ABSTRACT

Points and Spheres: Cosmological Innovation in Dante's *Divine Comedy*

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This thesis analyzes the cosmology of Dante's *Divine Comedy*, with particular focus on the ways in which Dante deviated from contemporary paradigms (and even from his own paradigms as expressed in his earlier *Convivio*) regarding the universe. Dante's fictional universe is constructed in a way that resolves certain inconsistencies in medieval understanding and that reconciles Christian theology with Aristotelian and Ptolemaic cosmological thought. I argue that this was one of Dante's conscious objectives in writing the *Divine Comedy*. This conclusion is then used to support a second, more specific theory: that Dante's universe behaves as the surface of a hypersphere. Not only do I endorse this interpretation; I argue that modern scholars have been too quick to reject the possibility that Dante intended for his universe to be understood as a hypersphere. Although it can never be definitively proven, there is evidence to suggest that Dante was aware of the physical consequences of a hyperspherical universe, including the necessity for elliptical non-Euclidean space.

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POINTS AND SPHERES:

COSMOLOGICAL INNOVATION IN DANTE'S *DIVINE COMEDY*

A Thesis Submitted to the Faculty of

Baylor University

In Partial Fulfillment of the Requirements for the

Honors Program

By

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Waco, Texas

May 2015

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ACKNOWLEDGEMENTS

Rather than thank only the people who have contributed to this project, I would also like to take this opportunity to thank some of the people who have defined my Baylor experience. Thank you for making these the best four years of my life.

Dr. David Ryden	for that time you directed my thesis
Dr. Jeff Hunt and Dr. Lorin Matthews	for that time you were on my thesis defense committee
Dr. Alden Smith	for that time I broke a gate
Courtney DePalma	for that time you bought literally hundreds of orange bandanas
Clayton Mills	for that time we made a flag
William Stöver	for that time we made a quick getaway in Mobile Command Center
Drake Gates	for that time we were off by a factor of 2
Sean O'Connor	for that time we sang *Welcome to the Black Parade*
Parker Dalglish	for that time you won a children's game
Paul Williard	for that time we won Southwest Regionals
Mom and Dad	for everything

INTRODUCTION

This thesis presents a cosmological study of Dante's *Divine Comedy*. It analyzes the ways in which Dante's fictive universe deviated from accepted medieval doctrine and from the ideas put forth in Dante's earlier work *Convivio*. At first glance, it seems that Dante allowed the story of the poem to completely determine the nature of his fictive universe, in which physical consistency is subordinated to deeper theological and philosophical meaning. If this is simply the case, then one would expect to see glaring physical inconsistencies in the universe of the *Divine Comedy* as Dante attempts to describe a physical journey through spiritual realms. On the contrary, this thesis finds that, although Dante's fictive universe is heavily inspired by Christian theology and Aristotelian philosophy, it is constructed in a way that actually resolves many of the physical inconsistencies of medieval cosmological thought. Dante may not have seriously believed the universe to behave the way he describes it, but he nonetheless presents a beautiful model of the universe, one that demonstrates his belief in the fundamental harmony of Christian theology, Aristotelian philosophy and metaphysics, and Ptolemaic astronomy.

As the centerpiece of its argument, this thesis endorses a theory dating to the 1920s: that the universe of the *Divine Comedy* behaves as the surface of a hypersphere. Despite its limited exposure within Dante scholarship, this interpretation is generally accepted as spatially accurate. Its great drawback is that the hypersphere and its accompanying non-Euclidean geometry would not be discovered until centuries after Dante wrote the *Divine Comedy*. For most scholars of the subject, this reduces the theory to a novel anachronism. That Dante understood and intended for his universe to be viewed as a hypersphere is considered highly improbable; that he recognized elliptical geometry as an alternative to Euclidean geometry is written off as an impossibility. Indeed, intentionality is impossible to prove, but should it be ruled out entirely?

This thesis distinguishes itself from prior scholarship by treating the hyperspherical universe within the context of Dante's other cosmological innovations. It begins with the more plausible argument that Dante wrote the *Divine Comedy* with an eye toward cosmological thought and that he intended for his fictive universe to resolve certain problems within medieval cosmology. The degree to which the hypersphere model accomplishes this objective while illustrating the poem's wider theme of the harmony between Christianity and Aristotelianism suggests that it could indeed have been Dante's intention. Furthermore, Dante's use of geometric imagery in the *Divine Comedy* seems to focus exclusively on the shortcomings of Euclidean geometry. This may indicate that Dante correctly recognized Euclidean geometry to be fundamentally incomplete. If this is the case, then it is conceivable that his fascination with curved geometry (which he makes abundantly clear in the *Convivio*) led him to experiment with elliptical geometry and to discover the hypersphere, in at least a topological sense.

The first chapter will discuss medieval cosmological thought, its sources, its basic assumptions, and its inherent problems. It will also briefly discuss the intellectual culture of the time and how this helped to inspire Dante's fictive universe. The second chapter will analyze the points at which the universe of the *Divine Comedy* deviates from accepted cosmological thought or from Dante's earlier cosmological thought (as expressed in the *Convivio*). It will also argue that each of these departures represents a conscious effort by Dante to further harmonize Aristotelian-inspired science with Christian theology.

The third and fourth chapters are both dedicated to the interpretation of Dante's universe as existing on the surface of a hypersphere. The third chapter will provide the reader with a topological overview of the hypersphere and of elliptical geometry. It is intended to be minimally technical and to give the reader a comparable understanding to what Dante may have possessed. The fourth chapter will explore the textual basis for interpreting Dante's universe as a hypersphere, and it will elaborate on Dante's use of geometric imagery in both the *Divine Comedy* and the *Convivio*. The chapter will ultimately suggest that Dante could conceivably have understood and intended to convey revolutionary discoveries in mathematical and cosmological thought.

Dante was certainly not an astronomer or a geometer. He did not possess vast technical knowledge on the subjects; rather, he considered himself a poet first and foremost. Nevertheless, he appears to have exhibited a high capacity for scientific and mathematical reasoning. It must be reiterated that intentionality is impossible to prove, and indeed that is not the goal of this thesis. It only argues that previous scholars have been too quick to dismiss the possibility that Dante conceived of his universe as hyperspherical and that he possessed a topological understanding of elliptical non-Euclidean space. Unfortunately, this is not the sort of question that can ever be definitively answered. Perhaps Dante made one of the greatest cosmological discoveries of all time, and perhaps he did not. Regardless of which is the truth, the universe of the *Divine Comedy* stands as a beautiful work of art and adds yet another facet of meaning to a masterful work of poetry.

CHAPTER ONE

Medieval Astronomy and Cosmology

When we gaze upward at the night sky, we cannot help but feel a sense of wonder. The Sun is just one of the hundreds of billions of stars that comprise our galaxy. Further beyond are hundreds of billions of other galaxies, spanning millions of light-years. The universe is unfathomably large, and yet it is characterized by a pervasive emptiness. Space is cold, silent, and dark. Earth is an unassuming pale blue dot in the midst of this great expanse.[1]

The medievals shared our sense of awe toward the cosmos, but they could not have disagreed more with our understanding of it. The medieval universe is vibrant, filled with heavenly light, music, and the mysterious substance called ether. Intelligent forces guide the motions of planets and stars, which in turn cast influence on earthly affairs. The medievals too respected the immense size of the universe, but they also knew it to be incontrovertibly finite. Earth's position within the cosmos is uniquely significant – both the center and the "bottom" of the world.

Dante's understanding of the universe was largely a product of the science of his age. In order to fully understand his ideas about the nature of Earth and the heavens, one must first understand the contemporary paradigms regarding the cosmos. This chapter will explore the core assumptions of late 13th century astronomers, with particular attention given to the problems that medieval scholars struggled to resolve. The chapter

[1] The description of Earth as a "pale blue dot" is taken from Carl Sagan's 1994 book *Pale Blue Dot: a Vision of the Human Future in Space*, which in turn takes its name from a photograph of Earth taken by *Voyager 1* in 1990.

will begin by discussing the sources of medieval cosmological thinking, as well as the intellectual atmosphere at the time of Dante's writing. It will then proceed in similar fashion to Dante's *Paradiso*, beginning with a discussion of Earth and then working upward through the medieval universe. This is meant to be more of a general overview, and indeed many of the models used in this section are oversimplified. The goal is to set the stage for the *Divine Comedy*, and this requires neither a technical nor an exhaustive understanding of medieval astronomy.

Sources of Early Medieval Cosmological Thought

The status of the medieval intellectual tradition prior to the 11th century, especially with regard to scientific thought, is a subject of debate. Some scholars have interpreted the period as an intellectual drought, producing minimal innovation and even losing much of the knowledge of previous ages. They also cite the intellectual culture of the time, which demanded strict religious orthodoxy, as generally hostile to scientific thought.[2] Others have argued that scientific thinking during this period, although primitive, was not completely lacking. They cite various commentaries written by medieval scholars, critiquing the works of Roman and Greek thinkers. Some have also observed that the medieval Catholic Church made a significant contribution to scientific thought by removing the influences of pagan mythology and superstition.[3] Both interpretations, however, point to a common theme: medieval knowledge and understanding were built upon an intellectual inheritance from the ancient Mediterranean tradition.

[2] Federn, *Dante and His Time*, 74-77.
[3] North, *Cosmos*, 234-237.

Among the limited number of ancient Mediterranean texts available to early medieval scholars was Plato's *Timaeus*. The work was translated into Latin first by Cicero in the 1st century BC[4] and again by Chalcidius (who also wrote a commentary on the work) during the 4th or 5th century AD.[5] In the *Timaeus*, Plato argued that the universe had been created (as opposed to existing eternally) by some intelligent being. The creator's affinity for aesthetic beauty was reflected in the spherical structure of the heavens and in the existence of natural laws (which the creator would never violate even though he had the ability to do so). Such ideas were particularly attractive to medieval Christians, and the Platonic view of the universe was easily adapted to fit Christian theology.[6]

Although some of Plato's ideas were rejected by Christian commentators (for example: that the world would last forever), the *Timaeus* would serve as the core of early medieval cosmological thought. That foundation was built upon by various Latin scholars. Boethius (early 6th century) attempted to reconcile Platonic and Aristotelian views of the cosmos, Martianus Capella (5th century) wrote a textbook on the seven liberal arts (which included astronomy), Macrobius (early 5th century) wrote a commentary on Cicero's *The Dream of Scipio* (in which Scipio journeys through the heavenly spheres), and Bede (early 8th century) wrote multiple treatises on measuring time from the motion of the heavens.[7] Typically at issue were astronomical questions rather than cosmological questions (for example: the ordering of planets). The Platonic

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[4] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 24.
[5] Moore, *Studies in Dante*, 156-157.
[6] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 22-29.
[7] North, *Cosmos*, 240-246.

understanding of an ordered and created universe remained unquestioned and would continue to define medieval cosmology.

The Reintroduction of Aristotle and Ptolemy

During the 12th and 13th centuries, the European intellectual tradition experienced a dramatic surge, brought about by an influx of Greco-Arabic influence. Many of the ancient Mediterranean texts lost to medieval Europeans had in fact been preserved by the Arabic tradition. Beginning in 750 and lasting through the 10th century, Baghdad emerged as the intellectual (and political) capitol of the Islamic world. Under the patronage of the city's elite, numerous scientific and mathematical texts were translated from Greek into Arabic. Among these texts were the works of Aristotle, Euclid's *Elements*, and Ptolemy's *Almagest*.[8] These, especially the works of Aristotle, had formed the core of Arabic scientific and mathematical thought during this time period.

During the 12th and 13th centuries, the ancient Mediterranean texts were reintroduced to the Christian world, along with the commentaries written by their Arabic translators. This was largely thanks to the efforts of Latin translators such as Gerard of Cremona (translating from Arabic) and William of Moerbeke (translating from Greek). The two of them alone accounted for most of the new Latin translations, and altogether the number of texts available to European scholars increased exponentially.[9] Included were the astronomical treatises written by Aristotle and Ptolemy. These would replace Plato's *Timaeus* as the core of European cosmological thought.

