Optimal Contract Under Asymmetric Information About Fairness

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(1) **Problem definition:** What is your research problem?

To improve the poor performance of supply chains caused by misaligned incentives under the wholesale price contract, theory proposes coordinating contracts. However, a common finding of experimental studies testing such contracts is that they tend to yield only a marginal, if any, performance improvement over wholesale pricing. These studies identify several behavioral factors that are at play, but none accounted for by the theory proposing coordinating contracts. Among them, identified as the single most detrimental for the supply chain performance, is incomplete information about preferences for fairness causing contract rejections. Can the supply chain performance be improved with a contract designed allowing for this type of information asymmetry? What does this contract (mechanism) look like?

(2) **Academic/Practical relevance:** How is your research problem relevant to the OM research/practice community?

The extant research characterized the optimal contracting mechanisms for such important practical cases as the suppliers' private information about production cost or the retailers' private information about the end-customer demand. The present study addresses the gap in another important practical case, when the source of information asymmetry is the private information about preferences for fairness.

(3) **Methodology:** What is the underlying research method?

Mechanism design.

(4) **Results:** What are your key findings?

We prove that the optimal mechanism consists of a single contract positioned on the Pareto frontier and characterize the optimal profit split between the supplier and the retailer. We show that, under a wide range of preferences for fairness, the efficiency loss due to private information is strictly positive, but exceptions are possible. We also show that the optimal mechanism can be implemented with a variety of commonly used in practice and widely studied in academic literature contracts including the minimum-order quantity and the two-part tariff ones.

(5) **Managerial implications:** How can academics/managers/decision makers benefit from your study?

We establish a direct link between a large volume of theoretical and empirical literature on social preferences with the research on supply chain contracts. Because rejections, that are due to incomplete information, are an important cause of contract inefficiency observed in the laboratory, managers should avoid take-it-or-leave-it offers when they negotiate contracts. Instead, the bargaining process should be geared towards discovering the extent of the fairness preferences of the contracting parties. Also, to the extent that contracting forms differ in their susceptibility to rejections, certain contract forms that are equivalent in theory, may differ in their effectiveness in coordinating supply channels.
1. Introduction

In today’s world, “... supply chain competes with supply chain and the success of any one company will depend upon how well it manages its supply chain relationships.”  (Christopher 2010). The main challenge facing supply chains is that decentralized decision-making by self-interested firms may well entail an overall poor supply chain performance due to misaligned incentives. In an early paper, Spengler (1950) demonstrates this under wholesale pricing. To ensure that incentives in a supply chain are aligned with the objective of the overall profit maximization, various coordinating contracts have been proposed, including block tariffs, two-part tariff, quantity discount, buyback, revenue-sharing, minimum-order quantity (MOQ), and many others (Jeuland and Shugan 1983, Moorthy 1987, Pasternack 1985, Cachon and Lariviere 2005, Cachon 2003).

However, when tested using controlled laboratory experiments with human subjects, those coordinating contracts fall short of delivering predicted benefits. Lim and Ho (2007), Ho and Zhang (2008), Katok and Pavlov (2013), Katok and Wu (2009) and others, test the performance of a number of different contracting arrangements in the laboratory, and find that the efficiency (a ratio of the realized profit to the theoretical maximum profit of a centralized supply chain) is only around 70%-80%, bringing only marginal, if any, improvement over the efficiency of wholesale pricing. We provide more specific details about the settings of the key experiments in Section 3.3. One reason for such a poor performance of coordinating contracts is that, depending on the study, around 20% of proposed contracts are rejected, while the efficiency of some of these contracts conditional on acceptance tends to be in the 90%-95% range. Thus, rejections are a significant cause of inefficiency. In this paper we provide a model that is based on incomplete information about preferences for fairness. This model explains rejections, and we use it to characterize an optimal contract when parties care about fairness.
Laboratory results that report coordinating contract inefficiency closely resemble outcomes of the *Ultimatum Game* experiments (Guth et al. 1982). In the Ultimatum Game, the proposer offers a division of a fixed amount of money between herself and the responder. If the responder agrees with the proposed division both players take their shares, and otherwise both earn nothing. A standard analysis predicts that the responder will not reject any offer, and anticipating this, the proposer will end up appropriating nearly the entire pie to herself. However, in laboratory experiments, about 20% of offers are rejected (Roth et al. 1991). Forsythe et al. (1994) suggest that rejections may occur because the proposer does not know the recipient’s aversion to inequitable outcomes, and, indeed, Andreoni et al. (2003) find a significant heterogeneity of inequity aversion among people. In supply chain contracting literature, Katok and Pavlov (2013) demonstrate that 2/3 of rejections observed in aforementioned contracting experiments are due to private information about preferences for fairness.

It is not surprising that fairness affects contracting outcomes. A large body of empirical evidence shows that fairness is virtually always at play, shaping interactions between people, between firms, and even between firms and their customers and employees (see Kahneman et al. 1986a, Fehr and Schmidt 1999, Cui et al. 2007, and references therein). A recent Wall Street Journal article (Benoit 2019) reports that the Business Roundtable, which consists of CEOs of nearly 200 of the largest US corporations, such as Amazon, Apple and J.P. Morgan Chase, signed a statement redefining the purpose of a corporation to include ”Investing in employees, delivering value to customers, dealing ethically with suppliers and supporting outside communities.” (Fitzgerald 2019).

