Data Analysis*, 23, 207–218.

Gustafson, P. (2003), *Measurement Error and Misclassification in Statistics and Epidemiology: Impacts and Bayesian Adjustments*, New York: Chapman & Hall /CRC.

Gustafson, P. and Greenland, S. (2006), "Curious Phenomena in Bayesian Adjustment for Exposure Misclassification," *Statistics In Medicine*, 25, 87–103.

Haines, L., Barrett, T. G., Wan, K. C., Shield, J. P. H., and Lynn, R. (2007), "Rising Incidence of Type 2 Diabetes in Children in the UK," *Diabetes Care*, 30, 1097–1101.

Heilbron, D. C. (1994), "Zero-Altered and Other Regression-Models for Count Data with Added Zeros," *Biometrical Journal*, 36, 531–547.

Hougaard, P., Lee, M. L. T., and Whitmore, G. A. (1997), "Analysis of Overdispersed Count Data by Mixtures of Poisson Variables and Poisson Processes," *Biometrics*, 53, 1225–1238.

Hur, K., Hedeker, D., Henderson, W., Khuri, S., and Daley, J. (2002), "Modeling Clustered Count Data with Excess Zeros in Health Care Outcomes Research," *Health Services and Outcomes Research Methodology*, 3.

Johnson, W. O., Gastwirth, J. L., and Pearson, L. M. (2001), "Screening Without a "Gold Standard": The Hui-Walter Paradigm Revisited," *American Journal Of Epidemiology*, 153, 921–924.

Joseph, L., Gyorkos, T. W., and Coupal, L. (1995), "Bayesian-Estimation of Disease Prevalence and the Parameters of Diagnostic-Tests in the Absence of a Gold Standard," *American Journal Of Epidemiology*, 141, 263–272.

Karlis, D. and Ntzoufras, I. (2006), "Bayesian Analysis of the Differences of Count Data," *Statistics In Medicine*, 25, 1885–1905.

Keeler, E. B. and Rolph, J. E. (1988), "The Demand for Episodes of Treatment in the Health-Insurance Experiment," *Journal Of Health Economics*, 7, 337–367.

Kuczmarski, R., Ogden, C., and Guo, S.S., e. a. (2002), "CDC Growth Charts for the United States: Methods and Development," *Vital Health Stat 11*, 246, 1–190.

Lambert, D. (1992), "Zero-Inflated Poisson Regression, with an Application to Defects in Manufacturing," *Technometrics*, 34, 1–14.

Lawless, J. F. (1987), "Negative Binomial and Mixed Poisson Regression," *Canadian Journal Of Statistics-Revue Canadienne De Statistique*, 15, 209–225.

Leonard, T. and Hsu, J. (2001), *Bayesian Methods: An Analysis for Statisticians and Interdisciplinary Researchers*, Cambridge: Cambridge University Press.

Liu, J. X., Gustafson, P., Cherry, N., and Burstyn, I. (2009), "Bayesian Analysis of a Matched Case-Control Study with Expert Prior Information on Both the Misclassification of Exposure and the Exposure-Disease Association," *Statistics In Medicine*, 28, 3411–3423.

Lunn, D. J., Thomas, A., Best, N., and Spiegelhalter, D. (2000), "WinBUGS - A Bayesian Modelling Framework: Concepts, Structure, and Extensibility," *Statistics And Computing*, 10, 325–337.

Mier, N., Piziak, V., Kjar, D., Castillo-Ruiz, O., Velazquez, G., Alfaro, M. E., and Ramirez, J. A. (2007), "Nutrition Provided to Mexican-American Preschool Children on the Texas-Mexico Border," *Journal Of The American Dietetic Association*, 107, 311–315.

Mier, N., Piziak, V., and Valdez, L. (2005), "Ultimate Nutrition Game for Mexican American Preschoolers," *Journal Of Nutrition Education And Behavior*, 37, 325–326.

Mosteller, F. and Wallace, D. L. (1964), *Inference & Disputed Authorship: The Federalist*, Addison Wesley.

Mwalili, S. M., Lesaffre, E., and Declerck, D. (2008), "The Zero-Inflated Negative Binomial Regression Model with Correction for Misclassification: An Example in Caries Research," *Statistical Methods In Medical Research*, 17, 123–139.

Nelson, J. F. (1985), "Multivariate Gamma-Poisson Models," *Journal Of The American Statistical Association*, 80, 828–834.

Ntzoufras, I. (2008), *Bayesian Modeling Using WinBUGS*, Chichester: Wiley.