Aristotle's understanding of the cosmos is revealed in his treatises *On the Heavens*, *Physics*, and *Metaphysics*. His universe is essentially Platonic, but with a few

[8] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 30-43.
[9] Grant, *Planets, Stars, and Orbs*, 12-14.

revisions. He keeps the spherical structure, but he does not believe the universe to have been created. Rather, he believes the universe to have existed eternally. He also theorizes that the heavens are made of some perfect fifth element, the "quintessence" or ether. Aristotle's overall approach is less abstract and more mechanical than Plato's. For example, he refines the Platonic understanding of a creator by arguing for the existence of a Prime Mover that imparts motion to the heavens. In addition, each of the heavenly spheres has its own Unmoved Mover that guides its motion.[10] Although his argument is based on a physical understanding of cause and effect, he ultimately accepted that these movers must be gods. Medieval scholars would reinterpret the movers as Intelligences or angels. The Prime Mover would be equated with God.[11]

Ptolemy's *Almagest*, which further built upon the Aristotelian and Platonic views of the universe, was viewed throughout the Middle Ages as the definitive work on astronomy.[12] The *Almagest* is almost entirely a technical treatise, concerned with cataloguing and mapping the exact motions of the planets and stars. Unlike the *Timaeus*, it is not concerned with the origin of the universe, and unlike the Aristotelian treatises, it is not concerned with the abstract causes of planetary and stellar motion. His catalogue of the stars includes 1022 stars, their positions, and their relative magnitudes. In order to map the seemingly erratic motions of the planets, he formalized the theory of epicycles (see Figure 1.1).[13]

By Dante's time, it was generally accepted that Ptolemy's observational astronomy was more accurate than Aristotle's (although Dante defends Aristotle by

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[10] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 44-45.
[11] North, *Cosmos*, 80-84.
[12] Orr, *Dante and the Early Astronomers*, 147.
[13] North, *Cosmos*, 110-118.

arguing that Ptolemy had the advantage of better measurements).[14] Nevertheless, Aristotle remained a paramount figure in cosmological thought because of his theory of Unmoved Movers and the Prime Mover. A similar attempt at understanding first causes was not to be found in the *Almagest*. Furthermore, the Platonic notion of a created universe held its ground throughout the surge of Greco-Arabic scholarship. The medieval understanding of the cosmos was thus built upon an inheritance from the ancient Greek tradition, relayed through the Arabic tradition.

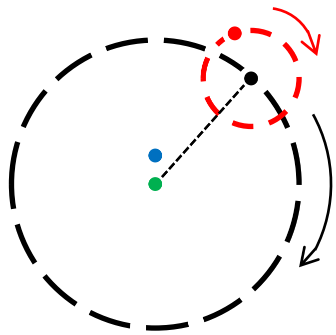

Figure 1.1 - This is a simplified model of an epicycle. The blue dot is the Earth, and the red dot is a planet. The planet orbits clockwise about its epicycle. The path of this orbit is denoted by the red dotted line, and the center point of the epicycle is denoted by the black dot. Simultaneously, the epicycle orbits clockwise about a second focal point, denoted by the green dot. The path of the epicycle is denoted by the black dotted line. Notice 1) that the resulting path of the planet is not circular, and 2) that neither the planet nor its epicycle exactly orbits the Earth.

The Reaction against Aristotelianism

During the 13th century, the growing influence of Aristotelianism began to meet opposition from the Catholic Church. A number of Aristotle's propositions directly challenged Christian teaching, the best example being his argument that the universe was not created. In addition, many Christian theologians feared that Aristotelian philosophy would be inappropriately applied to theology. Throughout the first half of the century,

[14] Dante, *Convivio*, II iii 1-12.

authorities at the University of Paris attempted to censure and even to ban outright certain works of Aristotle and his Arabic commentators. These efforts ultimately failed, and Aristotelianism established itself as a dominant force within the university and within the larger European intellectual tradition.[15]

The conflict renewed in 1270 when the bishop of Paris, at the behest of conservative theologians, issued a condemnation of thirteen Aristotelian propositions.[16] This was followed in 1272 by a decree of the University of Paris, which ordered that,

> No master or bachelor of our faculty should presume to determine or even to dispute any purely theological question… If any master or bachelor of our faculty reads or disputes any difficult passages or any questions which seem to undermine the faith, he shall refute the arguments or texts as far as they are against the faith or concede that they are absolutely false and entirely erroneous, and he shall not presume to dispute or lecture further upon this sort of difficulties, either in the text or in authorities, but shall pass over them entirely as erroneous.[17]

Finally, the strongest attack against Aristotelian philosophy came with the Condemnation of 1277, in which the bishop of Paris condemned an additional 219 propositions.[18]

This series of developments had emerged in response to a tendency among Aristotelian scholars to require that God obey natural laws. For example, medieval scholars agreed that the natural laws prevented the existence of a vacuum anywhere in the universe. Therefore, many had denied that God had the ability to create a vacuum. In

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[15] Grant, *Planets, Stars, and Orbs*, 52.
[16] Grant, *Planets, Stars, and Orbs*, 52-53.
[17] Thorndike, "University Records and Life in the Middle Ages," 64-65.
[18] Grant, *Planets, Stars, and Orbs*, 53.

their understanding, God's omnipotence was not in any way undermined because the natural laws represented a self-imposed restriction. This approach to questions of science and theology was overruled by the Condemnation of 1277. Although scholars were not required to believe that God did or would break natural law, they were forced to admit that He could if He so desired.[19]

The Condemnation did little to alter the commonly held assumptions and understandings regarding the universe, but the approach to scientific questioning was fundamentally altered. In a way, scholars were given greater creative freedom. They could now challenge popularly held scientific beliefs (especially those of Aristotle and his commentators) without attacking them outright. They were free and even encouraged to speculate about theories and ideas that, before, would have lacked any merit.[20] Returning to the earlier example, scholars could, in light of the Condemnation, entertain the theory that God created vacuums somewhere in space. Nobody would have seriously believed such a hypothesis, yet it would have held a degree of validity within the academic community.[21]

European cosmology at the end of the 13th century was firmly rooted in the ancient Greek tradition, yet the intellectual culture had evolved to permit and even encourage deviation from commonly held assumptions. This was the context within which Dante studied the cosmos. The next section is devoted to exploring the medieval universe, giving particular attention to the questions that divided medieval scholars. It is by no means an exhaustive discussion on medieval cosmological thought, but it will serve as an adequate introduction to Dante's personal ideas regarding the cosmos.

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[19] Grant, *Planets, Stars, and Orbs*, 53-54.
[20] Egginton, "On Dante, Hyperspheres, and the Curvature of the Medieval Cosmos," 213.
[21] Grant, *Planets, Stars, and Orbs*, 53-55.

The Medieval Earth

There is a misconception that Columbus discovered that the Earth is round. In reality, the Earth's spherical shape had been known even as far back as the ancient Greek tradition, and this knowledge had not been lost to the medievals. Furthermore, they astutely recognized that the Earth was not perfectly spherical (Its landscape features mountains and valleys.) but that this was a sufficient approximation.[22] A second aspect of medieval astronomy that is not so foreign to modern astronomy is the Earth's relatively small size compared to the heavens. There had been numerous attempts to approximately measure the Earth (and the surrounding heavenly spheres), but it was almost universally accepted that the Earth had "no appreciable magnitude" against the scale of the cosmos.[23]

Where medieval astronomy deviates significantly from modern astronomy is in the Earth's location. The medieval Earth, in keeping with Ptolemy's *Almagest*, was positioned at the absolute center of the universe. The idea of a heliocentric universe had indeed been considered by medieval thinkers (and even advocated for as early as the 3rd century B.C. by the Greek writer Aristarchus of Samos[24]) but had ultimately been rejected in favor of the Ptolemaic geocentric model.[25] Furthermore, despite contrary arguments, the Earth was generally agreed to be immobile at the center of the universe. It experienced neither rotational nor translational motion.[26]

It is not quite enough, however, to describe the Earth as being at the center of the medieval universe, for it was also understood to occupy the *bottom* of the universe. The medieval world was characterized by an absolute Up and Down, with movement toward

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22 Grant, *Planets, Stars, and Orbs*, 626-630.
23 Lewis, *The Discarded Image*, 97-98.
24 North, *Cosmos*, 84-86.
25 Grant, *Planets, Stars, and Orbs*, 672-673.
26 Grant, *Planets, Stars, and Orbs*, 624-626, 637-647.

the Earth being downward and movement away from Earth being upward.[27] One might then imagine the medieval universe as being shaped like a bowl. The center of the bowl is also its lowest point; movement away from this point is both outward and upward toward the rim. This also carries deep theological meaning, for Heaven is literally above the Earth, and fallen Man occupies the lowest point in the cosmos.

Although Columbus must be denied credit for discovering the Earth's roundness, he did make another lasting contribution to geography by discovering the Americas. Prior to his voyage, European scholars (and their Arabic counterparts) had denied the existence of a landmass on the opposite side of the globe. They argued that, if such a land existed, it could not be inhabited by people (all of whom had to be direct descendants of Adam and Eve). Instead, the Earth was thought of as having two hemispheres: a hemisphere of earth (the Afro-Eurasian supercontinent) and a hemisphere of water (the oceans). This sentiment was reflected in the primitive T-O maps of the period, which placed Jerusalem at the center of Earth's landmass.[28] Often, these maps also included Biblical and mythical locations such as the Tower of Babel, Sodom and Gomorrah, and Troy.[29] The location of the Earthly Paradise, the Garden of Eden, was a subject of debate, but it was generally believed to be somewhere in the East. Some envisioned a garden surrounded by fire and guarded by an angel. Others thought that Eden was an island or a mountain, tall enough to survive the Flood. The consensus though was that Eden was a physical place that was inaccessible to humans.[30]

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[27] Lewis, *The Discarded Image*, 98-99.
[28] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 49-60.
[29] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 118.
[30] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 58-60.

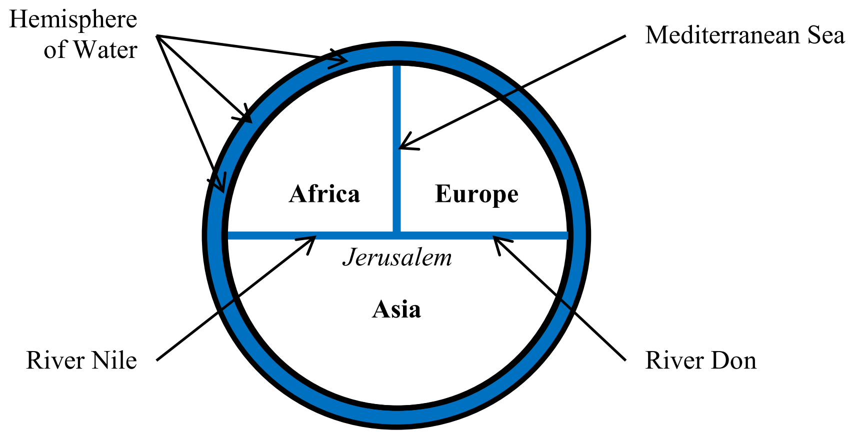

Figure 1.2 - A simplified example of a T-O map, oriented such that North points to the right side of the page. The three major landmasses as well as the dividing bodies of water are labeled. Jerusalem is situated at the center of the hemisphere of earth. This map is based on one found in Edson and Savage-Smith.[31]

The notion of elemental hemispheres was rooted in yet another inheritance from the ancient Mediterranean tradition. Aristotle conceived of the sublunary world as being composed of four elemental spheres. The lowest, inner-most sphere was the sphere of earth. Upon that sphere rested the sphere of water, beyond that sphere rested the sphere of air, and beyond that rested the sphere of fire (where comets and shooting stars originated). The boundaries between the spheres were not exactly dichotomous, but rather the spheres gradually transitioned into one another. The nature of each of the elements was such that it possessed a general inclination toward its appropriate sphere. Hence, earth tends to sink in water, and fire tends to rise through the air.[32]

This conception of elemental spheres was readily adopted and to some extent Christianized by medieval scholars. The Aristotelian model was applied to the Biblical account of creation, specifically to the third day of creation. On that day, God

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[31] Edson and Savage-Smith, *Medieval Views of the Cosmos*, 50.
[32] Orr, *Dante and the Early Astronomers*, 81-83.

commanded to "let the waters under the heaven be gathered together unto one place, and let the dry land appear."[33] Medieval scholars interpreted this as God reordering the spheres of earth and water.[34] There was, however, an obvious problem with the Aristotelian model of the Earth: it was unclear whether or not the spheres of earth and water were truly distinct. Some argued that earth and water comprised a single sphere. The act of the third day of creation was therefore to consolidate the two spheres together into one. Other scholars suggested that, in the beginning, the sphere of water enveloped the sphere of earth but that, on the third day of creation, God displaced the two spheres' centers of mass. This theory better explained the presence on Earth's surface of two distinct elemental hemispheres.[35] (This is demonstrated in Figure 1.3)

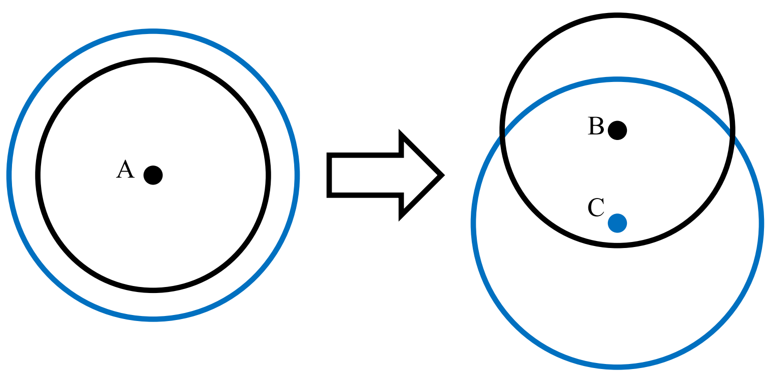

Figure 1.3 - This model demonstrates the theory that, on the third day of creation, God displaced the centers of mass of the earth and water spheres. In the model, the black circle represents the sphere of earth, and the blue circle represents the sphere of water. In the beginning, they share a common center of mass (Point A). God then displaces the two spheres such that the earth center of mass (Point B) and the water center of mass (Point C) no longer coincide. This produces on Earth's surface a hemisphere of land and a hemisphere of water, just as the medievals believed to be the case. This diagram is based on one found in Grant.[36]

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[33] Genesis 1:9 KJV.
[34] Grant, *Planets, Stars, and Orbs*, 630.
[35] Grant, *Planets, Stars, and Orbs*, 630-635.
[36] Grant, *Planets, Stars, and Orbs*, 634.

