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Note on a NAHE Variation

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Abstract

We present a variation of the NAHE-basis for free fermionic heterotic string models. By rotating some of boundary conditions of the NAHE periodic/anti-periodic fermions $\{y^m, \overline{y}^m, w^m, \overline{w}^m,\}$, for $m = 1$ to 6, associated with the six compact dimensions of a bosonic lattice/orbifold model, we show an additional method for enhancing the standard NAHE gauge group of $SO(10)$ back to E_6. This rotation transforms $(SO(10) \otimes SO(6)^3)_{rmobs} \otimes (E_8)_{\rm hid}$ into $(E_6 \otimes U(1)^5)_{\rm obs} \otimes SO(22)_{\rm hid}$. When $SO(10)$ is enhanced to E_6 in this manner, the $i^{\rm th}$ MSSM matter generation in the $SO(10)$ $\mathbf{16}_i$ rep, originating in twisted basis vector $\mathbf{b}_i$, recombines with both its associated untwisted MSSM Higgs in a $\mathbf{10}_i$ rep and an untwisted non-Abelian singlet ϕ_i, to form a $\mathbf{27}_i$ rep of E_6. Beginning instead with the E_6 model, the inverse transformation of the fermion boundary conditions corresponds to partial GUT breaking via boundary rotation.

Correspondence between free fermionic models with $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist (especially of the NAHE class) and orbifold models with a similar twist has received further attention recently. Our NAHE variation also involves a $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist and offers additional understanding regarding the free fermion/orbifold correspondence. Further, models based on this NAHE variation offer some different phenomenological features compared to NAHE-based models. In particular, the more compact $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist of the NAHE variation offers a range of mirror models not possible from NAHE-based models.

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1 NAHE Variation with a Geometric Twist

The parameter space of the weak coupled free fermionic heterotic string (WCFFHS) [1, 2] region of the string/M landscape has proven to be rich in quasi-realistic models containing the Minimal Supersymmetric Standard Model or its extensions. The WCFFHS region has produced a vast range of quasi-realistic (Near-)MSSM-like models [3, 4, 5, 6], semi-GUT models [7, 8, 9, 10], and GUT models [11]. The majority of these models are constructed as extensions of the NAHE set [12], with the 5 basis vectors of the NAHE set as their common core. Within the five basis vectors of the NAHE set, the 12 real free fermions representing the 6 compactified bosonic directions have boundary condition vectors equivalent to a $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ orbifold twist. While basis vector extensions to the NAHE set may or may not break this $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry, the quasi-realistic models consistently follow the latter scheme.

The phenomenological fruitfulness of the WCFFHS region of the string landscape continues to inspire attention. Recent random searches of the region have been performed [13, 14, 10] and systematic searches are underway [15, 16, 17, 18]. Distribution functions of various phenomenological features have been computed and are being further refined by the systematic searches.

Also of current focus is the correspondence between free fermionic and orbifold models [19, 20, 21, 22, 23]. In [19] a complete classification was obtained for orbifolds of the form X/G, with X the product of three elliptic curves and G an Abelian extension of a group of $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twists acting on X. This includes $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ orbifolds. Each such orbifold was shown to correspond to a free fermionic model with geometric interpretation. The NAHE basis and certain model extensions were shown to have geometric interpretation and thus, have orbifold equivalences. However, the general class of quasi-realistic models with a NAHE basis were shown not to have geometric interpretation–specifically, their Hodge numbers were not reproducable by any orbifold X/G. In other words, the beyond-NAHE basis vectors necessary to yield a quasi-realistic model (by reducing the number of copies of each generation from 16 to 1 and breaking $SO(10)$ to a viable sub-group* consistently break the $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry in a manner that also eliminates geometric interpretation.

The non-geometric feature of the quasi-realistic WCFFHS models inspired us to investigate variations of the NAHE set that might allow for quasi-realistic models with geometric interpretation, particularly with geometric $T^6/\mathbb{Z}_2 \otimes \mathbb{Z}_2$ interpretation. In the next section of this note, we construct a NAHE variation of this form by rotating (interchanging) the boundary conditions of a subset of the 12 real fermions in two of the twisted sectors. We conclude by considering some of the phenomenological aspects of our new model class, especially in comparison to those of the NAHE class.

2 Construction and Phenomenology of the NAHE Variation

The NAHE basis set contains 5 basis vectors: The-all periodic sector **1** (present in all free fermionic models, the supersymmetry generating sector **S**, and the three generation sectors $\mathbf{b}_{i=1,2,3}$: The NAHE set is depicted in Table 1 below (in which a "1" denotes a periodic

*$SO(10)$ must be broken via Wilson loop effects of basis vectors rather than by GUT Higgs, since adjoint or higher dimension scalars are not possible in Kač-Moody rank one models.

fermion and a "0" denotes an antiperiodic fermion), which highlights its cyclic permutation symmetry. In Table 1, the $(y,\, w)^m$, for $m = 1$ to 6, are the six pairs of real fermions that replace the left-moving bosonic scalar fields X_m for the six compactified direction and the corresponding $(\overline{y},\, \overline{w})^m$ are the six pairs of real fermions that replace the left-moving $\overline{X}_m$. All other fermions in Table 1 are complex.

The gauge group after the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ space–time supersymmetry. The matter content is 48 spinorial $\mathbf{16}$'s of $SO(10)$ matter states, coming from sixteen copies from each sector $\mathbf{b}_1$ sector, $\mathbf{b}_2$ and $\mathbf{b}_3$. The sixteen copies in each sector are composed of 2 copies of $(\mathbf{16}, \mathbf{4}_i)$ reps and 2 copies of $(\overline{\mathbf{16}}, \overline{\mathbf{4}}_i)$ reps of $SO(10) \times SO(6)_i$. The untwisted sector also contains six copies of a pair of Higgs for each generation in the form of $(\mathbf{10}, \mathbf{6}_i)$ reps of $SO(10) \times SO(6)_i$, in addition to a single $(\mathbf{6}_i, \mathbf{6}_j)$ rep of $SO(6)_i \otimes SO(6)_j$, for each case of $i, j \in \{1, 2, 3\}$ and $i \neq j$. In a real basis of the $\overline{y}$ and $\overline{w}$, the generators of $SO(6)_1$ are $(\overline{\eta}^1, \overline{y}^1,\, \overline{y}^2,\, \overline{w}^5, \overline{w}^6)$; of $SO(6)_2$ are $(\overline{\eta}^2, \overline{y}^3,\, \overline{y}^4,\, \overline{y}^5, \overline{y}^6)$; and of $SO(6)_3$ are $(\overline{\eta}^2, \overline{w}^1,\, \overline{w}^2,\, \overline{w}^3, \overline{w}^4)$.

The three sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ are the three unique twisted sectors of the corresponding $\mathbb{Z}_2^a \times \mathbb{Z}_2^b$ orbifold compactification. The $\mathbb{Z}_2^a \times \mathbb{Z}_2^b$ acts on the $(y,\, w)_i$ and $(\overline{y},\, \overline{w})_i$ in the $\mathbf{b}_i$ according to

$$\mathbb{Z}_2^a: \qquad (y,\, \overline{y})^{m=3\,\dots,6} \to\; (y+1,\, \overline{y}+1)^m \pmod 2 \tag{2.1}$$

$$\mathbb{Z}_2^b: \qquad (y,\, \overline{y})^{m=1,\,2};\; (w,\, \overline{w})^{n=5,\,6} \to\; (y+1,\, \overline{y}+1)^m;\; (w+1,\, \overline{w}+1)^n \pmod 2. \tag{2.2}$$

Thus, $\mathbf{b}_1$ is a $\mathbb{Z}_2^a$ twisted sector; $\mathbf{b}_2$ is a $\mathbb{Z}_2^b$ twisted sector, and $\mathbf{b}_3+\mathbf{1}$ is a $\mathbb{Z}_2^a \otimes \mathbb{Z}_2^a$ twisted sector. The $Z_2^a \times Z_2^b$ NAHE orbifold is special precisely because of the existence of three twisted sectors (one per generation), with a permutation symmetry with respect to the horizontal $SO(6)^3$ symmetries. This symmetry also enables $\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 + \mathbf{1}$ to generate the massless sector that produces the spinor components of the hidden sector E_8 gauge group.

