# ABSTRACT

## Astrophysical Applications of Dusty Plasma Physics

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An ionized gas is a plasma, a sea of ions and electrons. The introduction of dust into the ionized gas forms a dusty plasma, a colloidal mixture of micron- to submicronsized grains and charged particles. The dust becomes charged and then can be influenced by electric and magnetic fields. This thesis examines the role of dusty plasmas in two separate astrophysical environments: circumplanetary rings, specifically Saturn's F Ring, and protoplanetary disks. Numerical calculations of the orbits of grains in the F Ring are carried out in an effort to better understand the plasma parameters of this poorly understood system. Perturbing forces on a single grain, such as Saturn's magnetic field, radiation pressure, and gravitational interactions with shepherding moons, are calculated and numerical integrations are performed to find the orbital parameters in time. Within protoplanetary disks, fractal dust grains collide and stick, forming small-scale structures believed to be the seeds of planet formation. A numerical model is used to study the impact of charging on these fractal aggregates and how said charging impacts aggregate morphology at various locations within the protoplanetary disk. APPROVED BY DIRECTOR OF HONORS THESIS:

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# ASTROPHYSICAL APPLICATIONS OF

# DUSTY PLASMA PHYSICS

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### CHAPTER ONE

# Introduction

## 1.1 Dust

Interstellar space is a near perfect vacuum, having a particle density of approximately 10<sup>6</sup> m<sup>-3</sup> (Whittet 2002). While this may seem relatively large, in comparison, the atmosphere on earth at sea level is 10<sup>19</sup> times more dense. Thus, it suffices to say that the interstellar medium (ISM), the mixture of gas and dust that occupies interstellar space, is quite diffuse. However, although quite dispersed, dust in interstellar space has played a central role in the development of astronomy over the past several hundred years. When William Herschel first conducted visual surveys of the stars in the 18<sup>th</sup> century, he believed there to be holes in the sky where he observed large black patches. It was not until a few hundred years later that astronomers began to hypothesize that these were not punctures in the celestial sphere, but rather areas that were heavily obscured by dust (Evans 1993).

This of course posed a problem for astronomers attempting to study the chemical composition of stars via radiation received from these distant stellar objects. How were they to know whether the data they were gathering were accurate if it was potentially being affected along the way? This initiated research on dust particles in order to better understand their effects on astronomical observations, rather than to investigate the dust in its own right. It was not until the advent of radio astronomy that astronomers and physicists realized the importance of dust particles in the context of the Universe. Today,

the study of dust particles in many different extraterrestrial environments is a popular topic in astrophysics (Evans 1993).

In the ISM, the majority of the material is composed of oxygen (O), carbon (C), nitrogen (N), magnesium (Mg), silicon (Si), iron (Fe), and sulfur (S) (Sofia *et al.* 1994). This is due to the nuclear processes that occur in the cores of stars, with other more massive elements being produced as the result of supernovae, the violent death that occurs at the end of some stars lifetimes. Using the fact that these elements are the most abundant in the universe, it is thus possible to examine a gas-dust parcel and discern the dust composition based on which of these elements is not in the gaseous phase (Sofia *et al.* 1994). Knowing the size and composition of the dust in question is very important when talking about dust particles in the context of various astrophysical environments. In addition to being dispersed throughout the ISM, dust can be found in circumplanetary rings, such as those around Saturn, and in protoplanetary disks, the sites of planet formation.

# 1.2 Plasma

While micron- and submicron-sized dust particles are a ubiquitous component of the universe, they are without exception, always suspended in some often-tenuous and diffuse gas. Heating via radiation from such sources as x rays or gamma rays can lead to the excitation of electrons to higher energy levels and the subsequent ionization of these particles resulting in the formation of a plasma. A plasma is a sea of charged particles, mainly positively charged ions and negatively charged electrons. A plasma can also be referred to as an ionized gas. In a terrestrial environment, one can see examples of a plasma in fluorescent light bulbs, TV and computer monitors, and lightning. A flame is

also in the plasma state, and plasmas can be created in the laboratory under very specific conditions. However, in an astrophysical context, the plasma state is by far the most common state of matter. It is often said that 99% of the observable universe is in a plasma state. Nobel Laureate Hannes Alfvén famously coined the term *Plasma Universe* to express the dominance of plasma (Verheest 2000). A plasma forms in the interior of a star due to the extremely high temperatures that completely strip all the atoms of their electrons. Another way of stating this is to say that all of the atoms are *ionized*. (It should be noted, however, that a gas need not be *completely ionized* in order to be considered a plasma.) This then leads to a collection of negatively charged electrons and positively charged ions. Stellar interiors and atmospheres, protoplanetary disks, the interstellar medium and supernovae are just a few of the astrophysical environments in which plasmas can be found (Krishan 1999).

Because plasmas are dominated by charged particles, they are sensitive to electromagnetic forces with the motion of charged particles within the plasma leading to the production of electric currents. However, although ionization is a necessary condition for plasmas, an ionized gas is not necessarily a plasma. To meet this criteria, the ionized gas must also exhibit collective behavior as well as quasineutrality (Krishan 1999). To explain these conditions, first consider the Coulomb repulsive (attractive) force between two charged particles,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathcal{F}},\tag{1.1}$$

where  $q_{1,2}$  are the two charges, *r* is the distance between them, and  $\epsilon_0 = 8.854 \times 10^{-12}$  F/m is the permittivity of free space. When two charged particles experience a Coulomb force, there are many other charges surrounding them that also influence these

particles. Every charged particle is affected by nearby charged particles. Thus, there are no free charges in the plasma. All motion is determined *collectively* by the entire system. This is the first condition. As these charges move around in the plasma, charge separation on a microscopic scale can occur, resulting in groups of positively (negatively) charged particles. This results in the formation of electric fields. The fact that these separations can exist for only a very small amount of charge over a very small distance for a very small time is what is defined as *quasineutrality* (Krishan 1999). Mathematically this can be stated as  $n_i + n_e = 0$ , implying equal numbers of ions and electrons in a given volume, where  $n_{i,e}$  are the number densities of the ions and electrons respectively.

Perhaps the most important parameter affecting a plasma is the temperature, as the plasma considered in this work is only thermally ionized. This essentially is a measure of how much radiation heats the gas and thus determines the number of ions and free electrons in the plasma. The degree of ionization manifests itself in the fractional ionization  $X_{ion}$ , and the plasma density  $n_e/n_i$ , the number density of the electrons divided by that of the ions. The fractional ionization can be calculated using the Saha equation given by

$$X_{ion} = \frac{2kTZ_{ion}}{P_e Z_e} \left(\frac{2\pi m_e kT}{h^2}\right) e^{-\chi_e/kT},$$
(1.2)

where k is Boltzmann's constant, T is the temperature,  $Z_{ion,e}$  is the partition function for ions and electrons respectively,  $P_e$  is the electron pressure,  $m_e$  is the mass of an electron,  $\chi_e$  is the ionization energy, and h is Planck's constant (Carroll and Ostlie 2007).

#### 1.2a Dusty Plasma

Thus far, only plasmas containing two species, ions and electrons, have been discussed. However, in astrophysical environments, this is almost never the case. Rather,

one often encounters what is referred to as a complex plasma, or a *dusty plasma*, that is, a plasma that contains dust particles which can become charged through collisions with positively charged ions and negatively charged electrons. The field of complex plasmas is a discipline in its own right and requires the implementation of several branches of classical physics including electromagnetism and thermodynamics. Much experimental and theoretical work has been carried out in this area with important applications in both industry and engineering. However, the concern here is primarily dusty plasmas in an astrophysical context.

As mentioned above, in a dusty plasma, dust grains can acquire charge as a result of their interactions with the ions and electrons (Krishan 1999). In this context, the notion of quasineutrality is extended to include the dust such that  $Z_i n_i + Z_e n_e + Z_d n_d = 0$ , where  $n_d$  is the number density of the dust and  $Z_{e,i,d}$  are the charge numbers of the electrons, ions, and dust. It is important to note that more often than not, dust grains will be charged negatively because collisions with smaller, faster electrons are more probable (and thus more frequent) than collisions with heavier, slower ions (Verheest 2000). Thus, these charged grains are now susceptible to the influence of electromagnetic fields as they have some nonzero charge. The force due to said fields on some grain with charge qis given by the Lorentz force,

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})], \qquad (1.3)$$

where E is the electric field, B is the magnetic field, and v is the velocity of the grain. Eq. 1.3 can be rewritten to give the acceleration of the grain,

$$\vec{a} = \frac{q}{m} \left[ \vec{E} + \left( \vec{v} \times \vec{B} \right) \right], \tag{1.4}$$

where *m* is the mass of the grain. The quantity q/m is known as the charge-to-mass ratio

and it is this quantity that is used to measure the influence of the electromagnetic force as compared to the gravitational force. This influence tends to increase as the size of the grain decreases (Verheest 2000). The extent to which the gravitational force dominates over the Lorentz force (and vice versa) for these charged grains is the subject of the field of *gravitoelectrodynamics*. In general, however, the behavior of charged dust as a result of charging in an ambient plasma depends on the specific astrophysical environment.

# 1.3 Astrophysical Environments

# 1.3a Circumplanetary Rings: Saturn's F Ring

As discussed previously, dust is present throughout the Universe, so it should



Figure 1: Illustration comparing the four known planetary ring systems, each clearly separated by a solid dark line. The radii of the systems are scaled to a common planetary radius represented by the central dark circle. The small filled circles represent the moons of each planet and are shown at their respective distances from the planet. Figure taken from Burns et al. (2001).

come as no surprise that this remains true in the planetary environment. Large numbers of dust particles encircle the gas giants, and Earth is also known to have a tenuous circumplanetary debris ring, although it is the only known terrestrial planet to have circumplanetary dust. In general, circumplanetary or planetary rings, where a ring is defined as any collection of particles encircling a planet, are composed of particles with radii in the micron size regime (Burns *et al.* 2001). A schematic of the known planetary rings in our solar system is shown in Figure 1. These rings are, for the most part, circular, vertically thin, equatorial, and axisymmetric (Burns *et al.* 2001). In other words, the dust encircles the planet in a thin disk at the equator and only varies with the radius. These collections of innumerable dust grains, because of both their size and location, are subjected to a wide range of perturbing forces including the gravitational force, force due to radiation pressure, and the electromagnetic (Lorentz) force (Burns *et al.* 2001). In fact, it is these perturbations that make these dusty rings so interesting and yet so mysterious.

Perhaps one of the most easily recognizable planets in the Solar System is Saturn. This planet in particular stands out so prominently because of its visually stunning ring system that has captured the attention of astronomers for hundreds of years. In fact, Saturn's system of rings is the most expansive and most diverse of all planetary ring systems known to date. The term *ring system* is used because there is not simply one ring around Saturn (or for that matter any of the planets in the Solar System), but rather a series of concentric rings, as can be seen in Figure 1. The first documented observations of specific rings in the Saturnian system were made by Galileo Galilei in 1612. Using a very rudimentary telescope, he observed what are now known as the A and B rings. Several hundred years later, flybys by Pioneer 11, Voyager I and Voyager II revealed an

even larger amount of structure in this ring system (Burns *et al.*, 2001). Such missions, most recently Cassini, allow for more information concerning the rings in ever-increasing detail (Sergis *et al.* 2010). However, as advances in technology allow for closer inspection of this planet and its peculiar rings, the mystery behind their origin, composition, and location continue to increase.

On September 1<sup>st</sup>, 1979, Pioneer 11 completed its flyby of Saturn and was the first human-made object to do so. In addition to examining Saturn's atmosphere, measuring its temperature (approximately -180 °C) and confirming the existence of a magnetic field, Pioneer 11also discovered a new feature of Saturn's intricate ring system: the F Ring (Gehrels *et al.* 1980).

Located at a radial distance of 140 224 kilometers from Saturn, the F Ring sits between the A and G Rings. It is quite narrow, between 50 and 300 kilometers wide, and much fainter than many of Saturn's other rings. The slightly elliptical F Ring, which was observed at one time as being composed of four separate strands, is a so-called



Figure 2: (a) Voyager I image showing the strands and braids in the F Ring. (b) Cassini image that shows spokes on the inner edge as well as inner and outer moons, Prometheus and Pandora respectively. Image courtesy of NASA.gov

"shepherded" ring. This term simply refers to the fact that the narrow circumplanetary ring is confined by two of Saturn's moons: Prometheus and Pandora. Much of the F Ring is made up of very fine dust grains, these roughly spherical particles having on average a radial width of around 10 microns (Burns *et al.* 2001).