This also touches upon another problem of medieval astronomy and geography: what part of the Earth coincided with the center of the universe? Although scholars considered the Earth's geometric and volumetric centers as candidates, they found the most agreeable answer to be the Earth's center of mass. Nevertheless, this remained an incomplete solution. Were there, as suggested by the theory that God displaced the spheres of earth and water, two separate centers of mass for earth and for water? More plausibly, they reasoned, the elemental spheres were concentric, sharing a common center of mass that coincided with the center of the universe. They ultimately rejected the theory of displaced spheres, but they still struggled to accept the theory of a single earth-water sphere (which countered the Aristotelian model). Furthermore, although they recognized that the Earth's center of mass is inconstant, varying as Earth's surface landscape changes, they were forced to reason that these perturbations were negligible. This was the only way for the Earth to retain its status as immobile at the center of the universe.[37] It should be noted that many of the theories regarding the elemental spheres were not formalized until after the *Divine Comedy* was finished. For Dante, the nature of Earth's elemental spheres was a problem yet to be fully addressed, but it was only one of the many uncertainties in medieval cosmological thought.

The Medieval View of the Heavens

Despite the preeminence of Aristotelianism among medieval scholars, Ptolemy continued to be regarded as the superior astronomer, and the structure of the medieval universe closely adhered to the model of the *Almagest*, albeit with a few additions. The Ptolemaic universe could be described as a solid sphere (the Earth) surrounded by a series

[37] Grant, *Planets, Stars, and Orbs*, 622-637.

of nested spherical shells (the heavenly spheres).[38] The first seven spheres were the planetary spheres, each of which featured an epicycle (or epicycles), upon which the actual planet was fixed. (Refer back to Figure 1.1) In addition, the orbits of the planetary epicycles were eccentric – the Earth was not the center point of their orbits.[39] This seemed to challenge Aristotle's model of concentric spheres and, by extension, the understanding that the Earth was the center of the universe. Fortunately, Ptolemy was able to salvage the Earth's centricity by arguing that each of the heavenly spheres had an inner concave surface and an outer convex surface. Although the inner concave surfaces of the heavenly spheres were not concentric, the outer convex surfaces were concentric and had Earth as their common center point.[40] (See Figure 1.4) Finally, beyond the seven planetary spheres was the eighth sphere, the sphere of the Fixed Stars.

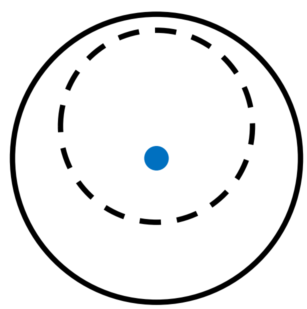

Figure 1.4 – This is a model of a single Ptolemaic planetary sphere. The inner concave part of the heavenly sphere (denoted by the dotted line) is eccentric and represents the path of the planet's epicycle. The outer convex part of the heavenly sphere (denoted by the solid line) has the Earth as its center point. The outer convex parts of all the heavenly spheres are concentric, and the Earth is therefore the center of the universe (even though it is not the center of each planet's orbit).

The medieval universe shared the observable qualities of the Ptolemaic model. Beginning from the Earth and moving outward, the planetary spheres were the Moon,

[38] Lewis, *The Discarded Image*, 96.
[39] North, *Cosmos*, 114-118.
[40] Grant, *Planets, Stars, and Orbs*, 277-287.

Mercury, Venus, the Sun, Mars, Jupiter, and Saturn. Following the planets was the eighth heaven of the Fixed Stars. The medievals also accepted Ptolemy's eccentric model over that of Aristotle.[41] Despite the apparent victory of Ptolemy's *Almagest*, Aristotle also exerted a high degree of influence on medieval cosmology. Whereas Ptolemy was more valued for his observational astronomy, Aristotle was more valued for his metaphysics. His proposed fifth element, the ether, remained the medievals' best guess as to the composition of the heavenly spheres.[42] Furthermore, his understanding that Intelligences moved the heavenly spheres was easily Christianized.[43] For the medievals, the heavenly spheres were moved by angels, and various angelic hierarchies were proposed in order to explain the correlation between different types of angels and the different heavens.[44] The medievals even believed that the movement of the heavenly spheres imparted certain influences to worldly affairs, and that the different planets possessed unique characters and domains of influence.[45]

One subject on which Aristotelian metaphysics seemed incompatible with observational astronomy was the Moon. Aristotle taught that the planets were homogenously composed of ether, yet the Moon seemed to defy this by having dark spots on its surface. Prior to the reintroduction of Aristotle's works, European scholars had theorized that the Moon was composed of the four terrestrial elements and that the dark spots were caused by an improper combination of the elements. With the rise of Aristotelianism, this interpretation was cast aside, and it became accepted that the Moon was composed of ether. This required scholars to readdress the question of the Moon's

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[41] Lewis, *The Discarded Image*, 96.
[42] Grant, *Planets, Stars, and Orbs*, 422-428.
[43] Grant, *Planets, Stars, and Orbs*, 526-528.
[44] North, *Stars, Minds, and Fate*, 190.
[45] Lewis, *The Discarded Image*, 103-109.

spots. Numerous theories were put forth (one of which correctly asserted that the Moon has landforms such as mountains and valleys), but the most appealing theory attributed the dark spots to "rarity" (having less substance) and "density" (having more substance) in the Moon's composition. Still, the answer remained unclear. Were the dark spots areas of greater rarity or were they areas of greater density? It should also be noted that such an interpretation was hardly less challenging to Aristotelian metaphysics, for it still characterized the Moon's composition as heterogeneous.[46] This discrepancy between Aristotelianism and observational astronomy was particularly troubling for Dante. He would address this question in both the *Convivio* and the *Divine Comedy*.

In addition to the eight heavens of Ptolemy's *Almagest*, medieval scholars added two additional heavens beyond the sphere of the Fixed Stars. The ninth heaven they called the Primum Mobile, and the tenth heaven they called the Empyrean. These two constructs were absolutely critical to the medieval understanding of the cosmos and yet were also the most problematic for medievals to rationalize.

The idea of the Primum Mobile, although incorrectly attributed by Dante to Ptolemy, was in fact produced by Arabic astronomers in response to the issue of precession.[47] This is the phenomenon by which the planets and stars appeared to revolve from East to West on a 24 hour period. (This is in fact caused by the Earth's daily rotation.) The Primum Mobile was believed to account for this, rotating from East to West once every 24 hours and imparting this motion to the eight spheres enveloped within it.[48] Once adopted into medieval Christian thought, the Primum Mobile received even greater distinction. The Biblical creation account describes how God commanded to

[46] Grant, *Planets, Stars, and Orbs*, 459-466.
[47] Orr, *Dante and the Early Astronomers*, 149-150.
[48] Lewis, *The Discarded Image*, 102.

"let there be a firmament in the midst of the waters, and let it divide the waters from the waters."[49] The most popular identity of the firmament was the sphere of the Fixed Stars. God's command to "let there be lights in the firmament of the heaven"[50] seemed to confirm the association of the firmament with the stars.[51] Medievals therefore reasoned that the Primum Mobile was composed of the waters above the firmament. This opened up two other problems. First, was the Primum Mobile literally composed of terrestrial elemental water, or was it composed of some other water-like substance? Second, was the Primum Mobile solid or fluid? Although it was often referred to as the "crystalline sphere," this did not necessarily imply hardness. Rather, it emphasized the sphere's transparency and luminosity.[52] An exact understanding of the Primum Mobile thus continued to elude medieval thinkers, yet its function as the universe's first mover was paramount.

The final, outermost sphere of the medieval universe is the Empyrean, the sphere of God's perfect light. This was the home of God, angels, saints, and other righteous souls. Unlike the other spheres, the Empyrean is immobile, for perfection cannot exist in a changing state. Although the Empyrean was completely unobservable, its existence was almost universally acknowledge by medieval Christian scholars. They pointed to the Biblical creation account, understanding the Empyrean to be the heaven created by God on the first day of creation.[53]

Despite the consensus on the existence of the Empyrean, medievals were at a loss to describe its exact nature. Aristotle had insisted that the universe was finite, and the

49 Genesis 1:6 KJV.
50 Genesis 1:14 KJV.
51 Grant, *Planets, Stars, and Orbs*, 95-103.
52 Grant, *Planets, Stars, and Orbs*, 332-334.
53 Grant, *Planets, Stars, and Orbs*, 371-378.

medievals held tightly to this assumption.[54] Aristotle also argued that there was nothing – neither space nor time – beyond the universe, and this too was accepted by the medievals as fact.[55] So then, medieval scholars were faced with a quandary: what happened at the edge of the universe? Was the universe like a snow globe, enveloped by a hard boundary that prevented anything from entering or exiting? Did it perhaps have an inexact boundary, with its edges gradually fading away into nothingness? Or was there actually some precise line that a person could cross and *fall out* of the universe? Some scholars avoided this question by suggesting that the Primum Mobile was the end of spatiality and corporeality. They described the Empyrean as a place of intellectual rather than material substance. Hence, it was not constrained by the human understanding of space and time.[56] When pressed though, this is hardly a satisfying answer as it only shifts the boundary of the physical universe to the Primum Mobile. Regardless of how one interprets the Empyrean, at some point the edge of the physical universe must be addressed. The medievals produced no answer to this question. They simply could not rationalize the boundary of the physical universe, yet they also could not reject the universe's finitude.

The medieval European universe was a product of influences from the ancient Mediterranean intellectual tradition, inherited through the Islamic world. The assumptions and paradigms that characterized medieval cosmology were in some parts mathematical or scientific, in other parts philosophical or metaphysical, and in other parts theological. The medievals were able to achieve an impressive amount of convergence between each of these disciplines. Nevertheless, certain inconsistencies and disputes continued to avoid resolution. It was here that Dante left his mark, attempting to solve

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[54] Grant, *Planets, Stars, and Orbs*, 106-113.
[55] Grant, *Planets, Stars, and Orbs*, 122-135.
[56] Lewis, *The Discarded Image*, 96-97.

these cosmological problems in a way that reconciled scientific, philosophical, and theological truths. Perhaps he was enabled to do this by an intellectual culture – characterized by the University of Paris' condemnations – that was favorable to theories supporting theological truth. It is not necessarily the case that Dante sincerely believed in the universe of the *Divine Comedy*; more likely, he was simply exploring a hypothetical world for the sake of telling a story. This should not, however, detract from the artistic beauty of his universe. The next chapter will explore certain ways in which Dante deviated from the commonly held notions of the cosmos, and it will show how his changes worked to resolve the shortcomings of medieval cosmology in a manner that respected God's omnipotence and providence.

CHAPTER TWO

The Cosmology of the *Divine Comedy*

Dante's *Divine Comedy* is the story of the Pilgrim's journey through the heavens. This journey is both spiritual and physical, and this same duality is present in Dante's cosmology. He founds his universe around contemporary cosmological ideas, but he makes certain alterations in order to create greater harmony between Ptolemaic astronomy, Aristotelian philosophy, and Christian theology. This chapter will begin with a brief discussion of Dante's primary sources for cosmological thought. It will then discuss four ways in which Dante deviated either from accepted views of the universe or from his own views of the universe, as described in his earlier work the *Convivio*. The first of these three subjects is the role of angels in guiding the motions of the heavens. The second subject is the nature of the Moon and the cause of its dark spots. The third subject (which will be discussed in greatest detail) is the physical nature of Hell and Purgatory and their relation to Earth. The fourth subject will only be introduced, for its discussion will comprise the following two chapters; it regards the nature of the Empyrean and the shape of the spatial universe.