Rejections may be inevitable in the Ultimatum Game because “... willingness to enforce fairness is common” (Kahneman et al. 1986b) and rejection is the only way to punish the proposer for an unfair offer, but they may be avoidable in contracting games. Katok et al. (2014), building on and generalizing some of the results of Cui et al. (2007), find also that the equilibrium rejection rate under wholesale pricing can be zero because retailers can “punish” the suppliers not by rejecting the contract but by choosing low order quantities (the stronger preferences the smaller orders).
That is, at one end, there are coordinating contracts that do not perform well because of rejections due to incomplete information regarding preferences for fairness, whereas at the other end, there is wholesale pricing that may not entail rejections but performs poorly anyway because its structure is not suited for aligning incentives due to double marginalization. Therefore, an important research question is whether there exists a better contract “in between”?

In the supply chain literature, optimal contracts have been characterized for cases when information asymmetry concerns technology or economic environment (e.g., production cost or demand forecast: Ha 2001, Cachon and Lariviere 2001, Corbett et al. 2004) but, to the best of our knowledge, the optimal contract under information asymmetry regarding preferences for fairness has not been characterized yet. However, considering that information asymmetry about inequity aversion can be detrimental to supply chain performance, and that fairness concerns are ubiquitous in the corporate world, it is important to use contracts designed in a way that allows for this type of information asymmetry. The main contribution of this study is that we characterize the optimal “second best” contract (menu), and find that under a broad range of conditions contract rejections are part of the optimal solution. We also show that it can be implemented with different types of the standard coordinating contracts (affine and non-affine) and illustrate how the behavioral factors affect the supply chain performance.

The rest of the paper is organized as follows. In Section 2 we introduce the model and derive the main result of the paper, a characterization of the optimal contract. In Section 3 we consider implications for practice, namely we show that the optimal contract can be implemented with a number of coordinating contracts, such as the minimum order quantity or the two-part-tariff. We conclude the paper with a “reality check” of our model predictions. To this end, we use the empirical distribution of the fairness parameter that controls disadvantageous inequality aversion (reported in Katok et al. 2014), numerically calculate the optimal rejection rate, and find it closely matches rejection rates observed in contracting experiments. We also summarize our findings and their implications in section 4.
2. The model

2.1. Economic environment, preferences and notation

Consider a dyadic supply chain composed of a supplier and a retailer, denoted by $S$ and $R$, respectively (also used as subscripts throughout the paper to indicate whether a given variable belongs to the supplier or the retailer). The supplier moves first and offers the retailer a menu of contracts. Taking the menu of contracts offered by the supplier as given, the retailer decides to either accept one contract or decline all contracts on the menu. Based on the retailer’s decision, the supplier and the retailer earn profits of $\pi_S$ and $\pi_R$, respectively. If the retailer rejects all contracts on the menu, both the supplier and the retailer earn zero profit, namely, $\pi_S = \pi_R = 0$. For simplicity, we assume the maximum achievable profit for the entire supply chain is 1 under the first best contract, so we present the model in terms of the profit split between the two parties instead of the standard way of presenting it in terms of contract parameters. Any contract implies the profits for the two parties, so recasting the contracting problem in terms of profits is equivalent to using contract parameters.

To capture the concerns for fairness, we model the fairness as inequity aversion following Fehr and Schmidt (1999) and adopt the utility functions in Cui et al. (2007). We assume the equitable profit for the retailer is $\gamma$ times the supplier’s profit, namely, $\gamma \pi_S$, where $\gamma$ captures the supply chain members’ contributions and is exogenous. Therefore, if the retailer’s profit is lower than the equitable profit, a disadvantageous inequality occurs, which will result in a disutility for the retailer in the amount of $\alpha_R$ per-unit difference the retailer’s profit and $\gamma$ times the supplier’s profit. If the retailer’s profit is higher than the equitable payoff, an advantageous inequality occurs in the amount of $\beta_R$ per-unit of difference. The supplier’s utility follows the same inequity aversion structure but has different disadvantageous and advantageous inequality parameters, $\alpha_S$ and $\beta_S$, respectively. Consistent with the literature (De Bruyn and Bolton 2008, Katok et al. 2014, Cui et al. 2007), we assume $\alpha \geq 0$, $0 \leq \gamma \leq 1$, $0 \leq \beta \leq \frac{1}{1+\gamma}$. Accordingly, the retailer’s and supplier’s utility functions are

$$U_R(\pi_R, \pi_S; \alpha_R, \beta_R) = \pi_R - \alpha_R (\gamma \pi_S - \pi_R)^+ - \beta_R (\pi_R - \gamma \pi_S)^+, \quad (1)$$

$$U_S(\pi_R, \pi_S; \alpha_S, \beta_S) = \pi_S - \alpha_S (\pi_R - \gamma \pi_S)^+ - \beta_S (\gamma \pi_S - \pi_R)^+. \quad (2)$$
We refer to the pair \((\alpha_R, \beta_R)\) as the retailer’s type, which is private information. Thus, \(\alpha_R\) and \(\beta_R\) are two random variables that are distributed according to publicly known distributions. At this moment, we do not impose explicit conditions on these distributions in the interest of generality, but will make some assumptions about the distribution of \(\alpha_R\) following Proposition 1. For the supplier, we treat \(\alpha_S\) and \(\beta_S\) as exogenous parameters because in our setting, the supplier is the first-mover.

2.2. The problem formulation

To characterize the supplier’s optimal contract when the uninformed party moves first we follow the standard principal-agent adverse selection problem approach based on the Revelation Principle (Hurwicz 1972, Gibbard 1973, Myerson 1979).

To streamline the notation, we formulate the problem in terms of profits, \(\pi_S(\cdot)\) and \(\pi_R(\cdot)\), where the profits are functions of the players’ types. Figure 1 graphically presents the key parts of the supplier’s problem of designing the optimal incentive-compatible menu of contracts. The Pareto frontier line corresponds to a set of coordinating contracts. The x- and y- axes measure the supplier’s and the retailer’s profits, respectively. Therefore, each point below the Pareto frontier line corresponds to a feasible contract, while the region above the Pareto frontier is infeasible. The “fair split” line has a slope \(\gamma\). As it immediately follows from (1), the retailer’s isoutility lines are piecewise-linear with kinks at the intersections with the fair-split line.