As discussed prior, the NAHE set is common to a large class of three generation free fermionic models. Model construction proceeds by adding to the NAHE set three or four additional boundary condition basis vectors which simultaneously break $SO(10)$ to one of its subgroups, $SU(5) \times U(1)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$, and reduce the number of generations to three chiral, one from each of the sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$. The various three generation models differ in their detailed phenomenological properties based on the specific assignment of boundary condition basis vector for the internal world–sheet fermions $\{y, w|\overline{y}, \overline{w}\}^{1,\cdots,6}$. This is one reason for our interest in examining the properties a new class of models based on a NAHE variation for which some of the boundary conditions of the $\{y, w|\overline{y}, \overline{w}\}^{1,\cdots,6}$ are exchanged.

Table 1. The NAHE Set

	ψ^μ	x^{12}	x^{34}	x^{56}	$\overline{\psi}^{1,...,5}$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0	0	0	0,...,0
$\mathbf{b}_1$	1	1	0	0	1,...,1	1	0	0	0,...,0
$\mathbf{b}_2$	1	0	1	0	1,...,1	0	1	0	0,...,0
$\mathbf{b}_3$	1	0	0	1	1,...,1	0	0	1	0,...,0

	$y^{3,...,6}$	$\overline{y}^{3,...,6}$	$y^{1,2}, w^{5,6}$	$\overline{y}^{1,2}, \overline{w}^{5,6}$	$w^{1,...,4}$	$\overline{w}^{1,...,4}$
1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
S	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}_1$	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}_2$	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
$\mathbf{b}_3$	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

The NAHE variation under discussion is produced by exchanging some of the periodic and anti-periodic boundary conditions in the second generation and third generation sectors as shown in Table 2: In $\mathbf{b}_2$ the boundary conditions of $(y, \overline{y})^{m=5,6}$ and $(w, \overline{w})^{m=5,6}$ are interchanged and in $\mathbf{b}_3$: the boundary conditions of $(y, \overline{y})^{m=1,2,3,4}$ and $(w, \overline{w})^{m=1,2,3,4}$ are interchanged. Under this exchange, both $\mathbb{Z}^a$ and $\mathbb{Z}^b$ now induce twists solely among the $(y,\, \overline{y})^m$ and no longer among the $(w,\, \overline{w})^m$. Further, $\mathbb{Z}^a \otimes \mathbb{Z}^b$ now corresponds exactly to $\mathbf{b}_3$, rather than to $\mathbf{b}_3 + \mathbf{1}$. The effect of the exchanged boundary conditions for the $\mathbb{Z}^a$ and $\mathbb{Z}^b$ twists is very non-trivial.

Table 2. A Variation on the NAHE Set

	$y^{1,2}$	$\overline{y}^{1,2}$	$y^{3,4}$	$\overline{y}^{3,4}$	$y^{5,6}$	$\overline{y}^{5,6}$	$w^{1,...,6}$	$\overline{w}^{1,...,6}$
$\mathbf{b}_1$	0,0	0,0	1,1	1,1	1,1	1,1	0,...,0	0,...,0
$\mathbf{b}_2$	1,1	1,1	0,0	0,0	1,1	1,1	0,...,0	0,...,0
$\mathbf{b}_3$	1,1	1,1	1,1	1,1	0,0	0,0	0,...,0	0,...,0

The observable gauge group is enhanced to $E_6 \otimes U(1)^5$ and the hidden sector gauge group transforms into $SO(22)$. The change in gauge group occurs because now it is the combination of $\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2+\mathbf{b}_3$, rather than of $\mathbf{1}+\mathbf{b}_1+\mathbf{b}_2+\mathbf{b}_3$, that forms a mass spinor gauge group sector. Thus, in the NAHE variation, there is a massless spinor sector involving the five complex $\overline{\psi}$ and the three complex $\overline{\eta}$ observable sector fermions rather than the eight complex $\overline{\phi}$ hidden sector fermions. This massless spinor sector enhances the $SO(10)$ symmetry generated into E_6. (The enhancement is into E_6 rather than E_8 because of the GSO constraints the $\mathbf{b}_i$ basis vectors place on the $\overline{\eta}^i$ spinors.)

The trace component of the 3 complex $\overline{\eta}$ fermions is also absorbed into the E_6, leaving $\overline{\eta}^1 - \overline{\eta}^2$ and $\overline{\eta}^1 + \overline{\eta}^2 - 2\overline{\eta}^3$ as generating 2 extra $U(1)$ charges, along with the 3 extra $U(1)$'s generated by the complex $\overline{y}^I = \overline{y}^1 + i\overline{y}^2$, $\overline{y}^{II} = \overline{y}^3 + i\overline{y}^4$, and $\overline{y}^{III} = \overline{y}^5 + i\overline{y}^6$.

Instead of producing 8 copies of non-chiral generations of $SO(10)$ $\mathbf{16}$'s in each $\mathbf{b}_i$ sector, this model produces 1 non-chiral generation of E_6 $\mathbf{27}$'s in each $\{\mathbf{1}, \mathbf{b}_i\}$ sector combination and an additional 4 non-chiral generations in each of the three $\{\mathbf{S} + \mathbf{b}_i + \mathbf{b}_j\}$, $i \neq j$ sector. (See Appendix A.) Thus, this model corresponds to $h^{1,1} = h^{2,1} = 15$. This model thus has the Hodge numbers and twisted sector matter distributions of the orbifold models $(1-2)$ and $(1-8)$ of [19] and may be the free fermionic equivalent of one of these.

The NAHE variation also contains 45 pairs of vector-like non-Abelian matter singlets (carrying $U(1)$ charges) with 9 pairs coming from the untwisted sector and 12 pairs from each of the three $\mathbf{b}_i + \mathbf{b}_j$ sectors. The untwisted sector also contains 6 copies of $\mathbf{22}$ reps of the hidden sector $SO(22)$, while each $\mathbf{S} + \mathbf{b}_i + \mathbf{b}_i$ sector produces an additional 8 copies of $\mathbf{22}$ reps of $SO(22)$. The third order components of the model's superpotential are given in Apendix B. (The next lowest order terms are fifth order–there are no fourth order terms.)

In concluding this section, we note that our NAHE variation has connection with another variation discussed in [8] that is formed form 6 basis vectors. In that model, the sector formed by the sum of the three $\mathbf{b}_i$ in our above variation was denoted as "X'' and was added to the NAHE group. In the latter, the observable sector GUT gauge group is also raised to E_6, with the same $U(1)$ enhancing $SO(10)$ to E_6. The total gauge group becomes $E_6 \otimes U(1)^2 \otimes SO(4)^3 \otimes E_8$, in contrast to our $E_6 \otimes U(1)^5 \otimes SO(22)$.

3 Discussion

In [15] we introduced a general algorithm for systematic generatation of the complete set of WCFFHS gauge group models up to a chosen layer L (number of basic vectors) and order N (the lowest common multiple of the orders N_i of the respective basis vectors $\mathbf{V}_i$, whereby N_i is the smallest positive integer such that $n_i \mathbf{V}_i = \vec{0} \,(\mathrm{mod}\, 2)$. (By gauge basis vectors, we mean those with all anti-periodic left-moving boundary conditions.) We have generalized our algorithm for systematical generation of models containing twisted matter sectors and, relatedly, have begun a systematic investigation of $SO(10)$ NAHE-based models [18]. Now, with the construction of the E_6 NAHE-variation presented herein, we are also initiating a parallel systematic investigation of models with the NAHE-variation as their core. The general phenomenology of this new class of models, and the particular characteristics of sub-classes of models defined by their observable gauge group will be presented in an upcoming series of papers.

One aspect of the NAHE-variation class of models that we will pursue are mirror models. That is, models with matching observable and hidden sector gauge groups and matter states. The possibility of NAHE-based mirror models was explored in [24], in which we show that since the charges of observable sector states in NAHE-based models are spread out beyond half (22) of the total number of right-moving complex fermions, GSO constraints imposed by the observable sector on the charges of hidden sector states significantly hinder realization of mirror models. In fact, in [24], we showed that in a large class (perhaps all) of NAHE-based models with mirror basis vectors, these GSO constraints enforce spontaneous breaking of an initial mirror symmetry of gauge groups.