But what makes the F Ring so interesting? Based on the images from Voyager I and II, and most recently Cassini, the strangeness of this Saturnian component is undeniable. As can be seen in Figure 2, both Voyager 1 and Cassini have revealed some unusual features in this faint object, including braids as well as spokes. Unexplained kinks can also occur in the ring. Clumps, which have very short lifetimes and often wash out quickly, have also been observed (Burns *et al.* 2001). These clumps have been tediously tracked and thoroughly detailed over a period of 1-7 weeks by Showalter (2004). As of yet, an exact explanation for any of these features has not been given and it is for precisely this reason that the F Ring is such an interesting object of study.

Because of the position of the ring, the dust particles that comprise it are subject to a wide variety of perturbing forces. It is also important to note that, unsurprisingly, the F Ring is a dusty plasma and so the particles of the ring become charged due to collisions with the charged particles (ions and electrons). Thus, dust grains in the ring can be perturbed by electromagnetic forces exerted by Saturn's axisymmetric magnetic field. Other perturbing forces include forces due to radiation pressure from the solar wind, and gravitational forces exerted by Saturn, the Sun, and the shepherding moons.

## 1.3b Protoplanetary Disk

In astronomy and astrophysics, protoplanetary disks are large, flat rotating disks of gas and dust that form around young stellar objects (YSO), stars in their earliest stages

of development. An example of a protoplanetary disk and its central YSO can be seen in Figure 3. These disks are believed to be the sites of planet formation, as implied by their name, and thus are popular topics for research in astronomy and astrophysics (Apai and Lauretta 2010). The initial seeds for planet formation are thought to form within the disk through collisions between dust particles. The resulting particles from these collisions become increasingly more complex as the particles grow and the disk evolves. Thus, it is important to have a sufficient understanding of both the macroscopic conditions affecting the protoplanetary disk as whole as well as the microscopic processes concerning dust particle interactions. However, it is important to note that there is a gap in the understanding between the transition from particles formed through collisions of constituent dust grains to the rocky Earth-sized planetesimals that precede many fullyformed planets (Apai and Lauretta 2010). Macroscopic conditions concerning the protoplanetary disk will first be addressed in order to better understand the environment in which these dust particles exist. A discussion of the dust particle's interactions in the context of this protoplanetary disk environment and why these processes are both complex and significant will follow in section 3.2.

# Disk Formation.

The formation of the protoplanetary disk, and thus by extension the process of planet formation, is generally believed to have its origins in star formation. Cold, dense molecular cloud cores within nebulae made up of dust (from the ISM) and gas collapse due to gravitational instabilities. In other words, these packets of gas and dust become too dense and collapse in on themselves due to self-gravity. Prior to collapse, these clouds are very cool, around T = 10 K. However, as the cloud begins to collapse, it undergoes

compressional heating due to increasing density. Because the cloud has now become more opaque (i.e. dust obscures the easy escape of radiation), heat cannot be radiated away as efficiently and so the temperature increases dramatically. At the core of the molecular cloud, diatomic hydrogen (H<sub>2</sub>) dissociates into monatomic hydrogen (H) which then becomes ionized (Apai and Lauretta 2010). This sets the stage for the ignition of nuclear processes involving hydrogen that lead to the formation of the YSO.

The formation of the disk structure following the collapse of the molecular cloud core is a consequence of the conservation of angular momentum. Prior to collapse, the molecular cloud has some rotational velocity. Recall that the angular momentum can be written as

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega},\tag{1.5}$$

where *I* is the moment of inertia and  $\omega$  is the angular velocity. Thus the conservation of angular momentum can be expressed as



Figure 3: This image taken in visible light by the Space Telescope Imaging Spectrograph on NASA's Hubble Space Telescope shows a nearly face-on view of gas and dust swirling around the developing star AB Aurigae. The thick dark lines visible in the image are caused by the windowpane-shaped occulting bar on the Hubble. Also visible in the image are clumps of gas and dust in the disk that may be the seeds of planet formation. Image courtesy of HubbleSite.org.

$$\vec{L} = 0, \tag{1.6}$$

where  $\dot{x}$  is used to represent dx/dt. Since L must remain constant, a change in I will cause a change in  $\omega$  and vice versa. So as the cloud collapses, the moment of inertia is greatly decreased as mass that is initially distributed over parsec scales (1 pc  $\approx$  $3 \times 10^{13}$  km) is compressed to AU scales (1 AU  $\approx$  150  $\times$  10<sup>6</sup> km). This in turn causes a dramatic increase in angular velocity which leads to the overall flattened structure of the disk. This process occurs very quickly after the cloud core collapses (Williams and Cieza 2011).

### Aggregation.

To study the aggregation of dust in a protoplanetary disk (PPD), it is not enough to simply study the behavior of a single spherical particle as in the Saturnian system. Previously the concern was only with the orbital motion of one particle. Now, in order to study the growth of dust grains into planetesimals, the individual structures of the grains and how they are affected by other dust grains must be considered.

Rather than simple spherical particles, the dust component of a dusty plasma is actually a collection of *aggregates*. An aggregate is defined simply to be a composition of *monomers* where a monomer in this case is a single spherical dust particle. These fluffy aggregates are heterogeneous collections of particles fused through inelastic collisions (Matthews 1998). When considering the interactions between dust particles in a dusty plasma, it is important to take into consideration the aggregate structure of the grains, as these grains will have a much larger surface area than a compact sphere, thus increasing the sticking probability in a collision. Additionally, the aggregate structure impacts the manner in which the grain will charge and the number and size of the monomers which will be incorporated into its structure.

As mentioned previously, protoplanetary disks are believed to be the sites of early planet formation, and it is actually the collisions and sticking of dust aggregates that provide the earliest seeds for this planet formation process. Thus understanding the growth and morphology of early aggregates is key to understanding the physical processes behind the formation of rocky terrestrial planets. Since these aggregates are immersed in a dusty plasma, collisions between the grains and charged particles (ions and electrons) occur and thus these aggregates can become charged. This charging then in turn affects how aggregates interact with each other, in particular how they stick together.

Matthews, Land and Hyde (2012) compared the growth and morphology of charged aggregates and neutral aggregates at a single location within a protoplanetary disk. They found that there is a significant difference between those aggregates that were charged and those that were neutral. In particular, the charged aggregates preferentially incorporated the largest monomers from the size distribution into their structure. This resulted in more highly charged aggregates being larger, more massive, and being much less compact. The neutral aggregates were the lightest and smallest aggregates in the population. Thus the charging of dust grains within a protoplanetary disk has been shown to affect the aggregate morphology.

Within in the disk itself, only about 1% of the total composition is solid particles (Williams and Cieza 2011). The rest is made up of partially ionized gas that, along with the solid material, plays an integral role in the process of planet formation. Dust particles in the protoplanetary disk are composed primarily of silicates (i.e. any compound

containing a silicate-bearing anion). Beyond the snow line, the inner edge of the region where the temperature falls below the condensation of water, many of these grains also bear an icy mantle composed of water-ice (Lecar *et al.* 2006). Grains located at radii within the snow line, calculated as 2.7 AU from the YSO using a model based on the Solar System, thus do not bear such a mantle due to their proximity to the YSO. The primary components of the disk are molecular and ionized hydrogen (H<sub>2</sub>, HI, HII). Other gaseous species that can be found in the disk include sodium (Na), magnesium (Mg) and carbon (C) (Semenov *et al.* 2004). The dust composition of the disk has its origins in dust from the ISM, and the extent to which dust from the ISM survives in the disk depends on the core collapse and formation of the disk (Apai and Lauretta 2010)

## 1.4 Summary

This thesis will explore two specific astrophysical applications of dusty plasma physics: the dust dynamics of circumplanetary dust in Saturn's F Ring as well as the growth of charged dust aggregates within a protoplanetary disk. In Chapter 2, the theory behind the dynamics and charging of dust grains will be discussed as this is key to understanding both the unusual features of Saturn's F Ring as well as the growth of dust grains in a PPD. In Chapter 3, the physical processes and numerical models for each astrophysical environment will be introduced and discussed. Concerning the former, a method that computes the forces acting on a singular dust grain within a circumplanetary system is used and which includes an ordinary differential equation solver to derive position and velocity data for the single grain. For dust within the protoplanetary disk, a numerical model called Aggregate\_Builder is used which builds and charges dust aggregates using parameters believed to be common to the protoplanetary disk at specific

locations. In Chapter 4, the initial conditions and results of our simulations for the F Ring will be detailed while Chapter 5 will cover the protoplanetary disk model in a similar context. Chapter 6 will contain discussion of results for both of these models as well as topics for future work.

## CHAPTER TWO

# Theory of Dust Grain Dynamics and Charging

## 2.1 Forces on dust

Because of their size and location, dust grains in astrophysical environments are subject to a wide range of forces. Circumplanetary dust grains, for example, are affected by forces from the planet they orbit, other satellites orbiting the planet, and even the Sun, while grains in a protoplanetary disk are impacted primarily by forces resulting from solar and disk gravity, electrostatic interactions with other grains and gas drag. The effects of these perturbations can be seen in deviations from the Keplerian motion of circumplanetary dust grains, or in the collisions between fractal dust aggregates in a protoplanetary disk.

## 2.1a Gravitational Force

The magnitude of the attractive gravitational force between two spherical bodies is given by Newton's law of gravitation,

$$F_G = G \frac{m_d M_P}{r^2},\tag{2.1}$$

where *G* is the gravitational constant,  $m_d$  is the mass of the dust grain,  $M_P$  is the mass of the other spherical body (i.e. the Sun, the planet, or a moon), and *r* is the distance between the dust grain and the other body. This equation can be used to calculate the perturbing gravitational forces due to the Sun and moons present in the system. However, the fact that the planet is not perfectly spherical must be taken into account when calculating the gravitational force. To do this, consider the gravitational potential  $\Phi$ . It can be shown that an arbitrarily shaped body satisfies Laplace's equation,  $\nabla^2 \Phi = 0$ . The solution to this equation in spherical coordinates is given by,

$$\Phi = -\frac{GM_P}{R_P} \sum_{j=0}^{\infty} \left(\frac{R_P}{r}\right)^{j+1} \sum_{k=0}^{j} \left[C_{j,k} \cos k\phi_R + S_{j,k} \sin k\phi_R\right] P_j^k(\cos\theta), \quad (2.2)$$

where  $M_P$  and  $R_P$  are the mass and radius of the planet,  $r, \theta, \phi_R$  are the usual spherical coordinates, and  $C_{j,k}$  and  $S_{j,k}$  are coefficients describing the mass distribution within the planet (Burns *et al.* 2001).  $P_j^k(\cos \theta)$  are the associated Legendre polynomials. These are given by the formula

$$P_j^k(x) = (1 - x^2)^{\frac{|k|}{2}} \left(\frac{d}{dx}\right)^{|k|} P_j(x), \qquad (2.3)$$

where  $P_i(x)$  is the set of Legendre polynomials defined by the Rodrigues formula

$$P_j(x) = \frac{1}{2^j j!} \left(\frac{d}{dx}\right)^j (x^2 - 1)^j$$
(2.4)

(Griffiths 2005). Assuming that the non-spherical deformation is axisymmetric, all terms where  $k \neq 0$  are small enough that they can be ignored. This combined with some simplification yields

$$\Phi = -\frac{GM_P}{R_P} \Big( 1 + \sum_{j=2}^{\infty} J_j (R_P/r)^j P_j(\cos\theta) \Big), \qquad (2.5)$$

where the  $J_j$  is a term that is related to the oblateness of the planet (Burns *et al.* 2001). Using this much simpler expression, and the fact that  $F_{grav} = -\nabla \Phi$ , and keeping a finite number of terms (typically four) in the expansion for the potential, the gravitational perturbing force including oblateness effects can be modeled. Taking oblateness, or planetary bulge, into account is important as this affect leads to precession in the orbits of nearby satellites of the planet. It can only be ignored for significant distances from the planet (~20 $R_p$  for Saturn). Most planetary rings reside only a few planetary radii from the planet itself. Thus, taking these corrections into account is important when determining the dynamics of a ring particle (Burns *et al.*2001).

## 2.1b Electrostatic Force

As discussed in the introduction, radiation in astrophysical environments can often lead to the ionization of gases and the subsequent formation of a plasma. A plasma is characterized by the fractional ionization,  $X_{ion}$ , given by Eq. 1.2. Because these ionized gases contain dust particles as well as charged particles, collisions occur between the free electrons and positively charged ions, leading to the dust grain accumulating some charge, a phenomenon that will be discussed in the following section. Taking each grain to be a point charge  $q_i$ , the electrostatic force between the two grains can be expressed as,

$$\vec{F}_{ij} = (1/4\pi\epsilon_0) q_i q_j / r^2 \hat{r},$$
 (2.6)

where  $q_{i,j}$  are the two charges and r is the distance between them.