For the retailer with a type \((\alpha_R, \beta_R)\), the slope of the isoutility line below the fair-split line is \(\frac{\alpha_R \gamma_R}{1 + \alpha_R}\), and above the fair split line is \(-\frac{\beta_R \gamma_R}{1 - \beta_R}\). The difficulty of our problem is that the isoutility lines do not satisfy the single-crossing property. As Figure 1 shows, the isoutility lines for types \(i\) and \(j\) retailers cross both below and above the fair-split line. This presents a challenge as, generally, such problems are not analytically tractable because any combination of constraints may prove binding at optimality (Laffont and Martimort 2002, p. 93). Proposition 1 addresses this difficulty.

Proposition 1. In the supplier’s optimal menu, no contract results in disadvantageous inequality to the supplier.
Figure 1  The contracting space is limited by the axes and the Pareto frontier. The fair split line (dashed) represents the location of contracts that incur no dis-utility to the players. In the region below the dashed line both parties experience dis-utility: the retailer due to the disadvantageous inequality and the supplier due to the advantageous. The situation is reversed in the region above the dashed line.

A simple intuition for Proposition 1 is that any incentive-compatible menu having contracts above the fair split line can be improved by moving all those contracts along the retailer’s isoutility lines to the intersection with the fair split line. Such a transformation preserves incentive-compatibility but increases the supplier’s utility as each of the new contracts gives the supplier a strictly higher profit and lower disutility from inequity than the original menu. The technical significance of Proposition 1 is that below the fair-split line the single-crossing property holds, and also we can simplify the original problem by ignoring $\alpha_S$ and $\beta_R$. Therefore, now information asymmetry only depends on $\alpha_R$. To simplify the notation, we drop the subscript of $\alpha_R$ and assume $\alpha$ is distributed according to a cumulative distribution function $F$ (with a continuous probability density function $f$) with finite support $[\underline{\alpha}, \bar{\alpha}]$. Where necessary, we assume $F$ is log-concave to ensure $\frac{f}{F'}$ is decreasing. Note that the class of log-concave distributions is broad; it includes such distributions as uniform, normal, logistic, exponential, extreme value, chi-square, Laplace, etc (see Bagnoli and Bergstrom (2005) for a more detailed list of these log-concave distributions). We further assume that $\pi_R(\alpha)$ and $\pi_S(\alpha)$
are differentiable with respect to $\alpha \in [\bar{\alpha}, \tilde{\alpha}]$. We obtain the supplier’s optimal menu of contracts by solving the following optimization problem:

$$\max_{(\pi_S, \pi_R)} \int_{\alpha}^{\tilde{\alpha}} \pi_S(\alpha) - \beta_s (\gamma \pi_S(\alpha) - \pi_R(\alpha)) dF(\alpha) \quad \text{(OB)}$$

s.t.

$$\pi_R(\alpha) - \alpha (\gamma \pi_S(\alpha) - \pi_R(\alpha)) \geq 0, \forall \alpha \in [\bar{\alpha}, \tilde{\alpha}] \quad \text{(IR)}$$

$$\pi_R(\alpha) - \alpha (\gamma \pi_S(\alpha) - \pi_R(\alpha)) \geq \pi_R(\tilde{\alpha}) - \alpha (\gamma \pi_S(\tilde{\alpha}) - \pi_R(\tilde{\alpha})), \forall \alpha, \tilde{\alpha} \in [\bar{\alpha}, \tilde{\alpha}] \quad \text{(IC)}$$

$$0 \leq \pi_S(\alpha) + \pi_R(\alpha) \leq 1, \quad (M_1)$$

$$\gamma \pi_S(\alpha) - \pi_R(\alpha) \geq 0. \quad (M_2)$$

(IR) and (IC) are Individual Rationality and Incentive Compatibility constraints for retailers. (M_1) is there because we normalized the first best channel profit to 1, and (M_2) directly follows from Proposition 1. Theorem 1 in the next section fully characterizes the structure of the supplier’s optimal menu of contracts.

2.3. The structure of an optimal contract (menu)

Before we state the main result of the paper, we must explain that we define the term pooling in the sense of that a single contract is offered to all types of retailers, but not all types of retailers will necessarily accept the contract.

**Theorem 1.** *The optimal contract is pooling and efficient.*

It is easy to see that if we know that the optimal contract is efficient then it has to be pooling simply because out of several efficient contracts, all retailer types would select the one that gives them the highest utility. If we know that the optimal contract is pooling, then it also has to be efficient, because for any contract below the Pareto frontier line, we can construct a contract on the Pareto frontier line by following the isoutility curve of the highest participating type retailer. This new contract will keep the most inequality averse retailer indifferent, and make the supplier and all less inequality averse retailer types strictly better off.
To better understand why the optimal contract is both pooling and efficient, let us consider in detail the simplest case, in which the population of retailers consists of only two types. Let $\rho_1$ be the proportion of $\alpha_1$ types in the population and $\rho_2$ be the proportion of $\alpha_2$ types. Also, let $\alpha_1 < \alpha_2$, meaning that retailer type $\alpha_1$ is less concerned with disadvantageous inequality than the retailer type $\alpha_2$. Before we proceed with writing down the supplier’s optimization problem, we note that it has to be the case that the more inequality-averse type $\alpha_2$ should receive zero utility from his contract, otherwise the supplier would be able to construct another contract that is acceptable to $\alpha_2$ and that has higher supplier’s profit. Also, the contract offered to the less inequality averse type $\alpha_1$ has to be efficient, because otherwise the supplier would be able to construct an efficient contract for $\alpha_1$ by following his isoutility curve to the Pareto efficient line, and this new contract will have higher supplier profit, the same utility for $\alpha_1$, and continue to be unacceptable for $\alpha_2$.