However, our variation on the NAHE set appears more condusive to mirror model construction, since the $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ twist in the NAHE variation allow observable sector states to carry charges within just the first 11 of the 22 right-moving complex fermions, allowing the additional 11 charges to be reserved for hidden sector states. Specifically, an additional three sectors denoted $\mathbf{b}'_{i=1,2,3}$ mirroring $\mathbf{b}_{i=1,2,3}$ in the hidden sector might be added to our NAHE variation to generate a $(E_6 \otimes U(1)^5)_{\rm obs} \otimes (E_6 \otimes U(1)^5)_{\rm hid}$ model with matching matter states.†

It should be noted that, nevertheless, the GSO projections between observable and hidden massless matter sectors can never be totally independent, since the observable and hidden matter sectors will always have a periodic complex spacetime fermion in common. Modular invariance constraints require that any pair of order-2 mirror matter sectors have at least one more non-zero complex fermion boundary condition in common, albeit the complex fermion can be either a left-moving or right-moving. Hence, for order-2 the modular invariant rules cannot be satisfied by simply adding an additional set of hidden sector mirror matter sectors $\mathbf{b}'_i$, for $i = 1$ to 3, with real right-moving components defined by $(\mathbf{b}'_i)^n = (\mathbf{b}_i)^{44-n}$. In this case, while $\mathbf{b}'_i \cdot \mathbf{b}_i$ satisfy modular invariance requirements, $\mathbf{b}'_i \cdot \mathbf{b}_{j\neq i}$ do not. As we will show in [25], for higher order basis vectors, this requirement is lifted–mirror observable/hidden matter sectors with either only a periodic spacetime boundary condition in common or else only a periodic spacetime and left-moving complex fermion x boundary condition in common are consistent with modular invariance.

Results of our full exploration of gauge and matter mirror models based on our NAHE variation will appear in [25]. Rather than discuss the range now, we conclude instead with an interesting NAHE variation-based example of a gauge (but not matter) mirror model that satisfies modular invariance requirements. The observable and hidden sector matter basis vectors are not completely mirrors among the $\{\overline{\eta}^{(')}, \overline{y}^{(')}, \overline{w}^{(')}\}$. Hence observable and hidden sector matter are not mirror images. The gauge group is $(E_6)_{\rm obs} \otimes (U(1)^7 \otimes SU(4) \otimes (E_6)_{\rm hid}$. The model is chiral with 21 **27** reps and 3 $\overline{\mathbf{27}}$ reps of $(E_6)_{\rm obs}$. (The untwisted sector provides 3 **27**'s and 3 $\overline{\mathbf{27}}$'s–the 18 net chiral reps are all from the twisted sectors.) The model also contains 12 **4** and 12 $\overline{\mathbf{4}}$ reps (not in vector-like pairs) of $SU(4)$ and 48 $U(1)^5$-charged non-Abelian singlets. There are neither **27** nor $\overline{\mathbf{27}}$ reps of $(E_6)_{\rm hid}$. A net $\mathbb{Z}_6$ twist from additional sectors is needed to simultanously (1) reduce $(E_6)_{\rm obs}$ to a (semi-)GUT that does not require adjoint or higher scalar reps to induce a spontaneous symmetry breaking to the MSSM at low energy, and (2) reduce the number of copies of each matter generation from 6 to 1. The basis vectors and GSO projection matrix are given in Tables 3a and 3b.

†Nevertheless singlet states carrying both observable $U(1)^5_{\rm obs}$ and hidden $U(1)^5_{\rm hid}$ charges are likely to exist, and therefore mix the observable and hidden sectors.

Table 3a. Basic Vectors for Mirror Gauge Group Model Based on NAHE Variation

	ψ^μ	x^{12}	x^{34}	x^{56}	$\overline{\psi}^{1,...,5}$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\eta'}^1$	$\overline{\eta'}^2$	$\overline{\eta'}^3$	$\overline{\psi'}^{1,...,5}$
1	1	1	1	1	1,...,1	1	1	1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0	0	0	0	0	0	0,...,0
$\mathbf{b}_1$	1	1	0	0	1,...,1	1	0	0	0	0	0	0,...,0
$\mathbf{b}_2$	1	0	1	0	1,...,1	0	1	0	0	0	0	0,...,0
$\mathbf{b}_3$	1	0	0	1	1,...,1	0	0	1	0	0	0	0,...,0
$\mathbf{b}_1^{'}$	1	0	1	1	1,...,1	0	1	1	1	0	0	0,...,0
$\mathbf{b}_2^{'}$	1	0	1	0	1,...,1	1	0	1	0	1	0	0,...,0
$\mathbf{b}_3^{'}$	1	0	0	1	1,...,1	1	1	0	0	0	1	0,...,0

	$y^{1,2}$	$\overline{y}^{1,2}$	$y^{3,4}$	$\overline{y}^{3,4}$	$y^{5,6}$	$\overline{y}^{5,6}$	$w^{1,2}$	$\overline{w}^{1,2}$	$w^{3,4}$	$\overline{w}^{3,4}$	$w^{5,6}$	$\overline{w}^{5,6}$
1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
S	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}_1$	0,0	0,0	1,1	1,1	1,1	1,1	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}_2$	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}_3$	1,1	1,1	1,1	1,1	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}^{'}_1$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}^{'}_2$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$\mathbf{b}^{'}_3$	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

Table 3b. GSO Projection Matrix for Mirror Gauge Group Model Based on NAHE Variation

$k_{i,j}$	**1**	**S**	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$	$\mathbf{b}_1^{'}$	$\mathbf{b}_2^{'}$	$\mathbf{b}_3^{'}$
1	0	0	1	1	1	1	1	1
S	0	0	0	0	0	0	0	0
$\mathbf{b}_1$	1	1	1	1	1	1	0	0
$\mathbf{b}_2$	1	1	1	1	1	0	1	0
$\mathbf{b}_3$	1	1	1	1	1	0	0	1
$\mathbf{b}_1^{'}$	1	1	0	0	0	1	0	0
$\mathbf{b}_2^{'}$	1	1	0	0	0	0	1	0
$\mathbf{b}_3^{'}$	1	1	0	0	0	0	0	1

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A $E_6 \otimes U(1)^5 \otimes SO(22)$ States

Note: all $U(1)$ charges below have been multiplied by a factor or 4 to eliminate fractions.

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(44)$
1	G_1	**27**	0	8	0	0	0	**1**
	G_2	**27**	4	-4	0	0	0	**1**
	G_3	**27**	-4	-4	0	0	0	**1**
	$\overline{G}_1$	$\overline{\mathbf{27}}$	0	-8	0	0	0	**1**
	$\overline{G}_2$	$\overline{\mathbf{27}}$	-4	4	0	0	0	**1**
	$\overline{G}_3$	$\overline{\mathbf{27}}$	4	4	0	0	0	**1**
$\mathbf{S} + \mathbf{b}_1 + \mathbf{b}_2$	G_4	**27**	0	-4	-2	-2	0	**1**
	G_5	**27**	0	-4	-2	2	0	**1**
	G_6	**27**	0	-4	2	-2	0	**1**
	G_7	**27**	0	-4	2	2	0	**1**
	$\overline{G}_4$	$\overline{\mathbf{27}}$	0	4	2	2	0	**1**
	$\overline{G}_5$	$\overline{\mathbf{27}}$	0	4	2	-2	0	**1**
	$\overline{G}_6$	$\overline{\mathbf{27}}$	0	4	-2	2	0	**1**
	$\overline{G}_7$	$\overline{\mathbf{27}}$	0	4	-2	-2	0	**1**
$\mathbf{S} + \mathbf{b}_1 + \mathbf{b}_3$	G_8	**27**	-2	2	-2	0	-2	**1**
	G_9	**27**	-2	2	-2	0	2	**1**
	G_{10}	**27**	-2	2	2	0	-2	**1**
	G_{11}	**27**	-2	2	2	0	2	**1**
	$\overline{G}_8$	$\overline{\mathbf{27}}$	2	-2	2	0	2	**1**
	$\overline{G}_9$	$\overline{\mathbf{27}}$	2	-2	2	0	-2	**1**
	$\overline{G}_{10}$	$\overline{\mathbf{27}}$	2	-2	-2	0	2	**1**
	$\overline{G}_{11}$	$\overline{\mathbf{27}}$	2	-2	-2	0	-2	**1**
$\mathbf{S} + \mathbf{b}_2 + \mathbf{b}_3$	G_{12}	**27**	2	2	0	-2	-2	**1**
	G_{13}	**27**	2	2	0	-2	2	**1**
	G_{14}	**27**	2	2	0	2	-2	**1**
	G_{15}	**27**	2	2	0	2	2	**1**
	$\overline{G}_{12}$	$\overline{\mathbf{27}}$	-2	-2	0	2	2	**1**
	$\overline{G}_{13}$	$\overline{\mathbf{27}}$	-2	-2	0	2	-2	**1**
	$\overline{G}_{14}$	$\overline{\mathbf{27}}$	-2	-2	0	-2	2	**1**
	$\overline{G}_{15}$	$\overline{\mathbf{27}}$	-2	-2	0	-2	-2	**1**