However, the approximation of the charged grain as a point charge is not always valid. This could be the case, for example, if the grain were non-spherical and had some non-uniform charge distribution across the surface. In this case it is better to say that the grain has some dipole moment  $\vec{p}$ , such that the electric potential is given by  $\Phi = 1/4\pi\epsilon_0(q/r + \vec{p}\cdot\vec{r}/r^3)$ , and some associated electric field  $\vec{E}$  given by  $\vec{E} = -\nabla\Phi$ . The dipole moment of each particle  $p_i$  can also interact with the electric field of another particle  $E_i$ , producing a torque about the center of mass of the *i*th particle,

$$\vec{\Gamma}_i = \vec{p}_i \times \vec{E}_j \tag{2.7}$$

(Matthews *et al.* 2007). The force due to the interaction between the dipole moment and the electric field can be expressed as

$$\vec{F}_E = (\vec{p} \cdot \nabla) \vec{E}, \qquad (2.8)$$

where  $\nabla$  is the del operator defined in the usual way (Griffiths 1999).

#### 2.1c Radiation Pressure

Dust grains can also experience a force due to radiation pressure from the Sun because photons carry momentum. When these photons interact with the dust, through absorption, emission, or scattering, a force is imparted to the dust grain as a result of the change in momentum of the photon. This perturbing force can often lead to a disruption in the normal elliptical orbit of the grain (Burns *et al.* 2001).

For a single spherical grain, the ratio of the force due to radiation pressure and the force of gravity due to the Sun is given by

$$\beta \equiv F_{RP}/F_G = (3L/16\pi GMc)(Q_{pr}/\varrho a) = 5.7 \times 10^{-5} Q_{pr}/\varrho a, \qquad (2.9)$$

where  $Q_{pr}$  is the radiation pressure efficiency factor ( $Q_{pr} = 1$  for a perfectly absorbing grain),  $\varrho$  is the density of the grain and a is the radius of the grain (Burns, Lamy and Soter 1979). Using the definition in Eq. 2.9 along with the gravitational force between the grain and the Sun, the force due to radiation pressure can be expressed as

$$F_{RP} = -\beta M_{sol} m_d / \rho^2 \,\hat{\rho} \tag{2.10}$$

where  $\rho$  is the grain-Sun distance,  $\hat{\rho}$  is the unit vector pointing from the dust grain to the Sun, and  $M_{sol}$  is the mass of the Sun (Hamilton 1993). Note the negative sign is included because this radiation force always directly opposes that of the force due to solar gravity. Additionally, another useful relation used for finding  $\beta$  is

$$2\alpha/3n = \beta M_{sol} r^2/M_P R^2, \qquad (2.11)$$

where  $\alpha$  is a parameter describing the strength of the radiation, *n* is the mean orbital motion of the dust grain about the Sun, and *r* is the orbital radius of the dust grain with respect to the planet, and *R* is the planet-Sun distance (Hamilton 1993).

## 2.1d Magnetic Force

As previously discussed, grains immersed in a plasma environment will accumulate charge as a result of collisions with both ions and free electrons. Thus, the grain will interact with the planet's magnetic field  $\vec{B}$  and be subject to the Lorentz force given by

$$\vec{F}_M = \frac{q}{c} \left( \vec{v}_{rel} \times \vec{B} \right), \tag{2.12}$$

where q is the charge on the grain, c is the speed of light, and  $\vec{v}_{rel}$  is the velocity of the grain relative to the magnetic field. This term is given by

$$\vec{v}_{rel} = \vec{v} - \left(\vec{\Omega}_P \times \vec{r}\right),\tag{2.13}$$

where  $\vec{v}$  is the inertial velocity of the grain and  $\vec{\Omega}_P$  is the constant rotation rate of the planet about its axis. In the context of circumplanetary dust,  $\Omega_P$  accounts for the fact that magnetic field lines corotate with the planet while grains are on roughly Keplerian orbits (Burns *et al.* 2001).

#### 2.1e Gas Drag

As a dust grain moves through the gas, its motion is impeded by the gas particles. This is analogous to the way the motion of a falling body is retarded by the force of friction due to the molecules in the air. For relatively small spheres, those that have radii much less than the mean free path of the gas particles, the force due to gas drag is given

$$\vec{F}_{g} = -\delta \frac{4}{3} \pi a^{2} m_{g} v_{th} n_{g} \vec{v}_{d}, \qquad (2.14)$$

where  $m_g$ ,  $n_g$ ,  $v_{th}$  are the mass, density, and thermal velocity of the gas and  $v_d$  is the velocity of the dust grain (Epstein 1924). The parameter  $\delta$  depends on the mechanism of collision between the gas particle and the dust grain and can take on values between 1.0 and 1.442 (Liu *et al.* 2003).

## 2.1f Turbulence

Turbulence is an oft-explored topic in the fields of biology, geophysics, and engineering and has applications to problems such as fluid flow in pipes, airflow around aircraft wings, and blood flow in circulatory systems. Flow is often found to be turbulent if its Reynolds number (Re), a dimensionless parameter used to describe the characteristics of the flow, is above a certain threshold. In the context of the PPD environment, turbulence is introduced because of the magnetorotational instability (MRI) (Balbus and Hawley 1991). In the ionized gas of the protoplanetary disk, free charges couple to magnetic field lines. When two parcels of gas on different Keplerian orbits couple to a given magnetic field line, differential rotation will lead to a separation between the two parcels. This differential rotation leads to the magnetic field lines being "dragged" along with the parcel of gas. The growing separation causes tension to develop in the field lines. The gas parcel on the smaller orbit will lose angular momentum and move inward, while the parcel on the larger orbit will move outward. This leads to angular momentum transport and subsequent dissipation of energy, the instability that is thought to lead to turbulence (Balbus and Hawley 1991, Ciesla and Dullemond 2010).

The motion of dust grains immersed in gas will be affected by turbulence. Because of its inertia, a grain will not immediately follow the motion of the gas, but rather requires some time interval in which to align its motion with the motion of the gas. This is known as the stopping time,  $t_s$ , given by

$$t_s = \frac{3}{4c_g \rho_g} \frac{m}{\sigma},\tag{2.15}$$

where  $\rho_g$  is the mass density of the gas and  $m, \sigma$  are the mass and collisional cross section of the grain (Ormel and Cuzzi 2007). Because of this inertial lag, dust grains develop relative velocities with respect to the gas as well as other grains. Assuming that the particles in question are very small and tightly coupled such that  $t_1, t_2 \ll t_\eta$ , where  $t_{1,2}$  are the stopping times of the first and second particles and  $t_\eta$  is the overturn time of the smallest eddy, the expression for the relative velocity between the first and second particles is given by

$$v_{1,2} = \sqrt{\frac{3}{2}} v_{\eta} \frac{(t_1 - t_2)}{t_{\eta}}$$
(2.16)

(Ormel and Cuzzi 2007). Here,  $v_{\eta} \sim L_{\eta}/t_{\eta}$ ,  $L_{\eta} = \text{Re}^{-3/4}L_L$ , the turnover velocity and length scale of the smallest eddy, and  $t_{\eta} = t_L \text{Re}^{-\frac{1}{2}}$ . The length scale and overturn time of the largest eddies, in addition to the Reynolds number, can all be defined in terms of parameters specific to the disk such that

$$L_L = \sqrt{\alpha'}H \tag{2.17}$$

$$t_L = \Omega_k^{-1} \tag{2.18}$$

$$\operatorname{Re} = \alpha' c_g H / \nu_m \tag{2.19}$$

where  $\nu_m$  is the kinematic viscosity of the gas and  $\alpha'$  is a parameter that describes the strength of the turbulent flow, here assumed to be 0.01 (Matthews, Land and Hyde 2012).

Additionally,  $c_g = \sqrt{k_B T/m_g}$  is the sound speed of the gas where  $k_B$  is the Boltzmann constant, T is the temperature, and  $m_g$  is the mass of the gas, and  $H = c_g/\Omega_k$  is the gas scale height where  $\Omega_k$  is the rotational velocity of the disk (Okuzumi *et al.* 2011).

#### 2.2 Grain Charging

As shown, dust grains in astrophysical environments are subject to a wide range of forces. Included are the Lorentz and electrostatic forces, both of which depend on the charge of the grain. These forces, however, become important only for micron- and submicron-sized grains. Consider the ratio of the gravitational and Lorentz forces,

$$\frac{F_G}{F_M} = \frac{Gm_d M_P / r^2}{qvB}.$$
(2.20)

Using parameters specific to Saturn's B Ring,

$$\frac{F_G}{F_M} = 53.8\frac{m}{q}.$$
 (2.21)

Treating the dust grain as a spherical capacitor (see Eq. 2.27 and section 2.2b), it can be shown that for a 1cm-sized particle,  $F_G/F_M \ge 10^{10}$ . Thus the impact of electromagnetic forces need not be taken into account for particles in the centimeter-size regime and larger (Matthews 1998, Mendis *et al.* 1984). Conversely, for a 0.5 µm-sized particle,  $F_G/F_M \ge 10^2$ . Thus for particles in this size regime, perturbations due to the magnetic force become significant.

Several different mechanisms can be responsible for the charging of dust grains in a plasma environment, including photodetachment, secondary electron emission, radioactive charging, triboelectric charging, as well as others (Mendis and Rosenberg 1994). In this study, charging of dust grains is limited to collisions between the dust grain and charged particles within the plasma. Incident ions and electrons may be thought of as charging currents to the grain (Matthews 1998). By computing these currents, the charge on the dust grain can be calculated.

## 2.2a Equilibrium Grain Charge

The charge Q on a dust grain varies with time due to the charging currents incident on the grain. The expression for the change in Q is given by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \sum_{\alpha} I_{\alpha}, \qquad (2.22)$$

where  $I_{\alpha}$  is the current due to an incoming particle  $\alpha$  (an electron or ion,  $\alpha = e$  or  $\alpha = i$ ). Additionally, the current density due to an incoming particle  $\alpha$  is given by

$$J_{\alpha}(t) = n_{\alpha} q_{\alpha} \iiint f_{\alpha}(v_{\alpha}) v_{\alpha} \cos \theta \, d^{3} \overrightarrow{v_{\alpha}}.$$
(2.23)

Here,  $n_{\alpha}$  is the plasma density far from the particle,  $q_{\alpha}$  is the charge on the impinging electron (or ion),  $f_{\alpha}$  is the velocity distribution function and  $v_{\alpha} \cos \theta$  is the velocity component of the electron (ion) perpendicular to the surface. One can assume a Maxwellian distribution for the speeds of the plasma species (ions and electrons). The distribution function in the vicinity of charged grain with potential  $\phi$  is given by

$$f_{\alpha} = \left(\frac{m_{\alpha}}{2\pi k T_{\alpha}}\right)^{\frac{3}{2}} \exp\left(-\frac{m_{\alpha}}{2k T_{\alpha}}v^2 - \frac{q_{\alpha}\phi}{k T_{\alpha}}\right), \qquad (2.24)$$

where  $m_{\alpha}$  and  $T_{\alpha}$  are the mass and temperature of the plasma respectively (Matthews and Hyde 2008). The grain can be said to be at equilibrium when the current incident on the grain is zero, i.e. when the charge of the grain is no longer changing with time. Thus, the equilibrium charge on the grains may be found by setting Eq. 2.22 to zero (Goertz 1989). These currents depend strongly on the surface potential,  $\phi_{surf}$ , of the dust grain.

## 2.2b Orbital Motion Limited (OML) Theory

Theories concerning the charging of dust particles were originally developed for the purposes of dealing with electrostatic probes used in laboratory plasmas. As the dust grain is simply a "probe" with no wires attached to it, a similar approach can be used. Currents to the grain are said to be "orbit-limited" if the condition  $a \ll \lambda_D \ll \lambda_{mfp}$  holds, where  $\lambda_{mfp}$  is the mean free path of a plasma particle. Currents are calculated by assuming that charged particles are collected by the surface of the grain if the collisionless orbits of the charged particles intersect the grain's surface (Liu *et al.* 2003).

Making the substitution  $d^3 \vec{v}_{\alpha} = v_{\alpha}^2 dv_{\alpha} d\Omega$  into Eq. 2.24, where  $d\Omega$  is the solid angle, yields

$$J_{\alpha}(t) = n_{\alpha}q_{\alpha}\int_{v_{m}(t)}^{\infty} f_{\alpha}(v_{\alpha})v_{\alpha}^{3}dv_{\alpha} \times \iint \cos\theta \,d\Omega$$
(2.25)

where  $v_m(t)$  is the minimum velocity a plasma particle with the same polarity of charge as the dust particle must have to reach the dust particle surface with potential  $\phi$ . (Matthews, Land and Hyde 2012). If the plasma particle and dust grain have opposite charges, the minimum velocity is zero.