Dante's Cosmological Sources

Dante was well versed in classical and contemporary scholarship, yet he was limited in the sources available to him. His "supreme authority" on astronomy and cosmology was Ptolemy's *Almagest*, but Dante had only limited exposure to the work. In fact, it is quite possible that he never read the *Almagest* directly. Dante references the

Almagest on three occasions in the *Convivio*, and on each of those occasions he is incorrect. First, he incorrectly attributes to Ptolemy a theory about the nature of the galaxy. (Ptolemy did not put forth any such theory.) Then, he credits Ptolemy with discovering precession. (Ptolemy was aware of precession but credited Hipparchus with its discovery.) And finally, Dante credits Ptolemy with the theory of the Primum Mobile. (In fact, the theory of the Primum Mobile was a contribution from the Arabic astronomers.)[57] Fortunately for Dante, the Ptolemaic system was widely known, and he would have at least had indirect access to it through the various second-hand commentaries.

Dante's textual authority on astronomy was likely *Elementa Astronomica*, written by the Arabic scholar Alfraganus. Numerous Latin translations (including one by Gerard of Cremona) were available, and Dante seems to have taken much of his astronomical data from the work. One such example is in *Paradiso* when Dante describes twenty-four brilliant stars: the fifteen brightest stars (which he does not name), the seven stars of Ursa Major, and two stars of Ursa Minor. In *Elementa Astronomica*, Alfraganus describes the fifteen first-magnitude stars and the nine second-magnitude stars that comprise the Arabic constellations Benet Naax and Alfarcatein, which correspond to the Western constellations Ursa Major and Ursa Minor.[58] Thus, it is likely that Dante was primarily exposed to Ptolemaic astronomy through the work of Alfraganus, rather than directly through the *Almagest*.

The previous chapter discussed how a significant amount of the medieval knowledge about cosmology was inherited through the Arabic tradition. Indeed, Dante's

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[57] Orr, *Dante and the Early Astronomers*, 149-150.
[58] Orr, *Dante and the Early Astronomers*, 150-151.

reliance on Alfraganus seems to highlight this. Oddly though, Dante seems to have made minimal use of Arabic scholarship. While it is highly unlikely that Dante could read Arabic, Latin translations were available. Dante was certainly aware of the Arabic commentators and even held some of them in high regard. For example, in the *Divine Comedy*, Avicenna and Averroës are in Limbo among the virtuous pagans, even though they lived during the time after Christ.[59] Nevertheless, Dante "seldom" references Arabic scholars.[60]

Among Christian scholars, Dante was particularly indebted to Thomas Aquinas. Dante was quite familiar with the latter's *Summa Theologica*, and he shared Aquinas' respect toward Aristotle and belief in the basic harmony between scientific reason and religious faith. Also of great influence were Albert of Cologne, Augustine, Peter Lombard, and Orosius, all of whom provided Dante with knowledge and commentary on astronomy, cosmology, and geography.[61]

Undoubtedly, Dante's single greatest influence was Aristotle. The number of times that Dante references Aristotle is staggeringly large, greater than the number of times Dante references any other author.[62] Edward Moore observes that "no other writer then, or at any other time, has surpassed Dante in the admiration and reverence expressed for Aristotle," and he describes Dante's knowledge of Aristotle as "almost encyclopaedic."[63] Unlike the limited and indirect means by which Dante studied Ptolemy, he had almost complete access to the works of Aristotle (excluding, of course, texts that

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[59] Dante, *Inferno*, iv 143-144.
[60] Orr, *Dante and the Early Astronomers*, 154.
[61] Orr, *Dante and the Early Astronomers*, 154-156.
[62] Refer to Moore's *Studies in Dante*, an exhaustive analysis of Dante's references to outside scholarship. A brief glance at the table of contents reveals the disproportionate amount of space dedicated to Aristotle.
[63] Moore, *Studies in Dante*, 92, 94.

have not survived). The only work of which Dante seems to lack any familiarity is Aristotle's *Poetics*.[64] Dante's commitment to Aristotle is visible in the universe of the *Divine Comedy* as Dante attempts to reconcile cosmological questions in a way that respects Aristotelian metaphysics.

The Angelic Intelligences

In the *Convivio*, Dante describes the commonly accepted universe, based on the Ptolemaic model. He enumerates the seven planetary spheres, followed by the sphere of the Fixed Stars, the Primum Mobile, and the Empyrean. He even describes and accepts the Ptolemaic theory of epicycles, using Venus as a specific example.[65] Although he accepts Ptolemy's observational astronomy (even incorrectly attributing to him the theory of the Primum Mobile), he continues to defend Aristotle's status. He contends, for example, that when Aristotle argued for the existence of only eight heavens, he was acting on incomplete research and the flawed assumptions by his contemporaries. As another example, Dante also suggests that Aristotle conceived of the Empyrean well before the Christian scholars.[66]

In metaphysics especially, Dante sought to preserve the position of Aristotle. One way of doing this was by adopting the Aristotelian theory that the heavens were moved by Intelligences. This idea had of course become Christianized by the understanding that the Intelligences were angels. Still, this was not a perfectly complete solution as it led to other interpretative problems.

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[64] Moore, *Studies in Dante*, 93.
[65] Dante, *Convivio*, II iii 7-17.
[66] Dante, *Convivio*, II iii 3-10.

One unavoidable question was to the exact number of angels. According to Dante, Aristotle had insisted that there could only be as many Intelligences as there were heavenly motions. Any additional Intelligences would not serve any purpose, and this would be an affront to Aristotle's teleological philosophy.[67] Dante challenged this position in the *Convivio* by arguing that there were in fact more angels than heavenly spheres and that these additional angels served contemplative, rather than active purposes.[68] This answer seems satisfactory, but it fails to address a second question which deals with precession. Did each planet have its own Intelligence to govern its daily motion, or was their one Intelligence that governed the daily motions of all the planets?[69] Essentially, this question asked whether daily motion was one type of motion or nine types. In the *Convivio*, Dante is unable to decide, and this question is left unresolved.[70] He is able to reason that there are additional angels with contemplative functions, but he is unable to determine how many angels have the active function of guiding heavenly motion.

One final question that emerges from the interpretation of angels as guiding heavenly motion deals with angelic hierarchy. What was the hierarchy of angels, and which angels were assigned to each type of heavenly motion? Aristotle had not imagined any sort of hierarchy for his Intelligences, yet this was a staple of medieval angelology. One theory had been put forth by pseudo-Dionysius in the work *On the Celestial Hierarchy*. (Dante misidentifies him as Dionysius the Areopagite, an Athenian who was

[67] Dante, *Convivio*, II iv 3.
[68] Dante, *Convivio*, II iv 8-15.
[69] North, *Stars, Minds, and Fate*, 190.
[70] Dante, *Convivio*, II v, vi.

converted by Paul.)[71] This theory was then slightly amended by Pope Gregory, whose interpretation proved rather popular (given the prestige of its author). In the *Convivio*, however, Dante rejects both of these theories and instead advocates for a third angelic hierarchy proposed by Bruno Latini.[72] He enumerates the nine classes of angels and their corresponding heaven: Angels (the Moon), Archangels (Mercury), Thrones (Venus), Dominations (the Sun), Virtues (Mars), Principalities (Jupiter), Powers (Saturn), Cherubim (the Fixed Stars), and Seraphim (the Primum Mobile). He also suggests that a select (but unspecified) number of angels from each class are charged with governing the heaven's motion.[73] For example, the various motions of the planet Mars are governed by a small group of Virtues; the unemployed Virtues serve some other contemplative function.

In the *Divine Comedy*, Dante changes his stance on the angelic hierarchy and opts for that of Dionysius. The order is Angels, Archangels, Principalities, Powers, Virtues, Dominations, Thrones, Cherubim, and Seraphim.[74] In addition, Dante alters his interpretation of angelic function. As the Pilgrim gazes into the Empyrean, he sees nine concentric rings of angels orbiting God, with the inner circles rotating faster and shining purer than the outer circles. Each ring is comprised of one class of angels and corresponds to one heavenly sphere. The innermost ring is the Seraphim, who correspond to the Primum Mobile; the second smallest ring is the Cherubim, who correspond to the Fixed Stars; and the pattern continues to the outermost ring of the Angels, who correspond to the Moon. Beatrice explains that the speed and brightness of each angelic

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[71] North, *Cosmos*, 248.
[72] North, *Stars, Minds, and Fate*, 190.
[73] Dante, *Convivio*, II v 4-18.
[74] Dante, *Paradiso*, xxviii 97-126.

class is a manifestation of its blessedness, which is caused by the act of vision. All of the angels direct their attention toward God, and their resulting blessedness causes the force that moves the heavens. The Seraphim, who are closest to God, are best able to behold his light. Hence, they are the most blessed of the angels, and the Primum Mobile is the most blessed of the heavenly spheres.[75]

Unlike the *Convivio*, the *Divine Comedy* makes no distinction between angels with active functions and angels with contemplative functions. All angels serve the contemplative function of observing God's light and the active function of moving the heavens.[76] Whereas the *Convivio* suggested that a select group within each class of angels is charged with moving a heaven, the *Divine Comedy* contends that each class of angels functions as a collective to move its corresponding heaven.[77] This preserves Aristotle's teleological approach to the problem without the need to enumerate the angels assigned to each heavenly motion. The angelic system of the *Divine Comedy* is more beautiful in that it is characterized by notably less uncertainty than the system of the *Convivio*.

Dante's decision to alter his angelic hierarchy seems more or less arbitrary. Indeed, it appears to be little more than a cosmetic change. It is possible that Dante actually changed his mind about the ordering of the angels. This would certainly point to Dante's intellectual maturing between authoring the *Convivio* and authoring the *Divine Comedy*. More likely though, Dante intended to draw parallelism between himself and Paul, the alleged source of Dionysius' angelology. Paul was thought to have journeyed through the heavens while still alive, in similar fashion to Dante the Pilgrim. (Dante

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[75] Dante, *Paradiso*, xxviii.
[76] Barsella, "Angels and Creation in *Paradiso* 29," S190.
[77] North, *Stars, Minds, and Fate*, 190.

references this parallelism early in the *Inferno*.)[78] In this case, the specific details of Dante's universe are subservient to the story. Dante may not have actually believed in Dionysius' angelic hierarchy, but he adopts it for the sake of its literary and theological implications. Here, Dante combined Christianity and Aristotelianism to solve a metaphysical problem of medieval cosmology. The following problems are of a more physical nature.

The Dark of the Moon

In the *Convivio*, Dante dismisses the dark spots on the Moon as being areas of lower density.[79] The understanding is that the Moon is composed of some substance that reflects sunlight back toward the Earth. Because this substance is rarer in the dark spots, light is not reflected. In *Paradiso*, Dante the Pilgrim suggests this same explanation but is refuted by Beatrice. She argues that, if the dark spots were such that they did not reflect sunlight, then it must be the case that they permit sunlight to pass through unobstructed. Thus, during a solar eclipse, one would expect to see the Sun through the Moon's dark spots. Clearly, this is not the case.[80]

Beatrice then preempts Dante's counterargument, which is to suggest that the dark spots are not rare all the way through, but are only rare to a certain depth. The understanding here is that sunlight penetrates into the dark spots but is then reflected at some point deeper within the Moon. Because the sunlight is being reflected at a farther point, it appears darker than the sunlight reflected at the Moon's surface. Beatrice uses a simple experiment to argue that this interpretation is also flawed. Holding a candle and

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[78] Dante, *Inferno*, ii 31-33.
[79] Dante, *Convivio*, II viii 9.
[80] Dante, *Paradiso*, ii 58-81.

placing a series of mirrors at staggered distances, one finds that the reflection of the candle has the same brightness in each of the mirrors (although the image will appear smaller in the mirrors that are farther away). Thus, the brightness of reflected light is not altered by distance. It therefore cannot be the case that the dark spots are caused by rarity.[81]

Instead of a physical solution to the problem, Beatrice offers a theological answer. The planets reflect God's light according to the blessedness of their corresponding angelic order. The Moon is moved by the lowest class, the Angels; therefore it is least perfect in its reflection of God's light. This, and not rarity or density, causes the Moon to have dark spots.[82] Once again, the physics of Dante's cosmology is subservient to his theology. Interestingly, although Beatrice's solution is hardly scientific, she uses scientific experimentation to counter the Pilgrim's theory. Dante the Poet imposes a theological answer on to a physical problem, yet by no means has he ignored the physical character of that problem. Rather than undermine scientific thinking, he hopes to demonstrate, much like Aquinas, that science and theology are compatible. Furthermore, Beatrice's theory of the Moon incorporates and converges with the angelology of the *Divine Comedy*, which itself harmonizes Christian theology and Aristotelian metaphysics.

Hell, Purgatory, and Earth

The most obvious change that Dante makes to the medieval universe is his interpretation of Hell and Purgatory as physical places on Earth. This notable feature of the *Divine Comedy* is not to be found in the *Convivio*, nor does it appear in contemporary

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[81] Dante, *Paradiso*, ii 85-105.
[82] Dante, *Paradiso*, ii 127-148.

scholarship. Indeed, it demonstrates Dante's imposition of theology onto questions of cosmology and geography, in order to harmonize scientific and theological truth. In addition, Dante's Earth is constructed so as to solve one of the more difficult quandaries of medieval cosmology: the nature of the Earth's elemental spheres and their center(s) of mass. Thus, although Dante's universe is primarily driven by its author's theology, it is designed to resolve rather than to ignore physical problems.