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(44)$
$\mathbf{1}$	ϕ_1 ($\overline{\phi}_1$)	$\mathbf{1}$	0	0	0	-4	-4	$\mathbf{1}$
	ϕ_2 ($\overline{\phi}_2$)	$\mathbf{1}$	0	0	0	-4	4	$\mathbf{1}$
	ϕ_3 ($\overline{\phi}_3$)	$\mathbf{1}$	0	0	-4	0	-4	$\mathbf{1}$
	ϕ_4 ($\overline{\phi}_4$)	$\mathbf{1}$	0	0	-4	0	4	$\mathbf{1}$
	ϕ_5 ($\overline{\phi}_5$)	$\mathbf{1}$	0	0	-4	-4	0	$\mathbf{1}$
	ϕ_6 ($\overline{\phi}_6$)	$\mathbf{1}$	0	0	-4	4	0	$\mathbf{1}$
	ϕ_7 ($\overline{\phi}_7$)	$\mathbf{1}$	4	-12	0	0	0	$\mathbf{1}$
	ϕ_8 ($\overline{\phi}_8$)	$\mathbf{1}$	4	12	0	0	0	$\mathbf{1}$
	ϕ_9 ($\overline{\phi}_9$)	$\mathbf{1}$	-8	0	0	0	0	$\mathbf{1}$
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2$	ψ_1 ($\mathbf{S}_1$)	$\mathbf{1}$	0	12	2	2	0	$\mathbf{1}$
	ψ_2 ($\mathbf{S}_2$)	$\mathbf{1}$	0	12	2	-2	0	$\mathbf{1}$
	ψ_3 ($\mathbf{S}_3$)	$\mathbf{1}$	0	12	-2	2	0	$\mathbf{1}$
	ψ_4 ($\mathbf{S}_4$)	$\mathbf{1}$	0	12	-2	-2	0	$\mathbf{1}$
	ψ_5 ($\mathbf{S}_5$)	$\mathbf{1}$	4	0	2	2	-4	$\mathbf{1}$
	ψ_6 ($\mathbf{S}_6$)	$\mathbf{1}$	4	0	2	2	4	$\mathbf{1}$
	ψ_7 ($\mathbf{S}_7$)	$\mathbf{1}$	4	0	2	-2	-4	$\mathbf{1}$
	ψ_8 ($\mathbf{S}_8$)	$\mathbf{1}$	4	0	2	-2	4	$\mathbf{1}$
	ψ_9 ($\mathbf{S}_9$)	$\mathbf{1}$	4	0	-2	2	-4	$\mathbf{1}$
	ψ_{10} ($\mathbf{S}_{10}$)	$\mathbf{1}$	4	0	-2	2	4	$\mathbf{1}$
	ψ_{11} ($\mathbf{S}_{11}$)	$\mathbf{1}$	4	0	-2	-2	-4	$\mathbf{1}$
	ψ_{12} ($\mathbf{S}_{12}$)	$\mathbf{1}$	4	0	-2	-2	4	$\mathbf{1}$
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_3$	ψ_{13} ($\mathbf{S}_{13}$)	$\mathbf{1}$	2	6	2	-4	2	$\mathbf{1}$
	ψ_{14} ($\mathbf{S}_{14}$)	$\mathbf{1}$	2	6	2	-4	-2	$\mathbf{1}$
	ψ_{15} ($\mathbf{S}_{15}$)	$\mathbf{1}$	2	6	2	4	2	$\mathbf{1}$
	ψ_{16} ($\mathbf{S}_{16}$)	$\mathbf{1}$	2	6	2	4	-2	$\mathbf{1}$
	ψ_{17} ($\mathbf{S}_{17}$)	$\mathbf{1}$	2	6	-2	-4	2	$\mathbf{1}$
	ψ_{18} ($\mathbf{S}_{18}$)	$\mathbf{1}$	2	6	-2	-4	-2	$\mathbf{1}$
	ψ_{19} ($\mathbf{S}_{19}$)	$\mathbf{1}$	2	6	-2	4	2	$\mathbf{1}$
	ψ_{20} ($\mathbf{S}_{20}$)	$\mathbf{1}$	2	6	-2	4	-2	$\mathbf{1}$
	ψ_{21} ($\mathbf{S}_{21}$)	$\mathbf{1}$	6	-6	2	0	2	$\mathbf{1}$
	ψ_{22} ($\mathbf{S}_{22}$)	$\mathbf{1}$	6	-6	2	0	-2	$\mathbf{1}$
	ψ_{23} ($\mathbf{S}_{23}$)	$\mathbf{1}$	6	-6	-2	0	2	$\mathbf{1}$
	ψ_{24} ($\mathbf{S}_{24}$)	$\mathbf{1}$	6	-6	-2	0	-2	$\mathbf{1}$
$\mathbf{S}+\mathbf{b}_2+\mathbf{b}_3$	ψ_{25} ($\mathbf{S}_{25}$)	$\mathbf{1}$	-2	6	-4	2	2	$\mathbf{1}$
	ψ_{26} ($\mathbf{S}_{26}$)	$\mathbf{1}$	-2	6	-4	2	-2	$\mathbf{1}$
	ψ_{27} ($\mathbf{S}_{27}$)	$\mathbf{1}$	-2	6	-4	-2	2	$\mathbf{1}$
	ψ_{28} ($\mathbf{S}_{28}$)	$\mathbf{1}$	-2	6	-4	-2	-2	$\mathbf{1}$
	ψ_{29} ($\mathbf{S}_{29}$)	$\mathbf{1}$	-2	6	4	2	2	$\mathbf{1}$
	ψ_{30} ($\mathbf{S}_{30}$)	$\mathbf{1}$	-2	6	4	2	-2	$\mathbf{1}$
	ψ_{31} ($\mathbf{S}_{31}$)	$\mathbf{1}$	-2	6	4	-2	2	$\mathbf{1}$
	ψ_{32} ($\mathbf{S}_{32}$)	$\mathbf{1}$	-2	6	4	-2	-2	$\mathbf{1}$
	ψ_{33} ($\mathbf{S}_{33}$)	$\mathbf{1}$	-6	-6	0	2	2	$\mathbf{1}$
	ψ_{34} ($\mathbf{S}_{34}$)	$\mathbf{1}$	-6	-6	0	2	-2	$\mathbf{1}$
	ψ_{35} ($\mathbf{S}_{35}$)	$\mathbf{1}$	-6	-6	0	-2	2	$\mathbf{1}$
	ψ_{36} ($\mathbf{S}_{36}$)	$\mathbf{1}$	-6	-6	0	-2	-2	$\mathbf{1}$

HWS Sector	State	E_6	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_5$	$SO(44)$
1	H_1	**1**	0	0	0	0	-4	**22**
	H_2	**1**	0	0	0	0	4	**22**
	H_3	**1**	0	0	0	-4	0	**22**
	H_4	**1**	0	0	0	4	0	**22**
	H_5	**1**	0	0	-4	0	0	**22**
	H_6	**1**	0	0	4	0	0	**22**
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2$	H_7	**1**	4	0	2	2	0	**22**
	H_8	**1**	4	0	2	-2	0	**22**
	H_9	**1**	4	0	-2	2	0	**22**
	H_{10}	**1**	4	0	-2	-2	0	**22**
	H_{11}	**1**	-4	0	2	2	0	**22**
	H_{12}	**1**	-4	0	2	-2	0	**22**
	H_{13}	**1**	-4	0	-2	2	0	**22**
	H_{14}	**1**	-4	0	-2	-2	0	**22**
$\mathbf{S}+\mathbf{b}_1+\mathbf{b}_3$	H_{15}	**1**	2	6	2	0	2	**22**
	H_{16}	**1**	2	6	2	0	-2	**22**
	H_{17}	**1**	2	6	-2	0	2	**22**
	H_{18}	**1**	2	6	-2	0	-2	**22**
	H_{19}	**1**	-2	-6	2	0	2	**22**
	H_{20}	**1**	-2	-6	2	0	-2	**22**
	H_{21}	**1**	-2	-6	-2	0	2	**22**
	H_{22}	**1**	-2	-6	-2	0	-2	**22**
$\mathbf{S}+\mathbf{b}_2+\mathbf{b}_3$	H_{23}	**1**	-2	6	0	2	2	**22**
	H_{24}	**1**	-2	6	0	2	-2	**22**
	H_{25}	**1**	-2	6	0	-2	2	**22**
	H_{26}	**1**	-2	6	0	-2	-2	**22**
	H_{27}	**1**	2	-6	0	2	2	**22**
	H_{28}	**1**	2	-6	0	2	-2	**22**
	H_{29}	**1**	2	-6	0	-2	2	**22**
	H_{30}	**1**	2	-6	0	-2	-2	**22**