For a negatively charged grain, positive ions will be attracted while electrons will be repelled. The opposite is true for positively charged grains. These primary electron and ion currents are given by

$$I_e = I_{0,e} e^{e\phi_{\text{surf}}/kT_e}, \qquad \phi_{\text{surf}} < 0, \qquad (2.26a)$$

$$I_e = I_{0,e}(1 + e\phi_{\text{surf}}/kT_e), \qquad \phi_{\text{surf}} > 0,$$
 (2.26b)

$$I_i = I_{0,i} e^{z_i e \phi_{\text{surf}}/kT_i}, \qquad \phi_{\text{surf}} < 0,$$
 (2.26c)

$$I_i = I_{0,i}(1 - z_i e \phi_{\text{surf}} / kT_i), \qquad \phi_{\text{surf}} > 0,$$
 (2.26d)

where *e* is the electron charge,  $z_i e$  is the ion charge,  $T_i$ ,  $T_e$  are the electron and ion temperatures, and  $I_{0,\alpha}$  is the charging current for  $\alpha$  when  $\phi_{surf} = 0$  (Goertz 1989).

The charge on the grain can be written in terms of the surface potential of the grain and the capacitance C such that

$$Q = C\phi_{\text{surf.}} \tag{2.27}$$

For an isolated spherical dust grain, an expression for the capacitance is trivial. However, in the presence of other grains or in the case of nonspherical dust particles, this calculation becomes much more complicated.

#### 2.2c Spitzer Result

For a single spherical grain immersed in a plasma, the capacitance is given by

$$C = 4\pi\varepsilon_0 a e^{-a/\lambda_D},\tag{2.28}$$

where *a* is the radius of the grain and  $\lambda_D$  is the screening or Debye length (Matthews 1998). Assuming that the primary ion and electron currents are the only currents present, the surface potential can be calculated from Eq. 2.22 resulting in the relation

$$1 - \frac{\phi_s}{k_B T_i} = \sqrt{\frac{m_i}{m_e}} e^{e\phi_s/k_B T_e}, \qquad (2.29)$$

where  $m_i, m_e$  are the electron and ion masses. Setting  $T_i = T_e = T$  and  $m_i = m_p$ , the mass of a proton, the well-known Spitzer result can be obtained such that  $\phi_s = 2.51 k_B T/e$ . (Spitzer 1978).

# 2.2d Aggregate Charge

For a single spherical particle, the integral over the solid angle  $d\Omega$  in Eq. 2.25 is trivial as each point on the surface is open to impinging plasma particles from direction  $0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi$ , describing a hemisphere. However, as can be seen in Figure 4, other monomers in the aggregate often block open lines of sight (LOS) to a given point on the aggregate surface. Thus the LOS\_factor is introduced by splitting the expression for  $J_{\alpha}(t)$  into two parts,

$$J_{\alpha}(t) = J_{\alpha}(v_{\alpha}, t) \times \text{LOS}_{\text{factor.}}$$
(2.30)

Here, the LOS\_factor is the numerical approximation to the solid-angle integral given the open and closed lines of sight for the aggregate. In order to calculate the LOS\_factor, the surface of each constituent monomer in the aggregate is divided into patches of equal area. Vectors are constructed between the center of the monomer and these surface patches and define the unit normal directions  $\hat{n}$ . Copies of these unit vectors originating at the surface point define test directions for the lines of sight  $\vec{t}'$ . The lines of sight are determined to be *blocked* if they intersect any other monomer in the aggregate or the surface of the monomer that the test direction resides on. Otherwise, the line of sight is determined to be *open*. Each separate line of sight is then assigned value of LOS(t')=0 if the line of sight is closed and 1 if it is open.

The total line of sight factor is determined for each patch using the expression



Figure 4: Illustration of the line of sight (LOS) approximation. Because of the structure of the aggregate, not all directions are open to allow for an impinging electron (or ion) to travel along them. The LOS approximation helps determine which lines of sight from a given point are blocked (those in the white space) and which are open (those in the shaded space). Figure taken from Matthews, Land and Hyde (2012).

$$LOS\_factor = \sum LOS(t') \times \cos \theta \times \Delta \Omega.$$
(2.31)

Here,  $\cos \theta$  represents the angle between the test line of sight  $\vec{t}'$  and the unit normal  $\vec{n}$  and thus the sum is over all test lines of sight.  $\Delta\Omega$  is the fractional solid angle subtended by any direction  $\vec{t}'$ . The net current for the specific species  $\alpha$  to a patch can be found by multiplying the current density by the area of the patch. By summing over the current species, the total surface charge accumulated on a particular patch can be found in some time interval. The change in the charge of the aggregate is then determined by summing over the charge accumulated on all of the patches on the aggregate surface. This process is carried out until the change in the charge of the whole aggregate is negligible, or in other words when the net current to the aggregate is zero (Matthews, Land and Hyde 2012). Thus the total charge and the dipole moment of the aggregate can be calculated.

### CHAPTER THREE

# Astrophysical Environments

Up to this point, the discussion of dust interaction within a plasma has not been specific to either of the astrophysical environments outlined in section 1.3. When discussing the methods implemented in this study, it is necessary to talk about the two astrophysical environments separately: the single-grain dynamics of Saturn's F Ring and the growth of fractal aggregates in a protoplanetary disk. A separate numerical model is used to study the behavior of the dust in each environment which incorporates forces specific to the environment.

## 3.1 Single-Grain Dynamics in a Circumplanetary Disk

#### 3.1a Forces on Spherical Grains

A single grain orbiting in a circumplanetary disk is subject to a wide range of forces. These include the gravitational force due to the planet and nearby moons, the force due to radiation pressure, and the force due to a planet's magnetic field.

Planetary ring systems can be perturbed by nearby moons orbiting the planet. Saturn's F Ring, situated between two of Saturn's moons, Prometheus and Pandora, is said to be a "shepherded" ring (Matthews and Hyde 2003). This simply means that ring particles are interacting gravitationally with the moons on either side of the ring and thus being *shepherded* by them. Since these moons are at different orbital radii and thus traveling at different angular velocities, the effect on the circumplanetary dust is not

simply one of confinement (the balancing of gravitational forces). Thus, these interactions with nearby moons gives rise to perturbations in the ring at different points in the orbital path of the grain around the planet. The force exerted by the moon on the grain is given by usual expression for gravitational force

$$\vec{F}_M = G \frac{m_M m_d}{\delta(t)^2},\tag{3.1}$$

where  $m_M$  is the mass of the moon and  $\delta$  is the time-dependent separation between the dust grain and moon. To find this separation, initial radial and angular positions are assigned to the moon. The moon is assumed to move around the planet on an elliptical orbit and so by simply keeping track of the time elapsed,  $\delta$  can be easily calculated.

In section 2.1a, the gravitational potential including a correction for oblateness is given by Eq. 2.5. Keeping four terms in our expansion of  $\phi$  and given that  $\vec{F} = -\nabla \phi$ , the gravitational force on the grain due to the planet is given by

$$\vec{F}_P = -G \frac{m_P m_d}{r^3} \left( 1 + J_2 \left(\frac{R_P}{r}\right)^2 P_2(\cos\theta) + J_4 \left(\frac{R_P}{r}\right)^4 P_4(\cos\theta) \right)$$
(3.2)

where *r* is the distance from the grain to the planet,  $J_{2,4}$  are the second and fourth order corrections for planetary oblateness and  $P_{2,4}(\cos \theta)$  are the second and fourth Legendre polynomials.

Perhaps the most difficult perturbation to account for in the planet-centered



Figure 5: Schematic of the Sun-dust-planet system. The radiation pressure due to the Sun must be taken into account for a circumplanetary ring particle, making the Sun-dust distance  $\rho$  important in considering perturbations of the grain.

reference frame considered here is that of the radiation pressure. As seen in Eq. 2.9, the force due to the radiation pressure is simply proportional to that of the gravitational force due to the Sun. Finding the Sun-grain distance is a more unique challenge as the Sun is not rotating in the planet-centered reference frame, unlike the dust grain or moons. A schematic of the geometry of this Sun-dust-planet system can be seen in Figure 4. Thus, the expression for the force due to radiation pressure is

$$\vec{F}_{RP} = -\beta \frac{GM_{sol}}{\rho^3} \vec{\rho}$$
(3.3)

where  $\vec{\rho} = \vec{R} + \vec{r}$  and  $\vec{R}$  takes into account the tilt of the planet out of the orbital plane of the solar system. To match the conditions in Burns *et al.* (2001), the parameter  $\beta$ , defined in Eq. 2.9, can be calculated using the relation in Eq. 2.11. This combined with the definition  $n = \sqrt{GM_P/r_0^3}$  yields

$$\beta = \frac{2}{3} \frac{\alpha R^2}{M_{Sun}} \sqrt{\frac{M_P}{Gr}}.$$
(3.4)

The parameter  $\alpha$  in general depends on both the size as well as the composition of the grain. Average values for the Sun-planet distance, *R*, and the planet-grain distance, *r*, are typically used.

Grains in Saturn's F Ring are immersed in a corotating plasma that surrounds the planet and so can become charged through collisions with ions and electrons (Matthews and Hyde 2003). As a result, these charged grains are subject to the force due to Saturn's nearly axisymmetric magnetic field. The force on the grain as a result of its interaction can be calculated using Eq. 2.12 where the strength of the magnetic field *B* is dependent on the position of the grain with respect to the planet.
#### 3.1b Numerical Model

The program used to study the dynamics of dust grains in circumplanetary ring systems is written using Matrix Laboratory Software (MATLAB). MATLAB is a programming environment that stores all data in matrices, thus allowing for the easy manipulation of large sets of data and plotting of results. MATLAB also has many builtin functions and operations as well as an environment that makes debugging relatively simple.

The initial position of the grain is simply set as the semi-major axis of the circumplanetary ring in question. Using the expression for gravitational potential, the initial velocity of the grain can be calculated as

$$\nu = \sqrt{\frac{GM_p}{r} \left(1 + \frac{3}{2} J_2 \left(\frac{R_p}{r}\right)^2\right)},\tag{3.5}$$

where  $M_p$  is the mass of the planet,  $R_p$  is the radius of the planet, r is the planet-dust distance, and  $J_2$  is the second term in the spherical harmonic expression for the gravitational potential (see section 2.1b). A total simulation time and time step are then specified, the former determining how long the simulation is run and the latter how often the position, velocity and acceleration of the dust grain are calculated. This simulation time is measured in Earth-years and is typically on the order of forty years. It should be noted that decreasing the time step, while often increasing the accuracy of the calculation, can be very expensive computationally.

This program uses a fourth-order Runge-Kutta method to compute the position and velocity of the single dust grain as it orbits the planet for each iterative time step. The time step is set at 50 seconds. Each of the perturbing forces listed in section 2.1b is computed based on the current position of the grain. Each of these forces can be turned on or off in order to determine the effects of individual forces on the orbit of the dust grain. The accelerations are summed to find the total acceleration of the grain due to these perturbations. After the numerical integration is carried out, the position, velocity, and acceleration are stored for each time step. These are all stored in terms of Cartesian coordinates in a planet-centered reference frame. Thus, conversion of these Cartesian coordinates into orbital parameters is necessary.

# 3.1c Orbital Parameters

Just as a planet travels in an elliptical orbit around the Sun, a dust grain follows an elliptical trajectory around a planet. When studying these elliptical orbits, it is often more useful to describe the trajectory of the grain in terms of six *orbital parameters* rather than giving the position, velocity, and acceleration in terms of Cartesian coordinates. Three of these parameters are used to describe the orientation of the grain's orbit while the other three are used to specify the position of the grain in its orbit (Burns *et al.* 2001). These parameters are the semi-major axis of the elliptical orbit, *a*, the eccentricity of the orbit, *e*, the angle of inclination, *i*, the longitude of the ascending node,  $\Omega$ , the argument of the pericenter,  $\omega$ , and the true anomaly,  $\nu$ . Use of these orbital elements allows for an easy way to describe not just the position or velocity of the grain, but the shape of its orbit as well. This description allows for a better of visualization of how forces affect the grain orbits.