With this in mind, let $\pi_{S_i} = \pi_S(\alpha_i)$ and $\pi_{R_i} = \pi_R(\alpha_i)$ for $i \in \{1, 2\}$, and the supplier’s optimization problem is as follows.

$$\max_{(\pi_{S_1}, \pi_{R_1}), (\pi_{S_2}, \pi_{R_2})} \rho_1 (\pi_{S_1} - \beta_s (\gamma \pi_{S_1} - \pi_{R_1})) + \rho_2 (\pi_{S_2} - \beta_s (\gamma \pi_{S_2} - \pi_{R_2}))$$

s.t.

\begin{align*}
\pi_{R_1} - \alpha_1 (\gamma \pi_{S_1} - \pi_{R_1}) & \geq 0, \\
\pi_{R_2} - \alpha_2 (\gamma \pi_{S_2} - \pi_{R_2}) & = 0, \\
\pi_{R_1} - \alpha_1 (\gamma \pi_{S_1} - \pi_{R_1}) & = \pi_{R_2} - \alpha_2 (\gamma \pi_{S_2} - \pi_{R_2}), \\
\pi_{R_2} - \alpha_2 (\gamma \pi_{S_2} - \pi_{R_2}) & \geq \pi_{R_1} - \alpha_2 (\gamma \pi_{S_1} - \pi_{R_1}), \\
\pi_{S_1} + \pi_{R_1} & = 1, \\
\gamma \pi_{S_1} - \pi_{R_1} & \geq \gamma \pi_{S_2} - \pi_{R_2} \geq 0, \\
\pi_{S_1} & \geq \pi_{S_2} \geq 0, \\
\pi_{R_1} & \geq \pi_{R_2} \geq 0, \\
\rho_1 + \rho_2 & = 1,
\end{align*}
where \((IR_1)\) and \((IR_2)\) are individual rationality constraints, and \((IC_1)\) and \((IC_2)\) are incentive compatibility constraints. The \(\alpha_2\)'s incentive compatibility constraint is implied by \((IC_1)\) and \((M_2)\), and thus can be removed. Constraint \((IR_2)\) has strict equality because with the optimal menu, the more inequality averse retailer has to derive zero utility from his contract. Constraint \((M_1)\) ensures that the contract for the less inequality averse retailer is efficient, and constraints \((M_2)\) and \((M_4)\) jointly ensure that at optimality, the amount of profit and the amount of inequality in the contract designed for the more inequality averse retailer does not exceed the amount of profit and inequality in the contract designed for the less inequality averse retailer (otherwise, the more inequality averse retailer would be sure to reject his contract in favor of the other one).

Further, note that \((IR_2)\) and \((IC_1)\) imply \((IR_1)\), which thus, can be removed. Note also that from three binding constraints, \((IC_1)\), \((IR_2)\), and \((M_1)\), we can express \(\pi_{S_2}\), \(\pi_{R_2}\), and \(\pi_{R_1}\) in terms of \(\pi_{S_1}\) as follows.

\[
\begin{align*}
\pi_{R_2} &= \frac{(-\alpha_1-1)\alpha_2}{\alpha_1-\alpha_2} + \frac{\alpha_2 \pi_{S_1} (\alpha_1(\gamma+1)+1)}{\alpha_1-\alpha_2}, \\
\pi_{R_1} &= 1 - \pi_{S_1}, \\
\pi_{S_2} &= \frac{(-\alpha_1-1)(\alpha_2+1)}{(\alpha_1-\alpha_2)\gamma} + \frac{(\alpha_2+1)\pi_{S_1} (\alpha_1(\gamma+1)+1)}{(\alpha_1-\alpha_2)\gamma}.
\end{align*}
\]  

(P-5)

We can now rewrite the objective function and the constraint \((M_2)\) in terms of \(\pi_{S_1}\), supplier’s profit from the contract offered to the less inequality averse retailer, by substituting expressions (P-5) for the other three decision variables and doing some algebraic manipulations. Out of constraints \((M_1)\), \((M_3)\) and \((M_4)\), one is redundant, because if we add \((M_3)\) and \((M_4)\) together, we get an expression that means that the less inequality averse retailer’s contract (which is 100% by constraint \((M_1)\)) is at least as efficient as the more inequality averse retailer’s contract. So constraint \((M_4)\) can be removed, and the rewritten formulation is:

\[
\max_{\pi_{S_1}} \quad \alpha_1 \left( (-\alpha_2 - 1) \rho_2 + \gamma (\rho_1 + \rho_2) \beta_s \right) \\
\quad + \frac{\alpha_2 (\rho_2 + \gamma \rho_1 \beta_s) + \rho_2 (\gamma \beta_s - 1)}{(\alpha_1 - \alpha_2) \gamma}
\]
\[
\pi_{S_1} \left( \frac{\rho_1 (\alpha_2 \gamma ((\gamma + 1) \beta_s - 1) + \alpha_1 (\gamma - \gamma(\gamma + 1) \beta_s))}{(\alpha_1 - \alpha_2) \gamma} \\
+ \frac{\rho_2 (\alpha_2 + \alpha_1 (-\gamma - 1)(-\alpha_2 + \gamma \beta_s - 1) - \gamma \beta_s + 1)}{(\alpha_1 - \alpha_2) \gamma} \right)
\] (OB)

s.t.
\[
\pi_{S_1} \leq \frac{\alpha_1 + 1}{\alpha_1 \gamma + \alpha_1 + 1}, ~ \text{(M}_2\text{)}
\]
\[
\pi_{S_1} \geq \pi_{S_2} \geq 0, ~ \text{(M}_3\text{)}
\]
\[
\rho_1 + \rho_2 = 1. ~ \text{(M}_5\text{)}
\]

Constraints, (IR\textsubscript{2}), (IC\textsubscript{1}) and (M\textsubscript{1}) in the original formulation are removed because they are used to calculate (P-5) which is now incorporated into the objective function. The new objective function is linear in single decision variable, \(\pi_{S_1}\), consisting of a constant term and \(\pi_{S_1}\) multiplied by a coefficient. Linearity of the objective functions implies that the optimal solution must be either a boundary solution (at the lower or upper bound of \(\pi_{S_1}\), depending on the sign of the \(\pi_{S_1}\) coefficient), or any feasible point (if the \(\pi_{S_1}\) coefficient is zero).