Dante's model of the Earth, like that of his contemporaries, is divided into two hemispheres. The hemisphere of earth is comprised of the known continents (Europe, Africa, and Asia) and has Jerusalem at its center. The hemisphere of water refers to the oceans covering the rest of the globe. Beneath the hemisphere of earth lies a vast cavern, conical in shape. The cave is widest just below the Earth's surface and tapers down to a point that coincides with the center of the Earth. This is Hell. On the other side of the Earth, directly opposite Jerusalem, a massive mountain thrusts upward from the sea and pierces the sky. This is Purgatory. The peak of Mount Purgatory, the highest point on Earth, is home to the Earthly Paradise (the Garden of Eden). The tip of Hell, the lowest point in the universe, is where Satan is confined.

There are numerous physical as well as theological implications resulting from the shape of Dante's Earth. First, Satan is imprisoned at the center of the Earth, which is the lowest point in the universe. This is appropriate as there is no deeper place into which he can fall. A closer reading, however, reveals another important detail. In the last canto of *Inferno*, Dante and Virgil must climb down Satan's body in order to escape from Hell. At "the point at which the thigh | revolves," the two are forced to reorient themselves so that they are ascending rather than descending.[83] In that moment, they have passed the center point of the Earth, the point "to which, from every part, all weights are drawn."[84] The center of the Earth is not just a general area; it is an exact identifiable point, implied to roughly coincide with Satan's pelvis. His upper body is oriented one way, and his lower body is oriented the other. This places him in a state of perpetual disorientation, which can also be described as a literal falling into himself. Satan is not just fallen; he is eternally falling. It is simply that there is nowhere further for him to fall.

Another unique characteristic of Dante's Earth is that Hell and Purgatory have physical locations. This seems to imply that a living person could travel to these locations. Indeed, this is explicitly mentioned when Dante the Pilgrim encounters Ulysses, and the latter explains the circumstances of his demise:

> Therefore, I set out on the open sea | with but one ship and that small company… At night I now could see the other pole | and all its stars; the star of ours had fallen | and never rose above the plain of the ocean… when there before us rose a mountain, dark | because of distance, and it seemed to me | the highest mountain I had ever seen. | And we were glad, but this soon turned to sorrow, | for out of that new land a whirlwind rose | and hammered at our ship, against her bow… until the sea again closed – over us.[85]

Ulysses and his men are able to sail to within eyesight of Purgatory, confirming the physical aspect of its existence. However, the spiritual aspect of its existence must remain intact, and Ulysses must perish before he can reach the island's shores. Although

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83 Dante, *Inferno*, trans. Mandelbaum, xxxiv 76-77.
84 Dante, *Inferno*, trans. Mandelbaum, xxxiv 111.
85 Dante, *Inferno*, trans. Mandelbaum, xxvi 100-142.

Purgatory is a physical place, it remains inaccessible to the physical bodies of men. Dante's placement of the Earthly Paradise at the peak of Mount Purgatory is therefore particularly appropriate as it is the most inaccessible place on the Earth.[86] The theological implications are just as clear. Ulysses, unlike Dante and Virgil, travels by his own merit and without any sort of divine sanction. His story shows that, without God's grace, even the best of men fail to reach Purgatory and eventually Paradise.

The Earth of the *Divine Comedy* is designed to synthesize cosmological and theological truth. This can be seen in Satan's placement at the center of the Earth (the lowest point in the universe) and in the physical nature of Hell and Purgatory. This theme is even further demonstrated by Virgil's account of how the Earth came to be shaped as such. Upon passing through the center of the Earth and crossing into Earth's opposite hemisphere, Virgil explains that,

> This {the hemisphere of the sea} was the side on which he {Satan} fell from Heaven; | for fear of him, the land that once loomed here | made of the sea a veil and rose into | our hemisphere; and that land which appears | on this side {Mount Purgatory} – perhaps to flee from him - | left here this hollow space {Hell} and hurried upward.[87]

Once again, a physical phenomenon is given a theological explanation. As Satan was cast out of the heavens and fell toward Earth, he scared Earth's land into gathering on the opposite hemisphere. This explains why there is a hemisphere of earth and a hemisphere of water, instead of an even surface distribution of both elements. In addition, Satan's

[86] Dante, *Purgatorio*, xxviii.
[87] Dante, *Inferno*, trans. Mandelbaum, xxxiv 121-126.

descent causes an amount of earth to rise up out of the ground and away from him. This land becomes Mount Purgatory, and the gap that it leaves behind becomes Hell.

Through his explanation of the Earth's geography, Dante defends Aristotle on two separate points. First, he preserves a teleological understanding of Hell and Purgatory. Both of these places are, in some capacity, defined by sin. The purpose of Hell is to hold the souls of unsaved sinners, and the purpose of Purgatory is to sanctify the souls of the righteous from their sins. From a teleological understanding, it would be inappropriate for either of these places to exist in a world without sin. Thus, in the universe of the *Divine Comedy*, Hell and Purgatory do not come into existence until Satan's rebellion introduces sin into the world. Aristotelian philosophy is therefore made to converge with Christian theology.

The second point on which Dante's Earth defends Aristotle is in regard to the question of Earth's elemental spheres and their center(s) of mass. Aristotle had envisioned an ideal model, in which a sphere of earth was enveloped by a sphere of water. In order to explain why the Earth's surface features land and water, he contended that the spheres overlapped near the Earth's surface. The medievals, however, were far from certain. They debated about whether or not earth and water were actually two separate spheres and, if they were, whether or not they were concentric. The existence of Earth's hemispheres was better explained by the nonconcentric theory, but the concentric theory ensured that Earth had one center of mass for all of its spheres. A careful analysis of Dante's Earth reveals how it beautifully resolves this problem while preserving the integrity of Aristotle's model.

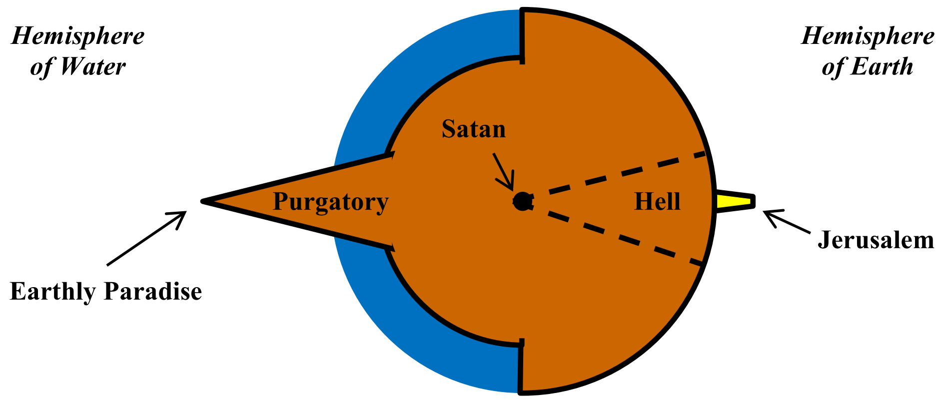

Figure 2.1 – Dante's Earth, altered by Satan's fall.

It is implied that prior to Satan's fall, the Earth was shaped as Aristotle envisioned – featuring separate but concentric spheres of earth and water. Presumably there would be, as Aristotle imagined, some degree of overlap near the Earth's surface, and this would cause earth and water to be evenly distributed along the surface. Furthermore, the two spheres would share a common center of mass, located near the Earth's geometric center. Satan's fall changes the Earth's geography by hollowing out the cavern that will become Hell, raising the mountain that will become Purgatory, and displacing Earth's surface lands toward Jerusalem. The sphere of water is unaffected, and it retains its shape and center of mass. By contrast, the sphere of earth is drastically altered.

As the Earth's lands are gathered toward Jerusalem, Earth's center of mass also shifts toward Jerusalem. The beauty of the model is that this shift is then offset by the creation of Purgatory and Hell (which shift the center of mass back away from Jerusalem). Although the sphere of earth loses its spherical shape, its center of mass is allowed to remain more or less in place. This also means that the Earth does not have to move in order to keep its center of mass properly situated at the center of the universe.

Dante's model therefore combines the advantages of both contemporary models. It explains why the Earth's lands are contained to a single hemisphere, and it maintains the condition that the Earth's elemental spheres have a single center of mass. Furthermore, it implies that the Earth, in its original created state, was as Aristotle envisioned it. Although Dante has once again used a theological event to explain a physical phenomenon, his model is physically sound. Once again, Dante has solved a cosmological problem by demonstrating the compatibility of science, philosophy, and theology.

The Empyrean and the Shape of the Physical Universe

Quite possibly, Dante's greatest and yet most subtle cosmological innovation is his depiction of the Empyrean. As previously discussed, the Empyrean was one of the most problematic subjects in medieval cosmology, for it required medievals to address the boundary of the spatial universe. Indeed, it seemed that the universe could not have a physical boundary, for this would imply that a person could fall out of the universe into nothingness (similar to the idea of falling off the edge of a flat world). However, this would also seem to imply an infinite universe, which the medievals rejected emphatically. How could the universe be both finite and unbounded? Dante's answer (as modern scholarship has interpreted) was revolutionary: his universe is hyperspherical in shape. The next two chapters are dedicated to exploring this aspect of the *Divine Comedy* as well as its physical and theological consequences within the story. They will demonstrate that the hyperspherical model of the spatial universe follows the same guiding principle as the rest of Dante's cosmological innovations, which is to resolve a problem of medieval cosmology in a way that demonstrates the basic harmony of science and theology within an Aristotelian context. These first two chapters have attempted to demonstrate that Dante was intentional with his other cosmological innovations. If this is indeed the case, then it is all the more plausible that he intended to depict and understood the consequences of a hyperspherical universe.

CHAPTER THREE

The Topology of a Hypersphere

Before the reader can understand and appreciate the interpretation of Dante's universe as a hypersphere, it is imperative that the reader have some familiarity with the topology of a hypersphere. That will be the focus of this chapter. The goal here is to give the reader a physical understanding of the construct, rather than a purely mathematical one. As such, the use of algebraic equations and analytical proofs will be avoided. Descriptive language and analogy will instead be the primary teaching tools. Particular emphasis will be placed on the methods by which a hypersphere can be constructed. (Here, "constructed" is used loosely. These are methods of abstract visualization, not of exact construction.) To further assist the reader, each method will first be demonstrated using more ordinary shapes: circles and spheres. By understanding the jump in complexity from a circle to a sphere, the reader will be better equipped to understand the leap from sphere to hypersphere.[88]

What is a Hypersphere?

A hypersphere, also called a 3-sphere by standard convention, is loosely defined as being one dimension higher than a sphere. This definition is simplistic (one might

[88] The methods for constructing a hypersphere are taken from Mark Peterson's article *Dante and the 3-Sphere*. His topological approach to visualizing a hypersphere is in keeping with the goal of this chapter. Also of great assistance was Edwin Abbott's fictional work *Flatland*. He too takes a topological rather than analytical approach to visualizing additional dimensions. Other consulted sources were primarily of an analytical character: Henry Parker Manning's *Geometry of Four Dimensions*, Rudolf v.B. Rucker's *Geometry, Relativity and the Fourth Dimension*, D.M.Y. Sommerville's *The Elements of Non-Euclidean Geometry*, I.M. Yaglom's *A Simple Non-Euclidean Geometry and Its Physical Basis*, and Marc Lachièze-Rey and Jean-Pierre Luminet's *Cosmic Topology*.

analogously define a sphere as one dimension higher than a circle), but it will serve as a useful starting point. By the same naming convention, a 2-sphere refers to an ordinary sphere, a 1-sphere refers to a circle, and a 0-sphere refers to a line segment[89]. Indeed, this highlights the dimensional progression between each construct. Furthermore, it points to a shared property, what might be termed sphere-ness, that can be used to create a more complete definition for the hypersphere.

A sphere of radius *R* is defined as the collection of points in a 3-dimensional space that are a distance *R* away from the center point. Likewise, a circle of radius *R* is the collection of points on a 2-dimensional plane that are a distance *R* away from the center. This formalism can even be extended to the 0-sphere, which is defined as the two points on a straight 1-dimensional line that are a distance *R* away from the center point.[90] From this, a universal definition can be derived. *An n-sphere of radius R is defined as the collection of points in (n+1)-dimensional space that are a distance R away from the center point*. The working definition of the hypersphere is appropriately amended to be the collection of points in 4-dimensional space that are a distance *R* away from the center point.