As mentioned previously, dust grains travel in elliptical orbits with the planet at one *foci* of the ellipse. The radial distance between the planet and the grain is given by r while half the length of the longest diameter of the ellipse is given by a, the semi-major axis. A schematic of this elliptical orbit is shown in Figure 6. Note that the center of this



Figure 6: Illustration showing the elliptical orbit of a dust grain, the semimajor axis *a*, semi-minor axis *b*, and radial distance *r*. Notice that for e = 0, the orbit becomes circular and in particular a = b = r at every point in the orbit. As *e* approaches 1, the orbit becomes more elliptical.



Figure 7: Illustration of an inclined elliptical orbit. Shown are four of the orbital elements discussed: *i*, the angle of inclination,  $\Omega$ , the longitude of the ascending node,  $\omega$ , the argument of the pericenter, and v, the true anomaly. The shaded plane is the equatorial plane of the planet, the white semicircle is the orbital plane of the grain, and the dotted line is the radial distance *r* to the grain. Figure taken from Hamilton (1993).

orbit is located where *a* and *b*, the semi-minor axis, intersect. The distance between the center of the orbit and the planet is given by *ae*, where *e*, the eccentricity, is the second orbital parameter used to describe dust grain orbits. The eccentricity of an ellipse may take on values  $0 \le e < 1$ , with e = 0 being a circular orbit. In this case, the eccentricity describes how elliptical the orbit is. Also shown in Figure 6 are the apocenter, at r = a + ae, and pericenter, at r = a - ae, the points in the orbit furthest from and closest to the planet. The third orbital element v, the true anomaly, is the angle between the radial position and the pericenter.

The three remaining orbital elements describe the orientation of the orbit itself. It is not necessary for the grain to be orbiting in the equatorial plane of the planet. The angle of inclination, *i*, measures the angle between the particle's orbital plane and the equatorial plane of the planet as illustrated in Figure 6. The *ascending node of orbit* is the point at which the orbit crosses the equatorial plane in the positive vertical direction and  $\Omega$ , the longitude of the ascending node, is the angle between this intersection and the arbitrarily chosen *zero point of longitude* (Burns *et al.* 2001). The argument of the pericenter,  $\omega$ , is the angle between the ascending node and the pericenter. It should also be noted that it is often useful to define the solar angle,  $\Phi_{sola}$ .

$$\phi_{sol} = \omega + \Omega - n_{sol}t - \delta, \qquad (3.6)$$

where  $n_{sol}$  is the mean motion of the planet about the Sun, *t* is time, and  $\delta$  is some constant. The solar angle is roughly the angle between the pericenter and the Sun as seen from the planet (Burns *et al.* 2001).

# 3.2a Motion of Dust in the Protoplanetary Disk

As discussed previously, aggregates in a PPD, because of collisions with other aggregates, have a fractal for fluffy nature, much like "dust bunnies" form under a bed (Matthews *et al.* 2007). These fluffy structures immersed in the plasma of the PPD follow the motion of the gas more easily than a compact sphere and so are subject to relative velocities between each other as well as the gas because of the turbulent flow in the disk (a process discussed in section 2.1f). Through this interaction with the gas, the charged fractal aggregates gain relative velocities with respect to one another. This is what allows two charged grains to overcome the Coulomb repulsion between them. Once these grains come into contact with each other, the van der Waals attraction dominates and causes the grains to stick.

#### 3.2b Forces on Dust Aggregates

In the context of dust interactions in a protoplanetary disk environment, the principal force taken into account is the electrostatic force between the two interacting charged dust aggregates. Unlike in the case of a single spherical circumplanetary dust grain, it is not sufficient to treat charged fractal aggregates (like the one seen in Figure 7) as point charges. Rather, the charge on each monomer and aggregate is used to calculate the electric field due to each monomer or aggregate. The dipole moment of each particle  $\vec{p}_i$  can also interact with the electric field of the opposite particle  $\vec{E}_j$  to produce a torque about the center of mass of the *i*th particle,

$$\overline{\Gamma_i} = \overline{p_i} \times \overline{E_j} \tag{3.7}$$

(Matthews *et al.* 2007). The motion resulting from these torques is governed by Euler's equations,

$$\lambda_1 \dot{\omega}_1 - (\lambda_2 - \lambda_3) \omega_2 \omega_3 = \Gamma_1, \qquad (3.8a)$$

$$\lambda_2 \dot{\omega}_2 - (\lambda_3 - \lambda_1) \omega_3 \omega_1 = \Gamma_2, \qquad (3.8b)$$

$$\lambda_3 \dot{\omega}_3 - (\lambda_1 - \lambda_2) \omega_1 \omega_2 = \Gamma_3. \tag{3.8c}$$

Here,  $\lambda_i$  are the principal moments of inertia and  $\vec{\omega}$  is the angular velocity with respect to a particle's body axes.

## 3.2c Numerical Model

To simulate the growth of dust aggregates within a protoplanetary disk, a numerical model called Aggregate\_Builder is used. Aggregate\_Builder, written in MATLAB, is based on box\_tree, an *N*-body code developed by Richardson (1995) to study planetesimal dynamics. The code was then modified to include the effects of charged particles and magneic fields (Matthews and Hyde 2003).

Aggregate\_Builder is used to study pairwise interactions of aggregates in a frame with the origin at the center-of-mass of the target particle. Three types of collisions can occur: particle-particle aggregation (PPA) in which two monomers collide and stick, particle-cluster aggregation (PCA) in which a single monomer collides with an aggregate, or cluster-cluster aggregation (CCA) in which two aggregates collide and stick together. Through successive PPA, PCA, and CCA collisions, aggregates are built up in size from two monomers to up to several thousand monomers. An example of an aggregate built using Aggregate Builder can be seen in Figure 8.

Aggregates in Aggregate\_Builder are built in three separate stages. First generation aggregates have a maximum of N = 20 monomers. All first generation

aggregates are characterized by PCA collisions with the exception of the first collision to create the initial dimer. Second generation aggregates have a maximum of  $N \approx 200$ monomers. The lack of strict equality here is due to the fact that 40% of the collisions that construct second generation aggregates are CCA collisions and 60% are PCA collisions. The aggregates chosen for the CCA collisions are the aggregates built in generation one. Thus, it may be the case that the target aggregate already has 185 monomers and undergoes a collision and sticking event with an aggregate that has 20 monomers. This then results in a generation two aggregate that has 205 monomers. Finally, generation three aggregates have a maximum of  $N \approx 2000$  monomers. In this third generation, 50% of the collisions are PCA, 30% are CCA with a first generation aggregate, and 20% are CCA with a second generation aggregate (Matthews, Land and Hyde 2012).

First, an initial monomer with a radius chosen randomly from the Mathis, Rumpl, and Nordsick (MRN) distribution (described in section 5.1b) is selected as the target particle. If generation two or generation three aggregates are being built, a generation one



Figure 8: Illustration of a generation three aggregate built through PPA and CCA collisions in Aggregate Builder. This aggregate has approximately 2000 monomers with radii selected from the polydisperse MRN size distribution. The structure of the aggregate differs greatly from that of a simple sphere of equal mass. It is important to take into consideration the "fluffy" structure of these aggregates when examining collisions between them.

or generation two aggregate, respectively, is randomly chosen as the target particle. The moment of inertia tensor is then calculated for the monomer or aggregate. Next, the incoming particle is selected based on the probabilities listed above. The position of the incoming particle is assigned. The radial distance of the incoming particle from the target particle is initially set as  $10(R_{AGG} + R_{NEW})$ , where the radius of the aggregate is defined as the maximum radial extent of the aggregate from the center of mass. The polar and azimuthal angles,  $\theta$  and  $\phi$ , are randomly assigned, taking on values between 0 and  $\pi$  and 0 and  $2\pi$ , respectively. The forces, as detailed in section 3.2b, are calculated and summed for the grain. A fifth-order Runge-Kutta method is utilized to determine the resulting position, velocity, and orientation of the target grain. These new parameters are then updated in the data structure that holds this information for the target and incoming particle. As the incoming particle moves closer to the target, the model begins checking for a collision. If the separation of the two particles' center-of-masses is less than the sum of the aggregate radii, then there is a possible collision. If the sum of the radii of two monomers is greater than the distance between the them, or in other words, if spheres in two different aggregates overlap, then there is a collision. If not, the forces, velocities, and positions are updated and the process continues until a collision does occur or a missed collision is determined.

When a collision occurs, due to low velocities, the particles are assumed to stick without restructuring (Matthews, Land, and Hyde 2012). The parameters of this new particle are calculated, including the radius, mass, moment of inertia, and angular momentum. These new parameters are then stored in the updated target particle data structure. The aggregate charge is determined using OML\_LOS, a code that utilizes OML

theory coupled with the LOS approximation, both of which are described in section 2.2c. All the date for the aggregate is then saved to a library that contains data for all intermediate aggregates in building up to the final aggregate size. This entire process then begins again for a new incoming particle. Particles are added to the target aggregate until the maximum number of monomers initially specified or maximum number of misses is reached. Then a new aggregate is initialized and built. Aggregates from these libraries are then used to build generation two and generation three aggregates.

#### CHAPTER FOUR

## Planetary Ring Results

### 4.1 Validating the F Ring Model

Little is known regarding the dynamics of charged dust within the F Ring, making it a very interesting object of study. The model being used to study the F Ring dynamics can be confirmed to be accurate. This indicates that the results produced by the code for the F Ring are physically realizable by first validating the model against the results for both non-charged and charged dust in other more well-studied ring systems.

## 4.1a Orbit Averaged Equations of Motion

In order to validate our model, comparison to the dust dynamics of more wellstudied systems, specifically the Martian system and Saturn's E Ring, are made. These simulations, carried out by Burns *et al.* (2001), calculate the orbital parameters directly through what are commonly referred to as *orbit-averaged equations of motion*. As outlined in section 3.1b, the methods for calculating orbital parameters as used in this study involve first summing all the forces on the grain and then numerically integrating to find the velocities and positions in Cartesian coordinates. Conversions are then made to orbital parameters.

The orbit-averaged method, outlined in Hamilton (1993), takes a more direct approach to the calculation of these orbital elements, although the expressions are certainly less intuitive than a Newtonian approach. As shown in Horanyi *et al.* 1992, the results of these two methods show very little variation between them, with the orbit-

averaged equations having the advantage of being much faster computationally, an important consideration when conducting simulations that cover several decades of simulated time.

Orbital elements are used primarily because, unlike Cartesian velocities and positions, they allow for direct visualization of a body's orbit. This also comes about because they are *slowly varying* in time, a property that grants a time-averaged approach much credence. This then allows for averaging of these parameters over a single Keplerian orbit (Hamilton 1993). Such calculations have been carried out by Burns *et al.* (2001), Burns *et al.* (1979), and Hamilton (1993) for the five osculating orbital elements ( $a, e, i, \Omega, \omega$ ) as well as the solar angle ( $\phi_{sol}$ ). Perturbations due to gravitational forces (including corrections for oblateness), electromagnetic forces, and the radiation pressure are taken into account. One example averaging method will be shown based on procedures given in Hamilton (1993) and then expressions for the summation of these perturbations for each of the orbital elements for all three perturbing forces will be given.

Consider the expression for the gravitational potential given in Eq. 2.5 including the  $J_2$  term in the expansion and thus taking into account the 2<sup>nd</sup> order correction for oblateness. By rewriting this in terms of the appropriate orbital elements and averaging it over time, an expression for the perturbation in terms of these more useful parameters is obtained. Substituting these into appropriate planetary equations, Hamilton (1993) obtains

$$\left\langle \frac{da}{dt} \right\rangle_{J_2} = 0, \tag{4.1a}$$

$$\left(\frac{de}{dt}\right)_{J_2} = 0, \tag{4.1b}$$

$$\left\langle \frac{di}{dt} \right\rangle_{J_2} = 0, \tag{4.1c}$$

$$\left(\frac{d\Omega}{dt}\right)_{J_2} = \frac{3nJ_2R_p^2}{2a^2(1-e^2)^2}\cos i,$$
(4.1d)

$$\left\langle \frac{d\omega}{dt} \right\rangle_{J_2} = \frac{3nJ_2R_p^2}{2a^2(1-e^2)^2} \left(2 - \frac{5}{2}\sin^2 i\right),\tag{4.1e}$$

$$\left(\frac{d\mathcal{M}}{dt} - n\right)_{J_2} = \frac{3nJ_2R_p^2}{2a^2(1 - e^2)^2} \left(2 - \frac{3}{2}\sin^2 i\right),\tag{4.1f}$$

where  $\mathcal{M} = nt$  is the mean anomaly, here used in place of v (or  $\phi_{sol}$ ). It should be noted that these expressions are trivially integrable and that, when combined, give the average rate at which a particle completes a single orbit. Similar methods are used to construct analogous equations for the perturbing electromagnetic forces as well as the radiation pressure. These orbit averaged expressions can be summed to find the total perturbation of each element such that  $\left\langle \frac{d\Psi}{dt} \right\rangle_{\text{total}} = \sum_j \left\langle \frac{d\Psi}{dt} \right\rangle_j$  where  $\Psi$  is any one of the orbital elements and the index *j* includes the gravitational and electromagnetic forces as well as the force due to radiation pressure. Summing these effects, Hamilton (1993) gives

$$\left\langle \frac{da}{dt} \right\rangle = 0, \tag{4.2a}$$

$$\left\langle\frac{de}{dt}\right\rangle = \alpha (1 - e^2)^{1/2} \sin \phi_{sol}, \qquad (4.2b)$$

$$\left\langle\frac{di}{dt}\right\rangle = Z\cos\omega,\tag{4.2c}$$

$$\left(\frac{d\Omega}{dt}\right) = Z \frac{\sin \omega}{\sin i} + \dot{\Omega}_{xy}, \qquad (4.2d)$$

$$\left\langle \frac{d\omega}{dt} \right\rangle = -Z \frac{\sin \omega}{\sin i} + \dot{\omega}_{xy},$$
 (4.2e)

where  $\dot{\Omega}_{xy}$ ,  $\dot{\omega}_{xy}$  are the precession rates arising from the included perturbing forces and Z represents the contributions of the two vertical forces: the out-of-plane component of the

radiation pressure and the force arising from the aligned gravitational quadrupole field. Thus, it can be seen that an orbit-averaged equation of motion approach certainly involves a higher degree of mathematical rigor than a simple summation of the perturbing forces on each grain. However, performing these time averages allows for smaller computing times, a worthwhile cause when performing lengthy simulations.