B $E_6 \otimes U(1)^5 \otimes SO(22)$ Third Order Superpotential (No Fourth Order Terms Exist)

$$\overline{G}_1\ G_2\ G_3 + G_1\ \overline{G}_2\ \phi_7 + G_1\ \overline{G}_3\ \overline{\phi}_8 + G_1\ G_4\ G_7 + G_1\ G_5\ G_6$$
$$+ G_1\ \overline{G}_4\ \overline{\psi}_1 + G_1\ \overline{G}_5\ \overline{\psi}_2 + G_1\ \overline{G}_6\ \overline{\psi}_3 + G_1\ \overline{G}_7\ \overline{\psi}_4 + G_2\ \overline{G}_1\ \overline{\phi}_7$$
$$+ G_2\ \overline{G}_3\ \phi_9 + G_2\ G_8\ G_{11} + G_2\ G_9\ G_{10} + G_2\ \overline{G}_8\ \overline{\psi}_{21} + G_2\ \overline{G}_9\ \overline{\psi}_{22}$$
$$+ G_2\ \overline{G}_{10}\ \overline{\psi}_{23} + G_2\ \overline{G}_{11}\ \overline{\psi}_{24} + G_3\ \overline{G}_1\ \phi_8 + G_3\ \overline{G}_2\ \overline{\phi}_9 + G_3\ G_{12}\ G_{15}$$
$$+ G_3\ G_{13}\ G_{14} + G_3\ \overline{G}_{12}\ \overline{\psi}_{33} + G_3\ \overline{G}_{13}\ \overline{\psi}_{34} + G_3\ \overline{G}_{14}\ \overline{\psi}_{35} + G_3\ \overline{G}_{15}\ \overline{\psi}_{36}$$
$$+ \overline{G}_1\ \overline{G}_2\ \overline{G}_3 + \overline{G}_1\ G_4\ \psi_1 + \overline{G}_1\ G_5\ \mathrm{s}2 + \overline{G}_1\ G_6\ \mathrm{s}3 + \overline{G}_1\ G_7\ \mathrm{s}4$$
$$+ \overline{G}_1\ \overline{G}_4\ \overline{G}_7 + \overline{G}_1\ \overline{G}_5\ \overline{G}_6 + \overline{G}_2\ G_8\ \psi_{21} + \overline{G}_2\ G_9\ \psi_{22} + \overline{G}_2\ G_{10}\ \psi_{23}$$
$$+ \overline{G}_2\ G_{11}\ \psi_{24} + \overline{G}_2\ \overline{G}_8\ \overline{G}_{11} + \overline{G}_2\ \overline{G}_9\ \overline{G}_{10} + \overline{G}_3\ G_{12}\ \psi_{33} + \overline{G}_3\ G_{13}\ \psi_{34}$$
$$+ \overline{G}_3\ G_{14}\ \psi_{35} + \overline{G}_3\ G_{15}\ \psi_{36} + \overline{G}_3\ \overline{G}_{12}\ \overline{G}_{15} + \overline{G}_3\ \overline{G}_{13}\ \overline{G}_{14} + G_4\ G_{10}\ G_{15}$$
$$+ G_4\ G_{11}\ G_{14} + G_4\ \overline{G}_7\ \overline{\phi}_5 + G_4\ \overline{G}_{10}\ \psi_{30} + G_4\ \overline{G}_{11}\ \psi_{29} + G_4\ \overline{G}_{14}\ \psi_{16}$$
$$+ G_4\ \overline{G}_{15}\ \psi_{15} + G_5\ G_{10}\ G_{13} + G_5\ G_{11}\ G_{12} + G_5\ \overline{G}_6\ \overline{\phi}_6 + G_5\ \overline{G}_{10}\ \psi_{32}$$
$$+ G_5\ \overline{G}_{11}\ \psi_{31} + G_5\ \overline{G}_{12}\ \psi_{14} + G_5\ \overline{G}_{13}\ \psi_{13} + G_6\ G_8\ G_{15} + G_6\ G_9\ G_{14}$$
$$+ G_6\ \overline{G}_5\ \phi_6 + G_6\ \overline{G}_8\ \psi_{26} + G_6\ \overline{G}_9\ \psi_{25} + G_6\ \overline{G}_{14}\ \psi_{20} + G_6\ \overline{G}_{15}\ \psi_{19}$$
$$+ G_7\ G_8\ G_{13} + G_7\ G_9\ G_{12} + G_7\ \overline{G}_4\ \phi_5 + G_7\ \overline{G}_8\ \psi_{28} + G_7\ \overline{G}_9\ \psi_{27}$$
$$+ G_7\ \overline{G}_{12}\ \psi_{18} + G_7\ \overline{G}_{13}\ \psi_{17} + G_8\ \overline{G}_6\ \overline{\psi}_{28} + G_8\ \overline{G}_7\ \overline{\psi}_{26}$$
$$+ G_8\ \overline{G}_{11}\ \overline{\phi}_3 + G_8\ \overline{G}_{13}\ \mathrm{s}8 + G_8\ \overline{G}_{15}\ \mathrm{s}6 + G_9\ \overline{G}_6\ \overline{\psi}_{27} + G_9\ \overline{G}_7\ \overline{\psi}_{25}$$
$$+ G_9\ \overline{G}_{10}\ \overline{\phi}_4 + G_9\ \overline{G}_{12}\ \mathrm{s}7 + G_9\ \overline{G}_{14}\ \mathrm{s}5 + G_{10}\ \overline{G}_4\ \overline{\psi}_{32} + G_{10}\ \overline{G}_5\ \overline{\psi}_{30}$$
$$+ G_{10}\ \overline{G}_9\ \phi_4 + G_{10}\ \overline{G}_{13}\ \psi_{12} + G_{10}\ \overline{G}_{15}\ \psi_{10} + G_{11}\ \overline{G}_4\ \overline{\psi}_{31} + G_{11}\ \overline{G}_5\ \overline{\psi}_{29}$$
$$+ G_{11}\ \overline{G}_8\ \phi_3 + G_{11}\ \overline{G}_{12}\ \psi_{11} + G_{11}\ \overline{G}_{14}\ \mathrm{s}9 + G_{12}\ \overline{G}_5\ \overline{\psi}_{14} + G_{12}\ \overline{G}_7\ \overline{\psi}_{18}$$
$$+ G_{12}\ \overline{G}_9\ \overline{\psi}_7 + G_{12}\ \overline{G}_{11}\ \overline{\psi}_{11} + G_{12}\ \overline{G}_{15}\ \overline{\phi}_1 + G_{13}\ \overline{G}_5\ \overline{\psi}_{13} + G_{13}\ \overline{G}_7\ \overline{\psi}_{17}$$
$$+ G_{13}\ \overline{G}_8\ \overline{\psi}_8 + G_{13}\ \overline{G}_{10}\ \overline{\psi}_{12} + G_{13}\ \overline{G}_{14}\ \overline{\phi}_2 + G_{14}\ \overline{G}_4\ \overline{\psi}_{16} + G_{14}\ \overline{G}_6\ \overline{\psi}_{20}$$
$$+ G_{14}\ \overline{G}_9\ \overline{\psi}_5 + G_{14}\ \overline{G}_{11}\ \overline{\psi}_9 + G_{14}\ \overline{G}_{13}\ \phi_2 + G_{15}\ \overline{G}_4\ \overline{\psi}_{15} + G_{15}\ \overline{G}_6\ \overline{\psi}_{19}$$
$$+ G_{15}\ \overline{G}_8\ \overline{\psi}_6 + G_{15}\ \overline{G}_{10}\ \overline{\psi}_{10} + G_{15}\ \overline{G}_{12}\ \phi_1 + \overline{G}_4\ \overline{G}_{10}\ \overline{G}_{15} + \overline{G}_4\ \overline{G}_{11}\ \overline{G}_{14}$$
$$+ \overline{G}_5\ \overline{G}_{10}\ \overline{G}_{13} + \overline{G}_5\ \overline{G}_{11}\ \overline{G}_{12} + \overline{G}_6\ \overline{G}_8\ \overline{G}_{15} + \overline{G}_6\ \overline{G}_9\ \overline{G}_{14} + \overline{G}_7\ \overline{G}_8\ \overline{G}_{13}$$
$$+ \overline{G}_7\ \overline{G}_9\ \overline{G}_{12} + \phi_1\ \phi_4\ \overline{\phi}_5 + \phi_1\ \phi_6\ \overline{\phi}_3 + \phi_1\ \psi_{25}\ \overline{\psi}_{26} + \phi_1\ \psi_{29}\ \overline{\psi}_{30}$$
$$+ \phi_1\ \psi_{33}\ \overline{\psi}_{36} + \phi_1\ H_2\ H_4 + \phi_1\ H_{23}\ H_{27} + \phi_2\ \phi_3\ \overline{\phi}_5 + \phi_2\ \phi_6\ \overline{\phi}_4$$
$$+ \phi_2\ \psi_{26}\ \overline{\psi}_{25} + \phi_2\ \psi_{30}\ \overline{\psi}_{29} + \phi_2\ \psi_{34}\ \overline{\psi}_{35} + \phi_2\ H_1\ H_4 + \phi_2\ H_{24}\ H_{28}$$
$$+ \phi_3\ \overline{\phi}_1\ \overline{\phi}_6 + \phi_3\ \psi_1 3\ \overline{\psi}_{18} + \phi_3\ \psi_1 5\ \overline{\psi}_{20} + \phi_3\ \psi_{21}\ \overline{\psi}_{24} + \phi_3\ H_2\ H_6$$
$$+ \phi_3\ H_{15}\ H_{19} + \phi_4\ \overline{\phi}_2\ \overline{\phi}_6 + \phi_4\ \psi_1 4\ \overline{\psi}_{17} + \phi_4\ \psi_1 6\ \overline{\psi}_{19} + \phi_4\ \psi_{22}\ \overline{\psi}_{23}$$
$$+ \phi_4\ H_1\ H_6 + \phi_4\ H_{16}\ H_{20} + \phi_5\ \overline{\phi}_1\ \overline{\phi}_4 + \phi_5\ \overline{\phi}_2\ \overline{\phi}_3 + \phi_5\ \psi_1\ \overline{\psi}_4$$
$$+ \phi_5\ \psi_5\ \overline{\psi}_{11} + \phi_5\ \psi_6\ \overline{\psi}_{12} + \phi_5\ H_4\ H_6 + \phi_5\ H_7\ H_{11} + \phi_6\ \psi_2\ \overline{\psi}_3$$
$$+ \phi_6\ \psi_7\ \overline{\psi}_9 + \phi_6\ \psi_8\ \overline{\psi}_{10} + \phi_6\ H_3\ H_6 + \phi_6\ H_8\ H_{12} + \phi_7\ \phi_8\ \phi_9$$
$$+ \phi_7\ \psi_{25}\ \psi_{32} + \phi_7\ \psi_{26}\ \psi_{31} + \phi_7\ \psi_{27}\ \psi_{30} + \phi_7\ \psi_{28}\ \psi_{29} + \phi_7\ H_{23}\ H_{26}$$
$$+ \phi_7\ H_{24}\ H_{25} + \phi_8\ \overline{\psi}_{13}\ \overline{\psi}_{20} + \phi_8\ \overline{\psi}_{14}\ \overline{\psi}_{19} + \phi_8\ \overline{\psi}_{15}\ \overline{\psi}_{18} + \phi_8\ \overline{\psi}_{16}\ \overline{\psi}_{17}$$
$$+ \phi_8\ H_{19}\ H_{22} + \phi_8\ H_{20}\ H_{21} + \phi_9\ \psi_5\ \psi_{12} + \phi_9\ \psi_6\ \psi_{11} + \phi_9\ \psi_7\ \psi_{10}$$
$$+ \phi_9\ \psi_8\ \psi_9 + \phi_9\ H_7\ H_{10} + \phi_9\ H_8\ H_9 + \overline{\phi}_1\ \psi_{28}\ \overline{\psi}_{27} + \overline{\phi}_1\ \psi_{32}\ \overline{\psi}_{31}$$
$$+ \overline{\phi}_1\ \psi_{36}\ \overline{\psi}_{33} + \overline{\phi}_1\ H_1\ H_3 + \overline{\phi}_1\ H_{26}\ H_{30} + \overline{\phi}_2\ \psi_{27}\ \overline{\psi}_{28} + \overline{\phi}_2\ \psi_{31}\ \overline{\psi}_{32}$$
$$+ \overline{\phi}_2\ \psi_{35}\ \overline{\psi}_{34} + \overline{\phi}_2\ H_2\ H_3 + \overline{\phi}_2\ H_{25}\ H_{29} + \overline{\phi}_3\ \psi_{18}\ \overline{\psi}_{13} + \overline{\phi}_3\ \psi_{20}\ \overline{\psi}_{15}$$
$$+ \overline{\phi}_3\ \psi_{24}\ \overline{\psi}_{21} + \overline{\phi}_3\ H_1\ H_5 + \overline{\phi}_3\ H_{18}\ H_{22} + \overline{\phi}_4\ \psi_{17}\ \overline{\psi}_{14} + \overline{\phi}_4\ \psi_{19}\ \overline{\psi}_{16}$$
$$+ \overline{\phi}_4\ \psi_{23}\ \overline{\psi}_{22} + \overline{\phi}_4\ H_2\ H_5 + \overline{\phi}_4\ H_{17}\ H_{21} + \overline{\phi}_5\ \psi_4\ \overline{\psi}_1 + \overline{\phi}_5\ \psi_{11}\ \overline{\psi}_5$$
$$+ \overline{\phi}_5\ \psi_{12}\ \overline{\psi}_6 + \overline{\phi}_5\ H_3\ H_5 + \overline{\phi}_5\ H_{10}\ H_{14} + \overline{\phi}_6\ \psi_3\ \overline{\psi}_2 + \overline{\phi}_6\ \psi_9\ \overline{\psi}_7$$