We now consider three cases based on the sign of the \(\pi_{S_1}\) coefficient in detail. If it is zero, the problem does not depend on \(\pi_{S_1}\), and any menu, including a menu that has a single contract, is an optimal menu. If the coefficient of \(\pi_{S_1}\) is positive, the optimal solution is \(\pi_{S_1}^* = \frac{\alpha_1 + 1}{\alpha_1 \gamma + \alpha_1 + 1}\) because constraint (M\textsubscript{2}) becomes binding, and using (P-5), \(\pi_{S_2} = 0\). This means that the optimal menu only includes one contract located on the Pareto frontier, \((\pi_{S}^* = \frac{\alpha_1 + 1}{\alpha_1 \gamma + \alpha_1 + 1}, \pi_{R}^* = \frac{\alpha_1 \gamma}{\alpha_1 \gamma + \alpha_1 + 1})\), and type \(\alpha_2\) retailer will reject this contract. Finally, if the coefficient of \(\pi_{S_1}\) is negative, the optimal solution is to set \(\pi_{S_1}\) to its lower bound \(\pi_{S_2}\) to satisfy the constraint (M\textsubscript{3}). Then, \(\pi_{R_1} = 1 - \pi_{S_1} = 1 - \pi_{S_2}\). Again, the optimal menu includes only one contract on the Pareto frontier, \((\pi_{S}^* = \pi_{S_2}, \pi_{R}^* = 1 - \pi_{S_2})\), and both retailer types will accept it.

So the optimal contract for the case of any two types of retailers is efficient and “pooling” in the sense that a single contract is offered to both types and it is either acceptable to both types, or the more inequality averse type rejects. In fact, because the menu of contracts for any two types always merges into one, the optimal contract when types are continuous is also pooling and efficient.
The full proof, which we present in the appendix, turned out to be rather technical because the optimization problem with continuous types is an optimal control problem. So the proof involves first, re-casting the problem as an optimal control problem, and then using several well-established techniques to solve it.

3. Implementation and testable predictions

In this section we show that the supplier who knows the distribution of the retailers’ aversion to disadvantageous inequality can use our results to construct the optimal contract. The supplier should start by computing the desired rejection rate, and then use this information to calculate the optimal profit split in a coordinating contract. We provide examples of our model’s recommendation for the two-part-tariff contract, which we compare to the Ho and Zhang (2008) data, and our model’s recommendation for the minimum order quantity contract, which we compare to the Katok and Pavlov (2013) data.

3.1. Optimal Rejection Rate

Having established that the optimal menu consists of a single contract located on the Pareto frontier, Theorem 1 reduced the supplier’s problem to that of finding the optimal cutoff type \( \hat{\alpha} \), which is the highest type willing to accept the contract. This optimal cutoff type determines the optimal desired rejection rate, given the distribution of \( \alpha \). The supplier finds the optimal cutoff type by solving the following problem

\[
\max_{\hat{\alpha}, \pi_S, \pi_R} U_S(\pi_S, \pi_R; \beta_S) F(\hat{\alpha})
\]

s.t.

\[
\pi_S + \pi_R = 1,
\]

\[
\pi_R - \hat{\alpha}(\gamma \pi_S - \pi_R)) = 0.
\]

Using (4) and (5) to eliminate \( \pi_S \) and \( \pi_R \), one can reduce the problem (3)-(5) to a single-variable maximization problem \( \max_{\hat{\alpha}} U(\hat{\alpha}) F(\hat{\alpha}) \). Once the optimal \( \hat{\alpha} \) is found, the supplier needs to plug it into (5) and find the two coordinates of the optimal contract, \( \pi_S \) and \( \pi_R \), by solving
two simultaneous equations (4) and (5). In case the optimal $\hat{\alpha} = \infty$ (see Proposition 2 and the discussion that follows) the optimal contract is at the intersection of the Pareto frontier and the fair split line $\gamma \pi_S = \pi_R$.

The next proposition characterizes the optimal cutoff type and helps illustrate the effect of two different aspects of inequity aversion on the supply chain performance.

**Proposition 2.** The optimal cutoff type, $\hat{\alpha}$, is the (smallest) solution to the equation

$$\frac{f(\alpha)}{F(\alpha)} = \frac{\gamma (1 - \gamma \beta_S - \beta_S)}{(\alpha + \alpha \gamma + 1)(1 - \gamma \beta_S + \alpha)},$$

(6)

if the solution exists, and $\hat{\alpha} = \infty$ otherwise.

The power of this proposition is that (6) separates the effect of the type distribution from the effect of the inequity-aversion scaling coefficients, helping understand the effect of both. As a starting (trivial) example, notice that $\beta_S = \frac{1}{1 + \gamma}$ makes the RHS of (6), which we further refer to as cutoff function (COF), equal to zero. Then, (6) has no solution regardless of the distribution of $\alpha$, implying the supplier offers a contract that all retailer types accept (in accord with Cui et al. 2007, Katok et al. 2014).