#### 4.1b Dust Dynamics around Mars and in Saturn's E Ring

To validate the results of simulations done using the F Ring parameters, the numerical model described in section 3.1b was used to recreate the results of Burns *et al.* (2001) for conditions specific to that of a dust grain orbiting Mars and in the E Ring of Saturn, as shown in Figures 9 and 10 respectively. Although the methods used are significantly different from those employed by the numerical model described in section 3.1a, the results should not differ greatly provided the same initial conditions are used. Several important features of the plot should be noted. The semi-major axis, *a*, is constant for both systems, a result that is consistent with the orbit-averaged equation for this parameter as seen in section 4.1a. Additionally, the eccentricity, *e*, and the angle of inclination, *i*, both have periodic behavior over the chosen timescale of 40 years. Figures 11 and 12 show the recreations of these plots using the integration of forces acting on the grain as it orbits Mars and Saturn respectively and the subsequent conversion from Cartesian coordinates to orbital parameters.

Comparing data from the two models for dust around Mars (Figures 9 and 11), similarities are immediately recognizable. The semi-major axis, a, in both the orbital parameter conversions method (OPC hereafter) as well as the orbit-averaged approach (OAE hereafter), seen in Figures 9 and 10, both yield a semi-major axis length that is



Figure 9: Plot of the orbital parameters of a 20  $\mu$ m dust grain orbiting Mars ( $J_2 = 0.001960$ ) taking into consideration the radiation pressure and oblateness correction where  $\alpha = 1.078$  rad/yr and  $\gamma = 25.2^{\circ}$ .



Figure 10: Plot of the orbital parameters of 1.2 µm dust grain orbiting in Saturn's E Ring ( $J_2 = 0.01630$ ) taking the radiation pressure, correction for oblateness, and electrostatic force into consideration where  $\alpha = 0.1169$  rad/yr and  $\gamma = 26.7^{\circ}$ , the planetary obliquity. Figures taken from Burns *et al.* (2001).

constant in time at approximately 7  $R_p$ . The frequency of modulation of the eccentricity, e, isroughly the same in both models although the OPC model (Figure 11b) yields a dip in magnitude approximately every one or two cycles (with the magnitude of the peaks decreasing to as low as 0.25 at some points). The general trend of the solar angle,  $\phi_{sol}$ , is also similar with the frequencies (approximately half a cycle per year) and the ranges  $(-90^{\circ} \le \phi_{sol} \le 90^{\circ})$  of the two models being approximately the same. Small perturbations are also seen in the OPC model and occur approximately every seven years. These deviations are mirrored in both the eccentricity and the angle of inclination (Figure 11). The change in the magnitude of the angle of inclination, i, is similar across the two simulations although the frequency in the OPC model (Figure 11) is much three times as great. Concerning the argument of the pericenter,  $\omega$ , the difference in range between the two models is simply a shift factor of 180°. The steady increase seen in the two distinct 20-year-long cycles of the OAE model (Figure 9e) is not seen in that of the OPC model (Figure 11e). The longitude of the ascending node,  $\Omega$ , varies over the same range in both models (Figures 9f and 11f), plus a shift factor of 180° between the OAE and OPC models. However, the gradual precession of approximately 15 deg/yr in the OAE model (Figure 9f) is replaced by a regression of about 25 deg/yr in the OPC model (Figure 11f).

Better agreement is seen between the two models when applied to Saturn's E Ring (Figures 10 and 12). Again, *a* (Figures 10a and 12a) is constant for the entire simulation time, this time having a magnitude of approximately  $4R_p$ . Nearly perfect agreement is seen in the change in eccentricity *e* (Figures 10b and 12b), with both models

having the same frequency and magnitude. Peaks in the eccentricity occur at approximately 10-year intervals beginning at about 5 years (in simulation time) in both models. The frequency of  $\phi_{sol}$  (Figures 10c and 12c) follows approximately the same trend in both the OAE and OPC models, both having a period of approximately 9 years. Similar trends are seen in *i* (Figures 10d and 12d) between the two models. However, for the entire simulation, the magnitude of *i* in the OPC model (Figure 12d) is double that of the magnitude seen in the OAE model (Figure 10d). In both models, the trends in  $\omega$ (Figures 10e and 12e) are similar, with the OPC model (Figure 12e) including the discontinuities that the OAE model (Figure 10e) excludes. The behavior of  $\Omega$  in both models (Figures 10f and 12f) is similar. The two models vary over approximately the same range, the OAE model (Figure 10f) simply shifted upward by 360°. Discontinuities in the OPC model can be accounted for by minor differences in the numerical calculation of  $\Omega$ .

Comparisons between the OAE and OPC models, especially those concerning dust in Saturn's E ring (Figures 10 and 12), yield reasonable agreement between them. This then validates the OPC model as a useful tool for studying the thus far poorly understood dust dynamics of the F Ring. To extend our model to this new ring system then, the gravitational effect of the shepherding moons Prometheus and Pandora must now be taken into consideration. This new gravitational perturbation leads to more direct comparison between gravitational and electromagnetic forces, a phenomenon captured in the charge-to-mass ratios of the grains used.



Figure 11: Plot of the orbital parameters for a Martian dust grain a period of forty years. Forces taken into account include gravitational effects due to the oblation, the planet and the moons (Deimos and Phobos) as well as radiation pressure from the Sun.



Figure 12: Plot of the orbital parameters for a Martian dust grain for a period of forty years. Forces taken into account include gravitational effects due to the oblateness and the planet as well as the radiation pressure and the Lorentz force due to the presence of Saturn's magnetic field.

## 4.2 New Results for Dust Dynamics in the F Ring

To extend the aforementioned model to the F Ring system, few changes need to be made outside of changing the initial conditions. The grain's initial position is set as the radial distance of the F Ring from Saturn, 140 000 km  $\approx 2.3 R_p$ . Now, an additional perturbing force must be accounted for: that of the gravitational effects of the shepherding moons. These additional perturbations are taken into account when calculating the acceleration on the grain during its orbit. The force due to planetary oblateness will remain the same, varying only with the dust-planet distance r as  $R_p$  and the  $J_2$  and  $J_4$  corrections will be the same. Concerning the force due to radiation, using  $\alpha = 0.1169 \text{ rad/yr}$  and Eq. 3.4, the ratio between the forces due to radiation pressure and solar gravity (as defined by Eq. 2.9) is  $\beta = 0.6648$ . The force due to radiation pressure can then be calculated using Eq. 2.10. Although there is some interval where the dust grain is shielded from the solar radiation pressure by the planet (the so-called shadow time), it has been shown that taking this into account does not significantly alter the dynamics of the grain (Burns et al. 2001). Additionally, the only difference in calculating the Lorentz force (Eq. 2.12) on the grain is the variability of the charge-tomass ratio that is introduced.

In the case of the F Ring, orbital parameters are computed for a range of chargeto-mass ratios with the goal of comparing orbital parameters to observations in order to more exactly probe the plasma parameters of the F Ring. Orbital parameters were calculated for grains having charge-to-mass ratios in the range  $-0.1 \le q/m \le 0.1$  C/kg in steps of 0.01 C/kg. Figure 13 shows only one plot in this range, that of q/m = -0.1 C/kg. As in Figures 9-12, *a* remains constant, here at approximately  $2.3R_p$  (Figure 13a). The eccentricity varies in time with a period of almost 1 year and a maximum eccentricity of  $e \approx 0.03$  (Figure 13b). The solar angle (Figure 13c) exhibits a saw tooth-like pattern for the entire simulation time and oscillates with a period of 1 year, matching that of the eccentricity. The angle of inclination (Figure 13d) ranges between 0° and 0.25° with a period of approximately 2.5 years. There are additional smaller magnitude variations which have a period of approximately 1 year. The variations seen in the argument of the pericenter,  $\omega$ , are quite rapid (Figure 13e). The "phase locking" seen for dust in Saturn's E Ring (Figures 10e and 12e) is not present here.

Figures 14 and 15 show results for two additional charge-to-mass ratio values: q/m = -0.0 C/kg (an uncharged grain) and q/m = 0.1 C/kg respectively. Some interesting features arise when comparing the negative, neutral, and positive cases. In Figures 13b, 14b, and 15b, a very obvious decrease by nearly an order of magnitude is seen in the maximum amplitude of the eccentricity *e* from 0.05 (Figure 13b) to 0.005 (Figure 15b) as q/m increases from -0.1 C/kg to 0.1 C/kg. An increase in the frequency of *e* by a factor of approximately 5 (from approximately 1 cycle per year in Figure 13b to nearly 5 cycles per year in Figure 15b) is also seen over this q/m range. Additionally, drastic differences in the angle of inclination can be seen as q/m is varied. In Figure 13d, *i* has a period of approximately 2.5 years that remains constant throughout the entire 40 year simulation time. Contrastingly, for the neutral case (Figure 14d), *i* increases almost linearly from 0 to 0.4 and exhibits no periodic behavior. For the positive case (Figure 15d), the periodicity is recovered with the period in *i* increasing to approximately 3.5 years. Thus, the Lorentz force has some sort of confining effect on the oscillation of the orbital plane of the dust grain out of the equatorial plane of Saturn. Additional results for  $-0.09 \le q/m \le 0.09$  C/kg are shown in the Appendix.



Figure 13: Plot of the orbital parameters of a dust grain orbiting in in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.1 C/kg. The initial radial distance between the grain and Saturn is set as the radius of the F Ring, 140 000 km  $\approx 2.3R_p$ . Perturbing forces include the Lorentz force, the force due to radiation pressure, as well as the gravitational forces due to the two moons (Prometheus and Pandora), the planet, and the correction for oblateness.



Figure 14: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.0 C/kg.



Figure 15: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.1 C/kg.

#### CHAPTER FIVE

## Protoplanetary Disk Results

### 5.1 Initial Conditions

As mentioned, protoplanetary disks are believed to be the sites of early planet formation and it is actually the collision and sticking of dust aggregates that provide the earliest seeds for this planet formation process. Thus, understanding the growth and morphology of early aggregates is key to understanding the physical processes behind the formation of rocky terrestrial planets. Since these aggregates are immersed in a dusty plasma, collisions between the grains and charged particles (ions and electrons) occur and thus these aggregates can become charged. This charging then in turn affects how aggregates interact with each other, in particular how they stick together.

Matthews, Land and Hyde (2012) compared the growth and morphology of charged aggregates and neutral aggregates for a single location within a protoplanetary disk. They found that there is a significant difference between those aggregates that were charged and those that were neutral. In particular, the charged aggregates preferentially incorporated the largest monomers from the size distribution into their structure. This resulted in aggregates that were more highly charged being larger, more massive, and being much more porous. The neutral aggregates were the lightest and smallest aggregates in the population. Thus the charging of dust grains within a protoplanetary disk has been shown to affect the aggregate morphology.