$+\ \bar{\phi}_6\ \psi_{10}\ \bar{\psi}_8$	$+\ \bar{\phi}_6\ H_4\ H_5$	$+\ \bar{\phi}_6\ H_9\ H_{13}$	$+\ \bar{\phi}_7\ \bar{\phi}_8\ \bar{\phi}_9$	$+\ \bar{\phi}_7\ \bar{\psi}_{25}\ \bar{\psi}_{32}$
$+\ \bar{\phi}_7\ \bar{\psi}_{26}\ \bar{\psi}_{31}$	$+\ \bar{\phi}_7\ \bar{\psi}_{27}\ \bar{\psi}_{30}$	$+\ \bar{\phi}_7\ \bar{\psi}_{28}\ \bar{\psi}_{29}$	$+\ \bar{\phi}_7\ H_{27}\ H_{30}$	$+\ \bar{\phi}_7\ H_{28}\ H_{29}$
$+\ \bar{\phi}_8\ \psi_{13}\ \psi_{20}$	$+\ \bar{\phi}_8\ \psi_{14}\ \psi_{19}$	$+\ \bar{\phi}_8\ \psi_{15}\ \psi_{18}$	$+\ \bar{\phi}_8\ \psi_{16}\ \psi_{17}$	$+\ \bar{\phi}_8\ H_{15}\ H_{18}$
$+\ \bar{\phi}_8\ H_{16}\ H_{17}$	$+\ \bar{\phi}_9\ \bar{\psi}_5\ \bar{\psi}_{12}$	$+\ \bar{\phi}_9\ \bar{\psi}_6\ \bar{\psi}_{11}$	$+\ \bar{\phi}_9\ \bar{\psi}_7\ \bar{\psi}_{10}0$	$+\ \bar{\phi}_9\ \bar{\psi}_8\ \bar{\psi}_9$
$+\ \bar{\phi}_9\ \bar{H}_{11}\ \bar{H}_{14}$	$+\ \bar{\phi}_9\ H_{12}\ H_{13}$	$+\ \psi_1\ \psi_{23}\ \psi_{36}$	$+\ \psi_1\ \psi_{24}\ \psi_{35}$	$+\ \psi_1\ \bar{\psi}_{19}\ \bar{\psi}_{30}$
$+\ \psi_1\ \bar{\psi}_{20}\ \bar{\psi}_{29}$	$+\ \psi_1\ \bar{H}_{21}\ \bar{H}_{30}$	$+\ \psi_1\ H_{22}\ H_{29}$	$+\ \psi_2\ \psi_{23}\ \psi_{34}$	$+\ \psi_2\ \psi_{24}\ \psi_{33}$
$+\ \psi_2\ \bar{\psi}_{17}\ \bar{\psi}_{32}$	$+\ \psi_2\ \bar{\psi}_{18}\ \bar{\psi}_{31}$	$+\ \psi_2\ \bar{H}_{21}\ \bar{H}_{28}$	$+\ \psi_2\ H_{22}\ H_{27}$	$+\ \psi_3\ \psi_{21}\ \psi_{36}$
$+\ \psi_3\ \psi_{22}\ \psi_{35}$	$+\ \psi_3\ \bar{\psi}_{15}\ \bar{\psi}_{26}$	$+\ \psi_3\ \bar{\psi}_{16}\ \bar{\psi}_{25}$	$+\ \psi_3\ \bar{H}_{19}\ \bar{H}_{30}$	$+\ \psi_3\ H_{20}\ H_{29}$
$+\ \psi_4\ \psi_{21}\ \psi_{34}$	$+\ \psi_4\ \psi_{22}\ \psi_{33}$	$+\ \psi_4\ \bar{\psi}_{13}\ \bar{\psi}_{28}$	$+\ \psi_4\ \bar{\psi}_{14}\ \bar{\psi}_{27}$	$+\ \psi_4\ H_{19}\ H_{28}$
$+\ \psi_4\ H_{20}\ H_{27}$	$+\ \psi_5\ \psi_1 7\ \psi_{33}$	$+\ \psi_5\ \psi_{25}\ \bar{\psi}_{20}$	$+\ \psi_5\ \bar{\psi}_{24}\ \bar{\psi}_{32}$	$+\ \psi_5\ H_2\ H_{14}$
$+\ \psi_5\ H_{21}\ H_{25}$	$+\ \psi_6\ \psi_1 8\ \psi_{34}$	$+\ \psi_6\ \psi_{26}\ \bar{\psi}_{19}$	$+\ \psi_6\ \bar{\psi}_{23}\ \bar{\psi}_{31}$	$+\ \psi_6\ H_1\ H_{14}$
$+\ \psi_6\ H_{22}\ H_{26}$	$+\ \psi_7\ \psi_1 9\ \psi_{35}$	$+\ \psi_7\ \psi_{27}\ \bar{\psi}_{18}$	$+\ \psi_7\ \bar{\psi}_{24}\ \bar{\psi}_{30}$	$+\ \psi_7\ H_2\ H_{13}$
$+\ \psi_7\ H_{21}\ H_{23}$	$+\ \psi_8\ \psi_{20}\ \psi_{36}$	$+\ \psi_8\ \psi_{28}\ \bar{\psi}_{17}$	$+\ \psi_8\ \bar{\psi}_{23}\ \bar{\psi}_{29}$	$+\ \psi_8\ H_1\ H_{13}$
$+\ \psi_8\ H_{22}\ H_{24}$	$+\ \psi_9\ \psi_1 3\ \psi_{33}$	$+\ \psi_9\ \psi_{29}\ \bar{\psi}_{16}$	$+\ \psi_9\ \bar{\psi}_{22}\ \bar{\psi}_{28}$	$+\ \psi_9\ H_2\ H_{12}$
$+\ \psi_9\ H_{19}\ H_{25}$	$+\ \psi_{10}\ \psi_{14}\ \psi_{34}$	$+\ \psi_{10}\ \psi_{30}\ \bar{\psi}_{15}$	$+\ \psi_{10}\ \bar{\psi}_{21}\ \bar{\psi}_{27}$	$+\ \psi_{10}\ H_1\ H_{12}$
$+\ \psi_{10}\ H_{20}\ H_{26}$	$+\ \psi_{11}\ \psi_1 5\ \psi_{35}$	$+\ \psi_{11}\ \psi_{31}\ \bar{\psi}_{14}$	$+\ \psi_{11}\ \bar{\psi}_{22}\ \bar{\psi}_{26}$	$+\ \psi_{11}\ H_2\ H_{11}$
$+\ \psi_{11}\ H_{19}\ H_{23}$	$+\ \psi_{12}\ \psi_1 6\ \psi_{36}$	$+\ \psi_{12}\ \psi_{32}\ \bar{\psi}_{13}$	$+\ \psi_{12}\ \bar{\psi}_{21}\ \bar{\psi}_{25}$	$+\ \psi_{12}\ H_1\ H_{11}$
$+\ \psi_{12}\ H_{20}\ H_{24}$	$+\ \psi_{13}\ \psi_{26}\ \bar{\psi}_4$	$+\ \psi_{13}\ \bar{\psi}_{12}\ \bar{\psi}_{30}$	$+\ \psi_{13}\ H_4\ H_{22}$	$+\ \psi_{13}\ H_{13}\ H_{28}$
$+\ \psi_{14}\ \psi_{25}\ \bar{\psi}_4$	$+\ \psi_{14}\ \bar{\psi}_{11}\ \bar{\psi}_{29}$	$+\ \psi_{14}\ H_4\ H_{21}$	$+\ \psi_{14}\ H_{13}\ H_{27}$	$+\ \psi_{15}\ \psi_{28}\ \bar{\psi}_3$
$+\ \psi_{15}\ \bar{\psi}_1 0\ \bar{\psi}_{32}$	$+\ \psi_{15}\ H_3\ H_{22}$	$+\ \psi_{15}\ H_{14}\ H_{30}$	$+\ \psi_{16}\ \psi_{27}\ \bar{\psi}_3$	$+\ \psi_{16}\ \bar{\psi}_9\ \bar{\psi}_{31}$
$+\ \psi_{16}\ H_3\ H_{21}$	$+\ \psi_{16}\ H_{14}\ H_{29}$	$+\ \psi_{17}\ \psi_{30}\ \bar{\psi}_2$	$+\ \psi_{17}\ \bar{\psi}_8\ \bar{\psi}_{26}$	$+\ \psi_{17}\ H_4\ H_{20}$
$+\ \psi_{17}\ H_{11}\ H_{28}$	$+\ \psi_{18}\ \psi_{29}\ \bar{\psi}_2$	$+\ \psi_{18}\ \bar{\psi}_7\ \bar{\psi}_{25}$	$+\ \psi_{18}\ H_4\ H_{19}$	$+\ \psi_{18}\ H_{11}\ H_{27}$