Ogden, C. L., Carroll, M. D., and Flegal, K. M. (2008), "High Body Mass Index for Age Among US Children and Adolescents, 2003-2006," *Jama-Journal Of The American Medical Association*, 299, 2401–2405.

O'Hagan, A., Buck, C., Daneshkhah, A., Eiser, J., Garthwaite, P., Jenkinson, D., Oakley, J., and Rakow, T. (2006), *Uncertain Judgements: Eliciting Experts' Probabilities*, Chichester: Wiley.

Papageorgiou, H. and Loukas, S. (1988), "Conditional Even Point Estimation for Bivariate Discrete-Distributions," *Communications In Statistics-Theory And Methods*, 17, 3403–3412.

Pena, A. S., Wiltshire, E., MacKenzie, K., Gent, R., Piotto, L., Hirte, C., and Couper, J. (2006), "Vascular Endothelial and Smooth Muscle Function Relates to Body Mass Index and Glucose in Obese and Nonobese Children," *Journal Of Clinical Endocrinology & Metabolism*, 91, 4467–4471.

Pietrobelli, A., Faith, M. S., Allison, D. B., Gallagher, D., Chiumello, G., and Heymsfield, S. B. (1998), "Body Mass Index as a Measure of Adiposity Among Children and Adolescents: A Validation Study," *Journal Of Pediatrics*, 132, 204–210.

Prescott, G. J. and Garthwaite, P. H. (2005), "A Bayesian Approach to Prospective Binary Outcome Studies with Misclassification in a Binary Risk Factor," *Statistics In Medicine*, 24, 3463–3477.

Preston, D. (1998), "Poisson Regression for Survival Data in Epidemiology," in *Encyclopedia of Biostatistics*, eds. Armitage, P. and Colton, T., Chichester: Wiley, vol. 4, pp. 3412 – 3416.

Reade-Christopher, S. and Kupper, L. (1991), "Effects of Exposure Misclassification on Regression Analyses of Epidemiologic Follow-up Study Data," *Biometrics*, 47, 535–548.

Schwarz, G. (1978), "Estimating Dimension of a Model," *Annals Of Statistics*, 6, 461–464.

Sharma, A., Grummer-Strawn, L., Dalenius, K., Galuska, D., Anandappa, M., Borland, E., Mackintosh, H., and Smith, R. (2009), "Obesity Prevalence Among Low-Income, Preschool-Aged Children United States, 19982008," *Morbidity and Mortality Weekly Report*, 58, 769–773.

Stamey, J. D., Seaman, J. W., and Young, D. M. (2005), "Bayesian Sample-Size Determination for Inference on Two Binomial Populations With No Gold Standard Classifier," *Statistics In Medicine*, 24, 2963–2976.

Veierod, M. B. and Laake, P. (2001), "Exposure Misclassification: Bias in Category Specific Poisson Regression Coefficients," *Statistics In Medicine*, 20, 771–784.

Vonesh, E. F. (1990), "Modeling Peritonitis Rates and Associated Risk-Factors for Individuals on Continuous Ambulatory Peritoneal-Dialysis," *Statistics In Medicine*, 9, 263–271.

Weiss, R., Dziura, J., Burgert, T. S., Tamborlane, W. V., Taksali, S. E., Yeckel, C. W., Allen, K., Lopes, M., Savoye, M., Morrison, J., Sherwin, R. S., and Caprio, S. (2004), "Obesity and the Metabolic Syndrome in Children and Adolescents," *New England Journal Of Medicine*, 350, 2362–2374.

Whittemore, A. S. and Gong, G. (1991), "Poisson Regression with Misclassified Counts - Application to Cervical Cancer Mortality Rates," *Applied Statistics-Journal Of The Royal Statistical Society Series C*, 40, 81–93.

Wolfe, R. A., Petroni, G. R., McLaughlinN, C. G., and McMahon, L. F. (1991), "Empirical-Evaluation of Statistical-Models for Counts or Rates," *Statistics In Medicine*, 10, 1405–1416.

Zaslavsky, B. (2009), "Empirical Bayes Models of Poisson Clinical Trials and Sample Size Determination," *Pharmaceutical Statistics*, Available at http://www3.interscience.wiley.com/journal/122445155/abstract.

Zeger, S. L. and Karim, M. R. (1991), "Generalized Linear-Models with Random Effects - A Gibbs Sampling Approach," *Journal Of The American Statistical Association*, 86, 79–86.