## 5.1a Locations

The purpose of this investigation is to examine the effect of charged aggregate growth and morphology at various locations within the PPD. Such an investigation could be important when studying why certain types of planets form at certain locations. The location within the PPD is characterized by the radial distance from the YSO, *R*, measured in astronomical units (AU), and *Z*/*H*, the elevation above the midplane, *Z*, measured in AU divided by the vertical scale height, defined as  $H = c_g / \Omega_K$ . Here  $c_g$  is the sound speed in the gas and  $\Omega_K$  is the rotational frequency of the disk, both of which depend on the radius *R*.

Using these two parameters, a location in the disk can be specified by measuring its distance from the YSO at the center of the disk and the elevation above the dense, dusty midplane. For the purposes of this study, two different disk radii are examined: R = 1, 2 AU as well as six separate values of Z/H = 0.4, 0.6, 0.7, 0.8, 0.9, 1.0, making a total of twelve different locations in the disk. (It should be noted that *H* is also a function of the radial distance *R* so, for example, Z/H = 0.8 at R = 2 AU is not the same height above the disk midplane as Z/H = 0.8 at R = 1 AU.)

#### 5.1b Plasma and Dust Parameters

Plasma parameters must be defined for each separate location in the disk. Such parameters include the fractional ionization, the number of ions divided by the total number of particles, electron depletion factor, the ratio of electron and ion number densities showing the extent to which dust density affects the number of electrons available for collision, temperature of the gas (see Table 1), dust density, gas density,

Table 1: Gas number density, electron depletion factor, fractional ionization and midplane temperature for various locations in the PPD. When calculating  $n_e/n_i$ , it is assumed that the dust-to-gas mass density ratio is 1%. At low elevations ( $Z/H \le 1.0$ ), the disk is assumed to be isothermal so the temperature remains constant across Z/H. Electron depletion factors are taken from Matthews, Land and Hyde (2012). Fractional ionization measurements are taken from Ruden and Pollack (1991). Temperature values are taken from Willacy *et al.* (1998).

Z/H		0.4	0.6	0.7	0.8	0.9	1.0
	$n_{gas} (cm^{-3})$ (10 <sup>14</sup> )	1.80	1.63	1.53	1.42	1.30	1.19
<i>R</i> = 1 AU	$n_e/n_i$	0.60	0.40	0.45	0.50	0.55	0.55
	$X_e$ (10 <sup>-10</sup> )	0.5	0.6	0.6	4.0	4.0	7.0
	$T_C(\mathbf{K})$	630					
	$n_{gas} ({ m cm}^{-3}) \ (10^{14})$	0.33	0.30	0.28	0.26	0.24	0.21
R = 2  AU	$n_e/n_i$	0.65	0.40	0.45	0.50	0.55	0.55
	$\frac{X_e}{(10^{-10})}$	0.501	3.0	5.0	25.0	40.0	50.0
	$T_C(\mathbf{K})$	300					

electron number density, and ion number density. All of these factors become important when calculating the impact of the plasma environment on the grain charging and in turn the effect on the size of the aggregates.

The mass density of the gas is calculated using the expression

$$\rho_g = \Sigma_g / (\sqrt{2\pi}H) e^{-(Z/H)^2/2}, \tag{5.1}$$

where  $\Sigma_g = 1.7 \times 10^3 R^{-3/2}$  g cm<sup>-2</sup> is the gas surface density (Okuzumi *et al.* 2011). The number density of the gas is given by  $n_{gas} = \rho_g/m_g$ , where the average mass of one gas particle is  $m_g = \mu m_H$  and  $\mu$  is the mean molecular weight, assumed here to be 2.34 (Matthews, Land and Hyde 2012). The ion number density can then be calculated as  $n_{\rm ion} = X_e n_{\rm gas}$ , where  $X_e$  is the fractional ionization. Values for  $X_e$  at different locations in the PPD are given in Table 1. It should be noted that the ion mass is assumed to be that of Mg<sup>+</sup>, such that  $m_{\rm ion} = 24m_H$  (Okuzumi *et al.* 2011).

Fits to spectral data yield the size distribution of grains in the ISM, the Mathis-Rumpl-Nordsieck (MRN) distribution, given by

$$n(a_0) \propto a_0^{\ \alpha} da_0, \tag{5.2}$$

where  $\alpha = -3.5$  (Mathis 1977). The dust distribution in a molecular cloud, the structure from which the PPD and YSO form, is very similar to that of the ISM and so this is a reasonable approximation for dust grains in a PPD simulation. The spherical particles used in this study have radii in the range of  $0.5 \ \mu m \le a_0 \le 10 \ \mu m$  such that the average radius computed for the MRN distribution over this range of radii,  $\langle a_0 \rangle = 0.83 \ \mu m$ , matches the average radius of dust grains found in a PPD,  $\langle a_0 \rangle \approx 1 \ \mu m$  (Matthews, Land and Hyde 2012).

## 5.1c Turbulence in the Protoplanetary Disk

Charged grains interacting with each other will experience a Coulomb repulsion between them given by Eq. 1.1. Consider the example of two grains, one with a radius of 0.5  $\mu$ m and one with a radius of 10  $\mu$ m. The relative velocity between the grains can be found by equating the kinetic energy of the two grains to the electrostatic potential between them

$$K = \frac{1}{2}\mu v_r^2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{\Delta} = U,$$
 (5.3)

where  $\mu = m_1 m_2 / (m_1 + m_2)$  is the reduced mass of the two grains,  $v_r$  is the relative velocity of the two grains,  $Q_1, Q_2$  are the charges on the two grains and  $\Delta = a_1 + a_2$  is the separation at contact with  $a_1$ ,  $a_2$  being the respective radii. The charge on each grain can be calculated by treating the grain as a spherical capacitor (Eq. 2.25) and using the Spitzer result (see section 2.2b) for the surface potential. As an example, assume a hot PPD environment ( $T \approx 1100$  K) and a Maxwellian hydrogen plasma such that the minimum relative velocity between the grains needed for a collision to occur is  $v_{r, \min} = 7$  cm s<sup>-1</sup>. Brownian motion, or the random movement of particles driven by the thermal motion of the gas, leads to a relative velocity of

 $v_{r,B} = \sqrt{8k_BT/\mu\pi} = 0.5 \text{ cm s}^{-1}$ . It is easy to see that since  $v_{r,B} < v_{r,\min}$ , Brownian motion alone is not enough to overcome this electrostatic repulsion (Matthews, Land and Hyde 2012). However, when turbulence in the protoplanetary disk is taken into account (see section 2.1f), an additional relative velocity between the grains must be taken into account. Using Eq. 2.16 and assuming the two particles are composed of silicates of equal density (2.5 g cm<sup>-3</sup>), a relative velocity of  $v_{turb} = 9.9 \text{ cm s}^{-1}$  can be calculated and so it can be seen that  $v_{turb} > v_{r,\min}$ . Thus, when turbulence in the disk is taken into account, grains can develop relative velocities large enough to overcome the Coulomb repulsion between them and collide.

#### 5.1d Compactness Factor

The main objective of this study is to examine the relative effect of charging on the morphology of grains at different locations in the PPD. To mathematically define the structure of an aggregate, a dimensionless parameter, the compactness factor ( $\Phi_{\sigma}$ ), is used to define how porous or "fluffy" the aggregate is (Paszun and Dominik 2009). To calculate  $\Phi_{\sigma}$ , the aggregate is projected on a plane and the projected surface area  $A_i$  is calculated. This area is then set equal to that of a circle such that  $A_i = \pi R_i^2$ , where  $R_i$  is the equivalent radius. By averaging this over a large number of orientations,  $R_{\sigma}$ , the average equivalent radius, can be found. The compactness factor then is the ratio of the sum of all the volumes of the constituent monomers in the aggregate to that of the volume of a sphere with radius  $R_{\sigma}$  such that

$$\Phi_{\sigma} = \sum_{j=1}^{N} r_j^3 / R_{\sigma}^3, \tag{5.4}$$

where *N* is the total number of monomers in the aggregate and  $r_j$  is the radius of the  $j^{\text{th}}$  monomer in the aggregate (Matthews, Land and Hyde 2012). An illustration of the compactness factor and equivalent radius for an example aggregate is seen in Figure 16.

# 5.2 Results

Figures 17a and 17b show the charge number calculated for aggregates at all scale heights for the two respective disk radii. The large spread in charged at both disk radii seen for those aggregates with N = 2 (dimers) is due to variation in size of the monomers in the initial small aggregates. A small monomer (with a consequentially small charge) collides with a much larger monomer and the two stick, forming an aggregate with a charge dominated by the initial charge of the larger monomer. It should be noted that



Figure 16: Illustration of the compactness factor and equivalent radius for one aggregate. The lighter circle indicates the maximum radial extent of the aggregate, R, while the darker circle indicates the average equivalent radius,  $R_{\sigma}$ . To calculate  $\phi_{\sigma} = 0.24$ , the volumes of each of the monomers are summed and then divided by the volume of a sphere with radius  $R_{\sigma} = 24.5 \,\mu\text{m}$ .

charge on the largest grains at R = 1 AU is over two times greater than the maximum charge on the largest aggregates at R = 2 AU. Some variation is seen in the relationship between  $Z_D$  and N for the differing heights above the midplane at R = 2 AU. For R =1 AU (Figure 17a), the minimum aggregate charge is found for the scale heights Z/H = 0.6, 0.7, though the difference between Z/H = 0.4 and Z/H = 0.6 is relatively small, their average charge numbers differing only by about  $Z_D = 235$ . For R = 2 AU aggregate charge tends to increase with scale height. For example, charge on the largest aggregates at Z/H = 0.4 is approximately 1200 while for Z/H = 0.8 it is approximately  $Z_D = 2200$ . This difference tends to decrease as scale height increases as can be seen in Figure 17b. An increase in charge with scale height is not surprising. Farther away from the dense midplane of the disk, electron depletion is less significant and so the ratio of electrons to aggregates is higher.

Figures 18a and 18b show the compactness factor  $\phi_{\sigma}$  (as defined by Eq. 5.4) for aggregates at all scale heights for radii of R = 1, 2 AU respectively. The data points for Z/H = 0.4 are shown while only the fit lines are shown for the remaining heights. The large spread in both plots is and indication of the variety of shapes that aggregates of even the same number of monomers may assume. For example, two aggregates composed of 20 monomers may exhibit differences as large as 0.3 in their respective values of  $\phi_{\sigma}$ . Aggregates composed of several thousand monomers have compactness factor values closer to 0.1. This behavior holds for aggregates at both disk radii. Slight differences in  $\phi_{\sigma}$  as a function of scale height are seen across various locations in the protoplanetary disk. In Figure 16a, aggregates at Z/H = 1.0 show slightly higher compactness factor values for  $N \gtrsim 100$ . Aggregates at Z/H = 0.7, 0.8, 0.9 show slightly



Figure 17: Charge number (or number of electrons) as a function of the number of monomers in the aggregate at all heights above the midplane for (a) R = 1 AU and (b) R = 2 AU. Only data points for Z/H = 0.4 are shown, while fit lines are shown for the other elevations. The blue points correspond to a total of (a) 21,496 and (b) 19,948 aggregates.

lower  $\phi_{\sigma}$  values for these larger aggregates, differing from the most fluffy aggregates at Z/H = 1.0 by approximately 0.01. In Figure 18b, aggregates at Z/H = 0.8 show slightly higher  $\phi_{\sigma}$  values beginning at  $N \approx 20$ . Minimum values for aggregates at Z/H = 0.8 lie just above  $\phi_{\sigma} = 0.2$  while those aggregates at the remaining scale heights have compactness factors just below 0.2.

Figures 19a and 19b show the relationship between the charge number and the equivalent radius (as defined in section 5.1d). The distribution seen in both of these plots is much narrower compared to that of Figures 17a and 17b, an indication that aggregate charge is more dependent on equivalent aggregate size than on the number of monomers in the aggregate. The differences with respect to the height above the disk midplane are more apparent in this figure. In Figure 19a, minimum aggregate charge occurs at Z/H = 0.6, 0.7 while maximum charge occurs at Z/H = 0.8, 0.9, 1.0 as in Figure 17a.

The differences in charging and compactness factor as functions of scale height are shown in Figures 20a and 20b. The average and maximum values of both parameters were calculated by finding the mean and maximum values of  $Z_D$  and  $\Phi_\sigma$  for aggregates with N > 200 for each location investigated. The behavior of  $Z_D$  with respect to Z/H is seen in both the average and maximum charge number for both disk radii: charge tends to increase with scale height at 2 AU while minimum charge occurs at Z/H = 0.6, 0.7 at 1 AU. Additionally, the large difference in charge number between 1 and 2 AU is now more apparent. As can be seen in Table 1, the electron depletion and fractional ionization are both significantly smaller for  $Z/H \leq 0.7$ . This means that the electron number density at these lower scale heights will be significantly lower as compared to higher elevations in the disk, thus leading to fewer collisions between aggregates and electrons.