$+\ \psi_{19}\ \psi_{32}\ \bar{\psi}_1$	$+\ \psi_{19}\ \bar{\psi}_6\ \bar{\psi}_{28}$	$+\ \psi_{19}\ H_3\ H_{20}$	$+\ \psi_{19}\ H_{12}\ H_{30}$	$+\ \psi_{20}\ \psi_{31}\ \bar{\psi}_1$
$+\ \psi_{20}\ \bar{\psi}_5\ \bar{\psi}_{27}$	$+\ \psi_{20}\ H_3\ H_{19}$	$+\ \psi_{20}\ H_{12}\ H_{29}$	$+\ \psi_{21}\ \psi_{25}\ \bar{\psi}_1 0$	$+\ \psi_{21}\ \psi_{27}\ \bar{\psi}_{12}$
$+\ \psi_{21}\ H_{13}\ H_{26}$	$+\ \psi_{21}\ H_{14}\ H_{24}$	$+\ \psi_{22}\ \psi_{26}\ \bar{\psi}_9$	$+\ \psi_{22}\ \psi_{28}\ \bar{\psi}_{11}$	$+\ \psi_{22}\ H_{13}\ H_{25}$
$+\ \psi_{22}\ H_{14}\ H_{23}$	$+\ \psi_{23}\ \psi_{29}\ \bar{\psi}_6$	$+\ \psi_{23}\ \psi_{31}\ \bar{\psi}_8$	$+\ \psi_{23}\ H_{11}\ H_{26}$	$+\ \psi_{23}\ H_{12}\ H_{24}$
$+\ \psi_{24}\ \psi_{30}\ \bar{\psi}_5$	$+\ \psi_{24}\ \psi_{32}\ \bar{\psi}_7$	$+\ \psi_{24}\ H_{11}\ H_{25}$	$+\ \psi_{24}\ H_{12}\ H_{23}$	$+\ \psi_{25}\ H_6\ H_{30}$
$+\ \psi_{25}\ H_8\ H_{20}$	$+\ \psi_{26}\ H_6\ H_{29}$	$+\ \psi_{26}\ H_8\ H_{19}$	$+\ \psi_{27}\ H_6\ H_{28}$	$+\ \psi_{27}\ H_7\ H_{20}$
$+\ \psi_{28}\ H_6\ H_{27}$	$+\ \psi_{28}\ H_7\ H_{19}$	$+\ \psi_{29}\ H_5\ H_{30}$	$+\ \psi_{29}\ H_{10}\ H_{22}$	$+\ \psi_{30}\ H_5\ H_{29}$
$+\ \psi_{30}\ H_{10}\ H_{21}$	$+\ \psi_{31}\ H_5\ H_{28}$	$+\ \psi_{31}\ H_9\ H_{22}$	$+\ \psi_{32}\ H_5\ H_{27}$	$+\ \psi_{32}\ H_9\ H_{21}$
$+\ \psi_{33}\ H_8\ H_{18}$	$+\ \psi_{33}\ H_{10}\ H_{16}$	$+\ \psi_{34}\ H_8\ H_{17}$	$+\ \psi_{34}\ H_{10}\ H_{15}$	$+\ \psi_{35}\ H_7\ H_{18}$
$+\ \psi_{35}\ H_9\ H_{16}$	$+\ \psi_{36}\ H_7\ H_{17}$	$+\ \psi_{36}\ H_9\ H_{15}$	$+\ \bar{\psi}_1\ \bar{\psi}_{23}\ \bar{\psi}_{36}$	$+\ \bar{\psi}_1\ \bar{\psi}_{24}\ \bar{\psi}_{35}$
$+\ \bar{\psi}_1\ H_{15}\ H_{24}$	$+\ \bar{\psi}_1\ \bar{H}_{16}\ \bar{H}_{23}$	$+\ \bar{\psi}_2\ \bar{\psi}_{23}\ \bar{\psi}_{34}$	$+\ \bar{\psi}_2\ \bar{\psi}_{24}\ \bar{\psi}_{33}$	$+\ \bar{\psi}_2\ H_{15}\ H_{26}$
$+\ \bar{\psi}_2\ \bar{H}_{16}\ \bar{H}_{25}$	$+\ \bar{\psi}_3\ \bar{\psi}_{21}\ \bar{\psi}_{36}$	$+\ \bar{\psi}_3\ \psi_{22}\ \psi_{35}$	$+\ \bar{\psi}_3\ H_{17}\ H_{24}$	$+\ \bar{\psi}_3\ \bar{H}_{18}\ \bar{H}_{23}$
$+\ \bar{\psi}_4\ \bar{\psi}_{21}\ \bar{\psi}_{34}$	$+\ \bar{\psi}_4\ \psi_{22}\ \psi_{33}$	$+\ \bar{\psi}_4\ \bar{H}_{17}\ \bar{H}_{26}$	$+\ \bar{\psi}_4\ H_{18}\ H_{25}$	$+\ \bar{\psi}_5\ \bar{\psi}_{17}\ \bar{\psi}_{33}$
$+\ \bar{\psi}_5\ \bar{H}_1\ \bar{H}_7$	$+\ \bar{\psi}_5\ H_{16}\ H_{28}$	$+\ \bar{\psi}_6\ \psi_{18}\ \psi_{34}$	$+\ \bar{\psi}_6\ \bar{H}_2\ \bar{H}_7$	$+\ \bar{\psi}_6\ H_{15}\ H_{27}$
$+\ \bar{\psi}_7\ \bar{\psi}_{19}\ \bar{\psi}_{35}$	$+\ \bar{\psi}_7\ \bar{H}_1\ \bar{H}_8$	$+\ \bar{\psi}_7\ H_{16}\ H_{30}$	$+\ \bar{\psi}_8\ \bar{\psi}_{20}\ \bar{\psi}_{36}$	$+\ \bar{\psi}_8\ H_2\ H_8$
$+\ \bar{\psi}_8\ H_{15}\ H_{29}$	$+\ \bar{\psi}_9\ \bar{\psi}_{13}\ \bar{\psi}_{33}$	$+\ \bar{\psi}_9\ H_1\ H_9$	$+\ \bar{\psi}_9\ H_{18}\ H_{28}$	$+\ \bar{\psi}_1 0\ \bar{\psi}_{14}\ \bar{\psi}_{34}$
$+\ \bar{\psi}_1 0\ \bar{H}_2\ \bar{H}_9$	$+\ \bar{\psi}_1 0\ H_{17}\ H_{27}$	$+\ \bar{\psi}_{11}\ \bar{\psi}_{15}\ \bar{\psi}_{35}$	$+\ \bar{\psi}_{11}\ H_1\ H_{10}$	$+\ \bar{\psi}_{11}\ H_{18}\ H_{30}$
$+\ \bar{\psi}_{12}\ \bar{\psi}_{16}\ \bar{\psi}_{36}$	$+\ \bar{\psi}_{12}\ H_2\ H_{10}$	$+\ \bar{\psi}_{12}\ H_{17}\ H_{29}$	$+\ \bar{\psi}_{13}\ H_3\ H_{15}$	$+\ \bar{\psi}_{13}\ H_8\ H_{25}$
$+\ \bar{\psi}_{14}\ H_3\ H_{16}$	$+\ \bar{\psi}_{14}\ H_8\ H_{26}$	$+\ \bar{\psi}_{15}\ H_4\ H_{15}$	$+\ \bar{\psi}_{15}\ H_7\ H_{23}$	$+\ \bar{\psi}_{16}\ H_4\ H_{16}$
$+\ \bar{\psi}_{16}\ H_7\ H_{24}$	$+\ \bar{\psi}_{17}\ H_3\ H_{17}$	$+\ \bar{\psi}_{17}\ H_{10}\ H_{25}$	$+\ \bar{\psi}_{18}\ H_3\ H_{18}$	$+\ \bar{\psi}_{18}\ H_{10}\ H_{26}$
$+\ \bar{\psi}_{19}\ H_4\ H_{17}$	$+\ \bar{\psi}_{19}\ H_9\ H_{23}$	$+\ \bar{\psi}_{20}\ H_4\ H_{18}$	$+\ \bar{\psi}_{20}\ H_9\ H_{24}$	$+\ \bar{\psi}_{21}\ H_7\ H_{29}$
$+\ \bar{\psi}_{21}\ H_8\ H_{27}$	$+\ \bar{\psi}_{22}\ H_7\ H_{30}$	$+\ \bar{\psi}_{22}\ H_8\ H_{28}$	$+\ \bar{\psi}_{23}\ H_9\ H_{29}$	$+\ \bar{\psi}_{23}\ H_{10}\ H_{27}$