For the next example, consider an exponential distribution. In this case it is immediate to see that (6) has a finite solution regardless of the values of $\beta_S$ and $\gamma$ (unless $\beta_S = \frac{1}{1 + \gamma}$, of course) because the LHS decreases exponentially (fast) from infinity to zero (lim$_{\alpha \to 0} \frac{f(\alpha)}{F(\alpha)} = +\infty$ and lim$_{\alpha \to \infty} \frac{f(\alpha)}{F(\alpha)} = 0$), whereas its RHS is also continuous in $\alpha$ but goes to zero from a finite positive value and at a (slower) rate of a power function $\frac{1}{\alpha^2}$. A solution existence follows due to the intermediate value theorem applied to the difference of the LHS and RHS.

Further, interestingly, for some distributions the optimal rejection rate can be zero even if the upper bound of the support is arbitrarily large. For example, consider a uniform distribution $U(0, \bar{\alpha})$. The LHS of (6) is then $\frac{f(\alpha)}{F(\alpha)} = \frac{1}{\alpha}$ while on the RHS we have $COF(\alpha) = \frac{\gamma (1 - \gamma \beta_S - \beta_S)}{(\alpha + \alpha \gamma + 1)(1 - \gamma \beta_S + \alpha)} < \frac{\gamma}{\alpha + \alpha \gamma + 1} = \frac{1}{\alpha + \alpha \gamma + 1} \leq \frac{1}{\alpha}$, where the first inequality follows because $1 - \gamma \beta_S - \beta_S < 1 - \gamma \beta_S + \alpha < 1$. Thus, in this example, the LHS is always greater than the RHS and (6) has no solution, meaning the optimal rejection rate is zero despite the fact that $\bar{\alpha}$, the most inequity averse type, can be arbitrarily large.
Proposition 2 allows not only computing the optimal rejection rate but also makes intuitive some of the comparative statics. To illustrate this, Figure 2 presents a graph of \( f(\alpha)/F(\alpha) \) for the empirical distribution reported by Katok et al. (2014) based on their experimental data, and several \( COF(\cdot) \) curves for different values of \( \beta_S \). Note that since this empirical distribution has a mass point at zero, \( f(0) = 1 \). Since Katok et al. (2014) also estimate \( \gamma = 0.83 \) it is also the case that \( f(0)/F(0) > COF(0) \) because (one can verify) \( \lim_{\alpha \to 0} COF(\alpha, \beta, \gamma) = \gamma \frac{1-\beta \gamma - \beta}{1-\beta \gamma} < \gamma = 0.83 < 1 \).

Referring to Figure 2 consider an obvious case first. As \( \beta_S \) increases, the \( COF(\cdot) \) curves go down (as they should, a straightforward calculation shows that \( \frac{\partial}{\partial \beta_S} COF(\cdot) < 0 \)). As a result, the point of the first intersection of \( f/F \) and \( COF(\cdot) \) moves to the right, meaning the optimal \( \hat{\alpha} \) increases and the rejection rate decreases. This is highly intuitive as a supplier with a higher \( \beta_S \) should offer a contract that allocates more profit to the retailer and such a contract, of course, will be acceptable to more types, resulting in a lower rejection rate. The effect of \( \gamma \), instead, is rather complex, changing the direction depending on both \( \beta_S \) and \( \alpha \). However, when \( \beta_S \) is small, which is the most practical
case (De Bruyn and Bolton[2008]), \( COF(\cdot) \) increases in \( \gamma \) (a straightforward differentiation yields \( \frac{\partial}{\partial \gamma} COF(\alpha, \beta_S = 0, \gamma) > 0 \)) and so the optimal \( \hat{\alpha} \) decreases. At another extreme, when \( \beta_S \to \frac{1}{1+\gamma} \), \( \hat{\alpha} \) increases in \( \gamma \) (a straightforward differentiation yields \( \frac{\partial}{\partial \gamma} COF(\alpha, \beta_S = \frac{1}{1+\gamma}, \gamma) < 0 \)). In the latter, case, though, \( \gamma \) does not have much influence on the optimal contract as the supplier offers a very generous contract anyway, that almost all types accept.

3.2. Optimal Contract Implementation

The optimal contract characterized by Theorem[1] and Proposition[2] is a “point” contract. Expressing it in quantity-payment terms results in a single point \((q,t)\), whereas contracts used in practice such as minimum-order quantity (MOQ), two-part tariff (TPT), quantity-discount (QD), buyback (BB), revenue-sharing (RS), sales-rebate (SR), etc., provide the retailer a continuum of quantities to choose from. Therefore, a question of immediate interest is which of these contracts can implement the optimal contract. Cachon (2003) reviews a large body of literature on coordinating contracts noting that “In general, a contract coordinates the retailer’s and the supplier’s action whenever each firm’s profit is an affine function of the supply chain’s profit” (ibid., p. 245), and showing that, indeed, most of the aforementioned contracts coordinate a supply chain because, with a proper choice of contract parameters, they transform the supplier’s and retailer’s profits, making them a linear function of the total supply chain profit:

\[
\pi_R(q) = \lambda\pi_T(q) - C, \tag{7}
\]
\[
\pi_S(q) = (1 - \lambda)\pi_T(q) + C, \tag{8}
\]

where \( 0 \leq \lambda \leq 1 \), \( 0 \leq C \leq \lambda \) are scalars, and \( \pi_T(q) \) is the total profit of the supply chain (we impose restrictions on \( C \) to ensure non-negative profits; also recall that we normalized \( \pi_T^{FB} \equiv \pi_T(q^{FB}) = 1 \)). Parameter \( \lambda \) provides what is essentially a degree of freedom that allows the parties to split profits, and it is this degree of freedom that also makes it possible to construct a coordinating contract that is optimal for inequality averse parties. In what follows, we call such contracts affine coordinating contracts and prove that being affine is a sufficient (but not a necessary) condition for a contract to be able to implement the optimal contract when preferences for fairness are private information.
Proposition 3. Affine coordinating contracts implement the optimal contract when preferences for fairness are private information.