This formalism seems, however, to be inconsistent with the standard naming convention. If a hypersphere exists in 4-dimensional space, then why is it called a 3-sphere instead of a 4-sphere? The answer is that the *n* of an *n*-sphere refers to the dimensionality of the object's surface. A sphere exists in 3-dimensional space, but its surface has a 2-dimensional area. Likewise, a circle encloses a 2-dimensional space, but its surface is simply a curved line. A hypersphere occupies a 4-dimensional space, but its

[89] Peterson, "Dante and the 3-sphere," 1031-1032.
[90] Sommerville, *The Elements of Non-Euclidean* Geometry, 51-53.

surface is a 3-dimensional volume. This distinction is critical with regard to cosmology. When one speaks of a hyperspherical universe, what one really means is that the universe exists on the surface of a hypersphere. Hence, the existence of a fourth dimension is not necessarily implied.[91]

As a further illustration of this point, consider Edwin Abbott's *Flatland*. In his fictitious novel, there is a flat world occupied by 2-dimensional people. These people have no concept of up or down, only of the four cardinal directions.[92] Now suppose that this flat 2-dimensional world is stretched and curved into the shape of a sphere. The world is no longer flat, but the 2-dimensionality of the world is not violated. The residents of Flatland are still restricted to motion along the four cardinal directions. Indeed, there are other consequences of the resulting curved space, but in general the existence of an n-sphere does not necessitate the existence of (n+1) dimensions. It does, however, necessitate the existence of n dimensions; hence the naming convention is appropriate.

Before continuing, one common misconception must be dispelled. This misconception arises from the description of the hypersphere as occupying 4-dimensional space. Often, "4-dimensional space" is taken to be synonymous with "space-time," whereby there are 3 spatial dimensions and 1 temporal dimension. These two spaces, however, are not identical. The n-sphere occupies (n+1) spatial dimensions and results in a surface geometry that is elliptical.[93] The addition of a temporal dimension carries with it certain restrictions. First, time can only move in one direction. Second, objects moving

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[91] Sommerville, *The Elements of Non-Euclidean Geometry*, 199-201.
[92] Abbott, *Flatland*.
[93] Manning, *Geometry of Four Dimensions*, 59-62. Sommerville, *The Elements of Non-Euclidean Geometry*, 89-90.

through space-time cannot exceed the speed of light. These restrictions cause space-time to be conical in shape.[94] Hence, any reference to "4-dimensional space" should be read as "four spatial dimensions" and not as "three spatial dimensions plus time."

The Method of Cross Sections

Suppose that one wanted to view a sphere in 2-dimensional space. Following the thought experiment put forth in *Flatland*, suppose that a sphere wanted to show itself to the residents of Flatland. How might it do this? Furthermore, what exactly would the residents of Flatland see? In the story, the sphere accomplishes this by passing through Flatland's 2-dimensional plane. The flat onlooker first sees a single point (at the moment when the sphere is tangent to Flatland). From this point, a circle expands outward (as the sphere passes through Flatland). The circle reaches a maximum size (at the moment when Flatland perfectly bisects the sphere) and then begins to diminish. The circle shrinks back to a single point (at the moment when the sphere's opposite pole is tangent to Flatland) and then vanishes.[95]

This thought experiment perfectly illustrates that the parallel cross sections of a sphere are circles. One can therefore project a sphere on to a 2-dimensional space as a series of concentric circles. Similarly, the parallel cross sections of a circle are line segments. In general, *the parallel cross sections of an n-sphere are a series of (n-1)-spheres*[96]. Later, this general formula will be applied to the construction of a hypersphere.

The weakness of this model is that it is temporally dependent. It requires the viewer to recognize the series of circles as a time lapse, rather than as existing

[94] Rucker, *Geometry, Relativity and the Fourth Dimension*, 57-67 and 100-116.
[95] Abbott, *Flatland*, 76-78.
[96] Peterson, "Dante and the 3-sphere," 1031-1032.

simultaneously. One might imagine a flip book with each successive page containing the next circular cross section. By flipping through the book, one sees the sphere mapped out in two dimensions. The sphere's true nature is obscured, however, if all of the circular cross sections are superimposed so as to be viewed simultaneously.

The Method of Suspension

The second method of constructing an *n*-sphere involves building upward from a single (*n*-1)-sphere. By this method, the (*n*-1)-sphere is "suspended" from two external anchor points. The resulting construct is topologically an *n*-sphere.[97] This method will now be demonstrated in two ways: 1) with the construction of a circle from a line segment and 2) with the construction of a sphere from a circle.

The 0-sphere (a line segment) serves as the starting base. Two anchor points are placed, one above and one below. Both anchor points are then connected to the surface of the 0-sphere. (The "surface" of a 0-sphere is the endpoints of the line segment.) The resulting construct (Figure 3.1) is topologically a circle.

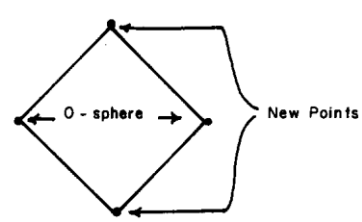

Figure 3.1 – Construction of a 1-sphere by suspending a 0-sphere from two anchor points.[98]

The newly constructed 1-sphere now serves as the starting base for the construction of the 2-sphere. Once again, two anchor points are placed, one above and

[97] Peterson, "Dante and the 3-sphere," 1032.
[98] Peterson, "Dante and the 3-sphere," 1032.

one below. And once again, both anchor points are connected to the surface of the 1-sphere. (The "surface" of a 1-sphere is the circumference of the circle.) The resulting construct (Figure 3.2) is topologically a sphere.

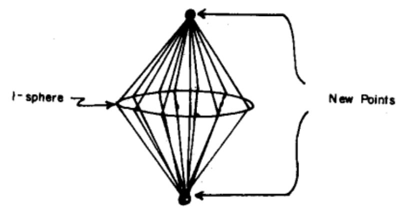

Figure 3.2 – Construction of a 2-sphere by suspending a 1-sphere from two anchor points.[99]

One might object to the claim that Figure 3.2 is topologically spherical (and likewise to the claim that Figure 3.1 is topologically circular). A simple thought experiment is provided so that the reader might better understand what is meant by "topologically circular/spherical." Suppose that a Traveler is surveying the 1-sphere pictured in Figure 3.1. The solid lines represent the allowable paths along which the Traveler can move. If he starts at the upper anchor point, he can either travel clockwise or counter-clockwise. Either route will cause him to arrive at the opposite anchor point. In addition, if he were to continue travelling in the same direction for long enough, he would eventually return to his starting point. This is motion along the circumference of a circle.

Similarly, if the imagined Traveler was to survey the 2-sphere pictured in Figure 3.2, he might once again begin at the upper anchor point. From there, he can proceed along any heading. Every route leads directly to the opposite anchor point and eventually back to the starting point. This is motion along the surface of a sphere. This should not be

[99] Peterson, "Dante and the 3-sphere," 1032.

too foreign to the reader, for it is analogous to travel along the Earth's surface. This thought experiment also illustrates the elliptical geometry of spherical surfaces. Later, it will be shown how the hypersphere can be constructed by "suspending" the 2-sphere from two anchor points. Furthermore, the consequences of elliptical space will be further explored.

The Method of Joining Hemispheres

The final method of constructing an *n*-sphere involves the joining of two (*n*-1)-spheres, each of which can be thought of as a single "hemisphere."[100] To demonstrate, the method will first be used to construct a circle from two line segments. Then, it will be demonstrated by constructing a sphere from two circles.

As pictured below in Figure 3.3, two 0-spheres will serve as the foundation. The endpoints are labeled *A*, *B*, *C*, and *D*.

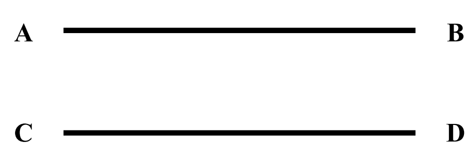

Figure 3.3 – Two 0-spheres. These will be joined as the two hemispheres of a 1-sphere.

The next step is difficult to visualize: the 1-sphere is constructed by treating the two 0-spheres as tangent at all points along their surfaces. (Recall that the "surface" of a 0-sphere is the two endpoints of the line segment.) Thus, the 1-sphere is formed by treating points *A* and *C* as connected and points *D* and *B* as connected. It is incorrect to simply draw lines connecting *A* to *C* and *D* to *B* (as is done below in Figure 3.4). Rather, one

[100] Peterson, "Dante and the 3-sphere," 1032-1033.

must treat the endpoints as overlapping. This can be accomplished by curving the lines toward each other, but this is only partially correct. It is more precise to describe a curving of space into an elliptical shape. This is demonstrated below in Figure 3.5, wherein the blue field lines denote the space containing the 0-spheres. Topologically, the resulting configuration is a circle. Furthermore, it is easy to see how the 0-spheres act as literal hemispheres for the resulting 1-sphere.

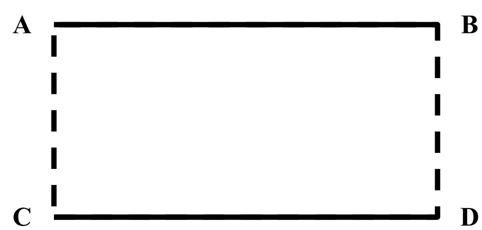

Figure 3.4 – An incorrect way of visualizing the method of joining hemispheres.

At this point, the method of joining hemispheres can be readily applied to the construction of a 2-sphere. This time, two 1-spheres will serve as the foundation. As before, it must be that the two 1-spheres are tangent at every point along their surfaces. (The "surface" of a 2-sphere is the circumference of the circle.) This of course requires a similar curving of space, such that each circle takes on a bowl-like shape. The two hemispheres are then attached at their rims (the circumferences of the circles). The ring on which the two hemispheres meet is the equator of the resulting 2-sphere. (This process is not pictured.) One can also observe yet another general rule: that *an n-sphere is comprised of two (n-1)-spheres, each acting as a single hemisphere, joined at all points along their surfaces*. By applying this rule to the hypersphere, one will see that the 3-sphere is comprised of two 2-spheres, joined along their surface areas. In the following section, each of the three methods will be applied toward the construction of the hypersphere.

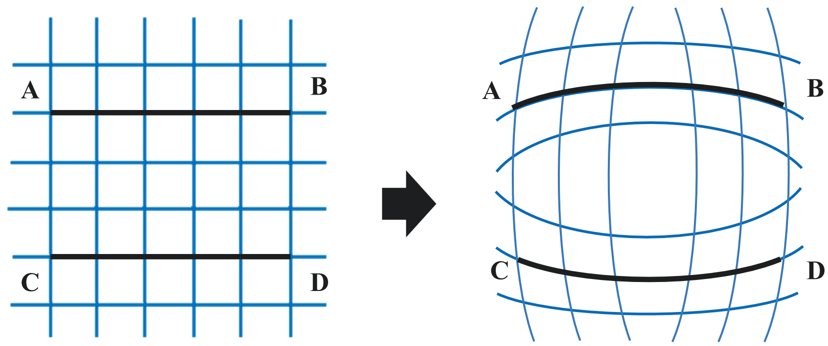

Figure 3.5 – This shows how it is space, and not the 0-spheres, that is bent. In fact, the line segments remain "straight" within the space around them. Further curvature would eventually produce the desired 1-sphere.

Applying the Methods of Construction to the Hypersphere

The discussion of the first method yielded the general rule that an *n*-sphere has parallel cross sections that are (*n*-1)-spheres. Thus, the parallel cross sections of a hypersphere are spheres. A hypersphere can then be visualized in 3-dimensional space as a series of concentric spheres, expanding outward from a point, reaching a maximum sized sphere, and then receding back to a single point.[101] As noted before, this model is temporally dependent, but it also does not require any curving of space.

According to the second method, an *n*-sphere can be constructed by "suspending" an (*n*-1)-sphere from two anchor points. With a 2-sphere as the foundation, the question becomes where to place the two anchor points. The solution is to place one anchor point at the center of the sphere and the second anchor point somewhere outside the sphere.[102] The result (shown in Figure 3.6) appears to resemble a teardrop. This should not be

[101] Peterson, "Dante and the 3-sphere," 1031-1032.
[102] Peterson, "Dante and the 3-sphere," 1032.

thought of as the actual shape of a hypersphere. As was the case with the 1-sphere and the 2-sphere, the constructed figure is only accurate in a topological sense. It does, however, reveal an important relationship between the two anchor points. Each anchor point is the farthest point away from the other. (Again, consider the analogy of the Traveler who is restricted to motion along the solid lines.) This can likewise be observed in Figures 3.1 and 3.2.