Figure 18: The compactness factor of aggregates for all heights above the midplane at (a) R = 1 AU and (b) R = 2 AU. Only data points for Z/H = 0.4 are shown while the other elevations are represented only by fit lines.

For aggregates at R = 2 AU, the trend of increasing charge with increasing scale height is more obvious, though the largest aggregates are not as highly charged as those at 1 AU (see Figure 20a). Again, an increase in fractional ionization is seen for Z/H > 0.7 (from  $X_e = 5 \times 10^{-10}$  to  $25 \times 10^{-10}$ ) with an additional increase between Z/H = 0.4 and 0.6 (from  $X_e = 0.501 \times 10^{-10}$  to  $3 \times 10^{-10}$ ). Differences in  $\Phi_{\sigma}$  as a function of radius and scale height as seen in Figure 20b are not nearly as pronounced as those differences seen in  $Z_D$  (Figure 20a). There is in fact no discernible difference in the average value of the compactness factor between the two radii at Z/H = 0.4 where  $\langle \Phi_{\sigma} \rangle \approx 0.24$  for both locations (as indicated by the dashed blue lines in Figure 20b). As Z/H increases, the difference in  $\Phi_{\sigma}$  at R = 1,2 AU increases with  $\langle \Phi_{\sigma} \rangle$  at R = 1 AU being less than that of  $\langle \Phi_{\sigma} \rangle$  at 2 AU. However, for R = 2 AU,  $\Phi_{\sigma}$  decreases for  $Z/H \ge 0.9$  and is in fact smaller than the values seen at R = 1 AU. The largest aggregates at both R = 1 AU and 2 AU have very little difference in  $\Phi_{\sigma}$ . These are the fluffiest aggregates with the smallest values of  $\Phi_{\sigma}$  ( $\Phi_{\sigma,min} \approx 0.14$ ). A small deviation is seen for R = 2 AU, Z/H = 0.8where  $\Phi_{\sigma,min} \approx 0.17$ . Thus, it can be seen that differences in aggregate charge persist not only across scale heights, but across radii as well while only small differences in  $\Phi_{\sigma}$ are manifest across disk radius and scale height.

Figures 21a and 21b show the distribution of monomer sizes for generation three aggregates, those with  $200 \leq N \leq 2000$ , for R = 1 AU and R = 2 AU. The straight dashed line indicates the MRN distribution described in section 5.1b. If the charging were to impact the size of the monomers incorporated into the aggregates, monomers of larger radii would be expected to be overrepresented as compared to the MRN distribution (Matthews, Land and Hyde 2012). Significant deviations from the MRN distribution are



Figure 19: The aggregate charge number versus the average equivalent radius measured in microns at all heights above the midplane for (a) R = 1 AU and (b) R = 2 AU. Note the difference in vertical scale for the two plots. Data points are for aggregates at Z/H = 0.4, while fit lines are shown for aggregate populations at other scale heights.

really only seen for Z/H = 0.4 at R = 1 AU (Figure 21a) and Z/H = 0.4, 0.9 at R = 2 AU (Figure 21b). Large deviations from the MRN distribution seen at larger radii are most likely due to small number statistics and are exacerbated by the log scale used here. These distributions shown for different locations in the PPD indicate that the difference in charging between locations is not significantly large enough to affect the distribution of monomer radii within the aggregate.



Figure 20: (a) Average and maximum values for the charge number  $(Z_D)$  at R = 1, 2 AU as functions of scale height. (b) Average and minimum values for the compactness factor  $(\Phi_{\sigma})$  at R = 1, 2 AU as functions of scale height. Average and maximum (minimum) values of both  $Z_D$  and  $\Phi_{\sigma}$  are calculated only for aggregates containing more than 200 monomers.



Figure 21: The distribution of monomers contained within generation three aggregates (200  $\leq N \leq$  2000) at (a) R = 1 AU and (b) R = 2 AU. At all heights above the midplane for both radii, the monomers within the aggregates follow the initial distribution with the exception of a few at larger radii.
#### CHAPTER SIX

## Conclusions

# 6.1 Discussion of Results

Two applications of dusty plasma physics in astrophysical contexts have been shown: (1) single-grain dynamics of charged dust in Saturn's F Ring and (2) charged aggregation of fractal dust grains at various locations within a protoplanetary disk. Extensive numerical simulations were carried out in each case. These individual studies were preceded by an extensive discussion of both the forces present on dust grains within astrophysical dusty plasmas as well as the mechanisms that lead to charging of dust grains in these environments. Perturbations of dust grain dynamics included (1) the gravitational force due to the planet (including the correction for planetary oblateness), the Sun, and other satellites that may be present (i. e. shepherding moons), (2) the electrostatic force that can occur between two charged dust grains, (3) the force due to radiation pressure from the Sun incident on the dust grain, (4) the Lorentz force on a charged grain resulting from a (planetary) magnetic field, (5) the force due to gas drag, and (6) relative velocities induced by turbulence in a gas. Charging currents to the grain were also discussed. While charge on a single spherical grain can be attained analytically with orbital motion limited (OML) theory, this approach in addition to a line of sight approximation were used in order to numerically compute the charge on fractal dust aggregates. Results of each of these simulations are discussed in the following sections.

#### 6.1a Circumplanetary Dust Results

A model was first tested to determine its validity for studying the dynamics of charged dust within Saturn's F Ring, an interesting and poorly understood component of Saturn's extensive ring system. Simulations by Burns *et al.* (2001) of the dust dynamics around Mars and in Saturn's E Ring using orbit averaged equations of motion were used in order to validate the results of the proposed model. The model proposed here uses a direct approach, numerically integrating the positions from the accelerations resulting from the perturbing forces acting on the grain. Comparisons between Figures 9(a-f), 10(a-f) and Figures 11(a-f), 12(a-f) reveal reasonably good agreement between the results of Burns *et al.* (2001) and this model. Slight deviations are seen in both the Martian and E Ring dust, but the differences are not such that significant changes to the model need be made.

Given this agreement, simulations were carried out for a range of charge-to-mass ratios for a single dust grain in the F Ring. Preliminary results indicate significant differences in the orbits of the grains as caused by changes to the charge-to-mass ratio. As seen in Figures 13-15, the differences due to varying q/m values are most manifest in the eccentricity e and the angle of inclination i. As the charge to mass ratio increases from -0.1 C/kg up to 0.1 C/kg, the magnitude of the eccentricity decreases by an order of magnitude while the frequency increases by a factor of 5 (see Figures 13b, 14b, and 15b). Additionally, negatively charged grains (see Figure 13 and Appendix) exhibit a periodicity in the angle of inclination i which steadily dies out as the grain charge gradually becomes neutral (Figure 14d). This periodicity is recovered once more as the grain charge increases (see Figure 15d and Appendix). The Lorentz force then has some

sort of confining effect on the grain as manifested by the limits placed on the angle of inclination. If the grain is uncharged, the Lorentz force has no effect and the angle of inclination can continue to increase linearly with time (see Figure 14d). This result is not consistent with measured values of the angle inclination of the F Ring thus leading to the conclusion that these grains must have some significant charge-to-mass ratio (Bosh *et al.* 2002).

## 6.1b Protoplanetary Disk Results

Simulations of the growth of charged dust at multiple locations within a protoplanetary disk environment were carried out. Matthews, Land and Hyde (2012) showed that for a particular spot in the PPD (R = 1 AU, Z/H = 0.5), charged aggregation, as compared to neutral aggregation, produces fluffier aggregates that preferentially incorporate larger monomers. Thus, the goal of this work was to determine whether the differences in conditions (i.e. temperature, fractional ionization, electron depletion, etc.) at differing locations in the PPD could impact the charging of aggregates so as to cause sufficient differences in their morphology and consequently changes in the rate of aggregate growth.

Simulations of dust growth at twelve locations (six different scale heights each for two different radii) revealed significant differences in the number of electrons per dust aggregate as a function of both disk radius and scale height. As seen in Figures 15a and 15b, higher charges were seen at R = 1 AU as compared to R = 2 AU for all six scale heights. Additionally, both mean and maximum charge number tend to increase as scale height increases (see Figure 18a). An exception occurs at R = 1 AU where a decrease in mean number of electrons is seen between Z/H = 0.4 and Z/H = 0.6, decreasing the

charge accumulated on the dust from approximately  $Z_D = 2500$  to  $Z_D = 2000$ . At R = 2 AU, the mean charge number tends to increase steadily as scale height increases, ranging from approximately 600 electrons per aggregate at Z/H = 0.4 up to nearly 1200 electrons per aggregate at Z/H = 1.0 (see Figure 18a).

Just as significant differences between aggregate charge number are seen between locations in the PPD, small differences in morphology between the chosen locations are also observed. As shown in Matthews, Land and Hyde (2012), charge is found to be a linear function of the aggregate equivalent radius (Figures 17a and 17b), as calculated in section 5.1d. The compactness factor, a numerical representation of how "fluffy" an aggregate is, decreases with number of monomers per aggregate at all radii and scale heights. However, differences between the compactness factors for the largest aggregates formed across locations in the PPD are slight.

An important result to note is the decrease in charge number on aggregates at R = 1 AU, Z/H = 0.6, 0.7 (Figure 18a). As stated previously, grains with small charges tend to be more compact and less fluffy, a result that is somewhat manifest in trends in the compactness factor as a function of scale height (see Figure 18b). The smaller charges allow for more frequent collisions between aggregates. Thus, aggregation will progress at a faster rate than at those locations where the average charge number is higher. This same implication applies to the charge numbers found at R = 1 AU as opposed to 2 AU. The significantly smaller charge per aggregate seen at the larger radial distance (see Figure 18a) will lead to the production of smaller, more compact aggregates at a faster rate as opposed to the larger, more fluffy aggregates that result at locations which have aggregates with higher  $Z_D$ .

#### 6.2 Future Work

Many points of charged dust dynamics in astrophysical systems still need additional research. In addition to the variation in q/m for a given  $\beta$ , several other values of  $\beta$ , reflecting differences in grain composition, need to be investigated. Matching the orbital characteristics of charged grains to those observed for the F Ring will allow constraints to be placed on the plasma environment, which has yet to be measured by Cassini. Allowances may also be made for the inclusion of the shadow time. More accurate calculations and checks of the orbital parameters are also needed. The greatest differences seen in the OAE and OPC models were in the magnitude of the angle of inclination, with the maximum simulated value of *i* being double that of the same parameter calculated by Burns *et al.* (2001). Explanations for such differences are sought.

A survey of a larger number of locations is needed in order to better examine the growth of fractal aggregates at various locations in the PPD. As it has been confirmed that differences in aggregate morphology exist between charged and neutral grains, it is logical that for large enough differences in grain charge, significant differences in aggregate structure will occur. An examination of PPD conditions at greater radii and higher scale heights would result in larger differences in temperature and fractional ionization leading to larger differences in aggregate charge and presumably more significant differences in morphology as seen by trends in the compactness factor. Computing parameters could also be used for the mapping out of a so-called "dead zone" in the disk, where it has been shown that aggregate growth is severely inhibited by increased grain charge (Okuzumi *et al.* 2011). This would involve a more global view of the protoplanetary disk and allow for more macroscopic conclusions concerning planet

formation. A more accurate method of determining the fractional ionization is also sought so as to include ionization rates due to x ray and cosmic ray ionization. This would allow for a more precise determination of charging conditions at given locations and allow for simpler addition of new locations in the PPD to the model. The inclusion of stochastic effects, variations in charge as a function of time as electrons and ions randomly impact the grain, would also be an important addition to this model. This is important for small grains because it leads to differences in morphology similar to those seen in more highly charged grains (Matthews, Shotorban, and Hyde 2012). APPENDIX



Figure A1: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.09 C/kg.



Figure A2: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.08 C/kg.



Figure A3: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.07 C/kg.



Figure A4: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.06 C/kg.



Figure A5: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.05 C/kg.



Figure A6: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.04 C/kg.



Figure A7: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.03 C/kg.



Figure A8: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.02 C/kg.



Figure A9: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = -0.01 C/kg.



Figure A10: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.01 C/kg.



Figure A11: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.02 C/kg.



Figure A12: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.03 C/kg.



Figure A13: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.04 C/kg.



Figure A14: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.05 C/kg.



Figure A15: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.06 C/kg.



Figure A16: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.07 C/kg.



Figure A17: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.08 C/kg.



Figure A18: Plot of the orbital parameters of a dust grain orbiting in Saturn's F Ring with a charge-to-mass ratio of q/m = 0.09 C/kg.

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