To illustrate this proposition, consider a TPT contract, which is an offer of a wholesale price, $w$, paid per unit, and a lump sum, $L$, which does not depend on the number of units the retailer orders and is paid if the retailer accepts the contract. Let $R(q), C(q)$ be the retailer revenue and the supplier cost functions, accordingly. If the supplier chooses $w = C(q)$ then the retailer utility becomes an affine function of the total profit of the supply chain, $U_R(q) = (1 + \alpha)(R(q) - C(q)) - (1 + \alpha + \alpha \gamma)\pi_T(q) - (1 + \alpha + \alpha \gamma)L$. The retailer, maximizing her utility, chooses $q = q^{FB}$ regardless of her type. If $\hat{\alpha}$ is the optimal cutoff type, then the supplier chooses $L = \hat{L}$ such that $(1 + \hat{\alpha})\pi_T^{FB} - (1 + \hat{\alpha} + \hat{\alpha} \gamma)\hat{L} = 0$, and implements the optimal contract.

Note that not all coordinating contracts are affine. Another important class includes contracts with price break points such as all-quantity discount, two-block tariff (not to be confused with two-part tariff, see Lim and Ho 2007), etc. With these contracts, implementation of the optimal contract is more straightforward than with affine coordinating contracts (therefore we decided not to put it as a proposition) because they can always be designed such that the retailer, fair-minded or not, will choose the quantity at a price break point. To see this, consider an extreme case of two-block tariff contracts, an MOQ contract, which, recall, is an offer of a fixed wholesale price, $w$, and a minimum order quantity, $q_{\text{min}}$ (as a side note, it is straightforward to verify that MOQ is not affine – attempting to make it affine one obtains an incremental quantity-discount contract). The retailer, in the event of accepting the proposed offer of $w, q_{\text{min}}$, has to order $q \geq q_{\text{min}}$. To implement the optimal contract, the supplier uses $q_{\text{min}} = q^{FB}$ and $w > C'(q^{FB})$. Any retailer type if accepts will order $q = q_{\text{min}}$ because ordering $q > q_{\text{min}}$ results in a smaller utility. The supplier then uses the wholesale price to set the optimal rejection rate.

3.3. Illustration by comparing with laboratory results

In this sub-section, we use the empirical distribution of the retailer’s fairness parameter $\alpha$ and an estimate of $\gamma = 0.86$ reported in Katok et al. (2014) (pp. 295-296) to derive the optimal desired
rejection rate. We then use this rejection rate to determine the optimal contracts, and compare our predictions to the data reported by Ho and Zhang (2008) in their two-part-tariff (TPT) treatment and by Katok and Pavlov (2013) in their minimum order quantity (MOQ) treatment.

Both studies are experimental and involved a linear demand function \( p = A - q \), in which \( A \) is the market base, and \( q \) and \( p \) are the quantity and the retail price that clear the market. The supplier has a constant production cost of \( c \) per unit. For each round of the experiment, a participant in the role of supplier was randomly matched with one of the participants in the retailer role and had to propose a contract to this retailer. In the TPT treatment of Ho and Zhang (2008), the contract offer specified a constant wholesale price \( w \) per unit and a fixed fee \( L \) (to be paid by the retailer to the supplier regardless of the number of units \( q \) ordered). In the MOQ treatment of Katok and Pavlov (2013), the supplier’s offer specified a wholesale price \( w \) per unit and a minimum quantity \( q_{\text{min}} \) that the retailer had to order if she accepted the contract; so \( q \geq q_{\text{min}} \) if the retailer accepted the contract and \( q = 0 \) if she rejected. In both of those studies, if the retailer accepted the contract the number of units ordered were immediately “sold” on the market and profits (\( \pi_S \) and \( \pi_R \)) accrued to the players based on the market price and the parameters of the accepted contract. If the retailer rejected, profits of both players were zero. The market parameters were \( A = 10, c = 2 \) in Ho and Zhang (2008) and \( A = 100, c = 20 \) in Katok and Pavlov (2013) so that the “first-best” contracts that allowed the supplier to extract the entire channel profit in these studies, were: TPT with \( (w = 2, L = 16) \) and MOQ with \( (w = 60, q_{\text{min}} = 40) \).

Given a distribution of \( \alpha \), one can determine the desired optimal rejection rate, as follows: Figure 2 presents the pdf and cdf of the empirical distribution of \( \alpha \), their ratio \( \frac{f(\alpha)}{F(\alpha)} \), and a family of cutoff functions (the right-hand side of (6)) parametrized by \( \beta_S \). Based on De Bruyn and Bolton (2008) finding from a meta-analysis of several bargaining experiments that \( \beta_S = 0 \), notice that the graph of \( \frac{f(\alpha)}{F(\alpha)} \) crosses the topmost COF curve (\( \beta_S = 0 \)) at around \( \hat{\alpha} = 0.39 \). Using the cdf graph, one finds that \( F(0.39) = 0.78 \), i.e., the optimal rejection rate is 22%.