$$
\begin{aligned}
&+ \overline{\psi}_{24}\ H_9\ H_{30} \quad + \overline{\psi}_{24}\ H_{10}\ H_{28} \quad + \overline{\psi}_{25}\ H_5\ H_{25} \quad + \overline{\psi}_{25}\ H_{14}\ H_{17} \quad + \overline{\psi}_{26}\ H_5\ H_{26} \\
&+ \overline{\psi}_{26}\ H_{14}\ H_{18} \quad + \overline{\psi}_{27}\ H_5\ H_{23} \quad + \overline{\psi}_{27}\ H_{13}\ H_{17} \quad + \overline{\psi}_{28}\ H_5\ H_{24} \quad + \overline{\psi}_{28}\ H_{13}\ H_{18} \\
&+ \overline{\psi}_{29}\ H_6\ H_{25} \quad + \overline{\psi}_{29}\ H_{12}\ H_{15} \quad + \overline{\psi}_{30}\ H_6\ H_{26} \quad + \overline{\psi}_{30}\ H_{12}\ H_{16} \quad + \overline{\psi}_{31}\ H_6\ H_{23} \\
&+ \overline{\psi}_{31}\ H_{11}\ H_{15} \quad + \overline{\psi}_{32}\ H_6\ H_{24} \quad + \overline{\psi}_{32}\ H_{11}\ H_{16} \quad + \overline{\psi}_{33}\ H_{11}\ H_{21} \quad + \overline{\psi}_{33}\ H_{13}\ H_{19} \\
&+ \overline{\psi}_{34}\ H_{11}\ H_{22} \quad + \overline{\psi}_{34}\ H_{13}\ H_{20} \quad + \overline{\psi}_{35}\ H_{12}\ H_{21} \quad + \overline{\psi}_{35}\ H_{14}\ H_{19} \quad + \overline{\psi}_{36}\ H_{12}\ H_{22} \\
&+ \overline{\psi}_{36}\ H_{14}\ H_{20}
\end{aligned}
$$

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