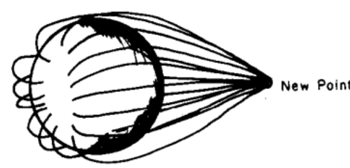

Figure 3.6 – Construction of a 3-sphere by suspending a 2-sphere from two anchor points.[103]

Further consideration of the Traveler analogy will even reveal another property of the anchor points. Suppose that the Traveler is positioned at the anchor point inside the sphere. (This designation is completely arbitrary. One must realize that the anchor points are in fact symmetrical.) Suppose also that he has infinite vision. In every direction that he looks, he will see the opposite anchor point. In fact, he might deduce that he is at the center of a sphere whose boundary is comprised of an infinitude of identical points (the opposite anchor point).[104] This leads to the rather poetic interpretation that each anchor point forms the boundary of a sphere, the center of which is the other anchor point. More succinctly: each anchor point is “both center and circumference”.[105]

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[103] Peterson, “Dante and the 3-sphere,” 1032.
[104] Egginton, “On Dante, Hyperspheres, and the Curvature of the Medieval Cosmos,” 214.
[105] Mazzotta, “Cosmology and the Kiss of Creation,” 9.

The final method of constructing a hypersphere involves the joining of two spheres along their surface areas.[106] It is quite simple to imagine two spheres that are tangent at a single point; it is quite another thing to suppose that the two spheres are tangent at every point along a mutual boundary. Once again, it will be helpful to return to the analogy of the Traveler. This time, suppose that the Traveler is inside sphere *A*. He reaches the boundary of sphere *A* and, upon passing through it, finds himself inside sphere *B*. The two spheres must therefore have been tangent at the point where he crossed between them. To capture the image of the spheres as being tangent at every point, one might imagine that the spheres roll alongside each other so that they are always tangent at the point closest to the Traveler. Thus, wherever and whenever the Traveler attempts to exit sphere *A*, he will invariably find himself inside sphere *B*. He is free to travel between the two hemispheres, but he can never exit the construct.

For an interpretation that is not temporally dependent, one might imagine that sphere *B* is turned inside out and then overlaid onto the surface of sphere *A* (as below in Figure 3.7). In adopting this interpretation, one must not forget that the two hemispheres are symmetrical. Both spheres are simultaneously the interior sphere.

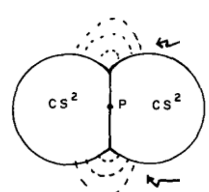

Figure 3.7 – Construction of a 3-sphere by joining hemispheres.[107]

[106] Peterson, "Dante and the 3-sphere," 1032-1033.
[107] Peterson, "Dante and the 3-sphere," 1032.

As in the other models, if the Traveler moved in a single direction for long enough, he would eventually return to his starting position. This model also reveals two new properties of the hypersphere. First, just as the equator of a sphere is the circle at which the two hemispheres are joined, the "equator" of a hypersphere is the sphere at which the two halves are joined. Second, the surface of a hypersphere has a finite volume (twice the volume of each sphere), but it has no physical boundary. It is analogous to the surface of a sphere, which has a finite surface area but no endpoints or edges.[108]

This chapter was by no means designed to make the reader an expert on the hypersphere. Indeed, much information was omitted for the sake of simplicity. The reader may still struggle to visualize all three models simultaneously. It is more important however that the reader understand each of the individual construction methods so as to be able to recognize them as they appear in Dante's *Divine Comedy*. Furthermore, the reader should recognize the basic properties of the hypersphere, as illustrated by the models. At this point, the reader should be well enough equipped to interpret Dante's universe as the surface of a 3-sphere.

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[108] Kuusisto, "The Limits of Geometry in the *Convivio* and their Inversion in the *Comedy*," 188.

CHAPTER FOUR

Dante and the Hypersphere

The interpretation of Dante's universe as a hypersphere was originally proposed by Andrea Speiser in his 1925 book *Klassische Stücke der Mathematik*.[109] Although the theory has gained sizeable recognition within mathematical scholarship, it has continued to receive only "marginal" exposure among Dante scholars.[110] This chapter will explore Speiser's theory by citing its evidence within the text of the *Divine Comedy*, explaining its physical and theological implications for the story, and addressing the most common criticism against the theory. Ultimately, this chapter not only suggests that the hypersphere model is a spatially accurate representation of Dante's universe, but also that Dante may have intended such an interpretation. Intentionality is impossible to definitively prove, but it will be argued that prior scholarship has been too quick to dismiss the possibility. There is reason to believe that the hypersphere interpretation is not as modern as it seems – that Dante was the first to untie this knot. Furthermore, his cosmological discovery may have also led him to an equally important mathematical discovery: elliptical non-Euclidean space.

Interpreting Dante's Universe as a Hypersphere

The previous chapter introduced three different methods of topologically constructing a hypersphere. This section will show how Dante employs each of these

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[109] Speiser, *Klassische Stücke der Mathematik*, 53-54.
[110] Kuusisto, "The Limits of Geometry in the *Convivio* and their Inversion in the *Comedy*," 187-188.

methods in the poetry of the *Divine Comedy*. The first method, the method of cross sections, is the most obvious within the story. In order to reach the Primum Mobile, Dante must ascend through a series of concentric spheres of increasing size (the planetary spheres and the sphere of the Fixed Stars). Upon reaching the Primum Mobile, Dante looks upward into the Empyrean and observes,

> A point that sent forth so acute | a light… Around that point a ring of fire wheeled, | a ring perhaps as far from that point as | a halo from the star that colors it… That ring was circled by a second ring, | the second by a third, third by a fourth, | fourth by a fifth, and fifth ring by a sixth. | Beyond, the seventh ring… The eighth and ninth were wider still.[111]

The Empyrean is not simply a tenth concentric sphere; it is another series of nine concentric spheres that parallels the physical universe.[112] In this *Empyrean* universe, travel toward God (the singular point of light) is through a series of concentric spheres of decreasing size. The Primum Mobile is then the largest of the spheres, the one which "no other heaven measures."[113] It may seem objectionable to speak of the angelic "rings" as spheres[114], yet Dante defends this interpretation in one of his footnotes to the *Convivio*. He clarifies that "[he uses] the word circle in a broad sense, to refer to anything that is round, whether a solid or a plane."[115] He thus does not distinguish between a 1-sphere and a 2-sphere. They are conceptually the same.

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[111] Dante, *Paradiso*, trans. Mandelbaum, xxviii 16-34.
[112] Mazzotta, "Cosmology and the Kiss of Creation," 7.
[113] Dante, *Paradiso*, trans. Mandelbaum, xxvii 115.
[114] Freccero, *Dante's Cosmos*, 8-9.
[115] Dante, *Convivio*, trans. Ryan, II xiii 26.

Despite the change in character from expanding to receding spheres, Dante's travel toward God continues to be an ascent, for God must retain his position as the highest point in the spatial universe. Thus, as Dante ascends upward from the lowest point in the universe, he encounters a singular point (Satan), then a series of expanding spheres (the planets and stars), then the sphere of maximum size (the Primum Mobile), then a series of receding spheres (the angels), and finally a second singular point (God).[116] This model perfectly represents the method of cross sections.

In the aforementioned model, God and Satan act as opposite poles of the universe, analogous to the Earth's north and south poles. In the second model, the method of suspension, God and Satan play a similar role. They are the anchor points from which the 2-sphere is suspended. The 2-sphere in question is the Primum Mobile, the equator of the universe.[117] Dante's descriptions of God, Satan, and the Primum Mobile reinforce this interpretation. Recall that Dante identified Satan as coinciding with a singular point, the bottom of the universe. God too is identified with an indivisible point as Beatrice says of him: "on that Point | depend the heavens and the whole of nature."[118] Offsetting Satan's position at the bottom of the universe, God occupies the highest point of the universe. Indeed, the analogy of the north and south poles is appropriate. Of the Primum Mobile, Beatrice states that "time has roots within this vessel and, | within the other vessels, has its leaves."[119] Time appears as the medium of suspension, connecting the Primum Mobile to its two exterior anchor points. This is appropriate, for traveling through the universe requires one to travel through time as well as space.

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116 Peterson, "Dante and the 3-sphere," 1033.
117 Peterson, "Dante and the 3-sphere," 1033-1034.
118 Dante, *Paradiso*, trans. Mandelbaum, xxviii 41-42.
119 Dante, *Paradiso*, trans. Mandelbaum, xxvii 119-120.

The final construction method involves the joining of two hemispheres. In Dante's model of the universe, the planetary spheres and Fixed Stars (with Satan as their focal point) comprise one hemisphere, and the Empyrean (with God as its focal point) comprises the other.[120] The two hemispheres meet at the Primum Mobile. This relationship between the two hemispheres is displayed when Dante reaches the Primum Mobile. He first looks downward upon the spheres he has already visited.[121] Then, turning his attention upward, he sees the angelic spheres which comprise the Empyrean.[122] Dante also observes of the Primum Mobile that "its parts were all so equally alive | and excellent, that [he] cannot say which | place Beatrice selected for [his] entry."[123] The Primum Mobile is uniform in such a way that it does not matter at which point Dante and Beatrice attempt to cross it. At any point along this ninth sphere, Dante can look downward on the planetary spheres and upward on the angelic spheres.[124] As Peterson observes, this indicates that the angelic spheres "are not just off to one side of the Primum Mobile but surround it."[125] The two hemispheres of Dante's universe must be tangent at every point along the Primum Mobile, perfectly adhering to the method of joining hemispheres. (See Figure 4.1)

As a final piece of evidence, Dante's universe features a fourth spatial dimension. Although not explicitly named, it is observed in the fact that Dante's entire journey from Satan to God is an ascent. This fourth dimension might then be termed cosmic height. In the story, this notion of cosmic height is associated with blessedness, with higher spheres

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[120] Peterson, "Dante and the 3-sphere." 1034.
[121] Dante, *Paradiso*, xxvii.
[122] Dante, *Paradiso*, xxviii.
[123] Dante, *Paradiso*, trans. Mandelbaum, xxvii 100-102.
[124] Egginton, "On Dante, Hyperspheres, and the Curvature of the Medieval Cosmos," 199.
[125] Peterson, "The Geometry of Paradise," 16.

being holier than their lower counterparts.[126] Dante also observes a physical manifestation of this notably abstract concept. He notes that, among the planets and stars, larger spheres move with greater speed, but among the Empyrean's angels, smaller spheres move with greater speed.[127] Thus, as he ascends through the universe, he sees ever increasing speed among the heavenly spheres.[128] Although he does not explicitly identify his fourth spatial dimension, he is clear that it has both a theological and a physical component.

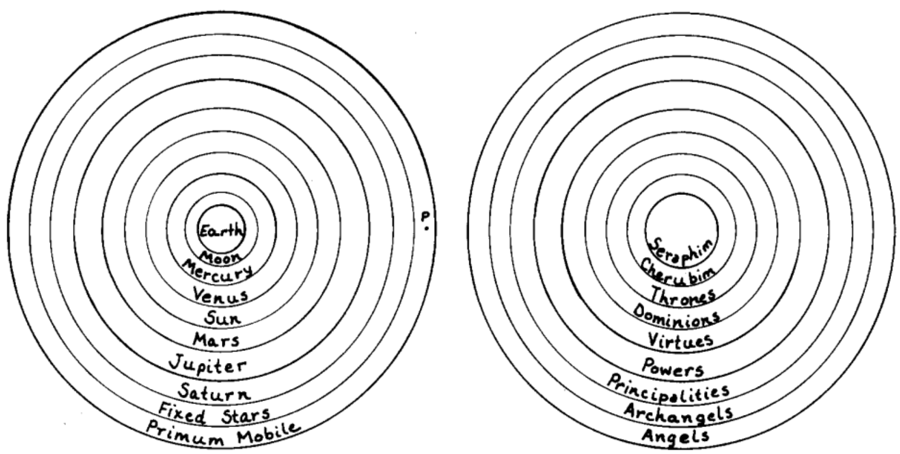

Figure 4.1 – A map of Dante's universe, as described in *Paradiso*. It begins at a single point (Earth), expands outward as a series of spherical cross sections, then recedes back to a second anchor point (God). This is the method of cross sections. The Primum Mobile acts as the equator of the universe. Just as the Earth's equator is its largest ring of latitude, the Primum Mobile is the universe's largest sphere. The map also illustrates the method of joining hemispheres. The left image illustrates the earthly hemisphere, and the right image illustrates the Empyrean hemisphere. According Dante's description in *Paradiso*, the two hemispheres must be tangent at all points along their surfaces. Specifically, the Primum Mobile must at all points be tangent to the sphere of the Angels. Thus, the Primum Mobile acts as the universe's equator in a second sense: it joins the two halves of the universe.[129]

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[126] Dante, *Paradiso*, xxviii 66-69.
[127] Dante, *Paradiso*, xxviii 22-78.
[128] Peterson, "Dante and the 3-sphere," 1033.
[129] Peterson, "Dante and the 3-sphere," 1034.

Consequences of a Hyperspherical Universe in the Divine Comedy

As in the case of Dante's other cosmological innovations, his creation of a hyperspherical universe results in numerous physical, philosophical, and theological implications. It resolves a specific problem of medieval cosmological thought (the problem of the edge of the universe) in a way that harmonizes science and theology. These appear to have been Dante's conscious objectives with regard to his other cosmological innovations. If that is indeed the case, then there is reason to believe that the creation of a hyperspherical universe was equally intentional.