In Table 1 we illustrate the use of our model to determine optimal contract parameters that suppliers should offer, and compare our model’s recommendations to the laboratory data. In the
first three columns, we analyze the TPT setting in [Ho and Zhang (2008)]. The number of independent observations in the TPT treatment of [Ho and Zhang (2008)] was \( N = 44 \), because the unit of analysis should be individual subject. The numbers in parentheses in the column labelled Observed are standard errors, which we calculate by taking the standard deviation reported in [Ho and Zhang (2008)] and dividing it by \( \sqrt{44} \). The first column tells us that the optimal TPT contract should have \( w = 2 \) and \( L = 12.93 \). [Ho and Zhang (2008)] do not observe these average contract offers, primarily because suppliers in their experiment offer wholesale prices that are too high (3.96 instead of 2), which causes offer efficiency to decrease to 94%. Given the 3.96 wholesale price, our model recommends that the fixed fee be set at \( L = 6.94 \). The average fixed fee they observe is \( L = 5.24 \), which is closer to 6.94 than to 12.93, although is significantly below 6.94 \( (t = 4.86, p < 0.0001) \). The average rejection rate of 25.76% that [Ho and Zhang (2008)] report is not significantly different from 22% that our model recommends \( (t = 0.569, p = 0.572) \). So the main difference between the TPT contract performance that [Ho and Zhang (2008)] report and the performance of the optimal contract that we recommend, is that proposed offers are less efficient. Offer efficiency under the TPT contract, which [Ho and Zhang (2008)] explain as the disutility from the fixed fee, is outside the scope of this paper because it cannot be explained by fairness ([Haruvy et al. 2020] analyse this issue in detail). The average fixed fee that [Ho and Zhang (2008)] observe, that is lower than what our model recommends, may be indicative of a slightly higher \( \alpha \) than what we are using.

In columns 4-6 of Table II, we analyze the MOQ setting in [Katok and Pavlov (2013)]. The number of participants in the supplier role in the MOQ treatment of [Katok and Pavlov (2013)] was \( N = 9 \), so we report standard errors, calculated by taking the standard deviation reported in [Katok and Pavlov (2013)] and dividing it by \( \sqrt{9} = 3 \). Column 4 tells us that the optimal MOQ contract should have \( q_{min} = 40 \) and \( w = 52.33 \). Average contract offers that [Katok and Pavlov (2013)] observed are quite close, but \( q_{min} = 37.03 \) is slightly (but not significantly) lower than 40 \( (t = 1.03, p = 0.33) \), which causes offer efficiency to slightly decrease to 99%. Given the 37.03 price break, our model recommends that the wholesale price be set at \( w = 54.73 \), which is higher than the observed average
Table 1  Comparison of our model recommendation with laboratory data.

<table>
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<tbody>
<tr>
<td></td>
<td>Optimal</td>
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<tr>
<td>Market base $A$</td>
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</tr>
<tr>
<td>Production cost $c$</td>
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<td>20.00</td>
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<td>Wholesale price $w$</td>
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<td>Order quantity $q$</td>
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<td>Retail price $p$</td>
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<td>6.98</td>
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<tr>
<td>Fixed Fee $L$</td>
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<td></td>
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<tr>
<td>Price break $q_{min}$</td>
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<tr>
<td>Retailer profit $\pi_R$</td>
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<tr>
<td>Supplier profit $\pi_S$</td>
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<tr>
<td>Channel efficiency (%)</td>
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<td>Rejection rate (%)</td>
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<tr>
<td></td>
<td>(6.60)</td>
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Note: Profits and channel efficiency are computed conditional on optimal order quantities.

wholesale price of 50.49, but only weakly so ($t = 1.91, p = 0.092$), and the average rejection rate of 19.52% that [Katok and Pavlov (2013)] report is not significantly different from our model’s prediction of 22% ($t = 0.664, p = 0.525$).

4. Discussion and summary

This study investigates how incomplete information regarding preferences for fairness affects the performance of a supply chain, contributing to a stream of research on the impact of behavioral
factors on supply chain coordination (Cui et al. 2007, Lim and Ho 2007, Ho and Zhang 2008, Katok and Wu 2009, Ho et al. 2014, Niederhoff and Kouvelis 2016 among others), and, more broadly, complementing studies on the role of other-regarding preferences in operations management contexts (Roels and Su 2013, Avci et al. 2014).

The novelty of this study, and its key unique contribution is that it characterizes the optimal contract under private information for inequity aversion. Perhaps the most significant implication for the contracting literature (as well as for contract designers) is that it helps organize results coming from different streams of bargaining research, and allows us to make both, recommendations and predictions. We showed how to use our model to construct optimal TPT and MOQ contracts, and demonstrated that, conditional on efficiency, results reported by Ho and Zhang (2008) and by Katok and Pavlov (2013) match our recommendations quite well.

Perhaps the most important “high-level” result of our study is that it establishes a direct link between the supply chain contracting literature and the literature on social preferences in behavioral economics. On one hand, this implies that rejections are inevitable and that the efficiency of around 75%–80% observed in contracting experiments is already close to the theoretical upper bound. On the other hand, it suggests that the more promising direction for improving supply chain efficiency is not searching for “better” contracts under the ultimatum bargaining protocol but exploring other bargaining formats, due to their potential to either reduce the share of total profit that the retailer considers fair prior to bargaining (Cui and Mallucci 2016), or cause beneficial behavioral changes that result from the bargaining format (e.g., see Haruvy et al. 2020).

On the technical side, one challenge was an intractability of the original problem due to the retailer’s isoutility curves not satisfying the single-crossing condition. Fortunately, it proved possible to reduce the original problem to one in which single-crossing holds. A small feature of our approach that may be useful for future studies, because we believe it helped make our exposition and results more intuitive, was the problem parametrization not in terms of standard quantity-payment but in terms of final outcomes (profits).
As any parsimonious model, ours has limitations. The same reasons that enable multi-stage bargaining to outperform the ultimatum protocol imply that the Revelation Principle does not hold in that case, and our model is not applicable to the multi-stage bargaining setting. Another limitation is that the technical results rely on the linearity of the Fehr and Schmidt (1999) model. Considering the evidence that some proportion of people have non-linear preferences (Bolton and Ockenfels 2000, Andreoni et al. 2003), both the Fehr and Schmidt (1999) model and ours should be interpreted as approximations.

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