The first consequence of Dante's universe is that it is both finite and unbounded. It has finite volume, in keeping with the teachings of Aristotle; however it does not have any sort of spatial boundary where a person can fall out of the universe.[130] This allows one to reconcile Aristotelian cosmology with a physical understanding of space. Furthermore, it highlights God's omnipotence by allowing that His created universe is pseudo-infinite. If the universe was truly infinite, it would seem to rival God's infinitude; however, a finite and bounded universe cannot be thought to physically exist. Part of the beauty of Dante's universe is that "only He who encloses understands [it]."[131] Our natural bias toward 3-dimensional Euclidean space inhibits us from a God-like understanding. This gives God's created universe an element of mystery, and it serves for Dante as a reminder of God's infinite majesty.

The placement of God and Satan as opposing poles or anchor points evokes a significant level of theological meaning. In one interpretation, God is the center of the universe, and Satan is the absolute periphery. The universe behaves "like | a wheel

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[130] Mazzotta, "Cosmology and the Kiss of Creation," 9.
[131] Dante, *Paradiso*, trans. Mandelbaum, xxvii 114.

revolving uniformly"[132] with God as its center point. This illustrates well the Aristotelian interpretation of God as the universe's Unmoved Mover, for the center point of a rotating wheel does not move, but causes the rest of the wheel to rotate about it.[133] In the opposing interpretation (which treats Satan as the center of the universe), God occupies the circumference of the universe.[134] This creates an image of the universe as literally existing within God's embrace. God's paradoxical ability to both occupy the center of the universe and contain the universe within His presence displays His characteristic omnipresence. Thus, Dante uses the topology of his universe to rationalize both an Aristotelian metaphysical understanding of God (as the Unmoved Mover) and a Christian theological understanding of God (as omnipresent).

In both interpretations of God's physical position (as the center and as the circumference), Satan occupies the exact opposite point. Thus, Satan is always the farthest point in the universe away from God. What is particularly noteworthy is that God and Satan mirror each other. Satan can be interpreted in the same way as God, as both the center and the circumference of the universe. There is a bitter irony at play here, in keeping with the *Divine Comedy*'s theme of *contrapasso* (counter-pass).[135] Satan attempted to usurp God's position and to become like God, yet his punishment is to be given exactly this. Satan is made to occupy an almost identical position to that as God, but it is a completely different experience for one who is not an Unmoved Mover. Dante's fourth dimension manifests itself physically as a measure of mobility. God, as

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[132] Dante, *Paradiso*, trans. Mandelbaum, xxxiii 143-144.
[133] Flosi, "Geometric Iconography in the *Commedia*," 39.
[134] Freccero, *Dante's Cosmos*, 9.
[135] *Contrapasso* refers to the phenomenon by which a punishment mirrors the crime or is an inversion of the crime. This theme is prevalent in the *Divine Comedy*. For example, in *Inferno* xx, the soothsayers who claimed to be able to see ahead into the future have their heads turned backward. See Kenneth Gross' "Infernal Metamorphoses" for a more detailed study on *contrapasso* as it appears in the *Divine Comedy*.

the highest point in the universe, represents unmatched mobility. Satan, as the lowest point in the universe, is incapable of movement.[136] God, like the center of a wheel, moves the universe about Him while Satan is trapped in a state of perpetual disorientation. Whereas God's anchor point is a seat of power, Satan's is a prison and a constant reminder of God's superiority.

Dante and Euclidean Geometry

The strongest counterargument against the theory that Dante intended for his universe to be interpreted as a hypersphere is that Dante, limited to Euclidean geometry, could never have conceived of elliptical space. Even proponents of the hypersphere model tend to dismiss the notion that Dante recognized, let alone understood non-Euclidean geometry. Euclid's *Elements* had remained unchallenged as the cornerstone of medieval mathematics, and its axioms precluded the possibility of a hypersphere. Indeed, later scholars such as Kant, Newton, and Leibniz refused to reject Euclidean geometry as they attempted to model the universe.[137] Why should Dante's approach have been any different? As it turns out, Euclid was not Dante's ultimate authority on geometry. Rather than learn geometry from the *Elements* and attempt to build a corresponding cosmological model, Dante appears to have based his ideas about geometry on his cosmological assumptions.[138] Although it would be a far cry to describe Dante's mathematical ideas as any sort of formalized theory, he offers a unique perspective on geometry that may have led him to pioneer an as yet undiscovered branch of mathematics.

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[136] Kuusisto, "The Limits of Geometry in the *Convivio* and their Inversion in the *Comedy*," 200.
[137] Callahan, "The Curvature of Space in a Finite Universe," 90.
[138] Callahan, "The Curvature of Space in a Finite Universe," 99.

In book II of the *Convivio*, Dante offers his thoughts on the seven liberal arts, of which geometry is one. He writes that,

> Geometry moves between two things antithetical to it, the point and the circle: as Euclid says, the point is the primary element in Geometry, and, as he also indicates, the circle is its most perfect figure, and must, therefore, be considered its end. So Geometry moves between the point and the circle as between its beginning and its end; and both of these are antithetical to the certainty characteristic of this science, for the point cannot be measured at all, since it cannot be divided, and the circle cannot be measured precisely, since, being curved, it cannot be perfectly squared.[139]

As was the case with his attempts to cite Ptolemy, Dante misattributes ideas to Euclid. Euclid never described the circle as the "most perfect figure" or as the "end" of geometry. Furthermore, the treatment of the point and the circle as antithetical to geometry is Dante's own idea, not to be found in Euclid's *Elements*. Mark Peterson observes that Dante "essentially ignores Euclid, even as [he] cites him."[140] It is quite possible that Dante did not read or have access to the *Elements*. At the very least, he was particularly uninspired by the work. It is therefore doubtful that Dante shared Kant, Newton, and Leibniz's bias toward Euclid's axioms.

In addition, the passage reveals Dante's fascination with a certain paradox of geometry. He describes geometry as possessing a characteristic certainty, yet he also observes that this certainty breaks down when geometry attempts to measure its "most

[139] Dante, *Convivio*, trans. Ryan, II xiii 26-27.
[140] Peterson, "The Geometry of Paradise," 16-17.

perfect figure." This appears to be a subtle jab at Euclidean geometry - that it fails when presented with its appropriate "end." It also reveals a fascination, not shared by Euclid, with curved (as opposed to linear) constructs. This idea becomes further developed as a motif of the *Divine Comedy*. Curved geometry is used to describe the divine, and linear geometry is used to describe the earthly. Hence, the heavens are spherical, but Hell and Purgatory are conical. Earth was spherical in its original created state, but it was corrupted by Satan's fall and the introduction of sin. Even the Trinity itself is described as a set of three circles.[141] The clearest appearance of the motif is when Dante compares the problem of squaring the circle to the Incarnation, literally the humanizing of the divine.[142] If anything, this seems to indicate Dante's natural bias toward elliptical geometry rather than toward Euclidean geometry.[143]

Further into the *Convivio*'s discussion of geometry, Dante observes a fundamental connection between geometry and "its ancillary science, called Perspective."[144] Both share the characteristic of certainty, at least in theory. In canto xxvii of *Paradiso*, Dante looks downward from beyond the Fixed Stars and is inexplicably capable of perceiving details on Earth's surface.[145] Later, upon entering the Empyrean and encountering the saints and the Celestial Rose, he recalls how,

> Nor did so vast a throng in flight, although | it interposed between the candid Rose | and light above, obstruct the sight or splendor, | because the light of God so

[141] Dante, *Paradiso*, xxxiii 115-132.
[142] Flosi, "Geometric Iconography in the *Commedia*," 32.
[143] Peterson, "The Geometry of Paradise," 16.
[144] Dante, *Convivio*, trans. Ryan, II xiii 27.
[145] Dante, *Paradiso*, xxvii 79-84.

> penetrates | the universe according to the worth | of every part, that no thing can impede it.[146]

These episodes are instances in which the physical science of perspective breaks down. Geometry's "ancillary science" fails when presented with the majesty of God's light, and this seems to hint at a similar weakness of geometry.

Additional jabs at Euclidean geometry are found throughout the *Divine Comedy*, indicating that Dante's sentiments toward the *Elements* have not improved. In canto xiii of *Paradiso*, Dante states that when Solomon asked for wisdom, he did not ask to know "if, within a semicircle, one | can draw a triangle with no right angle."[147] This is as if to say that Euclidean geometric truth does not constitute wisdom.[148] Then, in canto xvii of *Paradiso*, Dante observes how "two obtuse angles cannot be | contained in a triangle."[149] The final and most well-known geometric reference occurs at the end of *Paradiso*, when Dante encounters God. He compares the futility of attempting to describe the Trinity to the futility with which "the geometer intently seeks | to square the circle."[150] This referred to the impossibility of constructing, with a compass and straightedge, a square with the same area as an established circle.

All three of these geometric references are of a distinctly negative character. Each describes a task that is impossible under Euclidean geometry. Amazingly though, each of these geometric tasks – inscribing a non-right triangle inside a semicircle, constructing a triangle with two obtuse angles, and squaring the circle – is allowable under elliptical

[146] Dante, *Paradiso*, trans. Mandelbaum, xxxi 19-24.
[147] Dante, *Paradiso*, trans. Mandelbaum, xiii 101-102.
[148] Peterson, "The Geometry of Paradise," 16.
[149] Dante, *Paradiso*, trans. Mandelbaum, xvii 16-17.
[150] Dante, *Paradiso*, trans. Mandelbaum, xxxiii 133-134.

geometry.[151] Perhaps Dante recognized this, in which case his critique of Euclidean geometry is complete. He reveals specific failures of Euclidean geometry, which he then corrects by using a geometric theory (here "theory" is used very loosely – it would have been minimally formalized) based around his understanding of cosmology and space. The resulting elliptical space would permit these geometric impossibilities and allow Dante to conceive of the universe as existing on the surface of a hypersphere.

Concluding Thoughts

Dante wrote the *Divine Comedy* during a period of changing intellectual culture. The rising influence of Aristotelianism, reintroduced to the West by the Arabic commentators, inevitably conflicted with Christian theological doctrine on certain questions. The tendency of medieval scholars to Christianize ancient Greek scholarship had produced numerous inconsistencies and conundrums in the fields of astronomy and cosmology. Meanwhile, the academic culture of the time, typified by the Condemnations at Paris, gave thinkers like Dante the creative freedom to espouse ideas regardless of their credibility, provided that they acknowledged God's omnipotence.

The Condemnations had been intended to remove pagan and secular influences from Christian teaching, yet Dante (and indeed many other scholars) did not see these as irreconcilable. He used the *Divine Comedy* as a medium for creating a fantastic universe – one that, intentionally or not, resolves certain problems of medieval cosmology in a way that harmonizes Christian theology, Aristotelian philosophy, and observable science. Even if the universe of the *Divine Comedy* is nothing more than a creative fiction, it deserves recognition as a masterful work of art.

[151] Kuusisto, "The Limits of Geometry in the *Convivio* and their Inversion in the *Comedy*," 192.

This thesis has explored the interpretation of Dante's universe as a hypersphere, within the context of Dante's other cosmological innovations. It argues that Dante made a conscious effort to solve certain problems of medieval cosmology and demonstrate the compatibility of Aristotelianism with Christianity. If this is true, then it is conceivable that Dante intended to depict a hyperspherical universe, for such an interpretation accomplishes these objectives. Furthermore, it is apparent that Dante was not wholly content with Euclidean geometry, to the extent that later cosmological thinkers were. While this alone does not imply that he understood any sort of alternative to Euclidean geometry, such a possibility should not be ruled out entirely. Dante exhibited a fascination with curved geometry that possibly led him to conceive of elliptical non-Euclidean space. Although he certainly did not formalize these thoughts into any sort of mathematical theory, a basic topological understanding would have been sufficient for him to recognize the consequences of a hyperspherical universe.

Dante was not a mathematician. His chosen medium was the poem, in which he could describe his ideas with language rather than with equations. Indeed, the universe of the *Divine Comedy* is almost entirely hypothetical, yet Dante surely recognized that he was grappling with difficult cosmological questions. His ability to solve a number of them in a way that represents the poem's deeper theological and philosophical meaning is truly impressive. Although, Dante likely did not think of himself as a cosmologist, he was able to make a significant contribution to the field, even if on accident and even though it went unnoticed for 600 years. If Dante did in fact conceive of the hypersphere and of elliptical geometry, as modern scholars have largely dismissed, then he deserves credit as one of the most talented mathematical thinkers of his age – not as one who possessed a vast technical knowledge of mathematics, but as one capable of unmatched spatial reasoning. Although this can never be definitively proven, this thesis has argued that such a scenario exists within the realm of possibility. Even if it is not the case, and the hypersphere interpretation is simply an anachronism, Dante's creation itself deserves praise for its ability to keep pace with the evolution of cosmological understanding. That Dante's universe could be understood in both the medieval and the modern tradition is a testament to its beauty and to the skill of its creator.

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